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► **To cite this version:**

Ming Cheng, Weinong Chen. Modeling Transverse Behavior of Kevlar® KM2 Single Fibers with Deformation-induced Damage. *International Journal of Damage Mechanics*, 2006, 15 (2), pp.121-132. 10.1177/1056789506060733 . hal-00571152

**HAL Id: hal-00571152**

**<https://hal.science/hal-00571152>**

Submitted on 1 Mar 2011

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# Modeling Transverse Behavior of Kevlar<sup>®</sup> KM2 Single Fibers with Deformation-induced Damage

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**ABSTRACT:** A phenomenological continuum model is adapted to describe the transverse mechanical behavior of Kevlar<sup>®</sup> KM2 single fibers in compression. This model could be used for numerical simulations of the mechanical behaviors of fabrics made of Kevlar<sup>®</sup> KM2 fibers. An equivalent fiber model is used to form the phenomenological model in terms of nominal stress and nominal stretch ratio. This model includes the features of stress-softening and residual strain to account for the damage caused by transverse compression, making it more accurate to represent the fiber when the effects of physical crossover of warp and fill yarns are considered and multiple loadings are applied. It is demonstrated that this model provides a good agreement with experimental results on the transverse behavior of Kevlar<sup>®</sup> KM2 single fibers in compression.

**KEY WORDS:** Kevlar<sup>®</sup> KM2, stress-softening, residual strain, constitutive behavior, phenomenological model.

## INTRODUCTION

**T**HE PROTECTION OF military and law enforcement personnel from injury by high-velocity-object impact has created new challenges for fundamental scientific research in the mechanical response of textile materials.

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Fabrics and flexible fibrous composites have been widely used in bulletproof vests and other body armor systems. Recent use of these materials in personnel protection applications creates an urgent need to develop a better scientific understanding of the mechanical response of these materials and the components made of these materials.

The earliest efforts in the attempt to predict ballistic impact effects on fabrics were those of Roylance (1973) on biaxial fabrics. Since then, many research efforts have focused on computational models to predict the resistance of fabrics to a high-velocity impact. For example, Taylor and Vinson (1990) proposed a ballistic fabric model that simulates the transverse impact by treating the fabric as a homogeneous, isotropic, and elastic plate, which deforms into the shape of a straight-sided conical shell. However, such simple isotropic material models preclude the possibility of accurate simulation.

Fabrics are composite materials that offer significant computational challenges. The stress-strain behavior of these materials is generally nonlinear as well as anisotropic. Since there are few experimental investigations on these materials, many models assume a simple linear stress-strain relationship (Parga-Landa and Hernandez-Olivares, 1995; Roylance et al., 1995). In order to improve model accuracy, Lim et al. (2003) employed a three-element spring-dashpot model to describe a rate-dependent nonlinear stress-strain behavior of Twaron fabric in their finite-element modeling study. They also accepted a failure criterion (Shim et al., 2001) that specifies the rate-dependent failure strain.

During impact, yarns and also fibers, are subjected to transverse compressive loading while they are being extended (tensile loading). Therefore, it is essential to obtain the tensile behaviors of the yarns with transverse compressive loading superimposed on them. However, few numerical models include the transverse behaviors of the fabric materials into account because of the lack of experimental investigations on this issue. Cunniff and Ting (1999) recognized this problem. They developed a numerical model to characterize the ballistic behavior of fabrics with warp and fill yarn elements modeled independently as elastic rod elements. Coupling between these elements was modeled with transverse spring elements corresponding to physical crossover. A nonlinear model with three empirical constants was used to describe the transverse behavior of the yarns.

Kevlar<sup>®</sup> KM2 fabrics are widely used to produce personnel protection systems because of their high stiffness, lightweight, and high strength. Kevlar<sup>®</sup> KM2 fabrics and their ballistic applications have recently been a research focus for many investigators (Johnson et al., 1999; Johnson et al., 2002). In order to investigate the validities of the

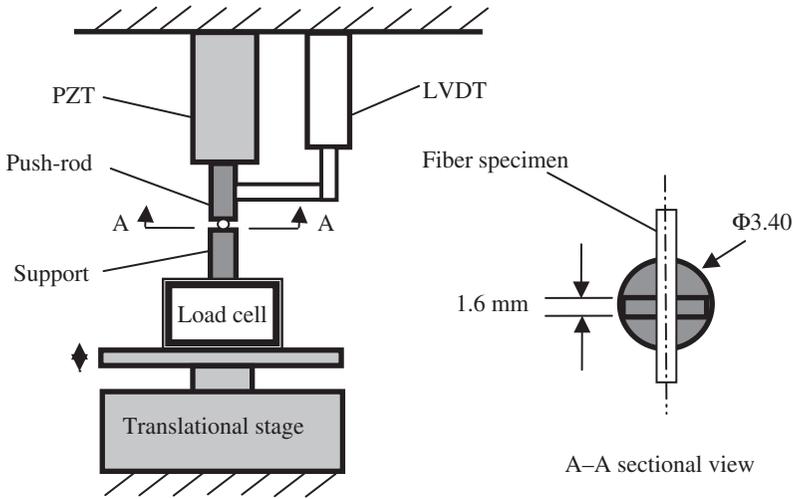
transverse spring elements, experimental studies must be performed on the yarns, which are bundles of fibers. Contact between individual fibers inside the yarns can be quite complicated and the behaviors of single fibers must be investigated first. Cheng et al. (2004) performed a systematic experimental study of single Kevlar<sup>®</sup> KM2 fibers. Their experimental results revealed that the transverse mechanical behavior of a single Kevlar<sup>®</sup> KM2 fiber is nonlinear and exhibits strong stress-softening phenomenon, which indicates deformation-induced damage in the fiber. We repeated the experiment over ten times on specimens with nominally identical properties and initial states, prepared from fibers in the same general location of the same tow from the same lot. The results overlap, which indicates the repeatability of the observed phenomenon. In this article, a phenomenological model that accounts for the damage effects is introduced based on the experimental results of Cheng et al. (2004). Since rate dependence of the mechanical behavior of Kevlar<sup>®</sup> KM2 single fiber has been reported insignificant (Cheng et al., 2004), this model takes no rate effects into consideration within the same strain-rate range as in the experiments.

## PHENOMENOLOGICAL MODELING

To investigate the transverse mechanical properties of Kevlar<sup>®</sup> KM2 fibers, Cheng et al. (2004) developed a new experimental device. Several series of periodic loading–unloading transverse compressive tests were carried out at room temperature to investigate the damage effects.

### Experimental Results

The main features of the new experimental setup are schematically shown in Figure 1. The loading system includes a piezoelectric translator (PZT). A push-rod is connected to the tip of the PZT. This applies the transverse compressive load on the fiber specimen lying on the flat end of a supporting rod. Both the push-rod and the supporting rod are made of silicon carbide (SiC). Young's modulus of silicon carbide is 420 GPa, much higher than that of general high-performance fibers, such as Kevlar<sup>®</sup> KM2 fibers, in transverse direction. Therefore, the deformation in the test jig is negligible compared to that of the fiber. The loading tip of the push rod is tapered from a circular cross-section to a rectangular shape to form a contact surface of 1.6 mm in width, as shown in the sectional view of Figure 1. This bottom contact surface of the push-rod and the top surface of the supporting rod are polished with a 0.3  $\mu$  de-agglomerated alumina polishing suspension.

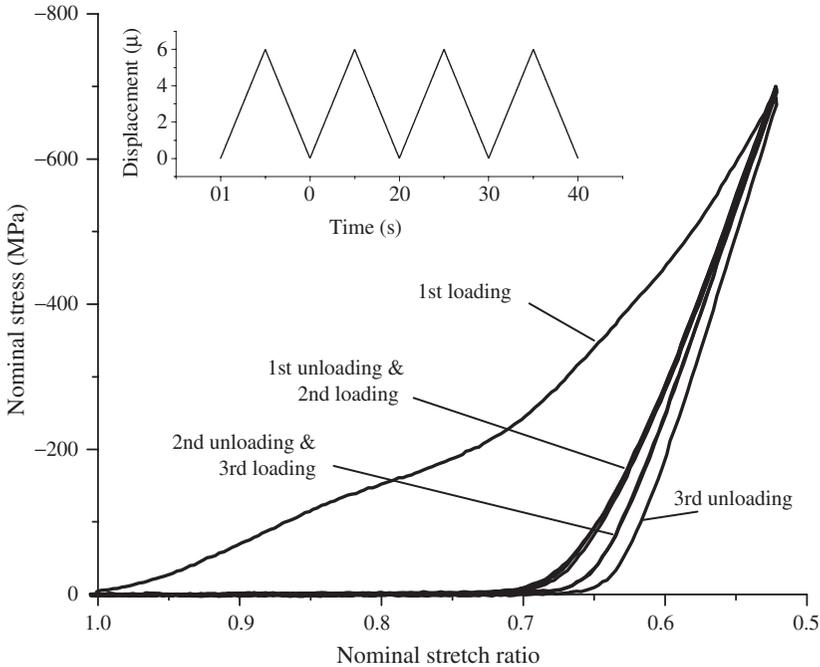


**Figure 1.** Schematic description of the experimental setup for transverse compressive tests.

The compressive load is sensed by a load cell, which is positioned under the supporting rod. The displacement is measured using a linear variable displacement transducer (LVDT). A vertical translational stage is used to facilitate the proper positioning and alignment of these components. The driving signal for the piezoelectric translator and data acquisition of load and displacement signals are controlled by a computer. This computer and the associated signal conditioners are not shown in Figure 1.

The specimens are 850 denier Kevlar<sup>®</sup> KM2 fibers made by DuPont. The density of these fibers is  $1440 \text{ kg/m}^3$ . The diameter ( $D$ ) of the Kevlar<sup>®</sup> KM2 fiber is determined using a scanning electronic microscope (SEM) and is  $12 \mu\text{m}$ .

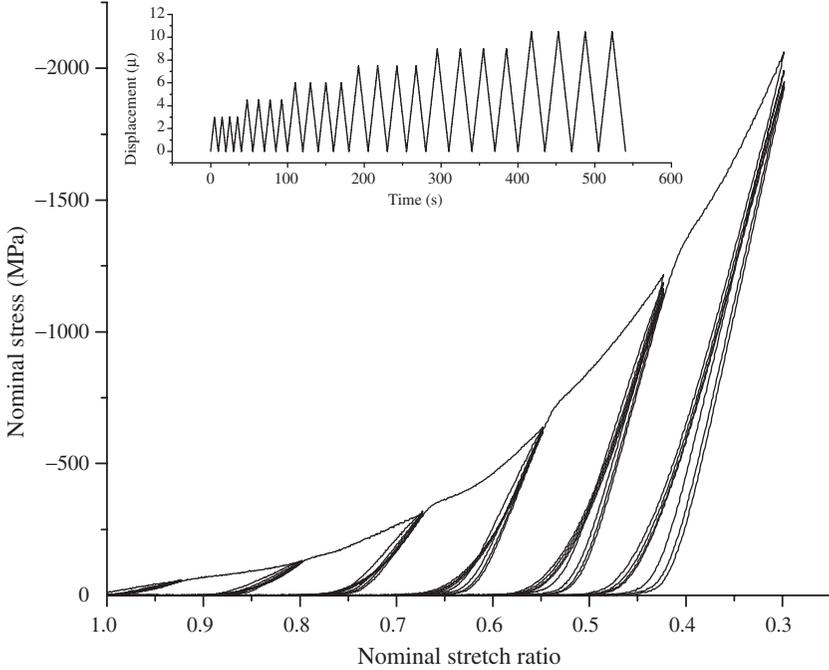
Figure 2 shows a typical load–deformation curve obtained from an experiment on a Kevlar<sup>®</sup> KM2 fiber with cycling of displacement. The displacement profile is repeated linear ramps with identical peaks. The insert in Figure 2 is the time history of the displacement. In Figure 2, the ordinate is nominal compressive stress, which is defined as push load per unit of length in longitudinal direction divided by the diameter of the fiber. Abscissa is the displacement of the bottom surface of the push-rod normalized by the diameter of the fiber specimen. In this study, the normalized displacement is called nominal strain when its value at the initial point of the push-rod touching the fiber is subtracted from data, so that this value is zero at the beginning of loading. The term, nominal stretch ratio, is the nominal strain plus one.



**Figure 2.** Cyclic transverse loading of a Kevlar<sup>®</sup> KM2 fiber at a nominal strain rate of 0.075/s.

A Kevlar<sup>®</sup> KM2 fiber may be approximately considered linear elastic in its transverse direction at an infinitesimal deformation range. However, the overall behavior in a large deformation range is obviously nonlinear and nonelastic. As shown in Figure 2, the first loading path from its virgin state to a nominal stretch ratio of about 0.52 is nonlinear. An unloading from this nominal stretch ratio hardly recovers its full deformation, leaving a large residual strain of about 0.70 in terms of nominal stretch ratio. Observation with an optical microscope shows that the fiber retains its deformed shape even two months after it is transversely pressed. Once this fiber is loaded again, the second and later loading and unloading paths follow the first unloading path with some small deviation from it.

From the experimental results described above, it is clear that the first nominal compressive stress versus nominal stretch ratio curve in transverse direction is much different from those curves obtained from reloading the same specimen. The first loading and unloading cycle is the most energy-absorbing cycle, and causes larger residual strain upon unloading. The differences observed in the following loading and unloading paths are not significant compared to differences observed between the first cycle and



**Figure 3.** Successive loading cycles with step-increasing nominal stretch.

the following loading cycles if their maximum nominal stretch does not exceed the first one. In the case of compression, the maximum stretch means the minimum value of stretch ratio because compressive stress and strain are both negative. In order to reveal the mechanical behavior of later loading cycles when their maximum nominal stretches exceed the previous maxima, a series of experiments were conducted with sets of four cycles where the nominal stretch amplitude of each set of cycles gradually increases. The results of these successive loading cycles are shown in Figure 3 with an insert to describe the time history of loading cycles. In these experiments, each loading-unloading cycle with same amplitude consists of four subcycles of loading and unloading.

The results in Figure 3 show that large residual strain exists even at a small strain level after the first subcycle in the first cycle. The amplitude of the first four loading and unloading subcycles is 0.925 in terms of nominal stretch ratio. The observed transverse mechanical behavior for the cycles is very similar to those shown in Figure 2. After these initial four successive sub-cycles, the next loading cycle can be divided into two stages. The first stage, with nominal stretch less than previous maximum, which is 0.925 in terms of nominal stretch ratio, follows the fourth loading subcycle.

However, once the first loading subcycle of the second loading cycle exceeds the previous maximum nominal stretch, it turns to a loading path that a virgin specimen is expected to follow. The fifth unloading path (unloading of first subcycle in the second cycle) leaves an even larger residual strain, causing large energy absorption. The later loading and unloading cycles follow the fifth unloading path until the ninth loading (first subcycle of the third loading cycle), which is another new loading with its maximum nominal stretch exceeding the previous maximum nominal stretch. This phenomenon is similar to the Mullins effect, which was found in many rubber materials (Mullins, 1969; Cheng and Chen, 2003). In the experimental results presented in this article, the degree of damage in specimen is exclusively determined by the maximum stretch the specimen experienced.

### **Pseudo-elastic Modeling**

Some finite elemental analysis (FEA) software, such as ANSYS® 6, have interface elements, which are based on the relative deformation of the top and bottom surfaces, offering a direct means to quantify through-thickness deformation of joints. It is therefore practical to use such solid continuum elements to effectively model the coupling between warp and fill yarn elements. A material model with stress-softening feature can readily be applied to this element.

Our purpose is to provide material properties for an interface element to represent the transverse coupling between warp and fill yarn elements. These material properties must be directly obtained from experimental results. Because of the circular cross-section of the fiber, it is impossible to obtain true material properties by performing transverse compressive tests. Stresses in the fiber are not uniformly distributed. However, in FEA simulations where the focus is on the fabric structural response instead of fiber behavior, we may represent a fiber's cross-section with a diameter-by-diameter square. With this equivalent fiber specimen, the transverse loading can be considered uniaxial, provided the engineering stress and strain are replaced with the nominal stress and nominal strain defined earlier.

There are many models for the description of the behavior of stress-softening. For example, Ogden and Roxburgh (1999) developed a phenomenological theory of pseudo-elasticity to model an idealized Mullins effect under quasi-static loading, which considers the stress-stretch behavior without hysteresis and permanent sets. Recently, this theory has been modified to accommodate permanent strain set (Dorfmann and Ogden, 2004) with success in describing the cyclic loading behavior of

rubber materials. Another modeling theory incorporating the permanent strain set was published by Holzapfel et al. (1999). This model provides a sharp unloading path and large residual strain, which is the case for most biological soft tissues rather than particle-reinforced rubber materials. The unloading paths of most rubber materials are like softened versions of their loading paths, leaving a relatively small residual strain in comparison to biological soft tissues. Although Holzapfel's modeling theory is not proper for the description of rubber behavior, it is good for the transverse behavior of the Kevlar<sup>®</sup> KM2 fiber. In this study, we use this modeling theory to develop a phenomenological model for the experimental results. Since, the mechanical properties of a single Kevlar<sup>®</sup> KM2 fiber has insignificant loading-rate effects (Cheng et al., 2004), loading-rate is not embedded into this model.

The stress–stretch behavior on the original loading path can be described using a strain-energy function in the conventional way for an incompressible hyperelastic material as follows,

$$\sigma_1 - \sigma_3 = \lambda_1 \frac{\partial W}{\partial \lambda_1}, \quad \sigma_2 - \sigma_3 = \lambda_2 \frac{\partial W}{\partial \lambda_2} \quad (1)$$

where  $\sigma_i$  ( $i = 1, 2,$  and  $3$ ) are the principal Cauchy stresses,  $\lambda_i$  ( $i = 1, 2,$  and  $3$ ) are principal stretch ratios, and  $W$  is a strain-energy function. Since incompressibility requires  $J = \lambda_1 \lambda_2 \lambda_3 = 1$ , the strain-energy function,  $W$ , is a function of  $\lambda_1$  and  $\lambda_2$  only. According to Holzapfel et al. (1999), the strain-energy function,  $W$ , for hyperelastic materials can be modified by incorporating into it three additional discontinuous variables  $\xi_i$  ( $i = 1, 2,$  and  $3$ ) that describe the internal damage state in the three principal directions and take the value of unity on the original loading path, with the form as

$$W(\lambda_1, \lambda_2, \lambda_3, \xi_1, \xi_2, \xi_3) = \tilde{W}(\lambda_1, \lambda_2, \lambda_3) + \Omega(\lambda_1, \lambda_2, \lambda_3, \xi_1, \xi_2, \xi_3) - p(J - 1) \quad (2)$$

where  $\tilde{W}(\lambda_1, \lambda_2, \lambda_3)$  is a strain-energy function characterizing the original loading path, which takes any standard form of the strain-energy function, such as the Neo-Hookean, Mooney-Rivlin, or Ogden forms and  $\Omega$  is a supplemental function describing the damage state in the material. A Lagrange multiplier  $p$  represents an arbitrary hydrostatic pressure. To represent the damage variables in terms of the principal stretch ratios, it is convenient to assume that

$$\frac{\partial \Omega}{\partial \xi_i} = 0, \quad (3)$$

which results in the following Cauchy stress expressions for the problem of homogeneous biaxial strain.

$$\sigma_\alpha = \lambda_\alpha t_\alpha = \lambda_\alpha \frac{\partial(\tilde{W} + \Omega)}{\partial \lambda_\alpha} \quad (4)$$

where  $t_\alpha$  are nominal (or Biot) stresses with  $\alpha = 1$  and 2. In the derivation of Equation (4), the stress  $\sigma_3$  is set to be zero without loss of generality (Dorfmann and Ogden, 2004).

Holzappel et al. (1999) proposed a function for  $\Omega$ ,

$$\Omega = \sum_{i=1}^3 [\xi_i (\lambda_i - \lambda_{i, \max}) + \phi(\xi_i)] \quad (5)$$

where  $\phi$  is a damage function and  $\lambda_{i, \max}$  are the three maximum principal stretches the material experienced. From Equation (3), we obtain

$$\phi'(\xi_i) = -(\lambda_i - \lambda_{i, \max}) \quad (6)$$

Assuming a linear function for the derivative of the damage function,

$$\phi'(\xi_i) = -c\xi_i \quad (7)$$

where  $c$  is a positive material constant to regulate the slope of the unloading path relative to the loading path. Therefore, we finally get the expressions for the damage variables:

$$\xi_i = \frac{1}{c} (\lambda_i - \lambda_{i, \max}) \quad (8)$$

Ogden's form of strain-energy function is then employed.

$$\tilde{W}(\lambda_1, \lambda_2, \lambda_3) = \sum_{m=1}^3 \frac{\mu_m}{\alpha_m} (\lambda_1^{\alpha_m} + \lambda_2^{\alpha_m} + \lambda_3^{\alpha_m} - 3) \quad (9)$$

In the case of uniaxial loading, we use the following notation,

$$\lambda_1 = \lambda, \lambda_2 = \lambda_3 = \lambda^{-1/2} \quad (10)$$

Therefore, we obtain the following stress-strain relationship for the uniaxial loading condition.

$$\sigma = \lambda t = \sum_{i=1}^3 \mu_i (\lambda^{\alpha_i} - \lambda^{-\alpha_i/2}) + \lambda \xi_1 - \lambda^{-1/2} \xi_2 \quad (11)$$

where

$$\xi_1 = \frac{1}{c} (\lambda - \lambda_{\max}), \quad \xi_2 = \frac{1}{c} (\lambda^{-1/2} - \lambda_{\max}^{-1/2}) \quad (12)$$

$\lambda_{\max}$  represents the maximum stretch the specimen experienced. In compression, it is the minimum value of the stretch ratio.

## RESULTS

With Equation (11), a loading and unloading cycle of a virgin specimen is used to identify material constants. The loading path is used to determine the six material constants associated with Ogden's model of strain-energy function, while the unloading path is used for the determination of the material constant,  $c$ , associated with the damage function. Table 1 lists the material constant set, curve-fitted from this loading and unloading cycle from one experiment on a virgin specimen.

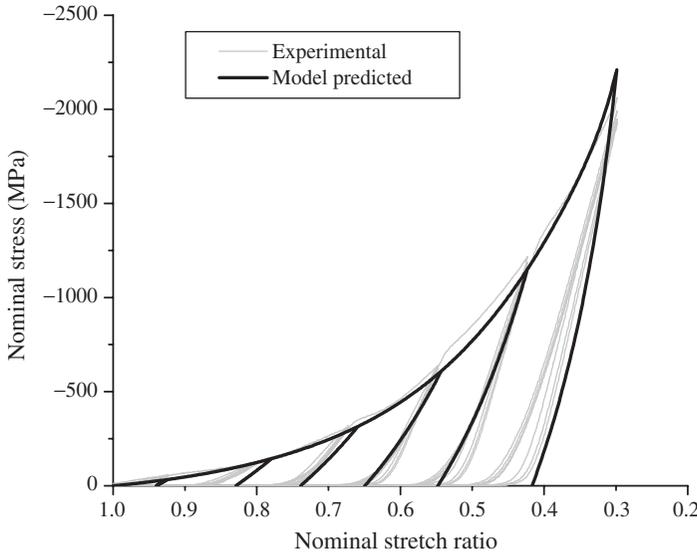
The material constants in Table 1 are based on nonlinear least-squares curve-fitting of only one loading and unloading cycle. With these constants in hand, it is possible to predict the transverse stress-stretch behavior (including the stress-softening effect) of the Kevlar<sup>®</sup> KM2 fiber. Figure 4 presents predictions of successive loading cycles with step-increasing nominal stretch ratios as shown in Figure 3. The solid gray curves are experimental results, exactly the same as shown in Figure 2. The solid black curves are predictions of the model (Equation (11)) based on the material constants listed in Table 1. The differences between successive loading and unloading in a certain loading section are not considered. There are six steps with maximum stretch ratios of 0.925, 0.78, 0.66, 0.544, 0.424, and 0.29875. The predictions provide a good description of the experimental results.

## CONCLUSIONS

A new phenomenological model with damage effects for the transverse stress-strain behavior of the Kevlar<sup>®</sup> KM2 single fiber is developed in this study. This theoretical modeling is based on the experimental results

**Table 1. Material parameters**

Index $i$	1	2	3
$\mu_i$	-155.3	1352	-1363
$\alpha_j$	-2.177	10.14	10.13
$c$		0.00098	



**Figure 4.** Comparison between experimental results and model description.

from Cheng et al. (2004). The degree of damage is a function of previous maximum stretch the specimen experienced.

The model development employs a pseudo-elasticity theory (Holzapfel et al., 1999) as modeling frame to describe the stress-softening effects and residual strains. The Ogden’s form of strain-energy function is used to describe the original stress–stretch behavior of the Kevlar® KM2 single fiber in transverse direction. This model exhibits good agreement with the experimental results over a wide range of nominal stretch ratios.

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