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Supervised super resolution to improve the resolution of hyperspectral images classification maps

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ABSTRACT

Hyperspectral imaging is a continuously growing area of remote sensing. Hyperspectral data provide a wide spectral range, coupled with a very high spectral resolution, and are suitable for detection and classification of surfaces and chemical elements in the observed image. The main problem with hyperspectral data for these applications is the (relatively) low spatial resolution, which can vary from a few to tens of meters. In the case of classification purposes, the major problem caused by low spatial resolution is related to mixed pixels, i.e., pixels in the image where more than one land cover class is within the same pixel. In such a case, the pixel cannot be considered as belonging to just one class, and the assignment of the pixel to a single class will inevitably lead to a loss of information, no matter what class is chosen. In this paper, a new supervised technique exploiting the advantages of both probabilistic classifiers and spectral unmixing algorithms is proposed, in order to produce land cover maps of improved spatial resolution. The method is in three steps. In a first step, a coarse classification is performed, based on the probabilistic output of a Support Vector Machine (SVM). Every pixel can be assigned to a class, if the probability value obtained in the classification process is greater than a chosen threshold, or unclassified. In the proposed approach it is assumed that the pixels with a low probabilistic output are mixed pixels and thus their classification is addressed in a second step. In the second step, spectral unmixing is performed on the mixed pixels by considering the preliminary results of the coarse classification step and applying a Fully Constrained Least Squares (FCLS) method to every unlabeled pixel, in order to obtain the abundances fractions of each land cover type. Finally, in a third step, spatial regularization by Simulated Annealing is performed to obtain the resolution improvement. Experiments were carried out on a real hyperspectral data set. The results are good both visually and numerically and show that the proposed method clearly outperforms common hard classification methods when the data contain mixed pixels.

Keywords: Super-resolution, mixed pixels, spectral unmixing

1. INTRODUCTION

The creation of thematic maps from remote sensing data is an important application of image analysis. Many practical applications, such as precision agriculture, monitoring of vegetation health, management of natural disasters, and issues related to security and defense take advantage of the creation of accurate classification maps. In the last few years, the development of always more sophisticated sensors allowed to collect data with a huge quantity of spectral information. Hyperspectral images are composed of hundreds of bands with a very high spectral resolution, from the visible to the infra-red region. Thanks to their characteristics, these images had a rapid development and an always growing interest around them. The continuously growing availability of hyperspectral data has opened new possibilities in the field of image analysis and classification.¹ The wide spectral range, coupled with the high spectral resolution give the possibility to solve problems which usually cannot be solved by multispectral images. In the case of classification, the higher detailed spectral information of the data increases the capability to detect and distinguish various classes with improved accuracy. On the other hand, several issues need to be considered when dealing with hyperspectral images. Beside the problem of handling very high dimensional data, the main issue to be considered is the low spatial resolution of such images, especially in the case of high altitude sensors or instruments which cover wide areas.³ There are many factors (such as imperfect imaging optics, atmospheric scattering, secondary illumination effects and sensor noise) that

degrade the acquired image quality and make the development of new technology to improve the spatial resolution a very challenging task. The low spatial resolution can be a major issue in the case of image classification, leading to the challenging problem of mixed pixels (Fig. 1), *e.g.*, pixels containing more than one land cover type.² In such a case, a pixel is a mixture of two or more "thematic classes", and cannot be considered as belonging to just one class. The assignment of the pixel to a class (*e.g.*, the class with biggest fraction within pixel) will inevitably lead to a loss of information, due to the neglect of some components of the mixture. A hyperspectral image usually contains both pure and mixed pixels in different ratio, also in case of high spatial resolution. As long as the spatial resolution decreases, a larger number of pixels containing mixtures of materials can be found.

Several techniques have been proposed in recent years for the classification of hyperspectral data: Maximum Likelihood (ML), Bayesian approaches, neural networks, kernel methods and Support Vector Machines.^{1, 5, 6} All these techniques are based on the assumption that each pixel corresponds to the spectral signature of one land cover type, and so that each pixel can be labeled as belonging to only one class. These techniques are not suitable for the analysis of mixed pixels and will inevitably lead to a high error rate when used for scenarios with a high number of sites with mixtures of land cover classes.

During the last few years, the issue of mixed pixels has received an increasing attention. Foody has investigated the use of mixed pixels to train a Support Vector Machine (SVM), for hard classification purposes, showing that an accurate design of the training stage by including mixed pixels can lead to an improvement of the overall classification accuracy.¹² However, the problem of the classification of mixtures of materials still is not addressed. A widely investigated approach is the use of soft classification techniques. These classifiers do not assign a pixel to only one class, but for each pixel their output is the degree of membership in the class in question.⁸ During the recent years, a number of soft classifiers have been proposed, making use of a soft version of ML,⁹ fuzzy logic,¹⁰ nonparametric methods.¹¹ These methods can give a useful insight of the 'purity' of a pixel. However, the membership degree does not necessarily reflect the fractional abundance of a class within a mixed pixel, and the hard labeling of a sample remains problematic, being usually performed by assigning the pixel to the class with *maximum* membership degree.

Linear Spectral Unmixing (LSU)¹⁸ is a technique explicitly designed to address the problem of mixed pixels. Following the linear mixing model, each pixel is considered as being the sum of a number of pure components, corresponding to the thematic class of the image. A number of techniques, exploiting both statistical and geometrical properties of the data, were proposed over the last few years.¹⁸ These techniques can partially overcome the weakness of full pixel classification methods when analysing mixed pixels, and they are suitable to be used for the analysis of these scenarios. However, when used to obtain crisp classification maps, the choice of the endmembers and the abundances determination are negatively affected by spectral variability, and common hard classification methods are more suitable in such a case.¹³

In this work, we propose a technique which can take advantage of the characteristic of both hard classification and spectral unmixing techniques. Being a hyperspectral images usually composed by area with pure pixels and other zones with mixtures of classes, the combination of these techniques is an interesting approach for the analysis of this kind of data. The aim is to create a method which exploits the state-of-the-art hyperspectral classifiers used to address the classification of pure pixels, and spectral unmixing to handle mixed pixels. We propose also a spatial regularization, so that we can obtain thematic maps with a finer spatial resolution. The method proposed in this paper is in three steps. In a first step, a coarse classification is performed, based on the probabilistic output of an SVM. Every pixel can be assigned to a class, if the probability value obtained in the classification process is greater than a chosen threshold, or unclassified. In the proposed approach it is assumed that the pixels with a low probabilistic output are either mixed pixels or pure pixels hard to classify and thus their classification is addressed in a second step. In the second step, spectral unmixing is performed on these pixels by considering the preliminary results of the coarse classification step and applying a Fully Constrained Least Squares (FCLS) method to every unlabeled pixel, in order to obtain the abundances fractions of each land cover type. Finally, in a third step, spatial regularization by SA is performed to obtain the resolution improvement. Experiments are carried out on a real hyperspectral data set. The results are excellent both numerically and visually and show that the proposed method clearly outperforms traditional hard classification methods when the data contain mixed pixels.

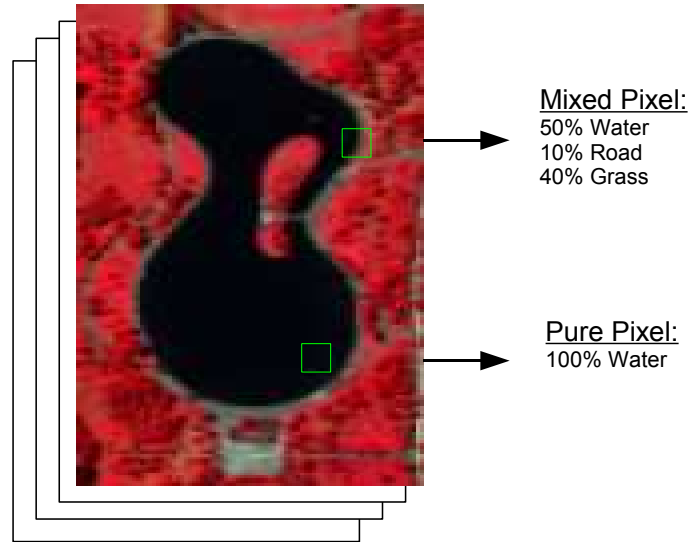


Figure 1. Illustration of the problem of mixed pixels: When the spatial resolution is not fine enough, several land cover classes lie in the same pixel. In this case, a hard classification process cannot give an accurate information about the pixel coverage, leading inevitably to a loss of information

The remainder of the paper is organized as follows. Section II presents in greater detail the proposed approach. Section III illustrates the experimental results on a real hyperspectral data set, and Section IV draws the conclusions.

2. METHODOLOGY

The proposed method is composed by three different steps. In a first step, a probabilistic classification is performed, in order to select a number of pixels which can be considered as "pure", with the lowest classification error. The labelling of the pixels not classified in this first step is then addressed by performing spectral unmixing. The final step is a spatial regularization obtained by applying Simulated Annealing. Based on the assumption of spatial correlation of the land cover classes, SA is used to optimize a function where spatial proximity of pixels belonging to the same land cover class are preferred to the opposite case.

2.1. Pixel-wise classification

The first step of the proposed method consists in performing a pixelwise classification of the hyperspectral image, in order to obtain, for every pixel, a probability value for it to belong to one of the land cover classes. The pixels with a probability higher than a chosen threshold are considered as pure pixels, and thus assigned to the considered class. These pixels are going to provide a preliminary classification map, where only the pixels containing a predominant land cover class are labeled. The classification of the unlabeled pixels will be addressed in a second step.

As a classifier, we have chosen the Support Vector Machine (SVM),¹⁴ which has proven to be extremely suitable to handle high dimensional data.⁷ Two outputs are considered after this first classification step:

- 1) A complete probability map, containing the probability estimates for each pixel to belong to the a certain class.
- 2) A coarse classification map of the pixels considered as "pure", containing class labels for the samples with a probability belonging to a certain class larger than a chosen threshold.

These outputs will be used as input of the spectral unmixing algorithm, in order to address the labelling of the remaining pixels.

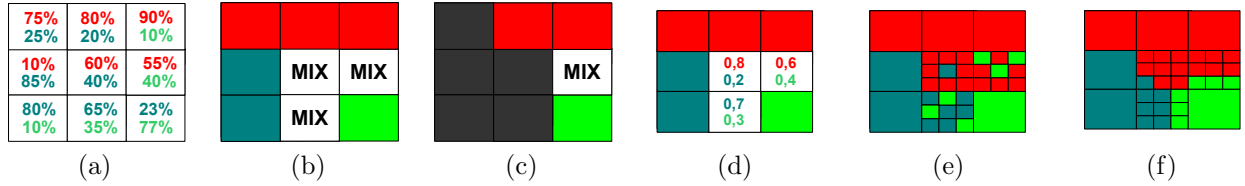


Figure 2. Basic steps of the proposed method: (a) A probabilistic classification map is computed for each class. (b) The pixels with highest probability greater than a chosen threshold are considered as pure and classified (in the figure, we set the threshold to 70%). The other pixels are considered as mixed (MIX in the figure). (c) For each mixed pixel, a suite of possible endmembers is selected, considering the results of the preliminary classification. The other pixels, pure or mixed, are just ignored. (d) Spectral unmixing provides information about the abundance fraction of a class within each pixel. (e) Pixels are split into n sub-pixels, according to the desired zoom factor, assigned to an endmember and randomly positioned within the pixel. The number of sub-pixels assigned to each class reflects the fractional value estimated in the previous step. (f) Simulated annealing performs random permutations of the sub-pixels position until *minimum* cost is reached.

2.2. Spectral Unmixing

After obtaining a coarse classification map, where the pixels considered as "pure" (due to the high probability to belong to a certain class) were classified, the labelling of the other pixels is addressed in the second step. They could be either pixels containing a mixture of pure classes or pure pixels which were not assigned to a class due to the high spectral difference with respect to the training samples.

Spectral mixture analysis (SMA) techniques have overcome some of the weaknesses of full pixel approaches by using linear statistical modeling and signal processing techniques.¹⁸ The key task in linear SMA is to find an appropriate suite of pure spectral constituents -called "endmembers" in hyperspectral analysis terminology-, which are then used to estimate the fractional abundances of each mixed pixel from its spectrum and the endmember spectra by using a linear mixture model.

In the Linear Mixture Model (LMM), the spectrum of a mixed pixel is represented as a linear combination of component spectra (endmembers). The weight of each endmember spectrum (abundance) is proportional to the fraction of the pixel area covered by the endmember. If there are M spectral bands, the spectrum of the pixel and the spectra of the endmembers can be represented by M -dimensional vectors. Therefore, the general equation for LMM is described as a linear regression form

$$\mathbf{z} = \sum_{i=1}^L \mathbf{a}_i \mathbf{s}_i + \mathbf{e} = \mathbf{A}\mathbf{s} + \mathbf{e} \quad (1)$$

where \mathbf{z} is an $M \times 1$ column pixel vector which describes the spectrum of the mixed pixel, $\mathbf{s} = [\mathbf{s}_1 \mathbf{s}_2 \dots \mathbf{s}_L]$ is an $M \times L$ endmember matrix of material signature, \mathbf{s}_i , ($i = 1, 2, \dots, L$) are the M -dimensional spectra of the endmembers, \mathbf{a} is an $L \times 1$ column vector and is composed of abundance coefficients \mathbf{a}_i , ($i = 1, 2, \dots, L$), \mathbf{e} is an M -dimensional error vector accounting for lack-fit and noise effects, and L is the number of the endmembers. Due to physical reasons, (1) has to respect the following constraint of non-negativity (abundance fractions within a pixel cannot be negative) and sum to one (the sum of all the abundances fraction within a pixel must have 1 as a result).

In recent years, several algorithms have been developed for automatic or semi-automatic extraction of spectral endmembers directly from the image data and to determine their fractional abundances within each pixel.¹⁸ Assuming that ground truth is available, we do not need to determine the endmembers composing the data, but simply the abundance of each land cover type within the pixels. In this case, a major issue is how to handle the spectral variability which affects the data. As a matter of fact, soft classification of hyperspectral images covering wide areas is negatively related to the intra-class spectral-variability, and the assumption that a single endmember could extensively represent a class is generally far from reality.²³ The choice of appropriate endmembers is very important in order to correctly estimate the fractional abundances. If the endmembers do

1.	Initial Temperature = T
2.	$StoppingCriterion = 0$
3.	if ($StoppingCriterion = 0$)
4.	Randomly change the configuration of sub-pixels within a mixed pixel
5.	Compute $NewCost$
6.	if ($NewCost < OldCost$)
7.	Accept the change
8.	else
9.	Reject the change with probability $(1-p)$
10.	end
11.	Update $T, p, Cost, StoppingCriterion$
12.	end

Table 1. Pseudo-code of the proposed spatial regularization approach based on Simulated Annealing.

not represent properly the land cover classes, the estimates of the sub-pixel coverage can be highly biased and lead to misclassification errors.

In order to overcome this problem, we propose an adaptive approach to select the best endmembers candidate for each pixel. This approach is based on two main assumptions:

1. The spatial correlation of the classes, *e.g.*, for each pixel, it is probable that the best endmember candidates lie in the spatial proximity of the considered pixel.
2. The probabilistic output provided by the SVM, *e.g.*, if a candidate is not spatially close to the selected pixel, but the probabilistic value of the class to which belongs is high, it is presumably a good candidate.

For each mixed pixel which has to be classified, we consider a set of 10 different spectra, that represent the endmember candidates. These candidates are chosen from the labeled samples of the training data and the results of the preliminary classification of step one, considered as a set of pure pixels correctly classified. If one of the land cover classes has a high probabilistic output (we consider a probabilistic output as high if its difference from the threshold chosen at step 1 is smaller than 5%), at least five spectra of this class are considered, otherwise all the 10 candidates are selected from the spectral signatures spatially closest to the considered pixel, after the coarse classification step. Once the spectral signatures representative of each class are extracted from the image, the abundance fraction of the elements within each pixel should be determined. Several algorithms have been developed for the linear mixing model according to the required constraint of abundances fractions. Due to its computational efficiency, we have chosen a fully constrained least squared (FCLS) unmixing algorithm, which satisfies both abundance constraints and is optimal in terms of least squares error.²⁰ After applying FCLS, we obtain the fractional abundances of each endmember. Due to the fact that in many cases several endmembers candidate represent the same class, by summing the fractional abundances of all the endmembers belonging to the same land cover class, we obtain the cover percentage of a class within a mixed pixel.

2.3. Improving Spatial Resolution

Spectral unmixing is useful to describe the scene at a sub-pixel level, but can only provide information about proportions of the endmembers within each pixel. Since the spatial location remains unknown, spectral unmixing does not perform any resolution enhancement. In this paper, we propose a sub-pixel mapping technique, which takes advantage of the information given by the spectral mixing analysis and uses it to enhance the spatial resolution of thematic maps. Our proposed approach is as follows: In a first step, each pixel is divided in a fixed number of sub-pixel, according to the desired resolution enhancement. Every sub-pixel is assigned to an endmember, in conformity with its fractional abundance within the pixel. In example, if we want to have a zoom factor of N , we have to divide each pixel into $N \times N$ sub-pixels. For each pixel, the number of subpixels to assign to each class i is computed according to the results of the previous step.

A Simulated Annealing (SA) mapping function is then used, to create random permutation of these sub-pixels, in order to minimize a chosen cost function. Relying on the spatial correlation tendency of landcovers, we assume that each endmember within a pixel should be spatially close to the same endmembers in the surrounding pixels. Therefore, the cost function to be minimized is chosen as the perimeter of the areas belonging to the same endmember.

SA is a well established stochastic technique originally developed to model the natural process of crystalization.²¹ This process is based on an analogy from thermodynamics where a system is slowly cooled in order to reach its lowest energy state. More recently, SA has been proposed to solve global optimization problems,²² and it has been used in various fields. The basic idea of the method is that, in order to avoid to be trapped in local minima, uphill movements, *e.g.*, points corresponding to worse values of the objective function could, sometimes, be accepted for the following iteration. As with a greedy search, it accepts all the changes that improve the solution. Changes degrading the solution can be accepted, but with a probability that is inversely proportional to the size of the degradation (small degradations are accepted with a higher probability). This probability also decreases as the search continues, or as the system cools, allowing eventual convergence to the optimal solution.

3. EXPERIMENT ON REAL HYPERSPECTRAL DATA SET

In this study, a segment of an hyperspectral image acquired by the HYDICE sensor the urban site over Washington, DC Mall, U.S, has been analysed in order to assess the effectiveness of the proposed method. The original size of the image is 1280×307 pixels, and we consider a segment of 300×300 in our study. Two hundred and ten bands are collected in the 0.4-2.4 μm region of the visible and infrared spectrum. Some water absorption channels were discarded, resulting in a total of 170 channels. The spatial resolution is 2.8 m. In the experiment, seven information classes, namely, Roof, Road, Trail, Grass, Tree, Water, and Shadow, were considered.

The assessment of the results when dealing with a mixed pixel is always a challenging task, due to the difficulty to have a perfect knowledge of the fractional abundances of each land cover type. Moreover, the hyperspectral data sets considered in our experiments have ground truth maps where all the pixels are considered as pure. In order to have the possibility to analyse data sets where the ground truth cover is known in detail, and to evaluate the obtained results from a quantitative point of view, we decided to use the original ground truth data only to compare the obtained results, and to decrease the spatial resolution of the image by applying a low pass filter. This way, we know exactly the quantity of each class within a pixel, and we can use the low resolution image obtained after filtering as input for the proposed method. In the following experiment, the spatial resolution was decreased by a factor 3. From the original segment we obtain a new image composed by 100×100 pixels, where each pixel corresponds to the average value of 9 pixels of the original image. The original high spatial resolution image and the low resolution image obtained after filtering can be seen in Fig. 3 (a-b). In order to compare the proposed approach with a common hard classification method, the same data were also classified with an SVM with Gaussian Kernel, One vs One multiclass strategy and 10 fold cross-validation strategy used to tune the parameters of the classifier.

Ten pixels per class, considered as "pure" in the low resolution image, were randomly chosen and used for training the SVM classifier.

The result of the classification obtained with a traditional SVM and with the proposed method are presented in Fig. 3 (c-d). The threshold selected to estimate if a pixel had to be labelled in the first step was set to 0.7. It can be noticed that the when applying the SVM to the low resolution image, it is hard to correctly define the borders between regions belonging to different classes. In this case, the results provided by the proposed method improve the classification results (see for example the border of the water region in the left side of the image, both at the the top and at the bottom, represented in dark blue in the thematic maps). The quantitative assessment of the results was also investigated. Due to the small number of pixels labelled as ground truth, we have compared the results of the proposed method with those obtained by applying an SVM trained with 40 pixels per class to the original high resolution data. By doing this, we have to keep in mind that this is not ground truth but only a reference data, which can however provide a useful indication of the effectiveness of the proposed method. The number of pixels correctly classified corresponds to the 75.05% of the total number of pixels. This percentage confirms the high potentiality of the use of spectral unmixing to obtain classification maps at a higher spatial resolution.

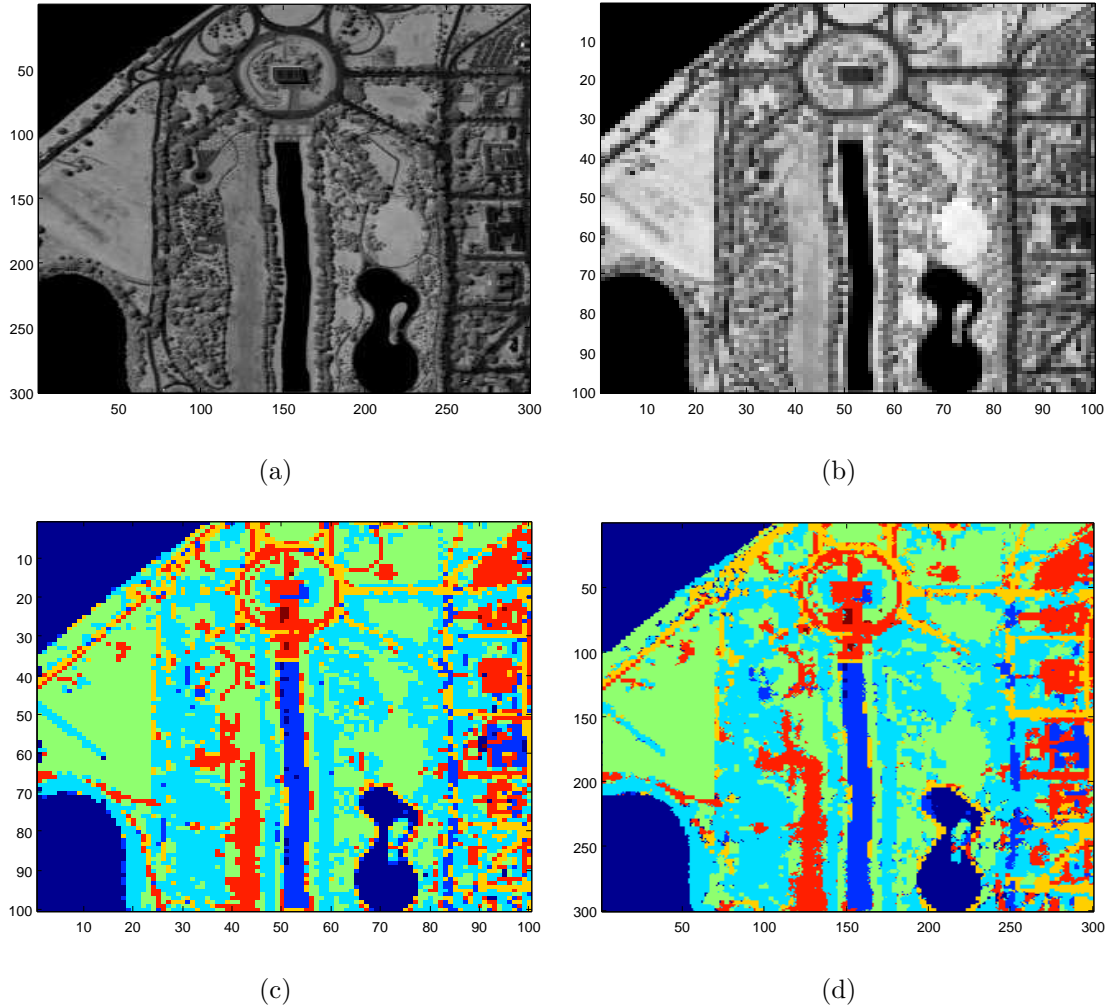


Figure 3. (a) HYDICE Washington DC Mall data set, band 90. (b) HYDICE Washington DC Mall data set with degraded spatial resolution, band 90. (c) Classification map obtained with a traditional SVM classifier. (d) Classification map obtained with the proposed approach.

3.1. AVIRIS complete

The second experiment was carried out on the well known AVIRIS Indian Pine data set. Nine land cover classes were considered, while seven were discarded due to the low number of training samples. The original image is composed by 145×145 pixels, and it was used as reference data. After applying a 2×2 low pass filter, an image composed by 72×72 pixels was obtained. The high resolution image and the land cover ground truth can be seen in Fig. 6, (a) and (d), respectively. In this case, thirty pixels per class, which were considered as "pure" in the low resolution image, were randomly chosen and used for training the SVM classifier. In order to have the possibility to compare the results of the proposed method with the available ground truth, we chose a zoom factor equal to 2, lower than in the previous case. However, the higher number of classes and their spectral similarity make this data set more challenging than the first one.

Figure 3 (b) and (c) shows the classification maps obtained with a conventional SVM and the proposed method. Also in this case, an improvement can be clearly seen in the classification maps, resulting in a less noisy map and an improve detection of the borders of spatial structures (in this case, agricultural fields). To have a quantitative comparison of the results obtained with the two methods, the overall accuracy of pixels correctly classified has been compared. The overall accuracy obtained with the SVM is 73.17%. As in the previous case, the low value of accuracy is due, to two main factors, which are the impossibility of a common hard classification technique to

distinguish different land cover classes at a sub-pixel level, and the difficulty to handle the high spectral variability. The proposed method obtained an overall accuracy of 90.23%, showing the capability of the proposed approach to better deal with the two main issues aforementioned.

4. CONCLUSIONS

The classification of hyperspectral images in presence of mixed pixels was addressed in this paper. A new method for the improvement of the spatial resolution of the classification maps was proposed. The method exploits the advantages of both soft classification techniques and spectral unmixing algorithms, in order to determine the fractional abundances of the classes at a sub-pixel scale. A spatial regularization by Simulated Annealing is finally performed to spatially locate the land cover classes within each pixel. Experiments were carried out on a real hyperspectral data set and show that the proposed method outperforms classical classification techniques when areas with mixtures of materials are located in the scene, providing good results both from a visually and quantitative point of view. Further research will be devoted to the investigation of advanced methods to better discriminate pure and mixed pixels, and of the possibility of alternative techniques of spatial regularization.

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