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ABSTRACT

In some applications as in biomedical analysis, we encounter the problem of estimating the common poles (angular-frequency and damping-factor) in a multi-channel set-up composed as the sum of Exponentially Damped Sinusoids. In this contribution, we propose a new subspace algorithm belonging to the family of the Orthogonal Vector Methods which solves the considered estimation problem. In particular, we expose a root-MUSIC algorithm which deals with damped components for an algorithmic cost comparable to the root-MUSIC for constant modulus components. Finally, we show by means of an example, that the proposed method is efficient, especially for low SNRs.

1. INTRODUCTION

Common poles estimation in a multi-channel Exponentially Damped Sinusoids (EDS) signal is an open problem since this problematic has been treated rarely. We can find some work for the two channels case [6, 7, 8] but, to our best knowledge, only a single method has been proposed so far for the case of more than two channels. This method, named MUSCLE [1] for MUlti-channel Subspace-based Common poLe Estimation, has been applied to the biomedical signal analysis and is based on shift-invariance of the signal subspace and Total-Least-Squares [6] technique.

In this paper, we propose a new solution to the common poles estimation problem which is based on the orthogonality of the signal and noise subspaces. Thus, the proposed method belongs to the family of the Orthogonal Vector Methods [4]. In addition, we propose an improved version of the DMUSIC algorithm [3] which is the MUSIC algorithm for damped sinusoids. This algorithm has been introduced in the context of the biomedical analysis processing and is based on a costly multidimensional search. In this paper, we propose a "root" version of the DMUSIC algorithm, called root-DMUSIC. More specifically, we avoid the costly multidimensional search by zeroing a polynomial form and by inspecting the orthogonality condition for preselected zeros. This is an efficient way to decrease the complexity burden of the method without additional assumptions on the damping-factor values. Finally, the overall complexity cost of the proposed method is comparable to the root-MUSIC [2] for constant modulus components. Consequently, this new algorithm allows the comparison with shift-invariant techniques as the MUSCLE algorithm. Moreover, we show on noisy multi-channel EDS signal example that in some situations, it is preferable, in terms of estimation accuracy and even w.r.t the computational cost, to use the root-DMUSIC algorithm instead of the MUSCLE algorithm.

2. COMMON POLE ESTIMATION FOR THE MULTI-CHANNEL EDS MODEL

The multi-channel EDS model is given for $k = 1, \cdots, K$ and $t = 0, \cdots, N/2 - 1$ by:

$$x_k(t) = Z_c a_k(t) + Z_k b_k(t) \ \in \mathbb{C}^{N/2}, \tag{1}$$

where K is the number of channels, N is the sample size,

$$a_k(t) = [\alpha_k(1) z_{1,c}^t, \cdots, \alpha_k(M_c) z_{M_c,c}^t]^T$$

$$b_k(t) = [\beta_k(1) z_{1,k}^t, \cdots, \beta_k(M_k) z_{M_k,k}^t]^T$$

and $\alpha_k = [\alpha_k(1), \cdots, \alpha_k(M_c)]^T$, $\beta_k = [\beta_k(1), \cdots, \beta_k(M_k)]^T$ are two vectors of complex amplitude,

$$Z_c = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \vdots & \vdots & & \vdots \\ z_{1,c}^{N/2-1} & z_{2,c}^{N/2-1} & \dots & z_{M_c,c}^{N/2-1} \end{bmatrix}_{N/2 \times M_c} \tag{2}$$

contains the common poles $z_{m,c} = e^{d_{m,c} + i\omega_{m,c}}$ and:

$$Z_k = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \vdots & \vdots & & \vdots \\ z_{1,k}^{N/2-1} & z_{2,k}^{N/2-1} & \dots & z_{M_k,k}^{N/2-1} \end{bmatrix}_{N/2 \times M_k} \tag{3}$$

is associated to the non-common poles $z_{m,k} = e^{d_{m,k} + i\omega_{m,k}}$ of the k-th channel. The processing window is chosen equal to $N/2$ as its optimal value belongs to the range $[N/3,\ 2N/3]$ as shown in [4, 5]. The common pole estimation problem can be algebraically described according to:

$$\bigcap_{k=1}^{K} \mathcal{R}\left(\begin{bmatrix} Z_c & Z_k \end{bmatrix}\right) = \mathcal{R}\left(Z_c\right). \tag{4}$$

where $\mathcal{R}(A)$ denotes the column range space of matrix A. So, we look for the intersection of K spaces.

3. MUSIC-LIKE CRITERION

3.1. First criterion

The $N/2 \times N/2$ Hankel matrix associated to the k-th channel admits a Vandermonde-type decomposition according to:

$$\begin{aligned} X_k &= [x_k(0), \cdots, x_k(N/2-1)] \\ &= \begin{bmatrix} Z_c & Z_k \end{bmatrix} \begin{bmatrix} A_k & 0 \\ 0 & B_k \end{bmatrix} \begin{bmatrix} Z_c^H \\ Z_k^H \end{bmatrix} \end{aligned} \tag{5}$$

where $A_k = diag(\alpha_k)$ and $B_k = diag(\beta_k)$. Now, consider K projectors on the noise subspace of $X_1, \cdots, X_K$, denoted, respectively by $\{P_1^\perp, \ldots, P_K^\perp\}$, i.e. $P_i^\perp = I - P_i$ where P_i is the orthogonal projector on $\mathcal{R}([Z_c\ Z_i])$. We can give the following result:

Theorem 1 *The MUSIC-like criterion defined by:*

$$f(z) = d(z)^H \left(\sum_{k=1}^{K} P_k^\perp\right) d(z) \tag{6}$$

where $d(z) = [1, z, \cdots, z^{N/2-1}]^T$ is zero (minimum) if and only if $z \in \{z_{1,c}, \cdots, z_{M_c,c}\}$, i.e. iff z is a common pole.

Proof: First note that:

$$d(z)^H P_k^\perp d(z) = 0 \iff d(z) \in \mathcal{R}([Z_c\ Z_k]),$$

and

$$d(z)^H P_k^\perp d(z) > 0 \quad \textit{otherwise}.$$

Hence

$$d(z)^H \left(\sum_{k=1}^{K} P_k^\perp\right) d(z) = 0$$

is equivalent to

$$d(z) \in \bigcap_{k=1}^{K} \mathcal{R}\left(\begin{bmatrix} Z_c & Z_k \end{bmatrix}\right) = \mathcal{R}\left(Z_c\right). \qquad \blacksquare$$

3.2. Another criterion

Now assume that we know that several poles are common in several channels (but not in all channels), in that case we can exploit this knowledge to reduce the complexity burden of the proposed method. The new algebraic problem is given by:

$$\bigcap_{i=1}^{T} \left\{ \bigcup_{k \in \mathcal{Q}_i} \mathcal{R}\left([Z_c\ Z_k]\right) \right\} = \mathcal{R}(Z_c). \tag{7}$$

where T is the number of partition of set $\{1, \ldots, K\}$. The expression below means that we associate channels for which we know that there is common poles with constraint $\bigcup_{i=1}^{T} \mathcal{Q}_i = \{1, \ldots, K\}$ and $\mathcal{Q}_i \neq \emptyset$. In that case, we can consider the following modified criterion:

$$g(z) = d(z)^H \left(\sum_{i=1}^{T} \Pi_i^\perp\right) d(z). \tag{8}$$

where $\Pi_i^\perp$ is the orthogonal projector on the noise subspace of matrix:

$$[X_{k_1}, \cdots, X_{k_i}], \text{ with } Q_i = \{k_1, \cdots, k_i\}. \tag{9}$$

or equivalently matrix

$$\sum_{k \in \mathcal{Q}_i} X_k X_k^H, \text{ for } i \in [1:T]. \tag{10}$$

So, we have to estimate T projectors $\Pi_i^\perp$ in comparison with K projectors $P_k^\perp$ in the general case, *ie.*, without additional knowledge. As $T \leq K$, we reduce the algorithmic burden.

For instance, we can give the following example. Consider three channels according to:

channel 1	z_1	z_2	z_3	
channel 2	z_1	z_2	z_4	z_5
channel 3	z_1	z_2	z_5	z_6

The common pole set is $\{z_1,\ z_2\}$ and if we assume that we know that z_5 belongs to channel 2 and 3. So, we associate channels 2 and 3 according to $\mathcal{Q}_1 = \{1\}$ and $\mathcal{Q}_2 = \{2,\ 3\}$. As, we have two partitions ($T = 2$), we need only two projectors $\Pi_2^\perp$ associated to $[X_2\ X_3]$ and $\Pi_1^\perp$ associated to X_1. This approach reduces the algorithmic burden since we have to consider only two SVDs in comparison to three in the general case.

Another situation of interest is when the common poles for any pair of channels are only those desired poles $\{z_{1,c}, \cdots, z_{M_c,c}\}$. In that case, one can partition the set of channels according to $\mathcal{Q}_1 = \{1\}$ and $\mathcal{Q}_2 = \{2, \cdots, K\}$ which leads again to 2 SVD computations instead of K.

3.3. Estimation of the projectors

The estimation of the noise projectors $P_k^{\perp}$ is classically obtained by the SVD [6] according to:

$$X_k = [\ \underbrace{U_k^{(S)}}_{M_c+M_k}\quad \underbrace{U_k^{(N)}}_{N/2-M_c-M_k}\ ]\Sigma_k V_k^H \tag{11}$$

where $U_k^{(S)}$ (respectively $U_k^{(N)}$) is the signal (respectively noise) part of the left singular basis. Then, the noise projector is $\hat{P}_k^{\perp} = U_k^{(N)} {U_k^{(N)}}^H = I - U_k^{(S)} {U_k^{(S)}}^H$. Note, that we use the same procedure for projector $\hat{\Pi}_k^{\perp}$ but we replace a single Hankel matrix by a sum of Hankel matrices.

4. ROOT-MUSIC ALGORITHM FOR DAMPED SINUSOIDS

The search procedure associated to the MUSIC criterion for damped sinusoids is multidimensional and therefore is a very costly operation. A MUSIC algorithm, called DMUSIC, has been presented in [3] for damped sinusoids. So in this section, we expose the "root" version of the DMUSIC algorithm which essentially consists of zeroing a polynomial form and next by inspecting the orthogonality condition for the preselected zeros. Thus, the overall complexity cost of the algorithm is comparable to the complexity cost of the root-MUSIC algorithm for undamped components.

The criterion of the root-MUSIC is given by $f(z)$ defined in expression (6) where we replace $d(z)^H$ by $d(z^{-1})$. All z, which are in the generator set of matrix Z_c, are zeros of the polynomial form $f(z)$. Due to the Vandermonde structure of vectors $d\,(.)$, the estimation of the poles can be formulated in term of finding the zeros of the following polynomial:

$$f(z) = z^{\frac{N}{2}-1} \sum_{\ell=-\frac{N}{2}+1}^{\frac{N}{2}-1} q_\ell z^\ell \quad \text{with} \quad q_\ell = q^*_{-\ell}. \tag{12}$$

The above polynomial is of degree $N-2$ and its explicit computation is given by summing along the diagonals of matrix $\sum_k \hat{P}_k^{\perp}$. As a consequence of the Hermitian character of $\sum_k \hat{P}_k^{\perp}$, $f(z)$ is a conjugate centro-symmetry polynomial. In addition, q_0 is real and equals to $\mathtt{Tr}(\sum_k \hat{P}_k^{\perp}) = 2N - (KM_c + \sum_k M_k)$. According to formulation (12), we can simply verify that:

$$f(z) = z^{N-2} f^* \left(\frac{1}{z^*}\right). \tag{13}$$

This property implies that if z_m is a zeros then $1/z_m^*$ is also a zero and therefore $(z_m, 1/z_m^*)$ occur in pairs. Note that for the M_c desired poles, we have not constraint $|z_m| = 1$, *ie.*, the zeros are inside, outside or on the unit circle. The factorized form of $f(z)$ is then:

$$f(z) = \prod_{m=1}^{N-2} (z - z_m)\left(z - \frac{1}{{z_m}^*}\right). \tag{14}$$

At this stage, we assume that all the zeros $\{z_1, \ldots, z_{N-2}\}$ of $f(z)$ are possible poles. To further reduce the number of zeros to M_c, we evaluate the orthogonality condition:

$$\arg \max_{m\in[1:N-2]} \frac{1}{d(z_m)^H \left(\sum_{i=1}^{T} \hat{\Pi}_i^{\perp}\right) d(z_m)}. \tag{15}$$

for the preselected zeros. The M_c pics in the above criterion provide the common poles.

Note also, that in most practical situation, the damping factor is non-positive and hence the desired poles are inside the unit circle. This means that one can limit the above selection only to halh of the zeros (only those inside the unit circle).

5. NUMERICAL SIMULATIONS

In this section, we compare the proposed method, called root-DMUSIC, with the one presented in [1] by means of 300 Monte-Carlo runs. The latter, named MUSCLE, is based on shift-invariance of the signal subspace. The considered test signal is a 4 channels with two EDS in each where one is common. In Fig. 1, we have drawn the zero locations of the different sinusoidal component. We can see that only the common pole is estimated.

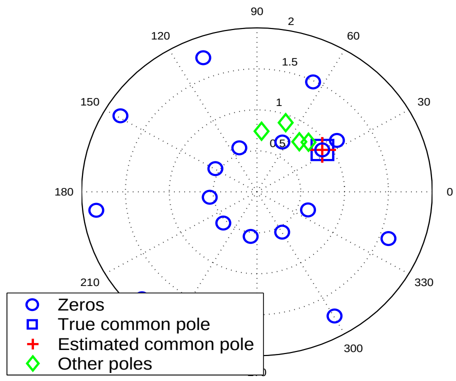
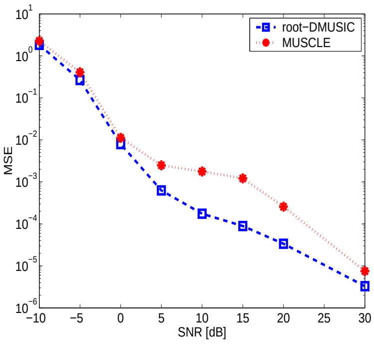


Fig. 1. Zero locations

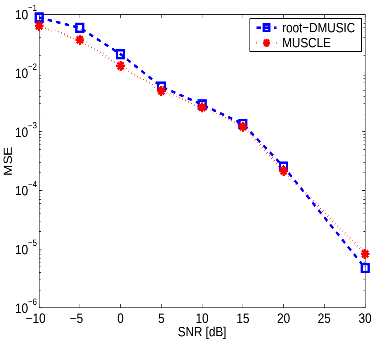
As a brief comparison, we have reported on Fig.2 and 3, the Mean Square Error (MSE) of the common angular-frequency and the damping-factorestimates with respect to the SNR for two situations: (1) four channels with spaced EDS and (2) two channels with closely-spaced EDS.

5.1. Four channels case

As we can see on Fig.2, the proposed method shows an interesting gain (compared to MUSCLE) for the angular-frequency and has a similar accuracy for the damping-factor estimation.



(a)



(b)

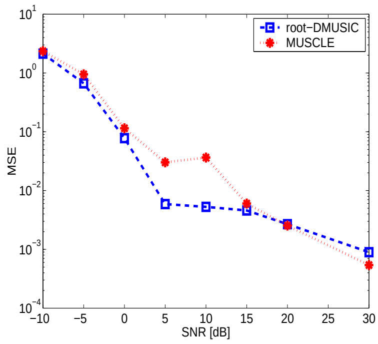
Fig. 2. MSE Vs. SNR, $N = 20$ (a) angular-frequency $\omega_c = 0.6$, $\omega_{1,1} = 1.2$, $\omega_{1,2} = 1.5$ and $\omega_{1,3} = 0.9$, (b) damping-factor $d_c = -0.1$, $d_{1,1} = -0.1$, $d_{1,2} = -0.3$ and $d_{1,3} = -0.25$.

5.2. Two channels case with closely-spaced sinusoids

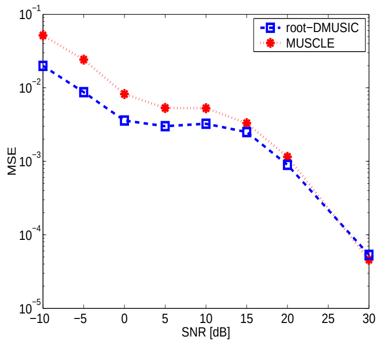
In this part, we deal with closely-spaced sinusoids. This situation is difficult since the components of each channel can potentially disturb the estimation of the component of interest. On Fig.3, we can see that in this scenario the root-DMUSIC can sligntly improve the estimation accuracy of the desired parameters.

6. CONCLUSION

In this paper, we have proposed a new orthogonal vector method for the common pole estimation of a multi-channel Exponentially Damped Sinusoidal (EDS) model. Our method is an evolution of the DMUSIC algorithm which is the MUSIC algorithm for damped sinusoids. In our solution, we avoid the costly multidimensional search by zeroing a polynomial form and by inspecting the orthogonality condition only for the preselected zeros. This is an efficient way to decrease the complexity burden of the method which allows possible



(a)



(b)

Fig. 3. MSE Vs. SNR, MSE Vs. SNR, $N = 20$ (a) angular-frequency $\omega_{1,c} = 0.6$, $\omega_{1,1} = 0.5$ and $\omega_{1,2} = 0.55$ (b) damping-factor $d_{1,c} = -0.2$, $d_{1,1} = -0.01$ and $d_{1,1} = -0.2$.

comparison with shift-invariant techniques as the MUSCLE algorithm.

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