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Creep Failure in Concrete as a Bifurcation Phenomenon

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ABSTRACT: Softening and time-dependence of fracture are two complex and coupled phenomena that have to be taken into account in order to simulate realistic concrete behaviour. Understanding the interaction between these two phenomena is important to design reliable civil engineering structures subjected to high level-loading for a long time. The aim of this paper is to develop a simple time-dependent softening model applied to concrete. Presentation is restricted to compression behaviour. A constitutive viscodamage model describes concrete phenomena like relaxation, creep and rate-dependent loading using a unified framework. The model could be viewed as a generalisation of a time-independent damage model and is based on strong thermodynamical arguments. The determination of the material parameters linked to the proposed constitutive equation results from constant strain rate experiments. Using these parameters values, creep and relaxation numerical tests give satisfactory qualitative responses. Phenomena as creep failure under high-sustained load are explained quite simply within stability theory. Creep failure appears as the manifestation of a bifurcation phenomenon. Conversely, for low-sustained load, the motion asymptotically converges towards an equilibrium configuration. Consequently, this model is able to predict creep failure for various stress levels. Implementation of this rheological model in a structural code is envisaged in the future: a non-local approach would be probably necessary in order to simulate objective structural behaviour.

KEY WORDS: damage, concrete, creep failure, thermodynamic modelling, bifurcation.

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INTRODUCTION

IN ORDER TO simulate realistic concrete behaviour, two complex and probably coupled phenomena namely softening and time-dependence of fracture, must be considered. Understanding the interaction between these two phenomena is important to design reliable civil engineering structures subjected to high-level and long-time loading. Softening is a subject which has been profited from a very rich literature since two decades (Jirasek and Bazant, 2002). The classical continuous softening models (which do not exhibit internal length), like damage or plasticity models, suffer from a pathological mesh-dependence in finite-element computations. This strong sensitivity is linked to the nature of the associated mathematical problem, which could be ill-posed during softening. Techniques known as regularisation were developed to restore the well-posedness of the problem. An internal length is generally introduced and constrains the new regularised system towards a family of imposed solutions.

One of the first models including internal length is probably the fictitious crack model (Hillerborg et al., 1976). It is natural that the pioneer's work dealing with time-dependence cracking (Hansen, 1992) extends this formulation by associating a standard model (model of Thompson-Poynting). This approach does not appear to be very satisfactory, mainly due to the selected negative viscosity which guarantees positive-time-constants delay. Another fictitious crack model combining time-dependent effect has been developed by Santhikumar and Karihaloo (1996) from micromechanical information. A more general approach has been proposed by Barpi and Valente (2001), using fractional rate laws. It makes possible to include a whole spectrum of dissipative mechanisms in a single viscous element. These models focussed on time-dependence tension softening. Moreover, tertiary creep seems not easy to obtain within these models.

Non-local damage models constitute another class of internal-length models, applied to the modelling of concrete cracking (Pijaudier-Cabot and Bazant, 1987). Theoretical connections exist between non-local approaches and fictitious crack models (or cohesive crack models) (Planas et al., 1993). A time-dependent extension of an isotropic elastic damage model (Mazars and Pijaudier-Cabot, 1996) has been proposed by Loukili et al. (2001). The creep or relaxation functions are expressed by Dirichlet series, introducing the generalised Kelvin or Maxwell models (Bazant and Wu, 1973). In the creep problem using generalised Maxwell model, damage appears using the effective stress concept: damage is a time-independent function. Such an approach, more general than the previous one, seems to be well correlated with experimental results for a wide range of sustained load. Nevertheless, it mobilises a large number of parameters. The model of Mazotti et al. (2001)

is quite analogous, adding the concept of effective strain. This model is based on the assumption that only a fraction of total creep strain contributes to damage evolution with time. Both families of models do not have the advantage of thermodynamical background. This is also the case of the viscoplastic model of Dragon and Mroz (1979) for rock-like materials accounting for the kinetics of fracture. This last model describes well steady-state and tertiary creep stages, upto failure.

The aim of this paper is to present a few parameters – Damage Mechanics model with thermodynamical arguments – which can be used in a broad range of loading paths. Covered loading paths are classically relaxation, creep and rate-dependent loading, as considered by Cernocky and Krempl (1979) for instance. The adopted framework could be perceived like another extension of Mazars time-independent damage model. The viscodamage model developed by Dubé et al. (1996) for high-rate concrete loading is selected and adapted to slow solicitation. In particular, the aptitude of the model to predict creep failure under high sustained load is an important detailed aspect. In this paper, presentation is restricted to the uniaxial compression case.

PHENOMENOLOGICAL ASPECTS

Compression concrete strength is usually determined with a monotonous increasing load or displacement test. Typically, these unconfined tests last several minutes and lead to the standard strength identification. As the rate of loading becomes slower, tests show that the ultimate stress decreases while the failure strain increases. Among other influences, it has to be mentioned that the exact behaviour depends on the moisture content and the age of concrete. Ultimate stress at very slow rate of loading (for a test being conducted over one to two years) decreases by roughly 10% compared to the standard strength (Rüsch, 1960). On the other hand, the brittleness observed for the standard test tends to vanish at slow loading rates. In this example, the decisive parameter is time to failure. Other experiments illustrate the time-dependence of concrete cracking. For instance, creep tests exhibit clearly such a dependence. Behaviour of concrete or rock-like material under high-sustained load can be classically subdivided into three stages:

- Primary creep or transitional creep where creep rate decreases steadily.
- Secondary creep or steady-state creep with a constant creep rate.
- Tertiary creep with rapidly increasing rate of creep until failure.

A powerful correlation between rates of steady creep and lifetime under high-sustained load has been observed.

The rate effect in fracture stems mainly from two sources. One source is obviously the creep which occurs in the whole volume of the structure. The second source is the time-dependence of the rupture bonds at the microscopic level. The classical fracture mechanics approach (i.e. time-independent) cannot describe such a sensitivity. Breakage of bonds, which is the origin of fracture, cannot appear instantaneously when the bond strength is exceeded, but occurs at a finite rate. From a physical point of view, this rate is governed by the statistics of the thermal vibration of atoms or molecules. The concept of activation energy, used in theories of kinetics of chemical reactions, could be mentioned, at least qualitatively, to explain creep in material sciences, of course at a different scale (Wittmann, 1984; Bazant, 1993). The phenomenon of mineral modification under sustained load is also controlled by a certain kinetics. Transition between the two scales could be managed from statistical considerations, assuming simple law at the microscopic scale. Following this idea, Mihashi and Izumi (1977) justify the increasing of strength as well as the variability of strength with the rate of loading. These tools are primarily qualitative.

The effect of strain rate for strain rate higher than 10^{-4} s^{-1} roughly is not studied here. It seems that strength increases in a more significant way in this range of behaviour (Bischhoff and Perry, 1991). It could be noted that the identification of rheological characteristics in the dynamic range, is associated with specific experimental difficulties.

VISCODAMAGE MODEL

General Framework

In small strain cases, the first principle of thermodynamics can be written as (Lemaitre and Chaboche, 1988):

$$\rho\dot{e} = \sigma\dot{\varepsilon} + r - divq \tag{1}$$

ρ is the mass density. e is the internal energy per mass unit, function of the strain ε, the entropy s (per mass unit) and some internal variables. Stress is classically denoted by σ. q is the heat flux and r is the volumetric density of internal heat production.

The second principle of thermodynamics could be expressed as follows:

$$\rho\dot{s} + div\left(\frac{q}{T}\right) - \frac{r}{T} \geq 0 \tag{2}$$

where T is the absolute temperature.

Equation (2) is equivalent, in case of isothermal processes considered in this study, to:

$$\rho T\dot{s} \geq \rho\dot{e} - \sigma\dot{\varepsilon} \tag{3}$$

Let us note that for:

$$(r - divq) \ll \sigma\dot{\varepsilon} \tag{4}$$

inequality (2) could be reduced to:

$$\rho T\dot{s} \geq 0 \tag{5}$$

The general framework is preserved without introducing this last assumption. The free Helmholtz energy ψ (per mass unit) is introduced as:

$$\psi = e - Ts \tag{6}$$

In cases of isothermal processes, the intrinsic dissipation density Φ is defined as:

$$\Phi = \sigma\dot{\varepsilon} - \rho\dot{\psi} \geq 0 \tag{7}$$

Φ is proportional to the entropy production rate when heat terms could be neglected in Equation (1). The latter dissipation turns out to be non-negative by the second principle of thermodynamics (Equation (3)). Inequality (7) represents the Clausius-Duhem inequality for isothermal processes.

For the considered visco-elastic damage process, internal variables are chosen as the strain ε, the damage variable denoted by D and an additional thermodynamic variable denoted by κ. κ could be understood as a memory variable. The damage variable describes the irreversible process of internal structure due to the growth of already existing microcracks and the apearance of new ones. This scalar variable evolves between 0 for a virgin state and 1 at the failure. κ is an internal variable which is analogous to the cumulated plastic strain for an elastoplastic process. The free Helmholtz energy is assumed to be the sum of two terms:

$$\psi(\varepsilon, D, \kappa) = \psi_0(\varepsilon, D) + \psi_1(\kappa) \tag{8}$$

with the free energy ψ_0, equal to:

$$\rho\psi_0 = \frac{1}{2}E_0(1 - D)\varepsilon^2 \tag{9}$$

E_0 is the Young's modulus of the virgin material.

The definite positive function ψ_1 is given later in the paper.

The associated thermodynamical "forces" could be deduced from:

$$\left|\begin{array}{l} \sigma = +\rho\dfrac{\partial\psi}{\partial\varepsilon} = +\rho\dfrac{\partial\psi_0}{\partial\varepsilon} \\ Y_0 = -\rho\dfrac{\partial\psi}{\partial D} = -\rho\dfrac{\partial\psi_0}{\partial D} \\ Y_1 = +\rho\dfrac{\partial\psi}{\partial\kappa} = +\rho\dfrac{\partial\psi_1}{\partial\kappa} \end{array}\right. \tag{10}$$

where Y_i are associated to damage energy release rates.

Using Equation (7), the rate of energy dissipated could be written with the new variables:

$$\Phi = Y_0\dot{D} - Y_1\dot{\kappa} \geq 0 \tag{11}$$

The first and the second conditions of Equation (10) lead to:

$$\left|\begin{array}{l} \sigma = E_0(1-D)\varepsilon \\ Y_0 = \frac{1}{2}E_0\varepsilon^2 \end{array}\right. \tag{12}$$

Thus, the classical relation of effective stress is obtained. This relation could be questionable in compression with uniaxial study. It appears that the damage energy release rate depends only on the strain ε. For the simple uniaxial case, it is possible to reason in an equivalent way with the variable $|\varepsilon|$, also called equivalent strain:

$$|\varepsilon| = \sqrt{\frac{2Y_0}{E_0}} \tag{13}$$

As Y_1 is also positive, it can be converted into a strain-homogeneous variable:

$$\chi = \sqrt{\frac{2Y_1}{E_0}} \tag{14}$$

Finally, the rate of energy dissipated takes the following form:

$$\frac{E_0}{2}\Phi = \varepsilon^2\dot{D} - \chi^2\dot{\kappa} \geq 0 \tag{15}$$

Thermodynamic Potential

The existence of a creep potential Ω is assumed, in order to satisfy the second principle of thermodynamics (Lemaitre and Chaboche, 1988). This potential is chosen to depend on the dual variables Y_0 and Y_1, or in an equivalent way, on the homogeneous strain variables $|\varepsilon|$ and χ.

$$\Omega = \frac{|\varepsilon_c|}{\eta(m+1)}\left\langle \frac{|\varepsilon| - (\chi + \varepsilon_D)}{|\varepsilon_c|} \right\rangle^{m+1} \quad \text{where } \langle x \rangle = \frac{x + |x|}{2} \tag{16}$$

ε_D which takes positive value, defines the size of the elastic strain domain. ε_c is a strain corresponding to the maximum of stress in a uniaxial compression test for an infinitely slow strain rate.

Two parameters have been introduced in the expression of the potential: a time constant η and a dimensionless parameter m.

The generalised normality rule is assumed:

$$\left| \begin{array}{l} \dot{D} = \dfrac{\partial \Omega}{\partial |\varepsilon|} \\ \dot{\kappa} = -\dfrac{\partial \Omega}{\partial \chi} \end{array} \right. \tag{17}$$

The rates of internal variables follow as:

$$\left| \begin{array}{l} \dot{D} = \dfrac{1}{\eta}\left\langle \dfrac{|\varepsilon| - (\chi + \varepsilon_D)}{|\varepsilon_c|} \right\rangle^{m} \\ \dot{\kappa} = \dot{D} \end{array} \right. \tag{18}$$

In particular, for the compression test only considered in the paper, the internal variable κ is simply related to the damage variable by:

$$\kappa = D \tag{19}$$

The relation (18) is similar to the one obtained for internal variables in viscoplasticity in the sense of Perzyna (1963). The main difference with a time-independent damage model is that points outside the elastic domain are acceptable within this model (as in viscoplasticity). The rate of damage can be interpreted as a function of the distance from the current point to the boundary of the elastic domain.

Using Equation (15), one obtains:

$$\frac{2}{E_0}\Phi = \varepsilon^2 \frac{\partial \Omega}{\partial |\varepsilon|} + \chi^2 \frac{\partial \Omega}{\partial \chi} = \left(\varepsilon^2 - \chi^2\right)\dot{D} \tag{20}$$

The following relation:

$$|\varepsilon| \geq (\chi + \varepsilon_D) \tag{21}$$

is necessarily verified on an equipotential outside the elastic domain. Thus, within the proposed creep damage model, the second thermodynamic principle is always valid:

$$\Phi \geq 0 \tag{22}$$

Equations (10), (13) and (14) yield the free energy ψ_1 as:

$$\psi_1'(\kappa) = \frac{E_0}{2\rho}[\chi(\kappa)]^2 \tag{23}$$

The thermodynamical variable χ is introduced, using the bijective function F for $\kappa \in [0; 1]$:

$$\chi(\kappa) = F^{-1}(\kappa) - \varepsilon_D \quad \forall \kappa \in]0, 1]; \quad \chi(0) = 0 \tag{24}$$

F is given later in the paper and results from a time-independent damage model. It is chosen such as:

$$\left| \begin{array}{l} \chi(0) = 0 \\ \chi(\kappa) > 0 \forall \kappa \in]0; 1] \end{array} \right. \tag{25}$$

Then, the free energy ψ_1 could be defined as:

$$\psi_1(\kappa) = \int_0^{\kappa} \frac{E_0}{2\rho}[\chi(s)]^2 ds \tag{26}$$

ψ_1 is a definite positive function.

NUMERICAL TEST WITH CONSTANT STRAIN RATE

Dimensionless Formulation

Let us consider the dimensionless and normalised formulation:

$$\left| \begin{array}{l} \varepsilon^* = \frac{\varepsilon}{\varepsilon_c} \\ \chi^* = \frac{\chi}{|\varepsilon_c|} \end{array} \right., \left| \begin{array}{l} \sigma^* = \frac{\sigma}{E_0 \varepsilon_c} \\ \tau = \frac{t}{\eta} \end{array} \right., \quad \text{and} \quad \varepsilon_D^* = \frac{\varepsilon_D}{|\varepsilon_c|} \tag{27}$$

The viscodamage equations are summarised:

$$\left| \begin{array}{l} \sigma^* = (1-D)\varepsilon^* \\ \dfrac{dD}{d\tau} = \left\langle \varepsilon^* - \left(\chi^*(D) + \varepsilon_D^*\right)\right\rangle^m \end{array} \right. \tag{28}$$

with: $\chi^*(D) = F^{*-1}(D) - \varepsilon_D^*$.

The function F^* results from the formulation of the time-independent Mazars model applied to uniaxial compression test (Mazars, 1986; Mazars and Pijaudier-Cabot, 1996). This function could be simplified in:

$$F^*(\varepsilon^*) = 1 + \frac{\varepsilon_D^{*2}}{\left(1-\varepsilon_D^*\right)\varepsilon^*} - \frac{1}{1-\varepsilon_D^*}\exp\left(\varepsilon_D^* - \varepsilon^*\right) \tag{29}$$

It is assumed that the model has no yield surface for the monotonic test, as for endochronic models for instance (Valanis, 1971).

$$\varepsilon_D^* = 0 \tag{30}$$

A numerical test at constant strain rate, denoted by α is considered:

$$\varepsilon = \alpha t; \quad \alpha \leq 0 \tag{31}$$

The response curve of the stress versus strain is now parameterised by the rate α:

$$\sigma^*(\varepsilon^*(\tau), \alpha) = \sigma_\alpha^*(\varepsilon^*) \tag{32}$$

Equations which govern such a numerical test are finally written as:

$$\left| \begin{array}{l} \varepsilon^* = \dfrac{\alpha\eta}{\varepsilon_c}\tau \\ \sigma^* = (1-D)\varepsilon^* \\ \dfrac{dD}{d\tau} = \left\langle \varepsilon^* - \left(\chi^*(D) + \varepsilon_D^*\right)\right\rangle^m \end{array} \right. \tag{33}$$

The material is assumed to be in a virgin state at the beginning of the test.

Simulations

Conventionally, the standard test corresponds to the high rate test:

$$\dot{\varepsilon}_c = -2 \times 10^{-5}\,\mathrm{s}^{-1} \tag{34}$$

Conversely, for an infinitely slow loading, the curve will tend towards the response of the time-independent model of Mazars:

$$\sigma_0^*(\varepsilon^*) = (1 - F^*(\varepsilon^*))\varepsilon^* \tag{35}$$

The values η and m control the transition between the extreme curves at high and slow strain rate. Identification of these parameters can be based on the analysis of experimental data, from one of the curves. Figure 1 represents the curves for various strain rate: the strain rate of each curve is equal to the strain rate of the standard test divided by different power of ten. The following parameters are adopted:

$$\varepsilon_c = -2 \times 10^{-3}; \quad m = 7; \quad \eta = 5 \times 10^{-4}\,\text{s leading to } \frac{\eta \dot{\varepsilon}_c}{\varepsilon_c} = 5 \times 10^{-6} \tag{36}$$

The asymptotic curves associated to both functions $\sigma_0^*(\varepsilon^*)$ and $\sigma_{\dot{\varepsilon}_c}^*(\varepsilon^*)$ are recognisable by heavy lines. The model reproduces the qualitative behaviour of strain-rate dependence as described by Rüsch (1960), Wittmann (1984) or Bischoff and Perry (1991). The ultimate stress decreases as the rate of loading becomes slower. Ductility follows this tendency. The main difference concerns the strain location at the ultimate stress. The limits of behaviour for high and slow strain rates are nevertheless realistic. It is remarkable that

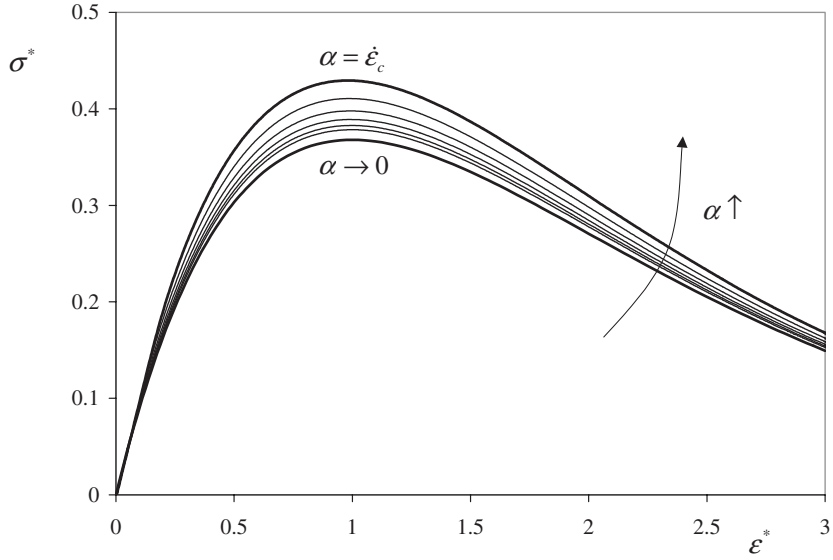

Figure 1. *Strain rate effect on the response of concrete in compression.*

the linear part of the curve tends to grow with the strain rate, suggesting that the apparent elastic domain evolves with the rate of loading.

CREEP FAILURE MODELLING

Description of the Creep Test

The potential of the model in the computation of creep tests is now studied. During the creep test, the stress is assigned to a constant value equal to:

$$\sigma^*(\tau) = \overline{\sigma}^* \; \forall \quad \tau > 0 \tag{37}$$

The differential equations which govern the evolution of the state variable are now written as:

$$\left| \begin{array}{l} \varepsilon^* = \dfrac{\overline{\sigma}^*}{1-D} \\ \dfrac{dD}{d\tau} = \left\langle \dfrac{\overline{\sigma}^*}{1-D} - \left(\chi^*(D) + \varepsilon_D^*\right) \right\rangle^m \end{array} \right. \quad \text{and} \quad \varepsilon_D^* = 0 \tag{38}$$

Formally, this equation is not so far to the specific creep Kachanov model (1958), which expresses the rate of damage only according to the effective stress (Rabotnov, 1969).

$$\frac{dD}{d\tau} = \left(\frac{\overline{\sigma}^*}{1-D}\right)^m \tag{39}$$

The second damage term in the bracket is nevertheless significant. The system of differential equations (38) can be viewed as an autonomous non-linear dynamics system (Lamnabhi-Lagarrigue, 1994). Of course, "Dynamics" here does not mean that inertial forces are involved during the motion. The damage differential equation is of degree one and the strain becomes a direct function of the damage with this model. Initial conditions result from an initial loading at a very high strain rate $\dot{\varepsilon}_c$. The initial damage value is assumed to be the smallest value of damage obtained for the constant stress value, reached with the initial loading (the solution is not unique in the case of softening models).

Creep Failure: A Bifurcation Phenomenon

The asymptotic behaviour of creep solutions is important to qualify: this is the information which guarantees the "timeless" stability of the creeping

material. The fixed point D_e of the autonomous non-linear system (38) is given by the equation:

$$\frac{dD}{d\tau} = 0 \tag{40}$$

When solutions of Equation (40) exist, the equilibrium damage D_e is obtained by the inequality:

$$\chi^*(D_e) \geq \frac{\overline{\sigma}^*}{1 - D_e} \tag{41}$$

The smallest value of this set is noted by D_e^-:

$$D_e^- = \min\left\{\chi^*(D_e) \geq \frac{\overline{\sigma}^*}{1 - D_e}\right\} = \min\left\{\chi^*(D_e) = \frac{\overline{\sigma}^*}{1 - D_e}\right\} \tag{42}$$

Reciprocally, if inequality (41) does not have any solution, the associated dynamic system does not possess a fixed point. It is the case when:

$$\overline{\sigma}^* \geq \beta \quad \text{where} \quad \beta = \sigma_0^*(\varepsilon^* = 1) = \frac{1}{e} \tag{43}$$

β corresponds to the maximal value of σ_0^*, obtained for an inifinitely slow strain rate.

In this last case, the motion evolves towards the failure, characterised by damage reaching the unity value.

The phases space in which the evolution problem is defined in a unique way has the dimension 1. The evolution can be characterised indifferently by the damage variable or by the strain variable. Considering the damage variable, the evolution could be understood by the diagrams of Figure 2:

This change of behaviour beyond the critical value:

$$\overline{\sigma}^* = \beta \tag{44}$$

is typically a bifurcation phenomenon.

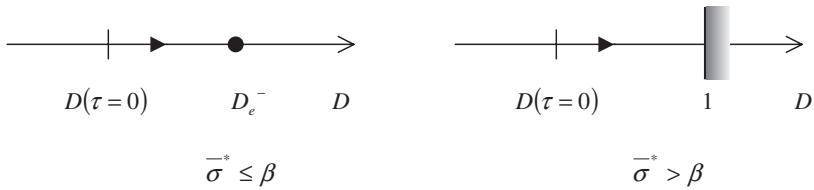

Figure 2. *Representation of the damage evolution in the phases space – creep test.*

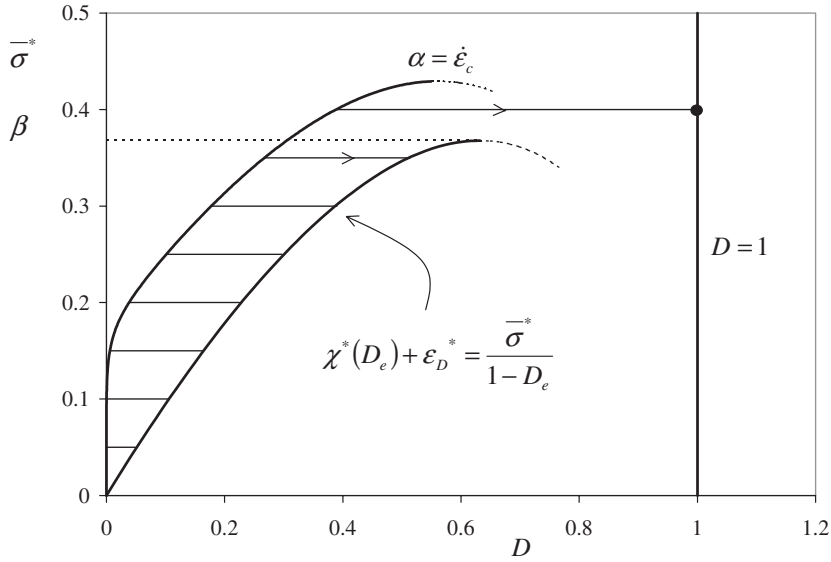

Figure 3. *Bifurcation diagram – creep tests.*

The bifurcation diagram (D, $\overline{\sigma}^*$) is shown in Figure 3. $\overline{\sigma}^*$ is the structural parameter. The equilibrium curve is defined from inequality (41).

$$\chi^*(D_e) = \frac{\overline{\sigma}^*}{1 - D_e} \tag{45}$$

A view using the co-ordinates (ε^*, $\overline{\sigma}^*$) is of course equivalent.

It is easy to show that existence of the fixed point ensures here the "timeless" stability of the material: the value of the equilibrium damage is lower than 1. Moreover, independently of the disturbance (thermodynamically admissible), asymptotic convergence towards the fixed point is necessarily reached when the stress is lower than the critical stress. It is clear that bifurcation at parameter β is associated to behaviour change of the non-linear system: it is also a sufficient and necessary condition of failure for this simple creep model.

Numerical Simulations

With the parameters chosen for the simulation, the bifurcation (or failure condition) is obtained for creep stresses higher than 86% of the maximum stress issued of a test at very high strain rate $\dot{\varepsilon}_c$. It is the order of magnitude

experimentally noted by Rüsch (1960) under high sustained load. The strain evolution is shown in Figure 4 for different sustained stress values.

Another representation of the creep tests is proposed in Figure 5 in the plan of the observable variables $(\sigma^*, \varepsilon^*)$.

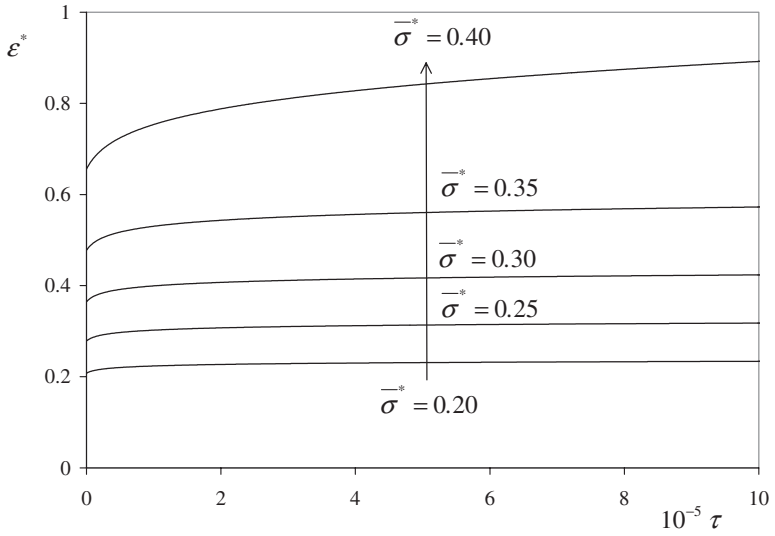

Figure 4. *Simulation of creep test for different sustained stress values .*

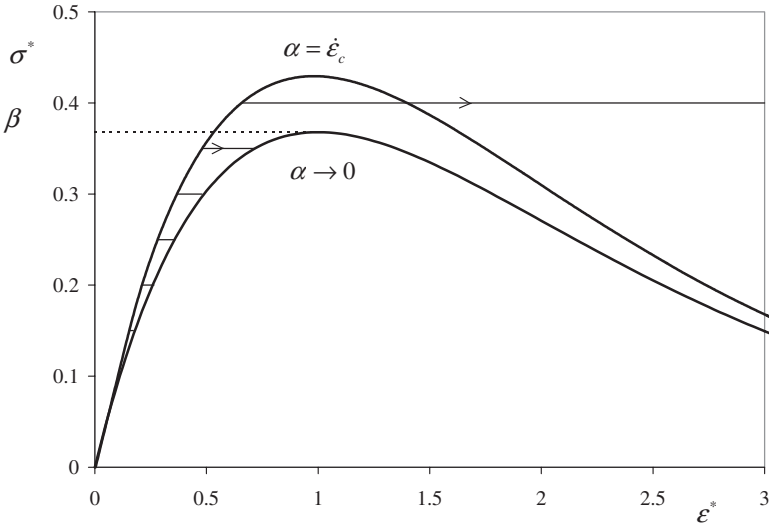

Figure 5. *Representation of the creep test, stress–strain diagram.*

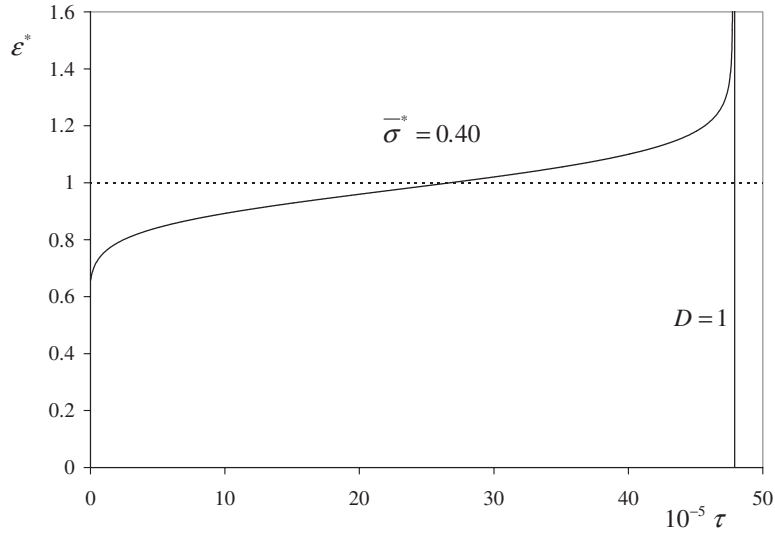

Figure 6. *Tertiary creep computation.*

An inflexion point appears clearly in the evolution of strain versus time for high stress level (Figure 6). This point is located in the vicinity of the characteristic strain $\varepsilon^* = 1$. Tertiary creep follows for high strain values (strain higher than this characteristic strain) and leads to material failure, i.e. D equal to 1. The model is thus able to predict the creep failure of this material under high sustained load.

OTHER LOADING PATHS

The Relaxation Test

Strain history is given this time by:

$$\varepsilon^*(\tau) = \overline{\varepsilon}^* \quad \forall \tau > 0 \tag{46}$$

The evolution problem is now governed by the following system of differential equations:

$$\left| \begin{array}{l} \sigma^* = \overline{\varepsilon}^*(1 - D) \\ \dfrac{dD}{d\tau} = \left\langle \overline{\varepsilon}^* - \left(\chi^*(D) + \varepsilon_D^*\right) \right\rangle^m \end{array} \right. \quad \text{and} \quad \varepsilon_D^* = 0 \tag{47}$$

System (47) is an autonomous system again. It could be seen in the second equation of (47) that the evolution is defined in phases space of dimension 1.

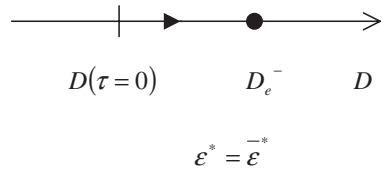

Figure 7. *Representation of the evolution in the phases space – relaxation test.*

Damage is the natural state variable but the evolution could also be expressed by the stress variable, which depends directly on the damage variable. As for the creep test, initial conditions are induced by loading at constant strain rate $\dot{\varepsilon}_c$. The equilibrium value of damage D_e is the result of the stationary evolution of damage:

$$\chi^*(D_e) \geq \overline{\varepsilon}^* \tag{48}$$

As for the creep test, the smallest value D_e^- is introduced:

$$D_e^- = \min\left\{\chi^*(D_e) \geq \overline{\varepsilon}^*\right\} = \min\left\{\chi^*(D_e) = \overline{\varepsilon}^*\right\} \tag{49}$$

This equation leads to the relation:

$$D_e^- = F^*(\overline{\varepsilon}^*) \tag{50}$$

which possesses always a solution.

Thus, contrary to the creep test, the relaxation test has a fixed point for every value of the structural parameter, which is the constant strain $\overline{\varepsilon}^*$ here. Moreover, the initial value of damage is lower than the value of the smallest equilibrium damage D_e. The rate of damage is positive and the motion asymptotically converges towards the fixed point (Figure 7). This fixed point is asymptotically stable with respect to the disturbance generated by the loading path at the constant strain rate $\dot{\varepsilon}_c$.

Numerical Simulations

The stress evolution is shown in Figure 8 for different sustained strain values.

It is interesting to note that the relaxation tests can be carried out for fixed strain value higher than the characteristic strain ($\varepsilon^* = 1$), corresponding to the maximum stress at the strain rate $\dot{\varepsilon}_c$ (Figure 9). Such tests are certainly difficult to achieve and remain quite theoretical.

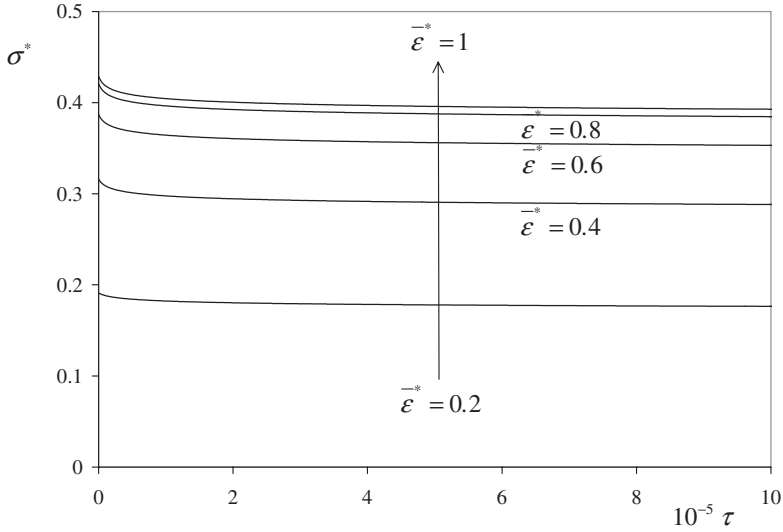

Figure 8. *Simulation of relaxation test.*

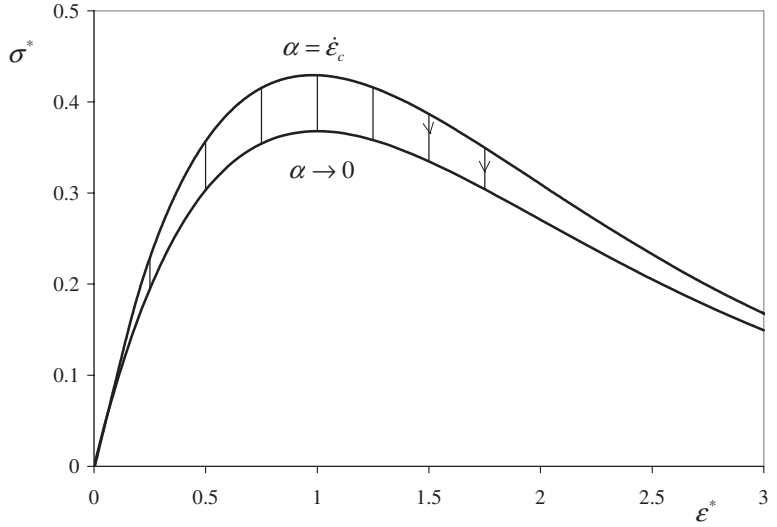

Figure 9. *Representation of the relaxation test, stress–strain diagram.*

Discussion on the Damage Parameter

The distinction between the viscoelastic contribution and the evolution of a certain irreversibility (here the damage variable) is difficult to exhibit

experimentally. For a damage model for instance, the unloading phase of the test is not so easy to interpret in order to extract the anelasticity contribution. In the case of the viscodamage model considered in the paper, the initial elastic domain is merged with the origin. The observed non-linearity of the concrete response in compression under very low strain rates corroborates such an assumption. Nevertheless, the size of the apparent elastic domain seems to increase with the strain rate. Damage is thus understood as a time-dependent deterioration of the mechanical characteristics of concrete: adaptation prevails at very low strain rate. Higher strain rates tend to increase the concrete strength as if strain rate acts as a time-hardening parameter. The model finally reveals the competition between the internal kinetics of damage (which appears under the time-constant η) and the strain rate imposed on the material. In an asymptotic way, for the dynamic regimes, concrete behaves as an elastic-brittle material.

It is known that the validity of a rheological model must be appreciated by experimental data. Unfortunately, contrary to the characterisation of steel creep (Rabotnov, 1969), few experimental data are available in the literature concerning the direct measure of creep damage of concrete. Damage can be evaluated during creep tests, using indirect measurements as sonic data (Lemaitre and Chaboche, 1988). It could be also planned to identify damage after a total unloading with a new high strain rate loading. The new effective modulus could be analysed in terms of damage. Another indirect way consists in measuring the strain directly: damage could be deduced by using the relation of effective stress (Piechnik and Pachla, 1980). This indirect technique is probably limited by the field of validity of effective stress relation to low values of damage. The questioning of such a relation (Kachanov, 1992) is one of the directions envisaged in the enrichment of the model presented in this paper.

CONCLUSIONS

A viscodamage model covering in a unified way phenomena like relaxation, creep and sensitivity to the rate of loading was presented. This model with few parameters is inspired of the basic formalism developed by Dubé et al. (1996) for dynamical regimes; it is adapted to slow loading. This model can be also viewed as an extension of the time-independent Mazars model (Mazars and Pijaudier-Cabot, 1996). The framework has thermodynamically strong arguments. Phenomena as creep failure under high-sustained load are explained quite simply within stability theory. Creep failure is the manifestation of a bifurcation phenomenon.

A non-local implementation of this model would be probably necessary in order to simulate objective structural behaviour. Following the

methodology of Pijaudier-Cabot and Bazant (1987) for a time-independent damage model, several simulations of realistic concrete structures would evaluate the effectiveness of the rheological model at the structural scale. The difficult problem of creep in statically indeterminate structures (Lévi, 1951; Salençon, 1983) could be studied within this formalism.

Complex loading paths could be formally described from a generalisation of this uniaxial model to the tension case and especially to multiaxial loading. Nevertheless, attention must be drawn on the strong experimental difficulties met in the conception of reliable creep tension tests.

It could be remarked that the model does not question the validity of viscoelasticity approaches for low level of solicitation. Transition between both mechanisms (viscoelasticity and viscodamage) is probably an important task in modelling time-dependent behaviour of concrete.

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