

Supplementary Material to “Dynamics and Thermodynamics of the Low-Temperature Strongly Interacting Bose Gas”

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CANONICAL TO GRAND-CANONICAL CORRESPONDENCE AND THE FIT TO THE LEE-HUANG-YANG PARAMETER

The pressure formula expresses the EoS $P(\mu)$ in the grand-canonical ensemble where μ is fixed. We perform a Legendre transform to the canonical ensemble (where the density n is fixed) to obtain $E(n)$ where E is the ground-state energy:

$$\frac{E}{N} = \frac{\hbar^2}{ma^2} \xi(y \equiv na^3) \quad (1)$$

where ξ is the dimensionless energy and y is the usual canonical interaction parameter. Similarly one writes the grand-canonical EoS:

$$P(\mu, a) = \left(\frac{\hbar^2}{ma^5} \right) h \left(\nu \equiv \frac{\mu}{g} a^3 \right), \quad (2)$$

Combining the Legendre transform $-PV = E - \mu N$ at zero temperature, with the Gibbs-Duhem formula $\partial P/\partial \mu = n$, one finds the following set of correspondence equations:

$$\nu(y, \xi) = \frac{1}{4\pi} (\xi(y) + y\xi'(y)) \quad (3)$$

$$h(y, \xi) = y(4\pi\nu(y, \xi) - \xi(y)) \quad (4)$$

The inverse transformation can be readily derived using $\partial E/\partial N = \mu$ instead of the Gibbs-Duhem relation. The measured EoS, $h(\nu)$, can thus be used to extract the value of the Lee-Huang-Yang parameter α_{LHY} , that is defined in the canonical ensemble as: $\xi(y, \alpha) = 2\pi y(1 + \alpha y^{1/2})$.

SELF-CONSISTENT RECONSTRUCTION OF THE EOS

The measurement of the EoS is based on the pressure formula [1, 2]:

$$P(\mu_z) = \frac{m\omega_r^2}{2\pi} \bar{n}(z) \quad (5)$$

The pressure can thus be directly measured from the doubly-integrated density profiles. The appropriate

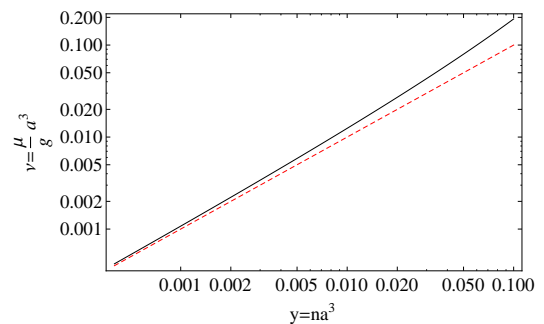


FIG. 1: (color online) Relation between the grand canonical $\nu = \frac{\mu}{g} a^3$ and the canonical gas parameters $y = na^3$. In solid black line, the MF relation $\nu = y$, in dashed red line $\nu(y)$ as predicted by the LHY equation of state.

thermodynamic variable is the chemical potential μ that varies in the trap according to $\mu_z = \mu_0 - \frac{1}{2}m\omega_z^2 z^2$ within the local-density approximation (LDA). The main issue lies in the determination of the global chemical potential μ_0 on each image. However, deducing μ_0 from the density profile is equivalent to the knowledge of $n(\mu)$, the EoS itself. In previous studies, the global chemical potential was determined using a known asymptotic behaviour of the EoS [3–5]. Because the mean-field (MF) limit is used to calibrate the density measurement, we cannot rely on a similar method. To overcome this issue, we have implemented a new scheme. The global chemical potential value at $T = 0$ is given by $\mu_0 = \frac{1}{2}m\omega_z^2 R_0^2$ where R_0 is the value at which the density profile vanishes. The determination of R_0 on a density profile with finite signal-to-noise ratio will in general depend on the choice of the fitting function to the profile. The simplest choice is the density profile expected by the MF prediction, the inverted parabola. However this leads to an EoS that is not self-consistent because the inverted parabola assumes the MF-EoS to deduce a different EoS. One can then implement an iterative scheme to obtain a self-consistent EoS. At the first step, one start with the density profiles $\bar{n}_i(z)$ (for $i = 1, \dots, M$) fitted with $\bar{n}^{(1)}(z) = n_0 \left(1 - \frac{z^2}{R^2} \right)^2$. From the values of R_i , one deduces $\mu_{0,i}^{(1)}$ and a first step EoS $P^{(1)}(\mu)$. From this

first step EoS, one can generate density profiles (using the pressure formula), using an interpolation function for $h(\nu)^{(1)} = 2\pi\nu^2(1 + \gamma_1\nu + \gamma_2\nu^2)$. We fit again all the density profiles but with fitting function deduced from $h^{(1)}(\mu)$, to get a second set of radii, and hence a new set of chemical potentials $\mu_{0,i}^{(2)}$ for each image. This procedure is iterated until a fixed point is reached and that the EoS no longer changes with additional iterations. The fixed point is a self-consistent EoS: the values of $\mu_{0,i}$ are determined using fitting function consistent with the EoS that is deduced.

We checked the convergence of this procedure to the correct solution and its robustness to the presence of noise in the density profiles. Using the Lee-Huang-Yang EoS, we generated a set of 15 density profiles, with a gaussian noise with a standard deviation σ normalized to the maximum amplitude of the profile at a value of $2150 a_0$ reached in the experiment. The EoS is then reconstructed using the iterative scheme. In practical, 10 iterations are sufficient to reach the fixed point. The self-consistent EoS obtained is then fitted with the grand-canonical transformation of the Lee-Huang-Yang energy $E(\alpha) = \left(\frac{\hbar^2}{ma^2}\right)^{-1} 2\pi na^3 \left(1 + \alpha\sqrt{na^3} + \dots\right)$. For a gaussian noise of the same amplitude as the experimental profiles ($\sigma = 0.06$), we find $\alpha = 4.7(3)$, in agreement with the theoretical value $\frac{128}{15\sqrt{\pi}} \approx 4.81$, demonstrating the validity of the procedure.

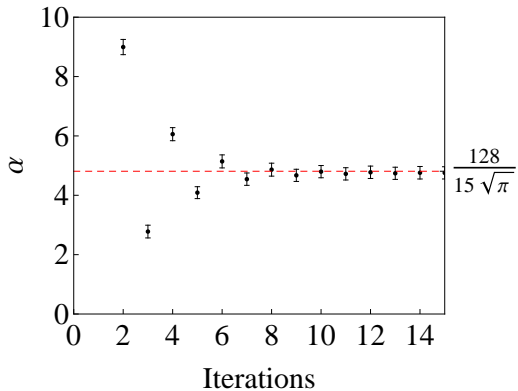


FIG. 2: (color online) Iterative construction of the EoS from images taken at $a/a_0 = 2150$. Parameter of the Lee-Huang-Yang beyond mean-field correction as a function of the iterations. Error bars represent the statistical error on the fits.

The Quantum Monte Carlo simulations provide an independent test on the determination of the chemical potential. The chemical potential associated with a given atomic distribution from the QMC is obtained from a fit of the wings of the density distribution with the mean-field prediction. The value of the global chemical potential can be used to check the experimental

one extracted from the self-consistent method. In the condition of Fig.1, the extrapolation to $T = 0$ of the QMC yields $\mu_0^{\text{QMC}}/k_B = 179.9$ nK, while we extract $\mu_0^{\text{exp}}/k_B = 177(2)$ nK from the density profile used for the χ^2 -test (inset of Fig.1).

RF-SPECTROSCOPY OF FESHBACH MOLECULES

The two-body interaction is described by the s -wave scattering length a that we vary by means of a magnetic Feshbach resonance. Its precise knowledge as a function of the magnetic field B is crucial for the accurate determination of the EoS. In [6] the scattering length $a(B)$ was deduced from the axial size of trapped clouds, assuming a gas at zero temperature, a method which by nature is afflicted with significant uncertainties. Here, we perform radio-frequency (RF) spectroscopy as in [7] to measure the binding energy of weakly-bound dimers from which we deduce the scattering length more precisely.

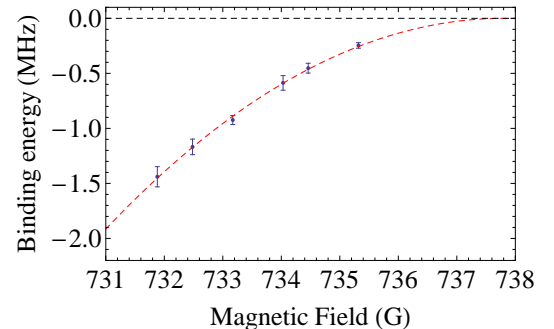


FIG. 3: (color online) Binding energy of the Feshbach molecules as a function of the magnetic field.

We prepare the atomic gas at a temperature of about $T/T_c = 1.2$ and at a magnetic field $B = 717$ G on the repulsive side of the Feshbach resonance near 740 G and apply an rf-field during 50 to 200 ms. When the frequency of the modulation is close to the binding energy of the two-body bound state, pairs of free atoms are associated to dimers that are lost via collisional relaxation into deeper bound states. We then obtain the binding energy from a Gaussian fit to the dip in the number of detected atoms. The binding energy E_b relates to the scattering length via the following formula:

$$E_b = \frac{\hbar^2}{m(a - \bar{a} + R^*)^2}, \quad (6)$$

where \bar{a} is a finite range correction [8] and R^* the effective range of the interatomic potential [9]. Since the values of these two parameters are not known accurately, we restrict our measurement for the EoS to magnetic fields higher than 733 G. In this range $a/a_0 > 700$,

much larger than $\bar{a} = 30 a_0$ and $R^* = 58 a_0$ [7]. We can compare our experimental data with the simpler relation $E_b = \hbar^2/(ma^2)$ to obtain the scattering length $a(B)$. Fitting the parametrization $a(B) = \frac{\Gamma}{B-B_0}$ to our experimental results yields the position $B_0 = 737.8(2)$ G and the width $\Gamma/a_0 = -3550(100)$ G of the Feshbach resonance. These values are in agreement with those found in [10] but differ from those in [6].

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