



HAL
open science

Density fluctuations in a very elongated Bose gas : crossover from weakly to strongly interacting regimes and 1D-3D dimensional crossover

Isabelle Bouchoule, Julien Armijo, Thibaut Jacqmin, Karen Kheruntsyan

► **To cite this version:**

Isabelle Bouchoule, Julien Armijo, Thibaut Jacqmin, Karen Kheruntsyan. Density fluctuations in a very elongated Bose gas : crossover from weakly to strongly interacting regimes and 1D-3D dimensional crossover. *Correlations, Fluctuations and Disorder*, Dec 2010, Grenoble, France. hal-00576406

HAL Id: hal-00576406

<https://hal.science/hal-00576406>

Submitted on 14 Mar 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Density fluctuations in a very elongated Bose gas : crossover from weakly to strongly interacting regimes and 1D-3D dimensionnal crossover

Isabelle Bouchoule, Julien Armijo, Thibaut Jacqmin and
Karen Kheruntsyan

Institut d'Optique, Palaiseau.

Grenoble, December 2010



Outline

- 1 Introduction
 - 1D interacting Bose gases
- 2 Density fluctuations in weakly interacting regimes
 - Experiment
 - Quasi-condensation
 - Quantum regime : sub-poissonian fluctuations
- 3 1D-3D Crossover
 - Quasi-bec
 - Quasi-bec transition in the dimensionnal crossover
- 4 Entering the strongly interacting regime

Outline

- 1 Introduction
 - 1D interacting Bose gases
- 2 Density fluctuations in weakly interacting regimes
 - Experiment
 - Quasi-condensation
 - Quantum regime : sub-poissonian fluctuations
- 3 1D-3D Crossover
 - Quasi-bec
 - Quasi-bec transition in the dimensionnal crossover
- 4 Entering the strongly interacting regime

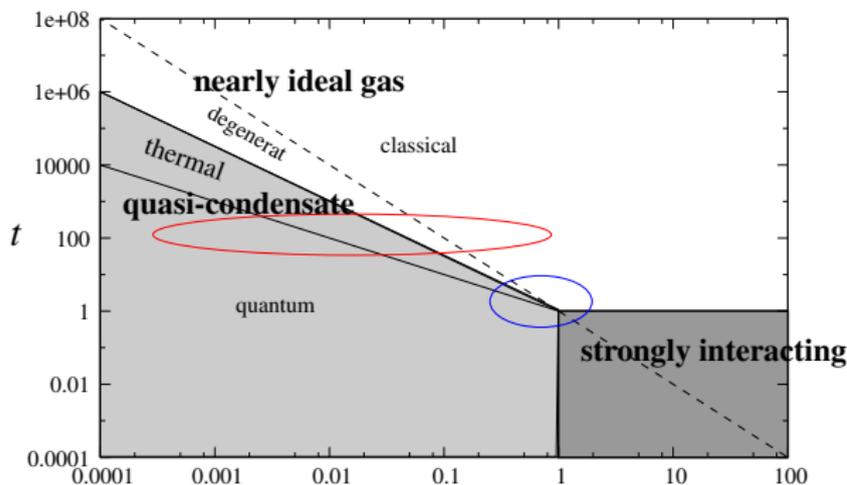
1D Bose gas with repulsive contact interaction

$$H = -\frac{\hbar^2}{2m} \int dz \psi^\dagger \frac{\partial^2}{\partial z^2} \psi + \frac{g}{2} \int dz \psi^\dagger \psi^\dagger \psi \psi,$$

Exact solution : Lieb-Liniger Thermodynamic : Yang-Yang (60') n, T

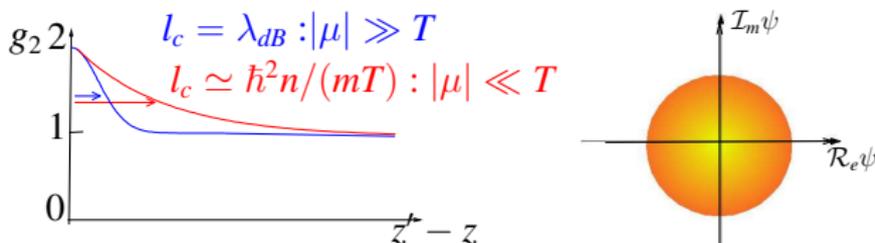
Length scale : $l_g = \hbar^2/mg$, Energy scale $E_g = g^2 m/2\hbar^2$

Parameters : $t = T/E_g$, $\gamma = 1/nl_g = mg/\hbar^2 n$



Nearly ideal gas regime : bunching phenomena

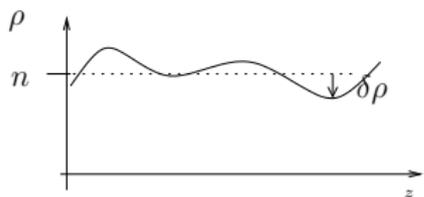
- Two-body correlation function $g_2(z) = \langle \psi_z^+ \psi_0^+ \psi_0 \psi_z \rangle / n^2$



- Bunching effect \rightarrow density fluctuations.
 $\langle \delta n(z) \delta n(z') \rangle = \langle n \rangle^2 (g_2(z' - z) - 1) + \langle n \rangle \delta(z - z')$
- Bunching : correlation between particles. Quantum statistic
- Field theory $\psi = \sum \psi_k e^{ikz}$, $n = |\psi|^2$: speckle phenomena

Transition towards quasi-condensate

- Repulsive Interactions \rightarrow Density fluctuations require energy



$$H_{int} = \frac{g}{2} \int dz \rho^2 \Rightarrow \delta H_{int} > 0$$

Reduction of density fluctuations at low temperature/high density

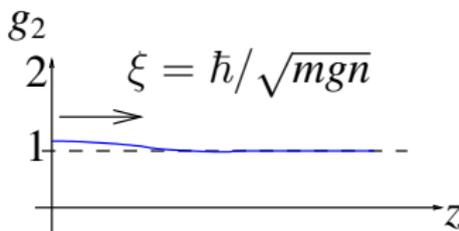
weakly interacting : $\gamma = mg/\hbar^2 n \ll 1$

Cross-over : $\frac{1}{N} H_{int} \propto gn \simeq |\mu|$

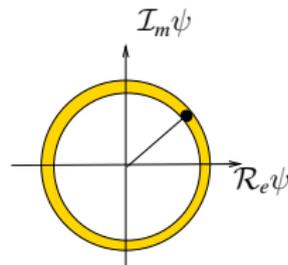
$$\mu = mT^2/2\hbar^2 n^2, \Rightarrow T_{c.o.} \simeq \frac{\hbar^2 n^2}{2m} \sqrt{\gamma}, \quad n_{c.o.} \propto T^{2/3}$$

Transition for a degenerate gas

- For $T \ll T_{c.o.}$: quasi-bec regime, $g^{(2)} \simeq 1$

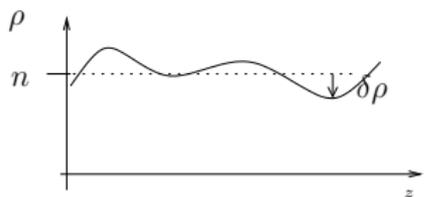


$$T > gn$$



Transition towards quasi-condensate

- Repulsive Interactions \rightarrow Density fluctuations require energy



$$H_{int} = \frac{g}{2} \int dz \rho^2 \Rightarrow \delta H_{int} > 0$$

Reduction of density fluctuations at low temperature/high density

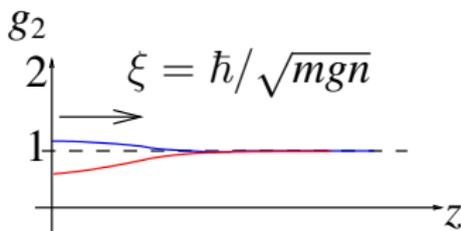
weakly interacting : $\gamma = mg/\hbar^2 n \ll 1$

Cross-over : $\frac{1}{N} H_{int} \propto gn \simeq |\mu|$

$$\mu = mT^2/2\hbar^2 n^2, \Rightarrow T_{c.o.} \simeq \frac{\hbar^2 n^2}{2m} \sqrt{\gamma}, \quad n_{c.o.} \propto T^{2/3}$$

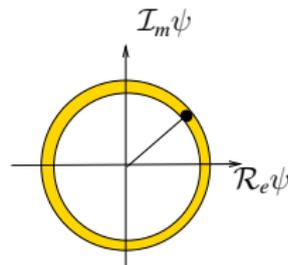
Transition for a degenerate gas

- For $T \ll T_{c.o.}$: quasi-bec regime, $g^{(2)} \simeq 1$

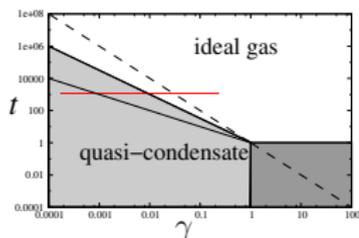


$$T > gn$$

$$T < gn$$

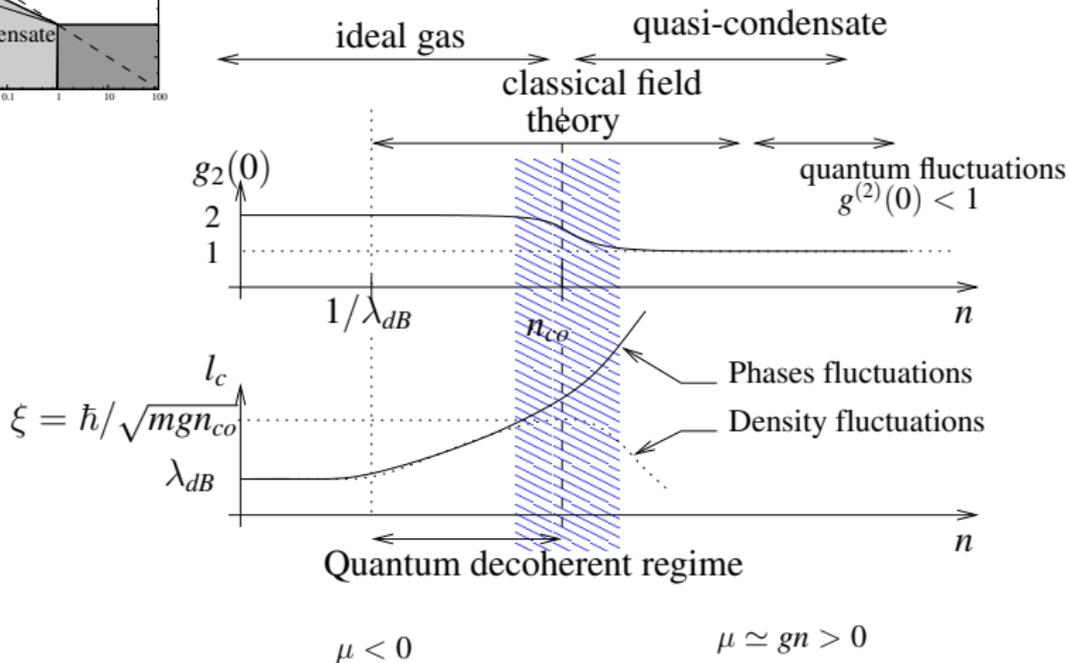


1D weakly interacting homogeneous Bose gas

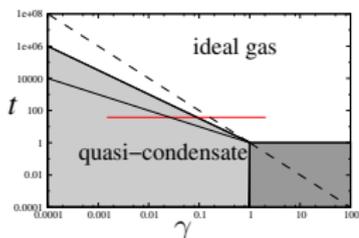


$$t = 2\hbar^2 T / (mg^2)$$

$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$



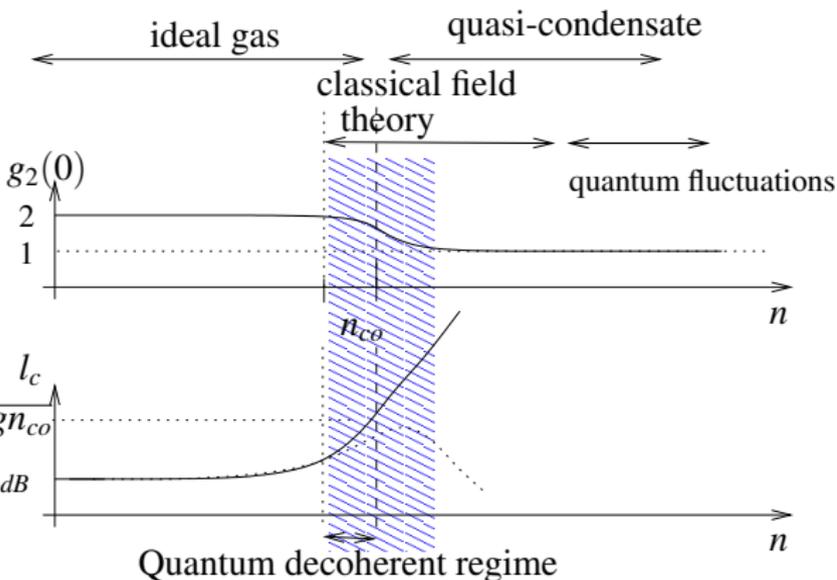
1D weakly interacting homogeneous Bose gas



$$t = 2\hbar^2 T / (mg^2)$$

$$n_{co} = \frac{1}{\lambda_{dB}} t^{1/6}$$

large transition
quantum decoherent
regime barely exists
for $t < 1000$



$$\mu < 0$$

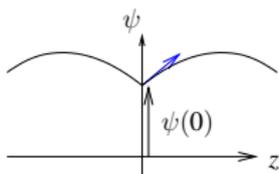
$$\mu \simeq gn > 0$$

Strongly interacting 1D Bose gas

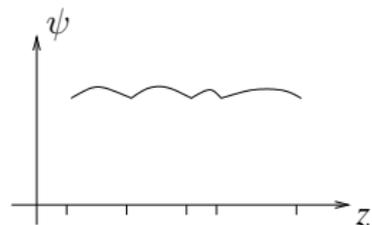
- Relative wave function

$$\text{If } E \gg E_g, |\psi(0)| \simeq 1$$

$$\text{If } E \ll E_g, |\psi(0)| \ll 1$$

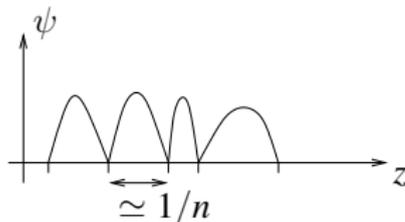


- Many body system



$$E = gn \gg E_g$$

$$\Rightarrow \gamma \ll 1$$



$$E = \hbar^2 n^2 / m \ll E_g$$

$$\Rightarrow \gamma \gg 1$$

$g^{(2)}(0) \simeq 0$
Fermionization

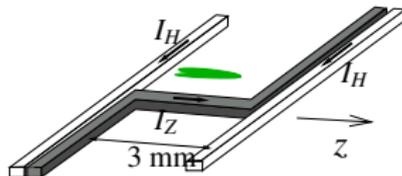
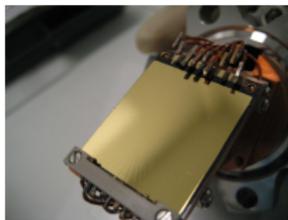
Strongly interacting regime : $\gamma \gg 1, t \ll 1$

Outline

- 1 Introduction
 - 1D interacting Bose gases
- 2 Density fluctuations in weakly interacting regimes
 - Experiment
 - Quasi-condensation
 - Quantum regime : sub-poissonian fluctuations
- 3 1D-3D Crossover
 - Quasi-bec
 - Quasi-bec transition in the dimensionnal crossover
- 4 Entering the strongly interacting regime

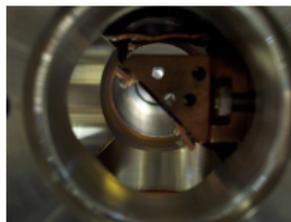
Realisation of very anisotropic traps on an atom chip

Magnetic confinement of ^{87}Rb by micro-wires
H-shape trap

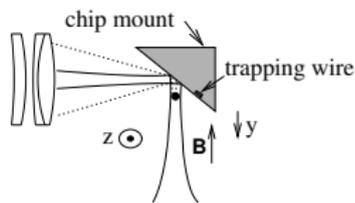


$$\begin{cases} \omega_{\perp}/2\pi = 3 - 4 \text{ kHz} \\ \omega_z/2\pi = 5 - 10 \text{ Hz} \end{cases}$$

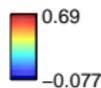
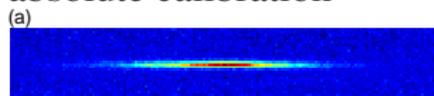
$$1\text{D} : T, \mu \ll \hbar\omega_{\perp} \\ g = 2\hbar\omega_{\perp}a$$



CCD camera



In-situ images
absolute calibration



$$T \simeq 400 - 15\text{nK} \\ \simeq 3.0 - 0.1\hbar\omega_{\perp} \\ N \simeq 5000 - 1000.$$

$t \simeq 80 - 1000$
**Weakly interacting
gases**

Density fluctuation measurements

- Statistical analysis over hundreds of images

$$l_c \ll \Delta \ll L$$

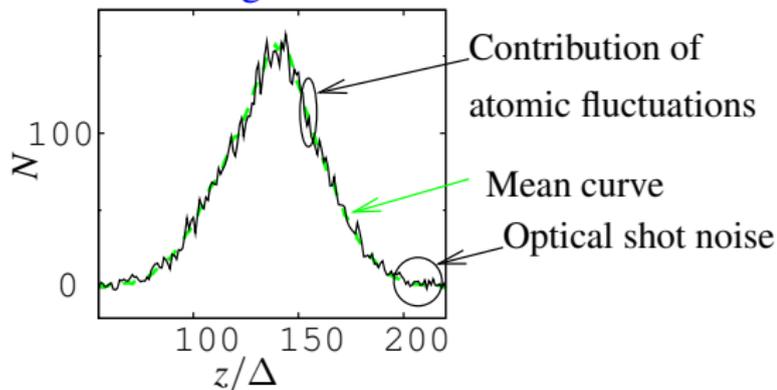
→ Local Density

Approximation valid

δN binned according to $\langle N \rangle$

$\langle \delta N^2 \rangle$ versus $n = \langle N \rangle / \Delta$

Optical shot noise
subtracted



→ Two-body correlation function integral

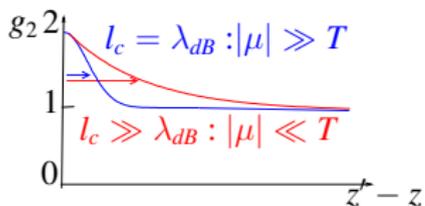
$$\langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z) - 1)$$

→ Thermodynamic quantity

$$\langle \delta N^2 \rangle = k_B T \frac{\partial n}{\partial \mu}$$

Expected behavior in asymptotic regimes

• Ideal gas regime



• Quasi-bec regime

$$H_{k=0} = g\Delta\delta n_0^2/2$$

$$\text{Equipartition : } T/2 = H_k$$

$$\Rightarrow \langle \delta N^2 \rangle = \Delta T/g$$

$$\text{Thermodynamic : } \mu \simeq gn$$

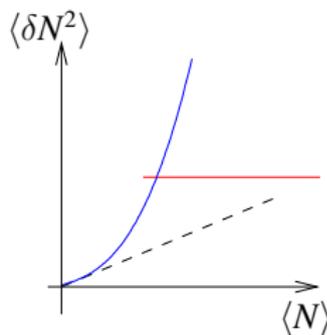
$$\langle \delta N^2 \rangle = \langle N \rangle + \underbrace{\langle N \rangle n \int dz (g_2(z) - 1)}_{l_c}$$

▷ Non degenerate gas : $nl_c \ll 1$

$$\langle \delta N^2 \rangle \simeq \langle N \rangle$$

▷ Degenerate gas : $nl_c \ll 1$

$$\langle \delta N^2 \rangle \simeq \langle N \rangle nl_c = \langle N \rangle^2 l_c / \Delta$$



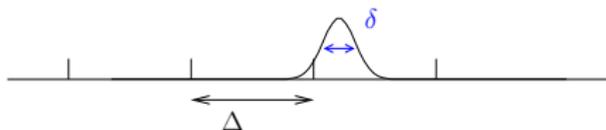
Effect of finite spatial resolution

- Absorption of an atom spreads on several pixels

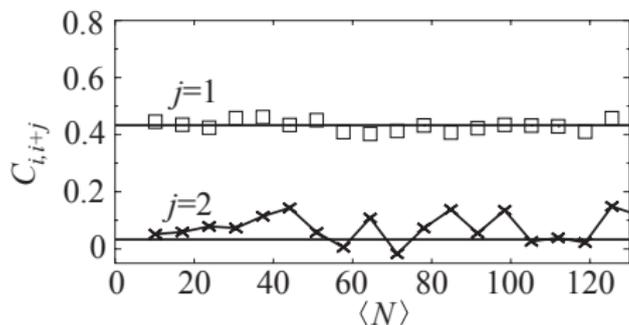
→ blurring of the image

Decrease of fluctuations :

$$\langle \delta N^2 \rangle = \kappa_2 \langle \delta N^2 \rangle_{\text{true}}$$



→ Correlation between pixels



δ deduced from measured correlations between adjacent pixels

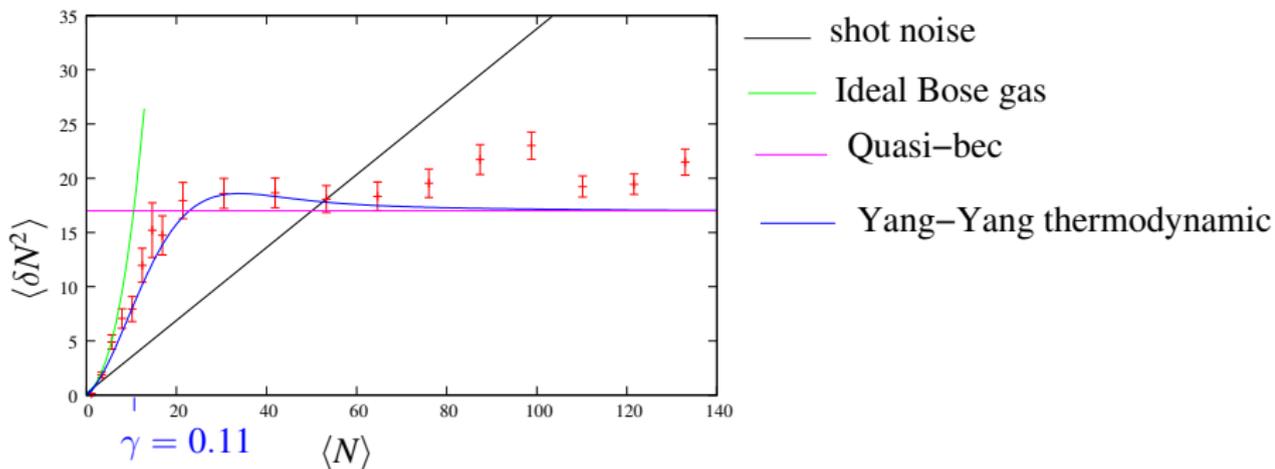
κ_2 deduced

Experimental results

$$T = 15 \text{ nK} \simeq \hbar\omega_{\perp}/10$$

$$t \simeq 80$$

$$\mu \simeq 30 \text{ nK} \simeq 0.2 \hbar\omega_{\perp}$$



- Strong bunching effect in the transition region
- Quasi-bec both in the thermal and quantum regime

Quasi-condensate in the quantum regime

We measure : $\langle \delta N^2 \rangle < \langle N \rangle$ (by more than a factor 2)

$$\langle \delta N^2 \rangle = \langle N \rangle + \langle N \rangle n \int dz (g_2(z) - 1)$$

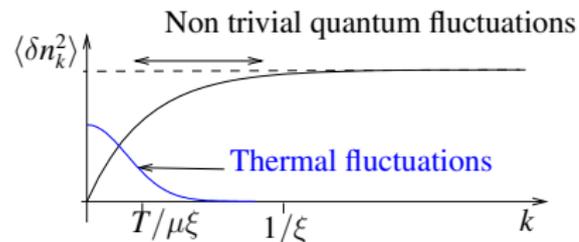
$\Rightarrow g^{(2)}$ function smaller than one \Rightarrow **Quantum regime**

$$\langle \delta N^2 \rangle = \Delta T / g \Rightarrow T < gn$$

- However we still measure thermal excitations

Low momentum phonons : high occupation number

Shot noise term, removed from the $g^{(2)}$ function, IS quantum.



Outline

- 1 Introduction
 - 1D interacting Bose gases
- 2 Density fluctuations in weakly interacting regimes
 - Experiment
 - Quasi-condensation
 - Quantum regime : sub-poissonian fluctuations
- 3 1D-3D Crossover
 - Quasi-bec
 - Quasi-bec transition in the dimensionnal crossover
- 4 Entering the strongly interacting regime

1D-3D Crossover in the quasi-bec regime

$\mu \ll \hbar\omega_{\perp}$ ($na \ll 1$) \rightarrow Pure 1D quasi-bec

$\mu \gtrsim \hbar\omega_{\perp}$: 1D \rightarrow 3D behavior

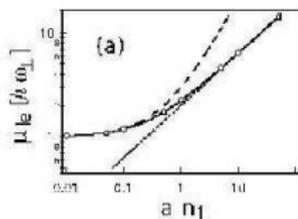
Transverse breathing associated with a longitudinal phonon has to be taken into account.

Thermodynamic argument : $\text{Var}(N) = k_B T \left(\frac{\partial N}{\partial \mu} \right)_T$

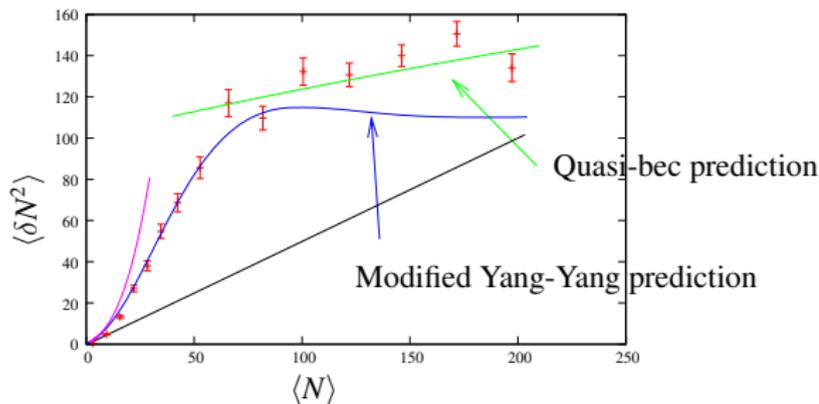
$$T = 96 \text{ nK} \simeq 0.5 \hbar\omega_{\perp}$$

Heuristic equation :

$$\mu = \hbar\omega_{\perp} \sqrt{1 + 4na}$$

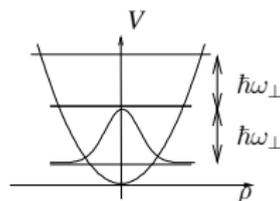
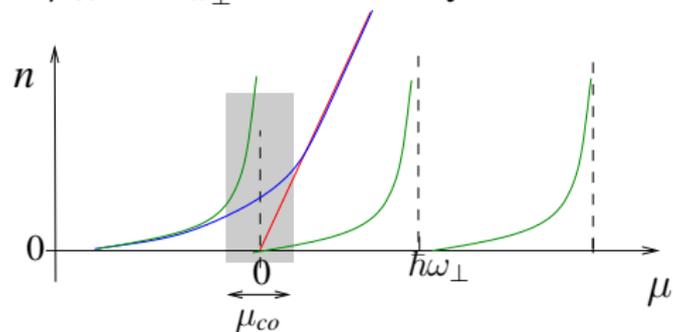


Efficient thermometry



Modified Yang-Yang model

If $\mu_{co} \ll \hbar\omega_{\perp}$, transversally excited states behave as ideal Bose gases.



$$\mu_{co} \simeq E_g^{1/3} T^{2/3} = (mg^2/\hbar^2)^{1/3} T^{2/3} \Rightarrow \mu_{co}/(\hbar\omega_{\perp}) \simeq \left(\frac{T}{\hbar\omega_{\perp}} \frac{a}{l_{\perp}} \right)^{2/3}$$

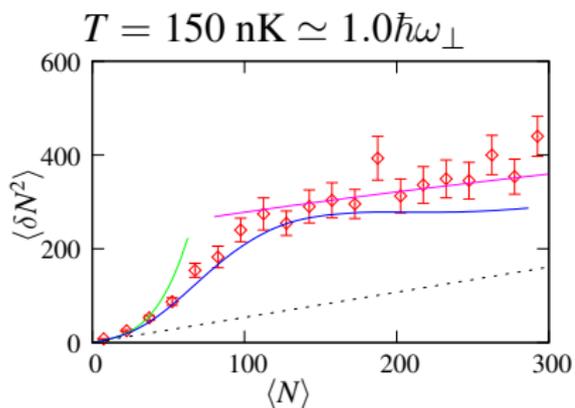
For our parameters : $a/l_{\perp} \simeq 0.025$

Modified Yang-Yang model :

- Transverse ground state : Yang-Yang thermodynamic
- Excited transverse states : ideal 1D Bose gases

First introduced by Van Druten and co-workers : Phys. Rev. Lett. 100, 090402 (2008)

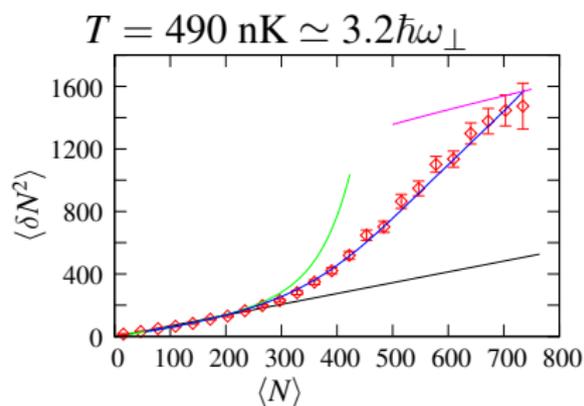
Modified Yang-Yang model



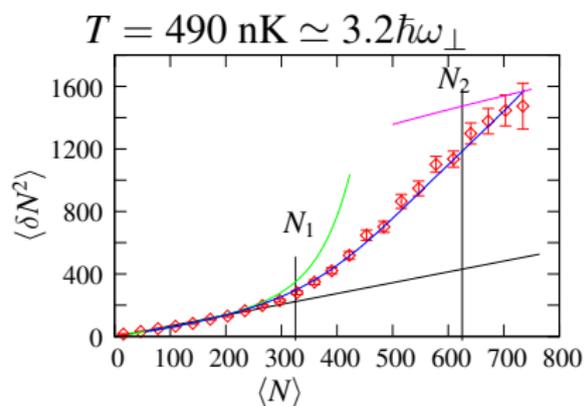
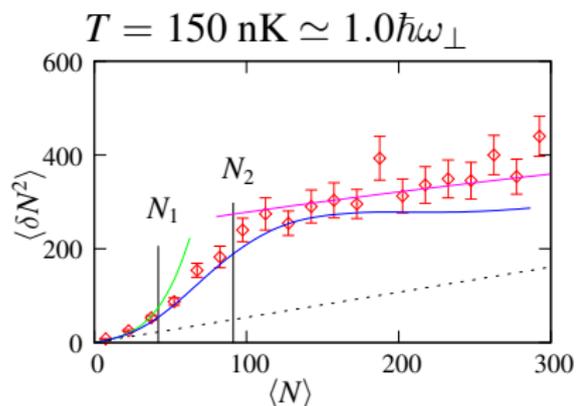
Very good agreement

N_1 (resp. N_2) :

deviation from the ideal gas (resp. quasi-bec) model by 20%



Modified Yang-Yang model

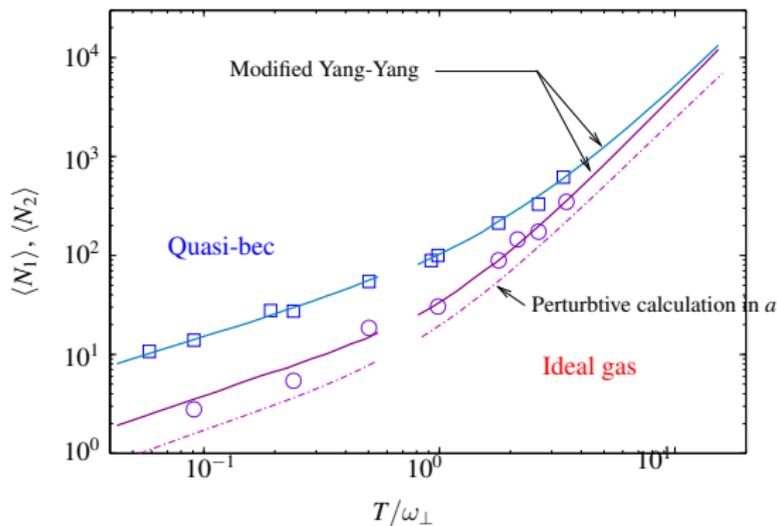


Very good agreement

N_1 (resp. N_2) :

deviation from the ideal gas (resp. quasi-bec) model by 20%

Quasi-bec transition in the 1D-3D crossover



Good agreement with MYY

Perturbation theory gives qualitative behavior for N_1

Weakly interacting Bose gases : 1D versus 3D behavior

1D physics

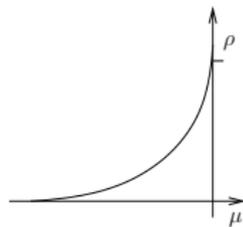
$$T \ll \hbar\omega_{\perp}$$

Weakly interacting gas :
Smooth transition
towards quasi-bec
driven by interactions

$$n_{co} \simeq (mT^2/(\hbar^2 g))^{1/3}$$

3D physics

$$T \gg \hbar\omega_{\perp}, \hbar\omega_{\perp} (a/l_{\perp})^{2/11}$$



Transverse BEC :

$$\rho \lambda_{dB}^3 = 2.612\dots$$

$$n_{c\perp} \propto T^{5/2}$$

If $n_{c\perp} \gg n_{co}$,

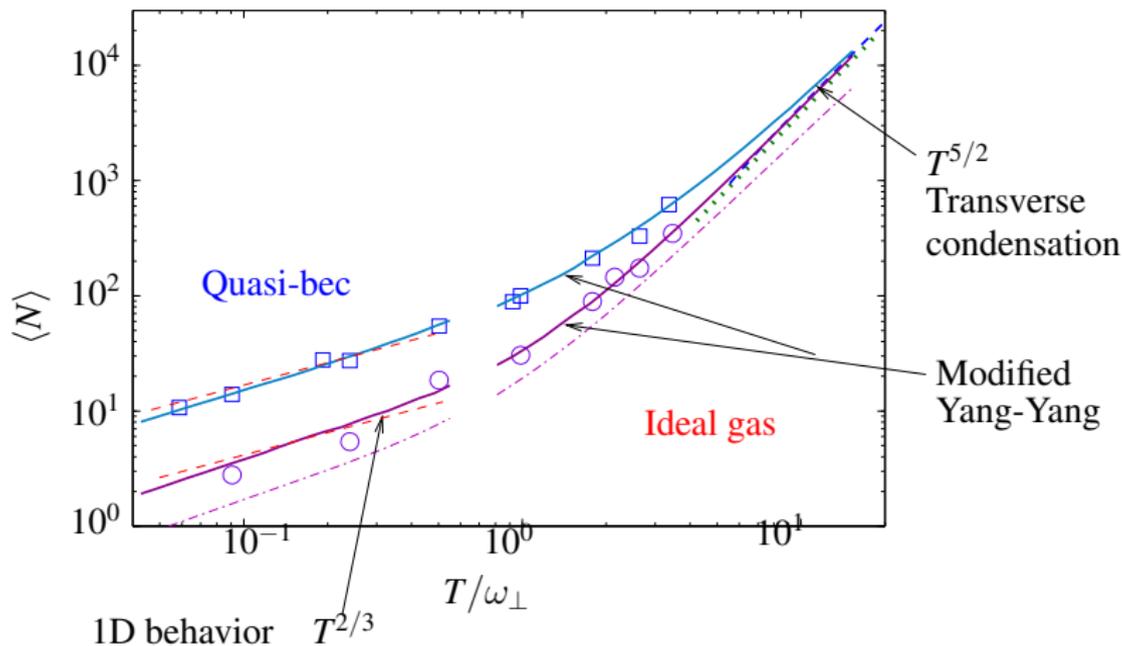
quasi-bec just after transverse BEC

$$n_{c\perp}/n_{co} \simeq ((T/\hbar\omega_{\perp}) \underbrace{(a/l_{\perp})^{2/11}}_{0.34})^{11/6}$$



dimensional crossover

Quasi-bec transition in the 1D-3D crossover



From a 1D behavior to 3D condensation phenomena

Outline

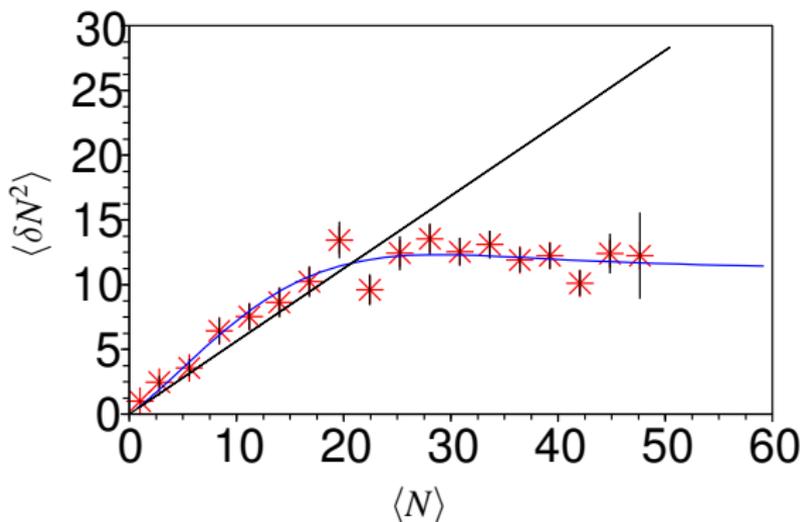
- 1 Introduction
 - 1D interacting Bose gases
- 2 Density fluctuations in weakly interacting regimes
 - Experiment
 - Quasi-condensation
 - Quantum regime : sub-poissonian fluctuations
- 3 1D-3D Crossover
 - Quasi-bec
 - Quasi-bec transition in the dimensionnal crossover
- 4 Entering the strongly interacting regime

Density fluctuations close to the strongly interacting regime

$$T = 40 \text{ nK} \simeq \hbar\omega_{\perp}/20$$

$$t \simeq 4$$

$$\mu/T \simeq 1.9$$



Almost no bunching anymore. Behaviour close to that of a Fermi gas

Conclusion

Summary

- Investigation of the quasi-bec cross-over
- 1D-3D dimensionnal crossover
- 1D quasi-bec in the quantum fluctuations regime (subshotnoise)
- Non gaussian atom number distribution measured \rightarrow 3 body correlations
- Evidence for three-body effect (strong cooling in the strongly 1D regime)

Prospect

- Study of correlation length of density fluctuations in 1D gases.
- 1D strongly interacting regime
- Localisation in strongly interacting 1D gases : in periodic and random potentials
- Momentum distribution/correlations