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# Control of a two-stage production/inventory system with products returns

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**Abstract:** We consider a two-stage production-inventory system with demand at the downstream stage and returns at each stage. We characterize the structure of the optimal policy which is a complex state-dependent Base-stock policy. We also investigate four classes of policies : Fixed buffer, Base-stock echelon, Kanban and Half-optimal. We compare the performances of these policies and exhibit that the maximal overcost for using the Half-optimal policy is of 0.35% on all the instances tested.

*Keywords:* Multi-echelon, Product returns, Production/Inventory system, Optimal Control, Control policies

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## 1. INTRODUCTION

Products are more and more returned in supply chains. Customers can return products a short time after purchase due to take-back commitments of the supplier. The proportion of returns is particularly important in electronic business where customers can not touch a product before purchasing it. Customers might also return used products a long time after purchase. This type of return has increased in recent years due to new regulations encouraging waste reduction, especially in Europe. Some industries also encourage it for economical and marketing reasons.

In this paper, we consider a two-stage production/inventory system with returns of products at each stage. Before presenting in detail our model, we briefly review the related literature of inventory control. We focus on models with several stages and / or returns. In their seminal work, Clark and Scarf (1960) studies a series inventory system with  $N$  stages, finite horizon, periodic review, linear holding and backorder cost, no setup cost and stochastic demand only on the downstream stage. They prove that a Base-stock echelon policy is optimal. These assumptions have been relaxed in several papers and we refer the reader to Iida (2001) for a review. For instance, Federgruen and Zipkin (1984) extend the results of Clark and Scarf (1960) to an infinite horizon. Parker and Kapuscinski (2004) add a constraint of capacity on the orders. They show that a Base-stock echelon policy is nearly optimal when the downstream echelon is not overloaded. In the contrary case, it is the Kanban policy which is nearly optimal. However, in general, a Base-stock echelon policy is not optimal when a capacity constraint is added.

In production-inventory systems, replenishment is modelled in a different way than in classical inventory systems. Items are produced by servers one by one, or possibly by batches. Each unit, or batch, requires a random lead-time to be produced. Implicitly, in production-inventory systems, replenishments are capacitated. Veatch and Wein (1994) consider a two-stage systems with exponential server at each stage. Otherwise, their assumptions are similar to Clark and Scarf (1960). Again, as in Parker and Kapuscinski (2004), they show on numerical experiments that the optimal policy is not a Base-stock echelon policy. They investigate several classes of policy (Base-stock, Kanban, Fixed Buffer and Conwip) and compare them to the optimal policy. Their conclusions are the same as those of Parker and Kapuscinski (2004) : the Base-stock policy is generally the nearest optimal policy but when the downstream station is overloaded Kanban policy is better. Dallery and Liberopoulos (2003) investigates other types of policies. In another paper, Veatch and Wein (1992) characterize the form of the optimal policy for a series systems with  $N$  stages and generalize their results to assembly systems, disassembly systems and to the routing problem. The form of the optimal policy is a state dependent Base-stock policy. It is optimal to produce at stage  $i$  when the inventory level at stage  $i$  is smaller than a Base-stock level, depending on inventory levels at all other stages. In this paper, we consider an extension of the model of Veatch and Wein (1994) where we add product returns at each stage.

Returns constitute a reverse flow, from the customer to the supplier, which complicates inventory control. An abundant literature is devoted to the inventory control of single echelon systems (see Ilgin and Gupta (2010) for a complete review). In several situations, the structure of

the optimal policy is similar to the case without returns. For instance, Fleischmann and Kuik (2003) consider a single inventory with stochastic demand and stochastic independent returns. To model the returns, they consider a demand that can be both positive or negative. They show average cost optimality of an  $(s, S)$  policy.

Fewer papers investigate the control of multiechelon systems with product returns. DeCroix et al. (2005) analyse a setting similar to Clark and Scarf (1960) except that demand can be negative. They prove that a Base-stock echelon is still optimal. They also propose a method to compute a near optimal policy, explain how to extend their model when returns occur at different stages and compare the Base-stock echelon policy to fixed buffer policies. With the same type of modelling, DeCroix and Zipkin (2005) characterize the optimal policy for an assembly system and DeCroix (2006) analyses a simple series model with only one return in any of the stages and evaluates whether the Base-stock model is optimal. His conclusion is that the Base-stock echelon policy is optimal if the return is made at the upstream station. Finally Mitra (2009) analyses a two echelon inventory system with returns and set-up costs.

The literature is very limited with respect to production-inventory systems with returns. In a single-echelon setting, Gayon (2009) studies an  $M/M/1$  make-to-stock queue with Poisson returns, linear holding and backorder costs. He finds that the Base-stock policy is optimal with a discounted or average costs. Furthermore, he provides an analytic formula for the optimal Base-stock level  $S^*$  (optimal Base-stock level) in both cases. With the same type of model, Zerhouni (2009) considers the case where the return product can be disposed with an additional disposal cost. In this case he finds that the optimal policy consists of two thresholds  $R$  (for the possibility of disposal) and  $S$  (for the classical production). It is optimal to produce (resp. return) when the inventory level is below  $S$  (resp.  $R$ ). He also models the case where the returns are linked with the demand and the case of advance information : when the system have the information that a return arriving before the return is arrived. In a multi-echelon setting, we are not aware of any production-inventory system. Our paper extends the work of Gayon (2009) to the case of two stages of production.

Our contributions are of two types. First, we show that the structure of the optimal policy shown by Veatch and Wein (1992) can be extended to the case when there are returns independent of demands. Second, we provide an extensive numerical study comparing the performances of different control policies. In particular, we show that a modified Base-stock echelon policy is nearly optimal in all the instances we have tested.

Next section details the model. Section 3 characterizes partially the structure of the optimal policy. Section 3 describes the procedures to compute policies. Finally, Section 4 compares the performance of the policies.

## 2. MODEL FORMULATION

We consider a two-stage production/inventory system in series which satisfies end-customer demand, see Figure 1.

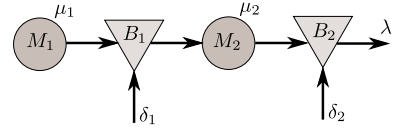


Fig. 1. Model

Station  $M_i$  produces items one by one. The production leadtime of station  $M_i$  is exponentially distributed with rate  $\mu_i$ . Preemption is allowed and station  $M_i$  can be started or stopped at any time. Produced items are stocked in a buffer  $B_i$  just after  $M_i$ . The end buffer  $M_2$  sees customer demands arriving according to a Poisson process with rate  $\lambda$ . We assume that backorders are allowed. At time  $t$ , the on-hand inventory at  $B_1$  is denoted by  $x_1(t)$  and the net inventory in  $B_2$ , possibly negative, is denoted by  $x_2(t)$ . When buffer  $B_1$  is empty, the production is blocked at station  $M_2$ . The novelty of this model, with respect to that of Veatch and Wein (1994), is to consider flows of returned products. Returns at buffer  $B_i$  are independent of demands and occur according to a Poisson process with rate  $\delta_i$ . To ensure stability of the system, we assume that :

$$\delta_2 < \lambda < \mu_2 + \delta_2 \quad (1)$$

$$\delta_1 + \delta_2 < \lambda < \mu_1 + \delta_1 + \delta_2 \quad (2)$$

Equation (1) (resp. (2)) ensures that demand is smaller than production capacity and can absorb all returns.

The state variable of our system can be described by  $x(t) = [x_1(t), x_2(t)]$ . We consider three types of costs. In state  $x$  the system incurs a cost rate  $c(x) = h_1 x_1 + h_2 x_2^+ + b x_2^-$  where  $h_i$  is the inventory holding cost per unit of time at buffer  $B_i$  and  $b$  is the backorder cost per unit of time. We also consider linear return cost  $c_i$  at buffer  $B_i$ . Note that the optimal production policy is independent of these return costs (a production policy defines when to produce or not). We do not include set-up costs.

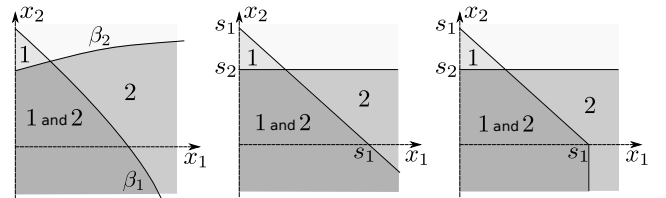


Fig. 2. Optimal policy (left), Base-stock policy (middle) and Kanban policy (right)

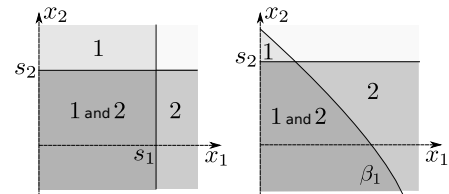


Fig. 3. Fixed buffer policy (left) and Half optimal policy(right)

Our objective is to compare the performances of several types of policies. With  $\mathcal{D}_i$  the domain of production of  $M_i$  and  $s_i$  the Base-stock level for buffer  $B_i$ , we investigate five class of policies (see Table 1). Figures 2 and 3 illustrate the different types of policies.

Table 1. Production control policies

Policy	$\mathcal{D}_1$	$\mathcal{D}_2$
Optimal ( $\pi^*$ )	$\{x : x_1 < \beta_1(x_2)\}$	$\{x : x_1 > 0, x_2 < \beta_2(x_1)\}$
Base-stock ( $BS$ )	$\{x : x_1 + x_2 < s_1 + s_2\}$	$\{x : x_1 > 0, x_2 < s_2\}$
Kanban ( $KB$ )	$\{x : x_1 + x_2^+ < s_1 + s_2\}$	$\{x : x_1 > 0, x_2 < s_2\}$
Fixed buffer ( $FB$ )	$\{x : x_1 < s_1\}$	$\{x : x_1 > 0, x_2 < s_2\}$
Half optimal ( $HO$ )	$\{x : x_1 < \beta_1(x_2)\}$	$\{x : x_1 > 0, x_2 < s_2\}$

### 3. CHARACTERIZATION OF THE OPTIMAL POLICY

A production policy  $\pi$  specifies when to produce or not at each stage. The discounted expected cost (with discount rate  $\alpha \in (0, 1)$ ) over an infinite horizon of a policy  $\pi$ , with initial state  $x = (x_1, x_2)$ , is given by

$$v^\pi(x) = E \left[ \int_0^{+\infty} e^{-\alpha t} c(X(t)) dt \mid X(0) = x, \pi \right] \quad (3)$$

where  $X(t)$  represents the state of the system at time  $t$ .

We want to find the optimal policy, denoted by  $\pi^*$ , that minimizes the expected discounted cost  $v^\pi(x)$ . We note  $v^*(x)$  the optimal value function :

$$v^*(x) = \min_{\pi} v^\pi(x) = v^{\pi^*}(x)$$

Let  $\tau = \lambda + \mu_1 + \mu_2 + \delta_1 + \delta_2 + \alpha$  be the uniformization rate. The optimality equations (Puterman, 1994) are then given by

$$v^* = T v^* \quad (4)$$

with

$$T v(x_1, x_2) = \frac{1}{\tau} \left( x_1 h_1 + x_2^+ h_2 + x_2^- b + \mu_1 T_1 v(x_1, x_2) + \mu_2 T_2 v(x_1, x_2) + \delta_1 T_3 v(x_1, x_2) + \delta_2 T_4 v(x_1, x_2) + \lambda T_5 v(x_1, x_2) \right) \quad (5)$$

and

$$\begin{aligned} T_1 v(x_1, x_2) &= \min(v(x_1, x_2), v(x_1 + 1, x_2)) \\ T_2 v(x_1, x_2) &= \min(v(x_1, x_2), v(x_1 - 1, x_2 + 1)) \\ T_3 v(x_1, x_2) &= v(x_1 + 1, x_2) + c_1 \\ T_4 v(x_1, x_2) &= v(x_1, x_2 + 1) + c_2 \\ T_5 v(x_1, x_2) &= v(x_1, x_2 - 1) \end{aligned} \quad (6)$$

These optimality equations are identical to the ones of Veatch and Wein (1994) when we set the return rates  $\delta_1 = \delta_2 = 0$ . As operators  $T_3$  and  $T_4$  preserve all submodularity and supermodularity properties, we can extend the results of Veatch and Wein (1994) to the case with product returns ( $\delta_1 > 0, \delta_2 > 0$ ).

*Theorem 1.* The optimal policy is a state-dependent Base-stock policy with switching curves  $\beta_1$  and  $\beta_2$  such that :

- Produce at stage 1 if and only if  $x_1 < \beta_1(x_2)$ . Moreover  $\beta_1(x_2) - 1 \leq \beta_1(x_2 + 1) \leq \beta_1(x_2)$ .
- Produce at stage 2 if and only if  $x_2 < \beta_2(x_1)$ . Moreover  $\beta_2(x_1) - 1 \leq \beta_2(x_1 + 1) \leq \beta_2(x_1)$ .

This theorem pertains to the average cost criterion (Puterman, 1994). In the rest of the paper, we focus on the average cost criterion.

## 4. PROCEDURES TO COMPUTE POLICIES

### 4.1 Computation of a given policy

To compute the optimal policy, we truncate the state space in three directions. Let  $\Gamma_1$  and  $\Gamma_2^+$  two positive integers and  $\Gamma_2^-$  a negative integer :

$$0 \leq x_1 \leq \Gamma_1 \text{ and } \Gamma_2^- \leq x_2 \leq \Gamma_2^+$$

We can then apply a value iteration algorithm to this truncated state space. Define the following sequence of value functions :

$$v_{n+1}(x_1, x_2) = T v_n(x_1, x_2), \forall x_1, x_2$$

The algorithm stops when the following condition is true :

$$\text{sp} |v^{n+1} - v^n| < \epsilon \quad (7)$$

with  $\text{sp}\{a\} = \max\{a\} - \min\{a\}$ . This algorithm guarantees that the computed policy is  $\epsilon_1$ -optimal (Puterman, 1994), with respect to the truncation vector  $(\Gamma_1, \Gamma_2^-, \Gamma_2^+)$ . We set  $\epsilon = 0.005$  in our numerical experiments.

We then increase simultenaously  $\Gamma_1, \Gamma_2^-$  and  $\Gamma_2^+$  as follows :

$$\Gamma_1 := \sqrt{2}\Gamma_1, \Gamma_2^+ := \sqrt{2}\Gamma_2^+, \Gamma_2^- := \sqrt{2}\Gamma_2^-$$

If we denote by  $C^i$  the average cost obtained at the  $i$ th iteration, we stop when the influence on the average cost of increasing the state space is less than  $\epsilon = 0.005\%$ , i.e. when

$$\frac{|C^{i+1} - C^i|}{C^{i+1}} < \epsilon \quad (8)$$

This procedure can be repeated for each type of policy by modifying the production operator  $T_1$ .

#### Base-stock

$$T_1 v(x_1, x_2) = \begin{cases} v(x_1, x_2) & \text{if } x_1 + x_2 \geq s_1 + s_2 \\ v(x_1 + 1, x_2) & \text{else} \end{cases} \quad (9)$$

#### Kanban

$$T_1 v(x_1, x_2) = \begin{cases} v(x_1, x_2) & \text{if } x_1 + x_2^+ \geq s_1 + s_2 \\ v(x_1 + 1, x_2) & \text{else} \end{cases} \quad (10)$$

#### Fixed buffer

$$T_1 v(x_1, x_2) = \begin{cases} v(x_1, x_2) & \text{if } x_1 \geq s_1 \\ v(x_1 + 1, x_2) & \text{else} \end{cases} \quad (11)$$

#### Half optimal

$$T_1 v(x_1, x_2) = \min(v(x_1, x_2), v(x_1 + 1, x_2)) \quad (12)$$

### 4.2 Optimization of parameters

For the heuristic policies described in Section 2, we want to find the parameters  $s_1, s_2$  that minimize the average cost function  $C(s_1, s_2)$ . This optimization problem is a non linear problem with integer variables that might be long to solve since evaluating a given policy might already take time. Therefore, we make the plausible assumption that the function  $C(s_1, s_2)$  is unimodal. A function  $f$  is unimodal if for  $x, y$  and  $z$  on a line and  $y$  between  $x$  and  $z$  :  $f(x)$  is finite and  $f(x) \leq f(y)$  implies  $f(y) \leq f(z)$ . This assumption has been validated on several instances.

Based on the unimodularity assumption, we can solve efficiently the problem with the Golden section search (Avriel and Wilde, 1968). This technique is optimal for an axis problem, so we need to search axis by axis. Another method which is easier to implement is the maximal gradient with constant step. This method is very efficient here because we can start the optimization with an approximate value of  $s_1$  and  $s_2$ , resulting from the calculation of the optimal policy.

## 5. NUMERICAL STUDY

For the next study we defined a nominal set of parameters, which forms a stable system :

$$\mu_1 = 1.5; \mu_2 = 1.5; \delta_1 = 0.3; \delta_2 = 0.3 \quad (13)$$

$$\lambda = 1; h_1 = 1; h_2 = 2; b = 4 \quad (14)$$

These parameters are changed one by one in the Appendix. According to the stability equations (1) and (2), we vary those parameters as defined in the equations below :

$$0.6 < \lambda < 1.8 \quad (15)$$

$$0.4 < \mu_1; 0.7 < \mu_2 \quad (16)$$

$$\delta_1 < 0.7; \delta_2 < 0.7 \quad (17)$$

Figures A.1 to A.8 presented in appendix show the influence of those parameters on the average cost.

**Stability bound** In figure A.1 to A.5 we can observe a divergence on the parameters bound stability. Furthermore, when the system is near to instability, the resolution of the system is longer. This phenomenon is like the classical M/M/1 queue, when the ratio between the arrival and the demand is near 1 ( $\rho \approx 1$ ), the steady-state probabilities spread. So we have to enlarge the state space to include the states which are further away (with high-value of  $x_1$  and  $|x_2|$ ) that are newly obtainable.

**Base-stock** The Base-stock policy is not optimal but is generally the best policy in comparison to the Fixed buffer and the Kanban if the second station (downstream) is not the bottleneck, see A.1, A.2, A.5. This result has been already found by Veatch and Wein (1994).

**Kanban** When the second station is bottleneck the Kanban policy is better than Base-stock policy.

**Fixed buffer** The Fixed buffer policy is never optimal, and generally worse than the other policies, but when the second station is very overloaded it is better than the Base-stock policy, see A.2.

**Half optimal** The Half optimal policy is not represented here because it is a very good approximation and, in our numerical study, give exactly the same results than the Optimal policy.

**Returns** Little return decreases the average cost because it helps to satisfy the demand, see A.3, A.4. However, when the quantity of return is significant, queues are overloaded and the average cost increased.

For a second analyses of our system, we compute all the possible and stable combinations of these values :

$$\lambda = \{1\}; \mu_1 = \{1; 1.5; 2\}; \mu_2 = \{1; 1.5; 2\} \quad (18)$$

$$\delta_1 = \{0; 0.3; 0.6; 0.8\}; \delta_2 = \{0; 0.3; 0.6; 0.8\} \quad (19)$$

$$h_1 = \{1\}; h_2 = \{0.5; 1; 10\}; b = \{0.5; 1; 10; 100\} \quad (20)$$

That represent 912 instances. In this case we obtain the results described in table 2. We can observe that the Base-stock policy is generally better than the other policies, and

in 75% this policy gives a result near to 5% of the optimal. Fixed buffer is the worst with near 80% of results with a deviation of more than 10% from the optimal. Kanban has received good results in some cases, with a result at 1% from the optimal in 25% of instances.

Table 2. Instances

(%)	FB	BS	KB	HO
Better than other policies (without HO)	0.1	68.0	31.4	
Minimal deviation from $\pi^*$	0.60	0.0	0.0	0.0
Maximal deviation from $\pi^*$	590	51.4	150	0.35
Average deviation from $\pi^*$	29.0	3.8	9.8	0.0
Quantity with deviation [0%; 1%[	0.4	45.6	25.4	100
Quantity with deviation [1%; 5%[	14.2	31.4	26.6	0.0
Quantity with deviation [5%; 10%[	5.6	11.2	13.7	0.0

Note that we compute all results with a compute convergence criteria  $\epsilon = 0.1\%$ . In this case too, the Half optimal policy is a very good approximation of the Optimal policy. The maximal deviation obtained is 0.35%. This result is obtained when servers are overloaded and  $h_1 \ll h_2$ . We observe every time that  $\beta_1^{\pi^*} = \beta_1^{HO}$ . Those observations permit to conclude that the information do not have to go from the upstream to the downstream to manage inventory very efficiently.

In Table 3, we analyse cases without returns i.e with  $\delta_1 = \delta_2 = 0$  (48 instances in our numerical study). In those cases the Base-stock policy is worse than Kanban policy. This result is predictable because when there is no returns servers are more in use, so more overloaded.

Table 3. Instances without returns

(%)	FB	BS	KB	HO
Better than other policies (without HO)	0.0	59.2	38.8	
Minimal deviation from $\pi^*$	1.34	0.0	0.0	0.0
Maximal deviation from $\pi^*$	42.2	24.1	151	0.0
Average deviation from $\pi^*$	22.8	5.0	15.6	0.0
Quantity with deviation [0%; 1%[	0.0	37.5	27.1	100
Quantity with deviation [1%; 5%[	12.5	27.1	27.1	0.0
Quantity with deviation [5%; 10%[	6.25	14.6	16.7	0.0

In this paragraph, we study more precisely the impact of the returns on the average cost. In Figure 4, we can observe that the more returns are sent to the first station, the more the average cost is big. This can be explained by the fact that a product arriving in the downstream station remains in the system longer than a product arriving in the upstream station. This result could be interesting for designers of supply chain, because the strategy between return downstream or upstream could be a compromise with cost of return products and inventory cost.

We finish this analysis by comparing the benefit of our model comparing to the case where the returns are neglected. If there is no return in this configuration :

$$\mu_1 = 1.5; \mu_2 = 1.5; \lambda = 1; h_1 = 1; h_2 = 2; b = 4 \quad (21)$$

the Base-stock policy is optimal with  $s_1 = 5$  and  $s_2 = 3$ , so we compute the model with this Base-stock level and we compare the average cost with the result obtained in figures A.3 and A.4. The result of this comparison is given in figure 5. We can observe that the gain for returns on the upstream station is lower than those on

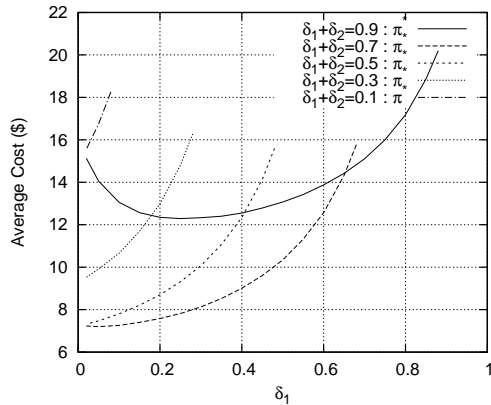


Fig. 4. Evolution of the average cost in function of the repartition of the returns

the downstream station. It could be explained by two phenomenons : the return on the second echelon decongest the downstream station and tends to improve the Base-stock policy. The other explanation is that the holding cost in 1 is lower, so the unwanted returns are less expensive. Another observation is very clear : the interest of the model is undoubted if the returns satisfy more than 20% of the demand. Finally, the relative gain decreases when the stability decreases. It could be explained by the relativity of the cost : with a lot of returns the main problem is not the Base-stock level but the stability of the queue.

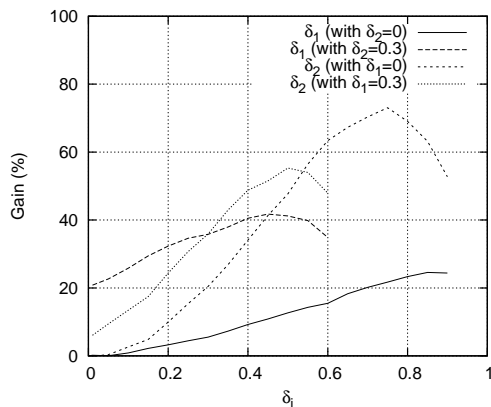


Fig. 5. Relative gain between neglecting the return and our model for the Base-stock policy

The same study for the 912 instances give us an average gain of 39%, for the instances who are stable when returns are neglected.

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## Appendix A. VARIATION OF THE PARAMETERS

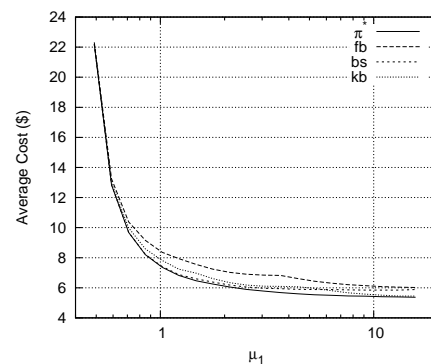


Fig. A.1. Variation of  $\mu_1$

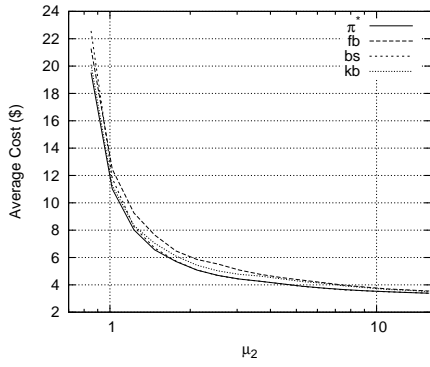


Fig. A.2. Variation of  $\mu_2$

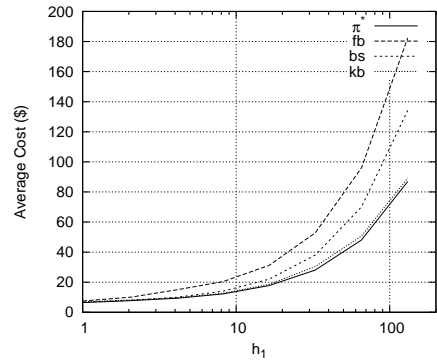


Fig. A.6. Variation of  $h_1$

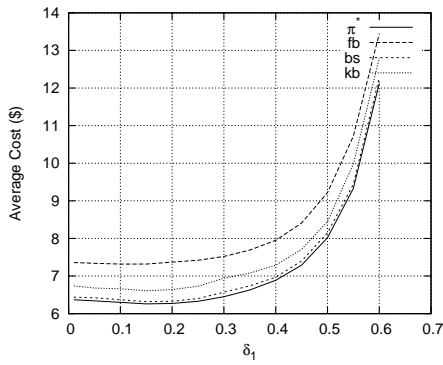


Fig. A.3. Variation of  $\delta_1$

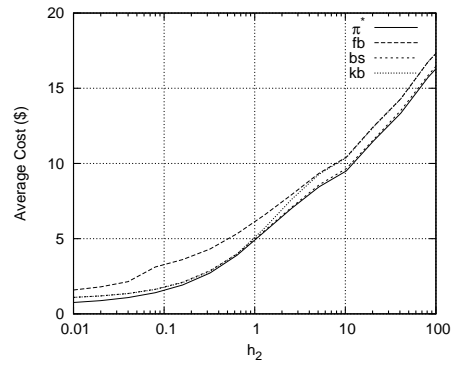


Fig. A.7. Variation of  $h_2$

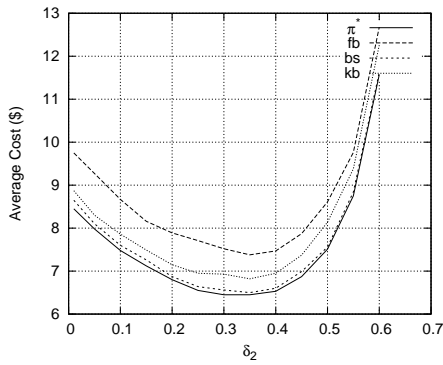


Fig. A.4. Variation of  $\delta_2$

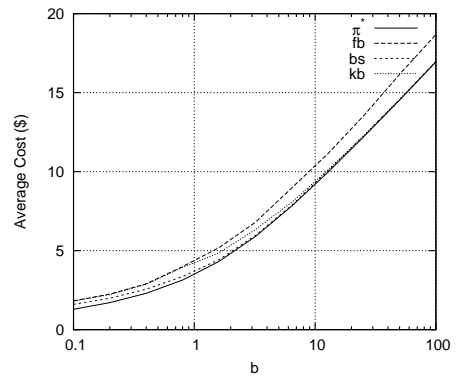


Fig. A.8. Variation of  $b$

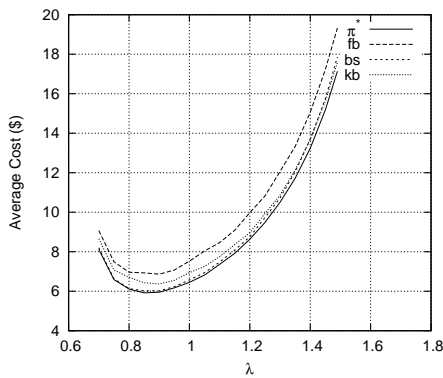


Fig. A.5. Variation of  $\lambda$