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Optimal Investment Across Different Aspects of Anti-predator Defences

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1 Abstract

2	We present a simple model of investment across a suite of different anti-predatory
3	defences. Defences can incur an initial construction cost and and/or may be costly
4	each time they are utilised. Our aim is to use a simple, but general, mathematical
5	model to explore when prey that face a single predatory threat where each attack is of
6	the same nature should invest only in a single defence, and when they should spread
7	their investment across more than one defence. This should help to explain the
8	observed variety of defences that a single prey individual may employ during repeated
9	attacks of a similar nature or even at different stages during one attack. Previous
10	verbal reasoning suggested that prey should specialise in investment in defences that
11	can be utilised early in the predation sequence. Our quantitative model predicts that
12	(depending of the relatively properties of different defences), there may be
13	concentrated investment in early-acting, or in late-acting defences, or a spread of
14	investment across both defence types. This variety of predictions is in agreement with
15	the variation in defences shown by natural organisms subjected to repeated predatory
16	attack.
17	

18 Keyword: predation, predator-prey, investment in defences, optimal strategy, costs of

19 defences

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22 Introduction

23	Predation is an almost ubiquitous process in the natural world, and very few animals
24	are immune to the risk of predation for at least part of their life history. Since
25	predation is responsible for a large proportion of mortality in many species, it is no
26	surprise that anti-predatory defences are also widespread and intensively studied by
27	behavioural ecologists (see Ruxton et al. 2004 and Caro 2005 for reviews). Caro
28	identified one of the ten most pressing questions in the study of anti-predatory
29	defences as "how can we explain patterns of morphological and physiological
30	defences across taxa?" There is indeed tremendous variety between species in the
31	forms of anti-predatory defences employed. However, there is even substantial
32	variability within an individual in the defences they employ against different attacks
33	(Buskirk 2001 and references therein). The conventional explanation of this variation
34	is that many individuals face risk of predation from a suite of different predatory types
35	for which different defences might be required. For example, a single caterpillar
36	might face attack by avian predators, predatory social wasps, parasitic wasps and flies,
37	ants, spiders, and even insectivorous rodents. Whilst visual crypsis may be an
38	effective defence against detection by birds, it is unlikely to be as effective against
39	ants that rely more on tactile, vibrational and olfactory cues to locate their prey.
40	However, variation in predatory threat (while certainly part of the answer) cannot be
41	the sole driver of within-individual variation in defences, since a single individual can
42	use different defences against the same type of predator in different attacks (reviewed
43	in section 13.6 of Caro 2005).

44

45 That a single individual can utilise a suite of different defences is noteworthy for at 46 least two reasons. Firstly, defences are likely to be costly and each defence added to 47 the prey's portfolio potentially adds an associated cost. Secondly, there can be 48 interference between different defences such that implementation of one may impair 49 the performance of another. The different costs of different defences are considered in 50 depth in Chapter 5 of Ruxton et al. 2004. For our purposes, we differentiate between 51 two general types of cost, those that are paid "up-front" such that the cost is paid 52 whether or not attacks occur and regardless of the number of attacks (often called 53 constitutive defences), and costs that are incurred each time the defence is used. For 54 example, a caterpillar that defends itself against birds by being difficult to detect 55 visually pays up-front costs. There may be physiological costs to the production of 56 pigments required to produce the desired appearance. Alternatively or additionally, 57 there may be opportunity costs associated with restricted use of microhabitats and 58 restricted movement required to maximise crypsis. These costs are paid regardless of 59 the number of attacks that an individual caterpillar experiences. In contrast some costs 60 occur whenever the defence is employed in a specific attack (Higginson & Ruxton 61 (2009) call these 'responsive' defences). For example, some insects (notably many 62 ladybirds) exhibit reflex bleeding where toxin-laced blood is exuded from joints in the 63 exoskeleton in response to handling by a potential predator. This blood may deter the 64 predator from pursuing its attack, but the exuded blood and its toxins have been lost, 65 and have to be replaced. Replacement of these is likely to be physiologically costly, 66 and the total cost of using this defence will increase with the number of times it is 67 deployed (Grill and Moore 1998). Notice that this reflex bleeding defence may also 68 incur up-front costs as well, since there may be physiological costs associated with the 69 storage of toxins in a way that avoids autotoxicity, and gaps in the exoskeleton that

allow reflex blood to leave may impose costs in terms of, for example, water loss or
risk of fungal disease. Other examples of responsive defences are regurgitation
(Bowers 1993), gland secretions (Eisner et al. 2004), urticating hairs (Bowers 1993),
ink release by cephalopods (Derby 2007), and the explosive defence of the
bombardier beetle (Eisner 1958).

75

76 Investment in, or deployment of, one defence can reduce the effectiveness of another 77 defence. For example investment in a tough spiny exterior may provide mechanical 78 protection against handling by predators, but it may make the prey individual more 79 conspicuous (decreasing the effectiveness of camouflage as a defence) and may make 80 it more difficult for the prey to choose to flee from predators that may be able to 81 circumvent the anti-handling defence. Flight and crypsis are two classically 82 interfering defences. If a predator has not yet detected a predator, the prey may 83 increase the chances of remaining undetected by remaining still and trusting to their 84 cryptic appearance. However, if the predator inadvertently comes close to the prey 85 before detecting it, the close spatial proximity of the two individuals will reduce the 86 effectiveness of fleeing by the prey to escape the predator. Conversely, if the prey 87 flees early in the predatory sequence before detection has occurred, then the 88 effectiveness of crypsis is likely to be greatly reduced as the fleeing animal is more 89 vulnerable to detection than when sitting still (Broom & Ruxton 2005).

90

91 Implicit in our description above is that an interaction between a prey individual and a 92 predatory individual is generally a sequence of different phases. The most commonly 93 used articulation of this is the sequence laid out by Endler (1991), who suggested that 94 a predation event involves a sequence of six stages: encounter, detection,

95	identification, approach, subjugation and consumption. Defences differ in which of
96	these stages they can be utilised in. Clearly crypsis only works in the initial
97	"encounter" stage prior to detection of the prey by the predator. In contrast, anti-
98	predatory vigilance to allow detection of the predator by the prey may be of use to the
99	prey throughout all of the first four of Endler's six stages. Here we will simplify
100	Endler's six stages: subsuming the first four into a "pre-capture" stage and the last
101	two into "post-capture. Our aim is to use a simple, but general, mathematical model to
102	explore when prey that face a simple predatory threat, where each attack is of the
103	same nature, should invest only in a single defence; and when they should spread their
104	investment across more than one defence.
105	
106	Methods
107	Model description
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119 these pre-capture defences in a single parameter D_1 . D_1 is a non-negative number

120 indicating the investment in pre-capture defences, with increasing values indicating 121 increasing investment. The probability that an encounter with a predator leads to 122 capture (P_1) is a decreasing function of D_1 . That is, increasing investment in pre-123 encounter defences reduces the probability that the prey individual will be 124 successfully captured by any predator that encounters it. 125 126 Post-capture defences involve such things as fighting ability, venomous stings, 127 armoured integument and production of mucus. All these defences have in common 128 the fact that they reduce the probability (P_2) that capture leads to the death (and thus 129 consumption) of the prey (i.e. capture leads to successful predation). We describe 130 increasing investment in post-capture defences by increasing values of D_2 , with P_2 131 declining with increasing values of D_2 . That is, increasing investment in post-capture 132 defences reduces the probability that a predator that physically captures the prey is

- able to subdue, kill and consume it.
- 134

135 We are interested in finding the optimal strategy in terms of investment in these two 136 defences. There are costs, as well as benefits, to investment in defence. We first of all 137 assume that there are fixed costs to the creation of the defences. Thus, even if the prey 138 individual never encounters a predator, it pays a cost for its investment in defences. 139 This occurs because these defences are constitutive (at least in part). The fecundity of 140 an individual that never encounters any predators is given by $F(D_1, D_2)$ where F 141 declines with increasing values of both D_1 and D_2 . This represents the fixed costs of 142 investment in the two types of defence. As investment in a defence increases so the 143 fitness cost increases.

144

145	However, as well as fixed costs, we assume that there can be additional costs every
146	time a defence is used. For example, for pre-capture defences, we can imagine that
147	there is a fixed cost to building and maintaining the muscle structure required for fast
148	escape, but there is an additional cost (say in energy expended and/or feeding time
149	lost) every time that defence is used and the prey has to flee. Specifically we assume
150	that if the prey encounters a predator on a number of occasions N_a , then it has to use
151	its pre-capture defence on N_a occasions. The cost of these encounters is represented
152	by multiplying the fecundity of the prey individual by $[C_1(D_1)]^{Na}$, where $C_1(0) = 1$
153	(that is unaffected if there is no investment in defence) and C_1 declines with
154	increasing values of D_1 (and thus fecundity declines both the investment in defence
155	and with how often the defences are used). This formulation captures the assumptions
156	that the costs of using pre-capture defences increase both with the number of times
157	these defences are used and with the extent of these defences. That is, greater
158	investment in pre-capture defence reduces the risk of capture, but also incurs higher
159	fixed costs and costs that increase with the number of time these defences are used.
160	
161	Let us assume that of these N_a encounters, a number N_c lead to capture of the prey and
162	a number N_n lead to no capture and the prey escaping. Thus, we assume that to
163	successfully survive these attacks the prey will have to successfully use its post-
164	capture defences on N_c occasions ($N_c \leq N_a$). The cost of these is represented by
165	multiplying the fecundity of the prey individual by $[C_2(D_2)]^{Nc}$.
166	
167	Where $C_2(0) = 1$ and C_2 declines with increasing values of D_2 . This formulation
168	captures the assumptions that the costs of using post-capture defences increases both
169	with the number of times these defences are used and with the extent of these

170 defences. That is greater investment in post-capture defence reduces the risk that

171 capture leads to death, but also incurs both higher fixed costs and higher costs each

- 172 time these defences are used.
- 173
- 174 If a number of encounters with predators occur, then the fecundity of the prey
- 175 individual is zero if it is killed in any one of these. The probability of it not being
- 176 killed in any of the encounters is $(1-P_2)^{Nc}$.
- 177
- 178 Thus, the total payoff to an individual playing a certain strategy (in terms of

investment across its two possible defences) is given by the fecundity multiplied by

- 180 the probability of surviving:
- 181

182
$$R = FC_1^{N_a}C_2^{N_c}(1-P_2)^{N_c}$$
$$= FC_1^{N_n}[C_1C_2(1-P_2)]^{N_c}$$

183 where for notational convenience, we have omitted writing out functional

184 dependences on D_1 and D_2 .

185

186 However, N_c (and so N_n) depends upon the value of D_1 . Let us assume that encounters 187 with a predator occur as a Poisson process at rate ε , and that reproduction requires that 188 an individual first avoid being consumed by a predator for a time period T. From the 189 general properties of the Poisson process, this can be broken down into two 190 independent Poisson processes. Namely, encounters that lead to capture (which are 191 Poisson with rate $\mathcal{E}P_I$ and encounters that do not lead to capture (which are Poisson 192 with rate $\mathcal{E}(1-P_1)$). Thus N_c takes values drawn from a Poisson distribution with mean 193 $\mathcal{E}P_1T$, and N_n from a Poisson distribution with mean $\mathcal{E}(1-P_1)T$.

Since N_c and N_n are generated by independent processes, we can write the payoff as $R = FE[C_1^{N_n}]E[\{C_1C_2(1-P_2)\}]^{N_c}],$ where E[] denotes the expectation of a stochastic process. From the properties of the Poisson process, if x is Poisson with mean λ , then SCÍ $E\left|t^{x}\right| = \exp(\lambda(t-1)).$ and using this, we can eliminate N_n and N_c from our expression for the payoff, giving $R = F \exp(\varepsilon(1-P_1)T(C_1-1))\exp(\varepsilon P_1T(C_1C_2(1-P_2)-1))$ If we take logs and divide by $\mathcal{E}T$, we can simplify this to $R_{l} = \frac{\ln(R)}{\varepsilon T} = \frac{\ln(F)}{\varepsilon T} - 1 + C_{1}(1 - P_{1}) + P_{1}C_{1}C_{2}(1 - P_{2})$ (1)Maximising the above function, maximises the payoff. In order to make further progress, we must now specify the following functional forms. $F(D_1, D_2) = F_{\text{max}} \exp(-f_1 D_1 - f_2 D_2)$. Thus, we assume that maximum fecundity (before any predator encounters) is a decreasing decelerating function of both D_1 and

- 218 D_2 . The higher the values of f_1 and f_2 , the higher the fixed costs of pre-capture and
- 219 post-capture defences respectively.
- 220 $P_1 = \max(0, 1 p_1 D_1)$
- 221 $P_2 = \max(0, 1 p_2 D_2)$
- 222 That is, we assume that the probability of an attack succeeding decreases linearly with
- investment in defences. The higher the values of p_1 and p_2 the higher the anti-
- 224 predatory efficacy of pre-capture and post-capture defences respectively.

225
$$C_1 = \max(0, 1 - c_1 D_1)$$

226
$$C_2 = \max(0, 1 - c_2 D_2)$$

- 227 We assume that the cost of defences increases linearly with the investment in
- defences. The higher the values of c_1 and c_2 the higher the per-use costs of pre-capture
- and post-capture defences respectively.
- 230
- 231
- 232 These are perhaps the simplest functions that have the required properties outlined in
- 233 the description above, Clearly, this restricts investment in defences such that p_1D_1 ,

234 p_2D_2 , c_1D_1 and c_2D_2 must all be less than one for the model to give sensible

- predictions. Thus as D_1 approaches $1/c_1$ for example, any use of the first defence
- would cost an individual almost all of its fitness. Provided that p_1 is not much smaller
- than c_1 , and p_2 not much smaller than c_2 , these upper boundaries will not be
- approached for any reasonable strategy.

239

240 Model predictions

We begin by identifying the types of different solutions (in terms of investment across

the two defences) that are optimal in different circumstances. We define the following

two terms that are central to the work that follows.

$$245 \qquad \beta_1 = \frac{f_1}{p_1 \varepsilon T} - 1$$

$$247 \qquad \beta_2 = \frac{f_2}{p_2 \varepsilon T} - 1$$

247	$p_2 = \frac{1}{p_2 \varepsilon T} - 1$	
248	C	
249	β can be thought of as the ratio of the constutive cost to the benefit of	defences. Note
250	that it immediately follows that $\beta_1 \ge -1$ and $\beta_2 \ge -1$. First, let us ask	t if it is ever
251	optimal for the prey never to make any investment in these costly def	ences. In all
252	analyses we explore the effect of f_1 and f_2 , which control the constitut	ive impact of
253	defences on maximum fecundity.	
254		
255	No investment: $(D_1, D_2) = (0, 0)$.	
256	This occurs when both the derivatives of equation (1) with respect to	D_1 and D_2 are
257	negative at $(0,0)$. It is easy to show that this occurs whenever we satisfy	sfy two
258	conditions:	
259		
260	$\beta_{\rm l} > 0$	(2)
261		
262	and	
263		
264	$\beta_2 > 0$	(3)

265	
266	For this solution, the stability condition is the same as the existence condition, so
267	whenever this solution is valid it is also stable. The above conditions are equivalent to
268	
269	$f_1 > p_1 \mathcal{E}T$ and $f_2 > p_2 \mathcal{E}T$.
270	
271	This is shown graphically in Figure 1 with example values for parameters. These
272	conditions make intuitive sense. Increasing the f values increases the constitutive cost
273	of defence. If the p value is small then the effectiveness of the associated defence is
274	low, if ε is small then attacks rarely occur, and if T is small then individuals only have
275	a short pre-reproductive period to survive: all of which should make investment in
276	defences less attractive. That is, if the constitutive cost of defence has a larger impact
277	on fitness than the increase in the probability of survival, prey should never invest in
278	defences. Notice, that the occurrence of this no-defence equilibrium is influenced only
279	by the constitutive costs of defence (f_1 and f_2) not by the costs of utilising the defences
280	$(c_1 \text{ and } c_2).$
281	
282	Investment only in pre-attack defences: $(D_1, D_2) = (D_1, 0)$.
283	This occurs when both the derivative of equation (1) with respect to D_1 is zero and the
284	derivative with respect to D_2 is negative at (D_1 ,0). Evaluation of these derivatives
285	leads to two conditions again:
286	
287	Firstly, the derivative with respect to D_2 gives
288	

289
$$\frac{-f_2}{\varepsilon T} + (1 - p_1 D_1)(1 - c_1 D_1)p_2 < 0$$

Expressed in terms of β_1 and β_2 this becomes

293
$$\beta_2 > \left(\frac{p_1 \beta_1}{4c_1}\right) \left(\beta_1 + 2 + \frac{2c_1}{p_1}\right) = \left(\frac{\beta_1}{4a_1}\right) \left(\beta_1 + 2 + 2a_1\right)$$
 (4)
294
295 where $a_1 = \frac{c_1}{p_1}$
296
297 The derivative with respect to D_I gives
298
299 $-\frac{f_1}{\epsilon T} + p_1 - 2p_1c_1D_1 = 0.$
300
301 Expressed in terms of β_I and β_2 this becomes
302
303 $D_1 = \frac{-\beta_1}{2c_1},$

295 where
$$a_1 = \frac{c_1}{p_1}$$

297 The derivative with respect to
$$D_1$$
 gives

299
$$\frac{-f_1}{\varepsilon T} + p_1 - 2p_1c_1D_1 = 0$$

Expressed in terms of
$$\beta_1$$
 and β_2 this becomes
 $D_1 = \frac{-\beta_1}{2c_1}$,

$$303 \qquad D_1 = \frac{-\beta_1}{2c_1},$$

which is positive providing $\beta_l < 0$.

307 For
$$(D_1,0)$$
 to be stable we need the second derivative of R with respect to D_1 to be

negative at $(D_1, 0)$. It is easy to demonstrate that this is always true. Thus, again this

- 309 solution is stable any time that it is valid. So, biologically we can conclude that it is
- 310 also possible for all investment to be concentrated in pre-attack defences.
- 311
- However there are further conditions on β_1 , since the optimal level of investment in
- 313 defence in this case is given by
- 314

315
$$D_1 = \frac{-\beta_1}{2c_1}$$

- 316
- 317 The condition $c_l D_l < 1$, together with $\beta_l \ge -1$, leads to the restriction $-1 < \beta_l < 0$;

(5)

- and the condition $p_1 D_1 < 1$ leads to the restriction $-2a_1 < \beta_1 < 0$.
- 319 This gives
- 320

321
$$-2\min(0.5,a_1) < \beta_l < 0$$

- 322
- 323 which in addition to (4) gives the conditions for the $(D_1, 0)$ solution.
- 324

325 These conditions are summarised graphically in Figure 1. Again, the results make 326 intuitive sense. From our arguments in the previous section, this solution is more 327 likely to occur under conditions where β_l is negative and β_2 is positive (or only just 328 negative). The other parameter group that affects the likelihood of obtaining this 329 solution is a_1 . a_1 takes a high value if the costs of using pre-attack defences is high 330 and/or if the effectiveness of such defences is low. Hence, it is logical that increasing 331 a_1 restricts the area of (β_1, β_2) space where such solutions occur. From the arguments 332 laid out already it is no surprise that the level of investment in pre-attack defences

benefits increase) and/or as c_1 is reduced (because responsive costs decrease). Investment only in post-capture defences: $(D_1, D_2) = (0, D_2)$ This occurs when the derivative of equation (1) with respect to D_1 is negative and the derivative with respect to D_2 is zero at $(0, D_2)$. Evaluation of these derivatives leads to two conditions again: Firstly, the derivative with respect to D_1 gives Firstly, the derivative with respect to D_1 gives Firstly, the derivative with respect to $D_1 p_2 D_2 < 0$ Firstly, the derivative with respect to β_1 and β_2 this becomes Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_2 D_2 < 0$ Firstly, the derivative with respect to $D_2 p_1 D_2 = \left(\frac{1+a_1}{4a_2}\right)\beta_2(\beta_2 + 2)$ (6) Firstly, the derivative with respect to $D_2 p_2 p_2 D_2 = 0$.	333	only increases as β_1 becomes more negative (because constitutive costs decrease or
335 336 337 Investment only in post-capture defences: $(D_{L}, D_{2}) = (\theta, D_{2})$ 338 This occurs when the derivative of equation (1) with respect to D_{1} is negative and the 339 derivative with respect to D_{2} is zero at $(0, D_{2})$. Evaluation of these derivatives leads to 340 two conditions again: 341 342 Firstly, the derivative with respect to D_{1} gives 343 344 $-\frac{f_{1}}{\epsilon T} + p_{1} - (p_{1} + c_{1})(1 - c_{2}D_{2})p_{2}D_{2} < 0$ 345 346 Expressed in terms of β_{1} and β_{2} this becomes 347 348 $\beta_{1} > \left(\frac{p_{2}}{4c_{2}}\right)(1 + \frac{c_{1}}{p_{1}})\beta_{2}(\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right)\beta_{2}(\beta_{2} + 2)$ (6) 349 350 where $a_{2} = \frac{c_{2}}{p_{2}}$. 351 352 The derivative with respect to D_{2} gives 353 354 $-\frac{f_{1}}{\epsilon T} + p_{2} - 2p_{2}c_{2}D_{2} = 0$.	334	benefits increase) and/or as c_1 is reduced (because responsive costs decrease).
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340two conditions again:341342Firstly, the derivative with respect to D_1 gives343344 $-f_1 + p_1 - (p_1 + c_1)(1 - c_2D_2)p_2D_2 < 0$ 345346Expressed in terms of β_1 and β_2 this becomes347348 $\beta_1 > \left(\frac{p_2}{4c_2}\right) \left(1 + \frac{c_1}{p_1}\right) \beta_2(\beta_2 + 2) = \left(\frac{1 + a_1}{4a_2}\right) \beta_2(\beta_2 + 2)$ 349350351352354 $-f_2 - p_2 - 2p_2c_2D_2 = 0.$	339	derivative with respect to D_2 is zero at $(0,D_2)$. Evaluation of these derivatives leads to
341 342 Firstly, the derivative with respect to D_1 gives 343 344 $-\frac{f_1}{eT} + p_1 - (p_1 + c_1)(1 - c_2D_2)p_2D_2 < 0$ 345 346 Expressed in terms of β_1 and β_2 this becomes 347 348 $\beta_1 > \left(\frac{p_2}{4c_2}\right)(1 + \frac{c_1}{p_1}\right)\beta_2(\beta_2 + 2) = \left(\frac{1 + a_1}{4a_2}\right)\beta_2(\beta_2 + 2)$ (6) 349 350 where $a_2 = \frac{c_2}{p_2}$. 351 352 The derivative with respect to D_2 gives 353 354 $-\frac{f_2}{eT} + p_2 - 2p_2c_2D_2 = 0$.	340	two conditions again:
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$ \begin{array}{ll} 344 & -\frac{f_1}{\epsilon T} + p_1 - (p_1 + c_1)(1 - c_2 D_2)p_2 D_2 < 0 \\ 345 \\ 346 & \text{Expressed in terms of } \beta_l \text{ and } \beta_2 \text{ this becomes} \\ 347 \\ 348 & \beta_l > \left(\frac{p_2}{4c_2}\right)\left(1 + \frac{c_1}{p_1}\right)\beta_2(\beta_2 + 2) = \left(\frac{1 + a_1}{4a_2}\right)\beta_2(\beta_2 + 2) \qquad (6) \\ 349 \\ 350 & \text{where } a_2 = \frac{c_2}{p_2} \\ 351 \\ 352 & \text{The derivative with respect to } D_2 \text{ gives} \\ 353 \\ 354 & -\frac{f_2}{\epsilon T} + p_2 - 2p_2c_2 D_2 = 0. \end{array} $	343	
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$\beta_{1} > \left(\frac{p_{2}}{4c_{2}}\right) \left(1 + \frac{c_{1}}{p_{1}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) $ $\beta_{2} = \left(\frac{p_{2}}{4c_{2}}\right) \left(1 + \frac{c_{1}}{p_{1}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) $ $\beta_{2} = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) $ $\beta_{2} = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) $ $\beta_{2} = \left(\frac{1 + a_{1}}{4a_{2}}\right) \beta_{2} (\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right) $	347	
349 350 where $a_2 = \frac{c_2}{p_2}$. 351 352 The derivative with respect to D_2 gives 353 354 $\frac{-f_2}{\epsilon T} + p_2 - 2p_2c_2D_2 = 0$.	348	$\beta_{1} > \left(\frac{p_{2}}{4c_{2}}\right)\left(1 + \frac{c_{1}}{p_{1}}\right)\beta_{2}(\beta_{2} + 2) = \left(\frac{1 + a_{1}}{4a_{2}}\right)\beta_{2}(\beta_{2} + 2) $ (6)
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351 352 The derivative with respect to D_2 gives 353 354 $\frac{-f_2}{\epsilon T} + p_2 - 2p_2c_2D_2 = 0.$	350	where $a_2 = \frac{c_2}{p_2}$.
The derivative with respect to D_2 gives $\frac{-f_2}{\varepsilon T} + p_2 - 2p_2c_2D_2 = 0.$	351	
353 354 $\frac{-f_2}{\varepsilon T} + p_2 - 2p_2c_2D_2 = 0.$	352	The derivative with respect to D_2 gives
354 $\frac{-f_2}{\varepsilon T} + p_2 - 2p_2c_2D_2 = 0.$	353	
	354	$\frac{-f_2}{\epsilon T} + p_2 - 2p_2c_2D_2 = 0.$

355	
356	Expressed in terms of β_1 and β_2 this becomes
357	
358	$D_2 = \frac{-\beta_2}{2c_2},$
359	
360	which is positive providing $\beta_2 < 0$.
361	
362	For this solution to be stable we need the second derivative of R with respect to D_2 to
363	be negative at $(0,D_2)$. It is easy to demonstrate that this is always true, and thus again
364	this solution is always stable when it exists. This means that it is possible to find
365	combinations of parameter values where investment in post-attack defences occurs
366	without any investment in pre-capture defence.
367	
368	Since this is the mirror image of the case above, it will not be surprising to find very
369	analogous conditions for this case. The conditions for the existence of this solution are
370	
371	$-1 < \beta_2 < 0$ $-2a_2 < \beta_2 < 0$
372	
373	giving $-2\min(0.5, a_2) < \beta_l < 0$ (7)
374	
375	which together with (6) gives the conditions for the $(0,D_2)$ solution.
376	
377	These limits can again be interpreted intuitively as above, and are shown graphically
378	in Figure 1.

380	Note, however, a significant difference between conditions (4) & (6); whilst a_2 had no
381	effect on the boundaries of the pre-attack only case, a_1 (as well as a_2) does affect the
382	boundaries of the post-attack only case. This asymmetry arises from the fundamental
383	asymmetry between the two types of defences, pre-attack defences occur before post-
384	attack defences, and thus influence the frequency with which post-attack defences are
385	used. However, post-attack defences do not affect the frequency with which pre-
386	attack defences are used. Increasing a_1 (increasing the costs of utilising a pre-attack
387	defence or decreasing its efficacy) increases the extent of (β_1, β_2) space where this
388	post-attack only investment strategy occurs – again this is just as we would expect.
389	
390	Non-zero investment in both forms of defence

At such a solution, the derivatives of equation (1) with respect to both D_1 and D_2 will be zero. Substituting the specific functional forms and differentiating gives:

$$394 \quad \frac{-f_1}{\epsilon T} + p_1 - 2p_1c_1D_1 + (2p_1c_1D_1 - p_1 - c_1)(1 - c_2D_2)p_2D_2 = 0$$
(8)

395

$$396 \quad \frac{-f_2}{\varepsilon T} + (1 - p_1 D_1)(1 - c_1 D_1)(p_2 - 2p_2 c_2 D_2) = 0 \tag{9}$$

398 These can be solved simultaneously for the non-zero combination of
$$D_1$$
 and D_2 that
399 maximises the payoff.

Using (9) we can get an expression for D_2 in terms of D_1 :

403
$$D_2 = \left(\frac{1}{2c_2}\right) \left(1 - \frac{f_2}{p_2 \epsilon T (1 - p_1 D_1) (1 - c_1 D_1)}\right)$$
 (10)

404

405 Using this substitution, and rearranging equation (8) gives

406

407
$$\left(\frac{p_2}{4c_2}\right)\left(1 - \left[\frac{f_2}{p_2 \varepsilon T (1 - p_1 D_1)(1 - c_1 D_1)}\right]^2\right) = \frac{\frac{f_1}{\varepsilon T} - p_1 (1 - 2c_1 D_1)}{c_1 + p_1 (1 - 2c_1 D_1)}$$
 (11)

408

409 It is relatively easy to show that as D_1 increases the RHS of this always increases,

410 because the nominator gets larger and the denominator gets smaller, and the LHS

411 always decreases (providing $c_1D_1 < 1$ and $p_1D_1 < 1$: which we have assumed to hold

412 earlier). Thus there can be at most one solution where the two sides balance and so at

413 most one solution with
$$D_1 > 0$$
 and $D_2 > 0$.

414

415 Thus, for any parameter value combination, if a solution (D_1, D_2) exists, then this

416 solution is unique.

417

418 Using the standard methodology of considering the determinant of the Hessian matrix,

419 the solution will be stable when this determinant is negative, so that

420

421
$$\frac{\partial^2 R_l}{\partial D_1^2} \frac{\partial^2 R_l}{\partial D_2^2} > \left(\frac{\partial^2 R_l}{\partial D_1 D_2}\right)^2, \frac{\partial^2 R_l}{\partial D_1^2} < 0, \frac{\partial^2 R_l}{\partial D_2^2} < 0$$

422

423 at the solution values (D_1, D_2) . We obtain

425
$$\frac{\partial^2 R_l}{\partial D_1^2} = 2p_1 c_1 (p_2 D_2 (1 - c_2 D_2) - 1)$$

426
$$\frac{\partial^2 R_l}{\partial D_2^2} = -2p_2c_2(1-p_1D_1)(1-c_1D_1)$$

427
$$\frac{\partial^2 R_l}{\partial D_1 D_2} = (2p_1 c_1 D_1 - p_1 - c_1)(p_2 - 2p_2 c_2 D_2)$$

428

- 429 It is clear that both second derivatives are negative and so we only need the first
- 430 condition to be solved. Substituting for D_2 in (8) and tidying yields

431
$$4p_{1}c_{1}p_{2}c_{2}v_{1} - p_{2}^{2}p_{1}c_{1}v_{1} + \left(\frac{f_{2}}{\epsilon Tv_{1}}\right)^{2}(p_{1}c_{1}v_{1} - (p_{1} + c_{1} - 2p_{1}c_{1}D_{1})^{2}) > 0$$
(12)

433 where
$$D_1$$
 is given by the solution of (11) and $v_1 = (1 - p_1 D_1)(1 - c_1 D_1)$.

434

435 Thus a stable non-zero investment occurs if the solutions of (10) and (11) yield values

436 of
$$D_1$$
 and D_2 such that

437
$$0 < D_1 < \min\left(\frac{1}{p_1}, \frac{1}{c_1}\right), \ 0 < D_2 < \min\left(\frac{1}{p_2}, \frac{1}{c_2}\right)$$

438

439 and (12) holds. An unstable solution occurs if (10) and (11) yield such values and (12)

440 does not hold. Unlike the other strategies discussed so far, this mixed-defences

441 strategy is not guaranteed to be stable whenever it exists.

442

443 **Co-existence of solutions**

- 444 We consider the five possible solutions Stable (D_1, D_2) ; Unstable (D_1, D_2) ; $(D^*_1, 0)$;
- 445 Stable $(0, D_2^*)$; (0,0) where we use D_1^* and D_2^* to indicate that if one of these single

446	defence solutions were to coexist for the same parameters as (D_1, D_2) , the levels
447	would in general be different to that of the same defence in the two-defence solution.
448	
449	It is clear from equations (2) , (3) , (5) and (7) that $(0,0)$ cannot co-exist with either of
450	the single defence strategies, and by noticing that the left hand side of equation (9)
451	decreases with increasing levels of either defence, it is also clear that it cannot co-
452	exist with either of the two-defence solutions (either the two-defence solution is stable
453	when it exists or it is unstable).
454	
455	We shall now consider the two single-defence solutions. The lower boundary to the
456	pre-attack only solution $(D_1,0)$ is given when the inequality in (4) is replaced by an
457	equality i.e.
458	$\boldsymbol{\beta}_2 = \left(\frac{\boldsymbol{\beta}_1}{4a_1}\right) (\boldsymbol{\beta}_1 + 2 + 2a_1). \tag{13}$
459	0
460	The left-most boundary of the post-attack-only solution in Figure 1 is attained using
461	(6) in a similar way to give:
462	G
463	$\beta_1 = \left(\frac{1+a_1}{4a_2}\right)\beta_2(\beta_2+2). $ (14)
464	
165	The endiant $d\theta/d\theta$ of the houndary of the (D, 0) solution evaluated at the endiant

465 The gradient $d\beta_2/d\beta_1$ of the boundary of the $(D_1,0)$ solution, evaluated at the origin is 466 simply $1/(2a_1)$. For the left-most boundary to the $(0,D_2)$ solution, the gradient $d\beta_1/d\beta_2$ 467 evaluated at the origin is simply $(1+a_1)/(2a_2)$. Thus, the gradient $d\beta_2/d\beta_1$ at this point 468 is $2a_2/(1+a_1)$.

469 470 From Figure 1, we can see that there will be an area of overlap of the two regions if 471 the gradient of the $(0,D_2)$ boundary is less steep than at the of the $(D_1,0)$ boundary at 472 the origin, i.e. if 473 $\frac{2a_2}{1+a_1} < \frac{1}{2a_1} \,.$ 474 (15)475 476 However if (15) is not satisfied for a particular set of parameter values then the two 477 single-defence-only solutions cannot co-exist. 478 479 From before, the conditions for a solution $(0,D_2)$ are given by (6) and (7) and the 480 conditions for a solution $(D_1, 0)$ are given by (4) and (5). If we look at the simplifying case where $a_2 = a_1 = 0.5$, then (4) becomes 481 , ec 482 $\beta_2 > \frac{\beta_1}{2} (\beta_1 + 3)$ 483 484 (5) becomes $-1 < \beta_l < 0$, 485 486 487 (6) becomes 488 $\beta_1 > \frac{3\beta_2}{4} (\beta_2 + 2),$ 489

490

491 and (7) becomes $-1 < \beta_l < 0$.

4	9	2
-	/	-

493	In general, $\beta_1 > -1$ and $\beta_2 > -1$ is the allowable region if $c_1 \ge 0.5 p_1$, $c_2 \ge 0.5 p_2$ and
494	$p_1\beta_1 > -2c_1$ (or $p_2\beta_2 > -2c_2$) if $c_1 < 0.5 p_1$ ($c_2 < 0.5 p_2$). So if $p_1 \le 2c_1$ and $p_2 \le 2c_2$ then
495	the full range of possible β s are valid (and so any parameter sets of this type give
496	solutions as above), but if $p_1 > 2c_1$ and/or $p_2 > 2c_2$ then there will be parameter
497	combinations that we cannot solve in this way; these cases correspond to defences
498	which are so effective that a reasonable investment can reduce the probability of
499	capture to zero, and we ignore this possibility here.
500	
501	We now show that (D_1, D_2) cannot occur with $(D^*_1, 0)$. For both of these to be
502	solutions, each must satisfy equation (8), and we can see from this that $D_{I}^{*}>D_{I}$.
503	For (D_1, D_2) to be a solution we need these values to solve (10) and (11). For $(D^*_1, 0)$
504	we need equation (11) to be solved with the left-hand side replaced by 0. However
505	this means that in this second solution the left-hand term is less than in the first, but
506	the right-hand term is greater than the first $(D*_{l}>D_{l})$ and the right-hand term increases
507	with D_1); which yields a contradiction. Thus the two solutions cannot occur for the
508	same parameters.
509	
510	Hence (D_1, D_2) can only occur together with $(0, D_2)$ (see Figure 1), or as a unique
511	solution.
512	
513	In all of our numerical calculations (see the following section) a stable solution of this
514	type (with investment in both defences) exists whenever none of the other three types
515	do, although we have been unable to prove that this must always be the case.

517	We have shown there are five possible equilibrium solutions, four of which are ESSs
518	$((0,0), (D_1,0), (0,D_2)$, stable (D_1,D_2) , together with the unstable (D_1,D_2) which is not
519	an ESS). Thus without restrictions, there would be 32 possible combinations of
520	solutions. However, we have shown that $(0,0)$ can only exist as the sole solution, and
521	that there can never be more than one solution from $(D_1,0)$, stable (D_1,D_2) and
522	unstable (D_1, D_2) . This leaves nine possible combinations, which we consider below.
523	The following five combinations are all observed in Figure 1:(0,0); $(D_1,0)$; $(0,D_2)$;
524	stable (D_1, D_2) ; $(0, D_2)$, stable (D_1, D_2) . In addition to some of these, both $(D_1, 0)$, $(0, D_2)$
525	and $(0,D_2)$, unstable (D_1,D_2) are observed in Figure 3. This leaves the two possibilities
526	of only unstable (D_1, D_2) and no solution, which are the two possibilities leading to no
527	ESS. We have been unable to prove that these cannot occur, but have not observed
528	them in any of our numerical investigations.
529	
530	Numerical Investigation of the Model

- Recall our definitions: $\beta_{1} = \frac{f_{1}}{p_{1}\varepsilon T} 1$ and

535
$$\beta_2 = \frac{f_2}{p_2 \varepsilon T} - 1.$$

- We know that (0,0) occurs when β_1 and β_2 are both positive, and under these
- circumstances (0,0) can be the only solution. This translates to conditions $f_1 > p_1 \epsilon T$
- and $f_2 > p_2 \mathcal{E} T$.

For all of the examples that follow we assume the values $p_1 = 0.1$, $p_2 = 0.1$, $\epsilon T = 10$ and $c_2=0.2$. Then (0,0) is the unique solution when $f_1 > 1$ and $f_2 > 1$. The boundaries of these conditions are shown by the dotted lines in Figure 1, and the region of no investment in defences lies above both these boundary values of f_1 and f_2 in the top right corner of Figure 1 (note the dotted lines will be partially covered by other lines). For the situation where the prey should invest only in pre-capture defences, there are

548 two restrictions on f_1 : $-2 < \beta_1 < 0$ and $-2a_1 < \beta_1 < 0$. As before, the prey should not

549 invest in post-capture defences where $f_2 > p_2 \epsilon T$. Under our default parameters

550 values, $c_1 = 0.2$, and thus $a_1 = 2$, since $a_1 = c_1/p_1$. and therefore the most restrictive

condition is that not involving a_1 . The region of (f_1, f_2) parameter space where only

552 pre-capture defences are predicted therefore occurs when β_1 is negative and (by re-

553 arrangement) $f_I > p \in T$. Since $f_1 = p_1 \in T \beta_1 + 1$, for our default values these fall at f_I

values of 1 and -1. However, we are only interested in non-negative values of f_1 and f_2 , so the critical f_2 values become 0 and 1.

556

557 There is also a restriction on f_2 given by

558

559
$$\beta_2 > \left(\frac{\beta_1}{4a_1}\right)(\beta_1 + 2 + 2a_1)$$

560

561 In order to find this line, we simply take a range of f_1 values, between the two extreme 562 values (0 and 1), convert these to β_1 values, and hence to β_2 values, before converting 563 these to f_2 values. Since

565	$f_2 = p_2 \varepsilon T \beta_2 + 1,$
566	
567	we need to specify the value of p_2 . We assume this takes the value 0.1. Thus, the
568	curved broken line and the vertical lines at f_1 equals 0 and 1 in Figure 1 enclose the
569	region where investment in pre-capture defences makes post-capture unnecessary.
570	This is the top, left region of Figure 1.
571	
572	A very similar situation occurs for the $(0,D_2)$ solution. Here, there are two restrictions
573	on f_2 : $-2 < \beta_2 < 0$ and $-2a_2 < \beta_2 < 0$. Let us assume that $c_2 = 0.3$, and thus $a_1 = 3$, since
574	$a_2 = c_2/p_2$. Thus, the most restrictive condition does not involve a_2 .
575	Since $f_2 = p_2 \varepsilon T \beta_2 + 1$, for our default values these fall at f_2 values of 1 and -1.
576	However, we are only interested in non-negative values of f_1 and f_2 , so the critical f_2
577	values become 0 and 1.
578	
579	There is then a restriction on f_1 given by
580	
581	$\beta_1 > \left(\frac{1+a_1}{4a_1}\right)\beta_2(\beta_2+2)$
582	
583	In order to find this line, we simply take a range of f_2 values, between the two extreme
584	values, convert these to β_2 values, and hence to β_1 values, before converting these to f_1

- values, convert these to β_2 values, and hence to β_l values, before converting these to f_l
- 585 values. Thus, the solid curved line and the horizontal lines at $f_2 = 0$ and $f_2 = 1$ to the
- 586 right of the curved line enclose the parameter values that yield this solution (in the
- 587 bottom, right part of Figure 1).

588

589	Lastly, we show the solution where investment is spread across both defences, these
590	must be solved numerically, and are only valid if both D values are positive, and that
591	all four P and C values are positive. All these solutions are shown in Figure 1; the
592	situation where non-zero investment in both defences is shown as a hatched region.
593	Notice that to the right of the solid curved lines there are parameter combinations
594	where the both-defences solution co-exists with another solution involving investment
595	only in post-attack defences.
596	
597	In order to explore the nature of the mixed solution, in Figure 2 we plot the D_1 and D_2
598	values for the mixed solution along a transect of f_1 values where we hold f_2 at the
599	value 0.6. We observe that for f_1 values close to zero there is no valid mixed solution,
600	since simultaneous solution for non-zero D_1 and D_2 values predicts a negative D_2
601	value. In this region the only solution is for zero investment in D_2 and all investment
602	to be channelled into D_1 . At around f_1 values of 0.4 we do begin to get a mixed
603	solution with initially very low investment in D_2 . The D_1 value at this point shown on
604	Figure 2 is close to the value 1.5 predicted for the solution with zero investment in D_2
605	at $f_1 = 0.4$. Generally as f_1 increases, so D_2 increases and D_1 decreases. Eventually, at
606	an f_1 value around 1.05 the D_1 value falls to zero, when this occurs, then Figure 2
607	predicts the D_2 value to be close to the 0.667 value predicted for the solution with no
608	investment in D_1 at this point.
609	

610 In Figure 3 we present the result of the same evaluation as in Figure 1 but where c_2 is 611 reduced six-fold to 0.05. Although the predictions are superficially similar, there are 612 several interesting differences. Firstly, there is now a region of f_1 - f_2 space where both

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613	the $(0,D_2)$ and $(D_1,0)$ solutions are valid and stable. We must also consider the
614	stability of the solution of investment across both defences. In figure 3, we only plot
615	such solutions where they are stable. In Figure 1, the interior solution is stable
616	whenever it exists. This is not true for the parameters chosen for Figure 3. As in
617	Figure 1, when $f_1 < 1$, the interior solution exists for all f_2 values below the broken
618	line marking the region of $((D_1, 0)$ solutions. However, it is only stable for a subset of
619	lower f_2 values. Thus, there can be a region where both $(0,D_2)$ and (D_1,D_2) are
620	potential solutions but only the first of these is stable. As we have seen in Figure 1, it
621	is possible also to find regions of parameter space where both solutions are stable
622	simultaneously.
623	
624	Discussion

624 Discussion

In his influential work, Endler (1991) suggested that it may be more attractive for prey 625 626 to interrupt the predation sequence as early as possible (see also Planque et al (2002) 627 who argue the same relating to successive defences against brood parasitism). He argues this for a number of reasons. Firstly, failure of a later defence may be more 628 629 likely to be catastrophic for the prey. Secondly, later in the sequence the performance 630 of the prey may be weakened or exhausted by implementation of defences earlier in 631 the sequence. Thirdly, the lost time and energy spent in repelling the attack will be 632 reduced if the attack is resolved early in the sequence. Finally, investment in defences 633 later in the sequence may be inefficient, if these defences are seldom used because 634 defences used earlier in the sequence are generally successful in repelling attacks 635 before the later-acting defences are implemented. The last point especially argues that 636 we should expect investment in early (pre-capture) defences, but not in later defences. 637

638 Furthermore, early defences are more often constitutive, in that the prey does not have 639 to detect the predator to be undetected, and so predator encounters might not affect the 640 cost of early defences . In contrast, post-encounter defences are more often 641 responsive, involving defensive behaviours or other physiological costs, such as 642 regurgitation or reflex bleeding that are triggered by a specific attack. 643 644 Our model predicts that there can be circumstances where it is optimal for the prey to 645 invest in neither defence. This occurs if the constitutive costs of the defences are high, 646 if the defences have low efficacy (per unit of investment), if attacks are rare and if the 647 period during which the prey is exposed to predation is short. There are also 648 circumstances where we predict investment in pre-capture defences but not post 649 capture defences. This solution is more likely to occur under conditions where β_l is 650 negative and β_2 is positive (or only just negative). The parameter β for a given defence can be seen as a description of the cost-benefit ratio of the defences: β 651 652 becomes larger (more positive) if the constitutive costs of the defence are high. β also 653 becomes larger if the attack rate, the period that must be survived by the prey and/or 654 the efficacy of the defence are low. Whether or not we predict investment in pre-655 capture defences only is also affected by the value of a_1 . This parameter takes a high 656 value if the costs of using pre-capture defences are high and/or if the effectiveness of 657 pre-attack defences is low. Increasing a_1 restricts the area of (β_1, β_2) space where 658 investment only in pre-capture defences is optimal. The level of investment in pre-659 capture defences increases as β_1 becomes more negative and/or as c_1 is reduced. 660 661 In circumstances where the asymmetry of the cost/benefit ratios are opposite (that is

662 β_s is negative and β_l is positive (or only just negative), then we get investment in

663 post-attack defences only. Where the two β values are generally similar and not too 664 large and positive, we predict investment across both defences. Indeed, such solutions 665 are likely where f_1 and f_2 are small, which might be reasonable for defences such as 666 colouration and sequestering of defences. Many defences serve other, perhaps primary 667 functions, such as catching or subduing prey, and so their marginal canonical costs in 668 terms of defence might be small. We find that the costs of one defence affects the 669 likelihood of investment in the other (cf Figure 3 and 1).

670

671 Further, there are combinations of parameter values for which different strategies are 672 both stable. In such circumstances the strategy to which the population settles will 673 depend on the history of the population. Thus, the model predicts that it is difficult to 674 make generalisations about how many and which defences a certain prey type will 675 invest in without quantitative evaluation of the different properties of each of the 676 alternative defences, as we discuss above. Our model might therefore provide insight 677 in to, for example, the evolution of aposematism in the lepidoptera. Many closely 678 related species have differing defensive strategies, with some investing heavily in pre-679 capture defences (excellent crypsis paying opportunity costs) and others having both 680 pre-capture (warning coloration) and post-capture (aversive chemistry) defences. Our 681 model shows how easily these might be equally fit, and so divergent selection might 682 occur based initially on very minor differences, or even genetic drift.

683

There is a significant difference between the two types of defence, in that pre-attack defences occur before post-attack defences, and thus influence the extent to which post-attack defences are used. However, post-attack defences do not affect the frequency with which pre-attack defences are used. This results in the co-existence of

solutions in both defences and in only post-attack defences, but not in both defencesand only pre-attack defences.

690

691	An interesting extension of our model would be to include a range of predator types
692	(or equivalently a range of different predatory situations), with different defences
693	differing in their rank order of efficacy against these different predators. In particular,
694	this would allow exploration of generalism and specialism in defences and in
695	particular the general trend (noted by both Endler 1991 & Caro 2005) that defences
696	used later in the predation sequence are more likely to be specialised for a particular
697	type of predator than more general defences used earlier in the sequence. Again, the
698	role of multi-functional defences was identified by Caro (2005) as another of his ten
699	most pressing questions in predator-prey interactions.
700	
701	Endler's (1991) categorisation of different anti-predatory defences in animals has
702	been highly influential, but development of theoretical underpinning for his
703	predictions and empirical testing of these predictions has been lacking. We hope our
704	work will be a useful early step in developing a more solid theoretical base for
705	understanding diversity of anti-predatory traits. There have been developments on the
706	empirical side. Low (1998) carefully demonstrated investment across a range of
707	defences by the leaf-mining larvae of a moth (Antispila nysaefoiella) against a
708	specialist parasitoid. Langridge et al (in prep) demonstrated that the anti-predatory
709	behaviours of cuttlefish (Sepai officinalis) accords with three of Endler's specific
710	predictions: that later in the sequences of an attack, defences become more predator-
711	specific, more risky if unsuccessful in deterring attack and more costly to mount.
712	More empirical and theoretical work to explore and develop these predictions would

be valuable. In this, an important resource may be the comparatively much more
extensive literature seeking to understand the diversity of anti-herbivore traits shown
by plants (e.g. Biere et al, 2004, Ode 2006). We suggest that a useful next
development for our understanding of anti-predator behaviours would be to explore
how applicable current understanding of anti-herbivore defences is to the different
system.
The diversity of possible solutions to investment across only two defences in our
simple model where prey face only one type of attack demonstrates that a general
understanding of the diversity of anti-predatory suites used by different prey will be a
challenge. However, to meet that challenge, we feel that we have demonstrated that
simple verbal reasoning will be insufficient and models that allow quantitative
evaluation of a complex of different costs and benefits (as well as interaction between
the effectiveness of defences, not considered here) will be required.
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728 **References**

729	Biere, A., Marak, H.B., van Damme, J.M.M. 2004. Plant chemical defense against
730	herbivores and pathogens: generalised defense or trade-off? Oecologia 140,
731	430-441
732	Bowers, M.D. 1993 Aposematic caterpillars: life-styles of the warningly colored and
733	unpalatable. In: Stamp, N.E., Casey. T.M. (eds) Caterpillars: ecological and
734	evolutionary constraints on foraging. Chapman & Hall, London, pp 331-371
735	Broom. M., Ruxton, G.D. 2005 You can run – or you can hide: optimal strategies for
736	cryptic prey against pursuit predators. Behavioural Ecology 16, 534-540
737	Caro, T 2005. Antipredator defenses in birds and mammals. Chicago University Press,
738	Chicago.
739	Derby, C.D. 2007. Escape by inking and secreting: Marine molluscs avoid predators
740	through a rich array of chemicals and mechanisms. Biol. Bull. 213:274-289
741	Eisner, T. 1958. The protective role of the spray mechanism of the bombadier beetle,
742	Brachynus ballistarius. J Ins Physiol 2:215-220
743	Eisner, T., Rossini, C., González, A., Eisner. M. 2004. Chemical defense of an
744	opilionid (Acanthopachylus aculeatus). J Exp Biol 207:1313-1321
745	Endler, J.A. 1991 Interactions between predators and prey. In Behavioural Ecology:
746	an evolutionary approach (eds. JA Krebs & NB Davies) pp 169-196.
747	Blackwell Scientific, Oxford.
748	Grill, C.P., Moore, A.J. 1998. Effects of a larval antipredator response and larval diet
749	on adult phenotype in an aposematic ladybird beetle. Oecologia 114, 274-282.
750	Higginson, A.D., Ruxton, G.D. 2009 Dynamic state-dependent modelling predicts
751	optimal usage patterns of responsive defences. Oecologia 160:399-410.

- 752 Low, C. 2008. Grouping increases visual detection risk by specialist parasitoids.
- 753 Behavioral Ecology 19, 532-538
- 754 Ode, P.J. 2006. Plant chemistry and natural enermy fitness: effects on herbivore and
- 755 natural enemy interactions. Annual Reviews in Entomology 51, 163-185
- 756 Planque, R., Britton, N.F., Franks, N.R., Peletier, M.A. 2002. The adaptiveness of
- 757 defence strategies against Cuckoo parasitism. Bulletin of Mathematical
- 758 Biology 64, 1045-1068.
- 759 Ruxton, G.D., Sherratt, T.N., Speed M.P. 2004 Avoiding attack: the evolutionary
- 760 ecology of crypsis, warning signals and mimicry. Oxford University Press,
- 761 Oxford.
- 762 Van Buskirk, J. 2001. Specific induced responses to different predator species in
- ary L 763 anuran larvae. Journal of Evolutionary Biology 14, 482-489.
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765	
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767	Figure captions
768	
769	Figure 1: Different optimal solutions in terms of investment in pre-capture and post-
770	capture defences for different combinations of values of the parameters f_1 and f_2 . The
771	higher the values of f_1 and f_2 , the higher the fixed costs of pre-capture and post-
772	capture defences respectively. Other parameter values: $\epsilon T = 10$, $p_1 = 0.1$, $p_2 = 0.1$, $c_1 =$
773	0.2, $c_2 = 0.3$. When both f_1 and f_2 are greater than 1 (top right quarter of this figure),
774	then zero investment in both defenses is predicted. There is a region where f_l is less
775	than 1 and f_2 is sufficiently high where investment only in pre-capture defences is
776	predicted. This region is bounded by the vertical dotted line at $f_1 = 1$ and the curved
777	broken line that separates this region from a hatched region below. The hatched
778	region indicates parameter combinations for which investment across both defences is
779	predicted. Note this solution can co-exist for some parameter combinations with
780	investment only in the later-acting (post-capture) defence. The both-defences solution
781	is plotted only when it is stable (although it is stable in all cases where it exists for this
782	figure, this is not true in general). The region where only investment in post-capture
783	defences is optimal occurs in the bottom right of the figure and is bounded by the
784	horizontal broken line at $f_2 = 1$ and the curved solid line that cuts through the middle
785	of the hatched region.
786	
787	Figure 2: D_1 (broken line) and D_2 (solid line) values for the mixed solution shown in

Figure 1 along a transect of f_1 values where we hold f_2 at the value 0.6. All other

- values are as in Figure 1. Clearly, only combinations where both D_1 and D_2 are nonnegative are valid.
- 791

792	Figure 3: Different optimal solutions in terms of investment in pre-capture and post-
793	capture defences for different combinations of values of the parameters f_1 and f_2 . The
794	higher the values of f_1 and f_2 , the higher the fixed costs of pre-capture and post-
795	capture defences respectively. The values of the other parameters are the same as
796	those used for Figure 1, except $c_2 = 0.05$. When both f_1 and f_2 are greater than 1 (top
797	right quarter of this figure), then zero investment in both defenses is predicted. There
798	is a region where f_1 is less than 1 and f_2 is sufficiently high when investment only in
799	pre-capture defences is predicted. This region is bounded by the vertical dotted line at
800	$f_1 = 1$ and the curved dotted line. The hatched region indicates parameter
801	combinations for which investment across both defences is predicted and this solution
802	is stable. However, this solution is not stable everywhere where it is valid, and so this
803	solution is substantially less commonly predicted than in Figure 1. Note that this
804	solution can co-exist for some parameter combinations with investment only in the
805	later-acting (post-capture) defence. The region where only investment in post-capture
806	defences is optimal occurs in the bottom right of the figure and is bounded by the
807	horizontal broken line at $f_2 = 1$ and the curved solid line. Between the two curved
808	lines there is a region labelled "both", where both the solution with investment only in
809	post-capture defences and the solution with investment only in pre-capture defences
810	are possible.
811	

812

813 Fig. 1



Fig. 2



Fig. 3

