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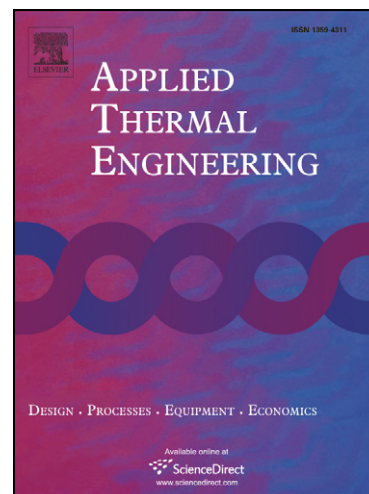
PII: S1359-4311(08)00208-1  
DOI: [10.1016/j.applthermaleng.2008.04.017](https://doi.org/10.1016/j.applthermaleng.2008.04.017)  
Reference: ATE 2494

To appear in: *Applied Thermal Engineering*

Received Date: 10 November 2006  
Revised Date: 14 April 2008  
Accepted Date: 20 April 2008

Please cite this article as: L. Malinowski, S. Bielski, Transient temperature field in a parallel-flow three-fluid heat exchanger with the thermal capacitance of the walls and the longitudinal walls conduction, *Applied Thermal Engineering* (2008), doi: [10.1016/j.applthermaleng.2008.04.017](https://doi.org/10.1016/j.applthermaleng.2008.04.017)

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**Transient temperature field in a parallel-flow three-fluid heat exchanger  
with the thermal capacitance of the walls and the longitudinal walls conduction**

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**Abstract**

Transient temperature response of a parallel-flow three-fluid heat exchanger with the thermal capacitance of the walls and the longitudinal heat conduction through the walls is investigated numerically, by the implicit MacCormack method, for a step change in flow rate of one fluid. The impact of thermal properties of the walls on temperature field is examined. The results of calculations show that as the thermal diffusivity of the walls decreases, the effect of the walls increases.

Keywords: Three fluid heat exchanger; Transient heat transfer; Implicit MacCormack method; Transient temperature field

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**Nomenclature**

- A** - diagonal matrix of thermal diffusivity,  $m^2/s$
- $A_i$  - cross-sectional area of channel no.  $i$ ,  $m^2$
- $a$  - thermal diffusivity,  $m^2/s$
- b** - interconnection vector,  $1/s$
- $c$  - specific heat at constant pressure,  $J/(kgK)$
- $l_w$  - characteristic dimension dependent on the shape of the wall,  $m$
- $L$  - length of channels,  $m$
- q** - heat flux vector,  $W/m^2$
- T** - temperature vector,  $^{\circ}C$
- $T$  - temperature,  $^{\circ}C$
- $t$  - time,  $s$
- V** - diagonal matrix of fluid velocity,  $m/s$
- $v_i$  - fluid velocity in channel no.  $i$ ,  $m/s$
- u** - interconnection vector,  $1/s$
- $w$  - coefficient resulting from replacing the heat conducted in the  $y$  direction by an equivalent heat source,  $1/s$
- $x$  - spatial co-ordinate,  $m$
- $X$  - dimensionless spatial co-ordinate
- $y$  - spatial co-ordinate,  $m$
- $\alpha$  - heat transfer coefficient,  $W/(m^2K)$
- $\delta$  - implicit temporal difference operator
- $\delta_w$  - wall thickness,  $m$

$\Delta$  - explicit temporal difference operator

$\Theta$  - dimensionless temperature

$\lambda$  - thermal conductivity,  $W/(mK)$

$\rho$  - density,  $kg/m^3$

$\phi$  - stability parameter,  $m/s$

$\tau$  - dimensionless time

$\Omega$  - heat transfer perimeter,  $m$

subscripts

$w$  - wall

superscripts

$a, b$  - labels of the nodes situated on the opposite surfaces of the wall

$o$  - initial steady state distribution of temperature

$x$  - in the  $x$  direction

$y$  - in the  $y$  direction

## 1. Introduction

In the general design process of multi-fluid heat exchangers, the stationary operating conditions are of decisive importance, however, in reality heat exchangers frequently undergo transients resulting from external load variations and regulations. Knowledge of dynamic behaviour of heat exchangers is necessary for designing control and regulation systems of different industrial processes and operations, such as in nuclear reactors, cryogenic and petrochemical process plants and HVAC systems.

A lot of attention has been given to mathematical modelling of three- and multi-fluid heat exchangers in the literature. However, in most cases, the solutions presented

refer to the determination of steady-state temperature fields [1 - 7]. According to the best knowledge of the authors, the available literature gives little information regarding mathematical modelling of transient behaviour of such exchangers [8-21]. Sekulic and Herman [8] solved numerically the set of partial differential equations describing temperature field in a counter-flow three-fluid heat exchanger. They used the Wendroff implicit finite difference approximation and steady state initial conditions. In Reference [9] Sekulic et al. studied experimentally and numerically transient temperature fields in a three-fluid heat exchanger with two thermal connections. The experiments confirmed the accuracy of the method used in Reference [8]. A semi-analytical solution for a three-fluid parallel-flow heat exchanger with two thermal communications and the steady state initial conditions was formulated by Bielski and Malinowski [10]. The authors applied the Laplace transform technique with numerical inversion. The same authors [11] derived fully analytical expressions for temperatures in a parallel-flow three-fluid heat exchanger with two heat connections between the fluids, constant temperature in one channel, the uniform temperature initial conditions, and a step increase in the inlet temperature of one fluid. Luo et al. [12] investigated semi-analytically and numerically dynamic responses to temperature and flow transients in multi-stream parallel- and counter-flow heat exchangers taking into account the heat capacities of walls. They used the uniform temperature initial conditions or steady-state conditions. For linear and linearized cases, the governing equations were solved by the semi-analytical Laplace transform method with numerical inversion. Non-linear cases were solved numerically. Malinowski [13] formulated equations for transient behaviour of multi-fluid heat exchangers and solved numerically, by the MacCormack explicit predictor-corrector method, a four-fluid case. He analysed the response of the exchanger

to a step change in temperature or flow rate of one fluid using the uniform temperature initial condition or steady state initial condition.

It is worth mentioning that an  $n$ -fluid heat exchanger characterizes itself by  $n$  separate channels in which can flow up to  $n$  different fluids. A two-fluid  $n$ -channel heat exchanger, e.g. multipass heat exchanger, can be considered mathematically as an  $n$ -fluid heat exchanger, i.e. the temperature field is described by  $n$  differential equations. Dynamic behaviour of  $n$ -channel two-fluid heat exchangers is studied in References [14-21]. Among References [1-21], only the authors of Reference [19] have accounted for the thermal capacity of the walls and the conduction of heat along the walls in their mathematical model of transient temperature field in a plate-type heat exchanger (they have, however, omitted the impact of heat resistance on the heat transfer across the walls).

In the present paper we solve numerically, by the implicit MacCormack predictor-corrector method, the system of partial differential equations describing the transient temperature field in a parallel-flow three-fluid heat exchanger with three heat connections between the fluids. The model accounts for the thermal capacitance of the walls, the longitudinal heat conduction through the walls, and the conductive resistance across the walls. Exemplary calculations are carried out to determine the unsteady response of the heat exchanger, initially at steady-state, to a step change in flow rate of one fluid.

## 2. Mathematical model

A parallel-flow three-fluid heat exchanger with three heat connections between the fluids is illustrated schematically in Fig. 1. For deriving the governing partial

differential equations describing the transient temperature field in such an exchanger, the following assumptions are made:

- physical properties of the fluids and walls materials are constant,
- the heat transfer coefficients are independent on temperature, but they depend on the flow rate,
- mass flow and fluid temperature in each channel are taken to be uniform over the cross-section perpendicular to the flow direction,
- no heat is lost to the ambient.

With these assumptions, the model equations for heat transfer can be stated as

$$\frac{\partial \mathbf{T}}{\partial t} + \mathbf{V} \frac{\partial \mathbf{T}}{\partial x} + \mathbf{b} = \mathbf{0} \quad (1)$$

$$\frac{\partial \mathbf{T}_w}{\partial t} + \mathbf{A}^x \frac{\partial^2 \mathbf{T}_w}{\partial x^2} + \mathbf{A}^y \frac{\partial^2 \mathbf{T}_w}{\partial y^2} + \mathbf{u} = \mathbf{0} \quad (2)$$

where

$$\mathbf{T} = \begin{bmatrix} T_1(x, t) \\ T_2(x, t) \\ T_3(x, t) \end{bmatrix} \quad (3)$$

$$\mathbf{T}_w = \begin{bmatrix} T_{w1}(x, y, t) \\ T_{w2}(x, y, t) \\ T_{w3}(x, y, t) \end{bmatrix} \quad (4)$$

$$\mathbf{V} = \begin{bmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{bmatrix} \quad (5)$$

$$\mathbf{A}^x = \begin{bmatrix} a_1^x & 0 & 0 \\ 0 & a_2^x & 0 \\ 0 & 0 & a_3^x \end{bmatrix} \quad (6)$$

$$\mathbf{A}^y = \begin{bmatrix} a_1^y & 0 & 0 \\ 0 & a_2^y & 0 \\ 0 & 0 & a_3^y \end{bmatrix} \quad (7)$$

$$a_i^x = \frac{\lambda_{w_i}^x}{\rho_{w_i} c_{w_i}} \quad (8)$$

$$a_i^y = \frac{\lambda_{w_i}^y}{\rho_{w_i} c_{w_i}} \quad (9)$$

$$\mathbf{b} = \begin{bmatrix} b_{1w1} [T_1(x,t) - T_{w1}(x,y,t)] + b_{1w3} [T_1(x,t) - T_{w3}(x,y,t)] \\ b_{2w1} [T_2(x,t) - T_{w1}(x,y,t)] + b_{2w2} [T_2(x,t) - T_{w2}(x,y,t)] \\ b_{3w2} [T_3(x,t) - T_{w2}(x,y,t)] + b_{3w3} [T_3(x,t) - T_{w3}(x,y,t)] \end{bmatrix} \quad (10)$$

$$\mathbf{u} = \begin{bmatrix} u_{1w1} [T_{w1}(x,y,t) - T_1(x,t)] + u_{2w1} [T_{w1}(x,y,t) - T_2(x,t)] \\ u_{2w2} [T_{w2}(x,y,t) - T_2(x,t)] + u_{3w2} [T_{w2}(x,y,t) - T_3(x,t)] \\ u_{1w3} [T_{w3}(x,y,t) - T_1(x,t)] + u_{3w3} [T_{w3}(x,y,t) - T_3(x,t)] \end{bmatrix} \quad (11)$$

$$b_{ijw} = \frac{\alpha_i \Omega_{ijw}}{\rho_i c_i A_i} \quad (12)$$

$$u_{ijw} = \frac{\alpha_i \Omega_{ijw}}{\rho_{wj} c_{wj} A_{wj}} \quad (13)$$

Anisotropic walls are assumed for calculations in order to allow for the impact of heat conduction across or/and along the walls being accounted for or not.

In this study a transient response of the heat exchanger is determined for a step change in flow rate of fluid no 1. For the case under consideration, the initial conditions are given by

$$\mathbf{T}(x,0) = \boldsymbol{\varphi}(x) \quad (14)$$

$$\mathbf{T}_w(x,y,0) = \boldsymbol{\varphi}_w(x,y) \quad (15)$$

where



$$\boldsymbol{\varphi}(x) = \begin{bmatrix} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \end{bmatrix} \quad (16)$$

$$\boldsymbol{\varphi}_w(x, y) = \begin{bmatrix} \varphi_{w1}(x, y) \\ \varphi_{w2}(x, y) \\ \varphi_{w3}(x, y) \end{bmatrix} \quad (17)$$

The  $\varphi_i(x)$  and  $\varphi_{wi}(x, y)$  are functions of spatial co-ordinates outlining the steady-state temperature field in the analysed heat exchanger at a start time  $t = 0$  s. The boundary conditions to be applied to set (1) – (2) are

- for fluids

$$\mathbf{T}(0, t) = \begin{bmatrix} T_1^{in} \\ T_2^{in} \\ T_3^{in} \end{bmatrix} \quad (18)$$

where the  $T_i^{in}$ , for  $i = 1, 2, 3$ , are the constant inlet temperatures of the fluids

- for walls

$$\dot{\mathbf{q}}(0, y, t) = -\lambda^x \frac{\partial \mathbf{T}_w(0, y, t)}{\partial x} = 0 \quad (19)$$

$$\dot{\mathbf{q}}(L, y, t) = -\lambda^x \frac{\partial \mathbf{T}_w(L, y, t)}{\partial x} = 0 \quad (20)$$

Conditions (19) and (20) mean that the edges of the walls are thermally insulated.

### 3. Numerical solution

The set of partial differential equations given by Eqs. (1) – (13) is solved numerically by means of the MacCormack implicit predictor-corrector method [22 - 24]. This two-step finite difference method of second order accuracy is very effective

for solving classical transient heat transfer problems in heat exchangers, both for a step change in flow rate and for a step change in inlet temperature [10, 11, 13].

While meshing the walls we used only two node points in the  $y$  direction, situated on the surfaces of the walls, because  $\delta_w \ll L$ . These nodes are designated as “ $a$ ” and “ $b$ ” (see Fig. 2). To simplify the solution of the problem given by Eqs. (1) and (2), we replaced the two-dimensional equation of heat conduction (2) with a one-dimensional equation in which the heat conducted in the  $y$  direction is considered as the heat generated by an equivalent heat source.

$$\frac{\partial \mathbf{T}}{\partial t} + \mathbf{V} \frac{\partial \mathbf{T}}{\partial x} + \mathbf{b}^* = \mathbf{0} \quad (21)$$

$$\frac{\partial \mathbf{T}_w}{\partial t} + \mathbf{A}^x \frac{\partial^2 \mathbf{T}_w}{\partial x^2} + \mathbf{u}^* = \mathbf{0} \quad (22)$$

Vector  $\mathbf{T}_w$  and matrix  $\mathbf{A}^x$  have now the following form, respectively

$$\mathbf{T}_w = \begin{bmatrix} T_{w1}^a \\ T_{w1}^b \\ T_{w2}^a \\ T_{w2}^b \\ T_{w3}^a \\ T_{w3}^b \end{bmatrix} \quad (23)$$

$$\mathbf{A}^x = \begin{bmatrix} a_1^x & 0 & 0 & 0 & 0 & 0 \\ 0 & a_1^x & 0 & 0 & 0 & 0 \\ 0 & 0 & a_2^x & 0 & 0 & 0 \\ 0 & 0 & 0 & a_2^x & 0 & 0 \\ 0 & 0 & 0 & 0 & a_3^x & 0 \\ 0 & 0 & 0 & 0 & 0 & a_3^x \end{bmatrix} \quad (24)$$

The source term,  $\mathbf{u}^*$ , in Eq. (22) takes into account the heat exchanged in the  $y$  direction both by conduction and by convection. A numerical scheme for solving Eqs. (21) and

(22) is developed on the basis of expressions formulated by MacCormack [22]. The finite-difference representations of Eqs. (21) and (22) are

- predictor

$$\left\{ \begin{array}{l} \Delta \mathbf{T}_j^n = -\frac{\Delta t}{\Delta x} \mathbf{V} (\mathbf{T}_j^n - \mathbf{T}_{j-1}^n) + \Delta t \mathbf{b}_j^{*n} \\ \delta \mathbf{T}_j^n = \left( \mathbf{I} + \mathbf{\Phi} \frac{\Delta t}{\Delta x} \right)^{-1} \left( \Delta \mathbf{T}_j^n + \mathbf{\Phi} \frac{\Delta t}{\Delta x} \delta \mathbf{T}_{j-1}^n \right) \\ \bar{\mathbf{T}}_j^{n+1} = \mathbf{T}_j^n + \delta \mathbf{T}_j^n \\ \Delta \mathbf{T}_{\mathbf{w}j}^n = \frac{\Delta t}{\Delta x^2} \mathbf{A}^x (\mathbf{T}_{\mathbf{w}j+1}^n - 2\mathbf{T}_{\mathbf{w}j}^n + \mathbf{T}_{\mathbf{w}j-1}^n) + \Delta t \mathbf{u}_j^{*n} \\ \delta \mathbf{T}_{\mathbf{w}j}^n = \left( \mathbf{I} + \mathbf{\Phi}_{\mathbf{w}} \frac{\Delta t}{\Delta x} \right)^{-1} \left( \Delta \mathbf{T}_{\mathbf{w}j}^n + \mathbf{\Phi}_{\mathbf{w}} \frac{\Delta t}{\Delta x} \delta \mathbf{T}_{\mathbf{w}j-1}^n \right) \\ \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} = \mathbf{T}_{\mathbf{w}j}^n + \delta \mathbf{T}_{\mathbf{w}j}^n \end{array} \right. \quad (25)$$

- corrector (final value)

$$\left\{ \begin{array}{l} \Delta \bar{\mathbf{T}}_j^{n+1} = -\frac{\Delta t}{\Delta x} \mathbf{V} (\bar{\mathbf{T}}_{j+1}^{n+1} - \bar{\mathbf{T}}_j^{n+1}) + \Delta t \bar{\mathbf{b}}_j^{*n+1} \\ \delta \bar{\mathbf{T}}_j^{n+1} = \left( \mathbf{I} + \mathbf{\Phi} \frac{\Delta t}{\Delta x} \right)^{-1} \left( \Delta \bar{\mathbf{T}}_j^{n+1} + \mathbf{\Phi} \frac{\Delta t}{\Delta x} \delta \bar{\mathbf{T}}_{j+1}^{n+1} \right) \\ \mathbf{T}_j^{n+1} = 0.5 (\mathbf{T}_j^n + \bar{\mathbf{T}}_j^{n+1} + \delta \bar{\mathbf{T}}_j^{n+1}) \\ \Delta \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} = \frac{\Delta t}{\Delta x^2} \mathbf{A}^x (\bar{\mathbf{T}}_{\mathbf{w}j+1}^{n+1} - 2\bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} + \bar{\mathbf{T}}_{\mathbf{w}j-1}^{n+1}) + \Delta t \bar{\mathbf{u}}_j^{*n+1} \\ \delta \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} = \left( \mathbf{I} + \mathbf{\Phi}_{\mathbf{w}} \frac{\Delta t}{\Delta x} \right)^{-1} \left( \Delta \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} + \mathbf{\Phi}_{\mathbf{w}} \frac{\Delta t}{\Delta x} \delta \bar{\mathbf{T}}_{\mathbf{w}j+1}^{n+1} \right) \\ \mathbf{T}_{\mathbf{w}j}^{n+1} = 0.5 (\mathbf{T}_{\mathbf{w}j}^n + \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1} + \delta \bar{\mathbf{T}}_{\mathbf{w}j}^{n+1}) \end{array} \right. \quad (26)$$

In equations (25) and (26),  $n$  is a time step,  $j$  is a grid point,  $\delta$  and  $\Delta$  are implicit and explicit temporal difference operators, respectively. The entries of matrixes  $\mathbf{\Phi}$  and  $\mathbf{\Phi}_{\mathbf{w}}$  are determined from the conditions for the method's stability formulated by MacCormack [22].

$$\Phi = \begin{bmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & \phi_3 \end{bmatrix} \quad (27)$$

$$\Phi_w = \begin{bmatrix} \phi_{w1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \phi_{w1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{w2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi_{w2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_{w3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \phi_{w3} \end{bmatrix} \quad (28)$$

$$\phi_i \geq \max \left\{ 0.5 \left( v_i - \frac{\Delta x}{\Delta t} \right), 0 \right\} \quad i = 1, 2, 3 \quad (29)$$

$$\phi_{wi} \geq \max \left\{ 0.5 \left( \frac{2a_i}{\Delta x} - \frac{\Delta x}{\Delta t} \right), 0 \right\} \quad i = 1, 2, 3 \quad (30)$$

Interconnections vectors  $\mathbf{b}^*$  and  $\mathbf{u}^*$  have the form

$$\mathbf{b}^* = \begin{bmatrix} b_{1w1} [T_{w1}^a(x,t) - T_1(x,t)] + b_{1w3} [T_{w3}^a(x,t) - T_1(x,t)] \\ b_{2w1} [T_{w1}^b(x,t) - T_2(x,t)] + b_{2w2} [T_{w2}^a(x,t) - T_2(x,t)] \\ b_{3w2} [T_{w2}^b(x,t) - T_3(x,t)] + b_{3w3} [T_{w3}^b(x,t) - T_3(x,t)] \end{bmatrix} \quad (31)$$

$$\mathbf{u}^* = \begin{bmatrix} u_{1w1} [T_1(x,t) - T_{w1}^a(x,t)] + w_1 [T_{w1}^b(x,t) - T_{w1}^a(x,t)] \\ u_{2w1} [T_2(x,t) - T_{w1}^b(x,t)] + w_1 [T_{w1}^a(x,t) - T_{w1}^b(x,t)] \\ u_{2w2} [T_2(x,t) - T_{w2}^a(x,t)] + w_2 [T_{w2}^b(x,t) - T_{w2}^a(x,t)] \\ u_{3w2} [T_3(x,t) - T_{w2}^b(x,t)] + w_2 [T_{w2}^a(x,t) - T_{w2}^b(x,t)] \\ u_{1w3} [T_1(x,t) - T_{w3}^a(x,t)] + w_3 [T_{w3}^b(x,t) - T_{w3}^a(x,t)] \\ u_{3w3} [T_3(x,t) - T_{w3}^b(x,t)] + w_3 [T_{w3}^a(x,t) - T_{w3}^b(x,t)] \end{bmatrix} \quad (32)$$

The value of coefficient  $w_i$  results from the equality of heat conducted to a cell and the heat generated in the cell by an equivalent volumetric heat source with capacity  $q_{vi}$ .

$$\frac{q_{vi}}{\rho_{wi} c_{wi}} = w_i (T^b - T^a) \quad (33)$$

$T^b$  and  $T^a$  are the temperatures of nodes  $a$  and  $b$ , respectively. The general expression for  $w_i$  is as follows

$$w_i = \frac{\lambda_{wi}^y}{\rho_{wi} c_{wi} \delta_w l_{wi}} \quad (34)$$

where  $l_{wi}$  is a characteristic dimension dependent on the shape of the wall. For a flat wall  $l_{wi} = \delta_w / 2$ .

The  $\bar{\mathbf{T}}_j^{n+1}$ ,  $\bar{\mathbf{T}}_w^{n+1}$  are predicted, and the  $\mathbf{T}_j^{n+1}$ ,  $\mathbf{T}_w^{n+1}$  are final values of temperatures at time step  $n+1$ . In the predictor equation we use a backward difference for  $\partial/\partial x$ , while in the corrector equation a forward difference is used. A second derivative,  $\partial^2/\partial x^2$ , is approximated with a central difference both in the case of predictor and corrector equations. In the implicit MacCormack method the calculation for each time step is divided into two stages. In the first stage, the explicit MacCormack method is used to calculate the changes in temperatures. In the second stage these changes are used in the implicit difference formula to determine the final temperatures for a given time step. Such two stages take place both at the predictor and corrector steps. During all calculations performed in this paper, we use the spatial step equal to  $\Delta x = 0.001 \text{ m}$  to achieve good accuracy of the results.

#### 4. Validation of the model and method

The correctness of the mathematical model adopted and the method of solution used is verified by comparing the results obtained from the numerical method with those from a semi-analytical solution for the case of a co-current three-fluid heat exchanger with two thermal couplings [10]. The present model with three thermal

couplings, accounting for the thermal capacity of the walls and heat conduction along the walls, is reduced to the model analysed in the work [10] by the adoption of the following data for calculations:  $c_{w_i}, \lambda_{w_i}^x \rightarrow 0$ , for  $i=1, 2, 3$  (heat is neither transferred along the walls nor accumulated in them),  $\lambda_{w_i}^y \rightarrow \infty$ , for  $i=1, 2$  (heat resistance of walls no. 1 and no. 2 in the direction perpendicular to the axis of the heat exchanger equals zero) and  $\lambda_{w_3}^y \rightarrow 0$  (wall no. 3 also does not conduct heat in the y direction).

Verifying calculations are carried out assuming that the system, initially at steady-state, undergoes an exponential rise of the inlet temperature of the fluid flowing through channel no. 1. The temperature profiles calculated using the semi-analytical method and the implicit MacCormack method are compared pictorially in Fig. 3. Very good consistency of the results is observed for the two methods applied.

## 5. Sample calculations

Sample calculations for the heat exchanger presented schematically in Fig. 1 are carried out using the implicit MacCormack method. The calculations account for the thermal capacity of the walls, longitudinal heat conduction through the walls, as well as the walls thermal resistance in the direction perpendicular to the heat exchanger axis. We consider a case where heat is transferred from a hot fluid to two cold fluids. Such a case is encountered in various technological processes where one heating agent is used to warm two different fluids (or the same fluids to different temperatures). Our calculations do not concern one specific practical process. The date was selected to be representative and to enable us to best illustrate the influence of walls on temperatures of fluids.

The steady-state distributions of fluids temperatures,  $\varphi_i(x)=T_i^o$ , and walls temperatures,  $\varphi_{wi}(x)=T_{wi}^o$ , for  $i=1, 2, 3$ , are assumed to be known. The following values of fluids temperatures at the exchanger inlet are chosen:  $T_1^{in} = 60^\circ C$ ,  $T_2^{in} = 20^\circ C$ ,  $T_3^{in} = 10^\circ C$ . The remaining data taken for the calculations are:  $D_1 = 0.05 m$ ,  $D_2 = 0.09 m$ ,  $L = 0.50 m$ ,  $\delta_w = 0.001 m$ ,  $v_1 = 0.20 m/s$ ,  $v_2 = 0.10 m/s$ ,  $v_3 = 0.15 m/s$ ,  $\alpha_1 = 3692 W/(m^2K)$ ,  $\alpha_2 = 2271 W/(m^2K)$ ,  $\alpha_3 = 3150 W/(m^2K)$ . The thermophysical parameters of the fluids correspond with the values for water. The thermophysical parameters of the walls have values corresponding to typical materials used for construction of heat exchangers: copper alloy C24000, stainless steel AISI 410 and titanium alloy Ti6Al4V. The thermal properties of these constructional materials are presented in Table 1.

In Figures 4-7 the transient response of the sample heat exchanger is presented for the case of a step-like twofold increase of fluid inlet velocity in channel no. 1. In our calculations we took into account the increase of heat transfer coefficient in channel no.1 resulting from the increase in the fluid velocity. Figure 4 shows the dimensionless exit temperatures of fluids,  $\Theta_i = [T_i(L, t) - T_{min}] / (T_{max} - T_{min})$ , for  $i = 1, 2, 3$ , versus dimensionless time  $\tau = t / t_{max}$ .  $T_{min}$  and  $T_{max}$  are the minimum and maximum temperatures in the exchanger, respectively,  $t_{max} = 10 s$  is the observation time. Time transients of the dimensionless average walls temperatures at the exchanger outlet,  $\Theta_{wi} = [T_{wi}(L, t) - T_{min}] / (T_{max} - T_{min})$ , are demonstrated in Fig. 5. The average wall temperature is calculated as  $T_{wi} = 0.5(T_{wi}^a + T_{wi}^b)$ . It is seen in Figs. 4 and 5 that as the time elapses, the fluids temperatures at the outlet and the walls temperatures at the outlet cross-section approach the steady-state values. It appears from Figs. 4 and 5 that

the durations of transients depend on the wall material. The exchanger reaches the steady state in about 8 - 10 s.

Figure 6 presents unsteady fluids temperature profiles,  $T_i(x, t)$ , for  $i = 1, 2, 3$ , for time  $t = 0.75$  s. Figure 7 presents unsteady average walls temperature profiles,  $T_{wi}(x, t)$ , for  $i = 1, 2, 3$ , for the same time  $t = 0.75$  s. The results of research into the impact of walls on the steady-state temperature field (unsteady temperature field for observation time  $t \rightarrow \infty$ ) in the analysed heat exchanger are shown in Fig. 8. The calculated temperature profiles are compared with those determined without taking into account the heat capacity of walls and heat conduction along and across the walls (solid lines). It is found that for the values of thermal diffusivity,  $a = \lambda_w / (\rho_w c_w)$ , characteristic for good heat conductors, such as copper alloy C24000 and stainless steel AISI 410, the impact of walls on the temperature field in the heat exchanger is negligibly small. The lower the value of thermal diffusivity (e.g. for titanium alloy Ti6Al4V), the higher the impact of walls on the temperature field in the heat exchanger.

## 6. Conclusions

Transient and steady state temperature fields have been determined, by the MacCormack implicit method, in a parallel-flow three-fluid heat exchanger with regard to the transverse resistance and thermal capacitance of the walls, as well as the longitudinal walls conduction. Impact of the walls material on the transient temperature profiles has been examined. This impact is not very large. The walls influence the fluids temperatures in three ways: (1) they resist heat conduction in the transverse direction, (2) they accumulate heat during transient process, (3) they conduct heat longitudinally. Good thermal conductivity of the walls increases the influence of longitudinal heat



conduction and at the same time it decreases the effect of resistance in transverse direction. An increase in specific heat always enlarges the effect of the walls on the transient temperature field in the exchanger. Low thermal conductivity and high specific heat result in low thermal diffusivity. It has been observed that the impact of walls on the temperature of fluids flowing in the channels of the exchanger is the higher, the lower is the value of thermal diffusivity of walls. Obviously, this impact increases with the increase of the wall thickness.

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**References**

- [1] D.D. Aulds, R.F. Barron, Three-fluid heat exchanger effectiveness, *International Journal of Heat and Mass Transfer* 10 (1967) 1457 – 1462.
- [2] J.C. Chato, R.J. Laverman, J.M. Shah, Analyses of parallel flow, multi-stream heat exchangers, *International Journal of Heat and Mass Transfer* 14 (1971) 1691 – 1703.
- [3] D.P. Sekulic, R.K. Shah, Thermal design theory of three-fluid heat exchangers, in: J.P. Harnett, T.F. Irvine (eds.), *Advances in Heat Transfer*, Vol. 26, Academic Press, New York, 1995.
- [4] D. Shrivastava, T.A. Ameel, Three-fluid heat exchangers with three thermal communications. Part A: general mathematical model, *International Journal of Heat and Mass Transfer* 47 (2004) 3855 – 3865.
- [5] D. Shrivastava, T.A. Ameel, Three-fluid heat exchangers with three thermal communications. Part B: effectiveness evaluation, *International Journal of Heat and Mass Transfer* 47 (2004) 3867 – 3875.
- [6] X. Luo, M. Li, W. Roetzel, A general solution for one-dimensional multistream heat exchangers and their networks, *International Journal of Heat and Mass Transfer* 45 (2002) 2695 – 2705.
- [7] L. Malinowski, Application of nonlinear programming methods for calculation of temperature profiles in parallel-flow multi-channel heat exchangers, *International Communications in Heat and Mass Transfer* 28 (2001) 59 – 66.
- [8] D.P. Sekulic, C.V. Herman, Transient temperature fields in a three fluid heat exchanger, *Proceedings of the XVIIth International Congress of Refrigeration B*, IIF, Vienna, Austria 1987, 833 - 837.

- [9] D.P. Sekulic, M. Dzolev, I. Kmecko, Dynamic behaviour of a three fluid heat exchanger: The experimental study, in: J.F. Keffer, R.K. Shah, E.N. Ganic (eds.), *Experimental Heat Transfer, Fluid Mechanics, and Thermodynamics*, Elsevier, New York, 1991, 1338 – 1343.
- [10] S. Bielski, L. Malinowski, A semi-analytical method for determining unsteady temperature field in a parallel-flow three-fluid heat exchanger, *International Communications in Heat and Mass Transfer* 30 (2003) 1071 – 1080.
- [11] S. Bielski, L. Malinowski, An analytical method for determining transient temperature field in a parallel-flow three-fluid heat exchanger, *International Communications in Heat and Mass Transfer* 32 (2005) 1034 – 1044.
- [12] X. Luo, X. Guan, M. Li, W. Roetzel, Dynamic behaviour of one-dimensional flow multistream heat exchangers and their networks, *International Journal of Heat and Mass Transfer* 46 (2003) 705 – 715.
- [13] L. Malinowski, Equations for transient behaviour of parallel-flow multichannel heat exchangers, *Heat and Mass Transfer* 39 (2003) 321 – 325.
- [14] M.N. Roppo, E.N. Ganic, Time-dependent heat exchanger modelling, *Heat Transfer Engineering* 4 (1983) 42 – 46.
- [15] D.J. Correa, J.L. Marchetti, Dynamic simulation of shell-and-tube heat exchangers, *Heat Transfer Engineering* 8 (1987) 50 – 59.
- [16] C.C. Lakshmanan, O.E. Potter, Dynamic simulation of plate heat exchangers, *International Journal of Heat and Mass Transfer* 33 (1990) 995 – 1002.
- [17] W. Roetzel, Y. Xuan, Transient behaviour of multipass shell-and-tube heat exchangers, *International Journal of Heat and Mass Transfer* 35 (1992) 703 – 710.

- [18] W. Roetzel, Y. Xuan, Analysis of transient behaviour of multipass shell and tube heat exchangers with the dispersion model, *International Journal of Heat and Mass Transfer* 35 (1992) 2953 – 2962.
- [19] S.K. Das, W. Roetzel, Dynamic analysis of plate heat exchangers with dispersion in both fluids, *International Journal of Heat and Mass Transfer* 38 (1995) 1127 – 1140.
- [20] S.K. Das, B. Spang, W. Roetzel, Dynamic behaviour of plate heat exchangers – experiments and modeling, *Transactions of the ASME. Journal of Heat Transfer* 117 (1995) 859 – 864.
- [21] W. Roetzel, S.K. Das, Hyperbolic axial dispersion model: concept and its application to a plate heat exchanger, *International Journal of Heat and Mass Transfer* 38 (1995) 3065 – 3076.
- [22] R.W. MacCormack, A numerical method for solving the equations of compressible viscous flow, *AIAA Journal* 20 (1982) 1275 – 1281.
- [23] D.A. Anderson, J.C. Tannehill, R.H. Pletcher, *Computational fluid mechanics and heat transfer*, Hemisphere Publishing Corporation, Washington, 1984.
- [24] C.A.J. Fletcher, *Computational techniques for fluid dynamics*, Springer – Verlag, New York, 1988.

**Figure captions**

Fig. 1

Schematic representation of the three-fluid heat exchanger under consideration.

Fig. 2

Cross section of the three-fluid heat exchanger.

Fig. 3

Comparison of transient temperature profiles calculated by the numerical and semi-analytical methods for  $t = 1.25 \text{ s}$ .

Fig. 4

Dimensionless outlet temperatures of fluids versus time in the sample three-fluid heat exchanger.  $\Theta_i = [T_i(L, t) - T_{\min}] / (T_{\max} - T_{\min})$ ,  $\tau = t / t_{\max}$ ,  $X = x / L$ .

Fig. 5

Dimensionless average temperatures of walls at the exchanger outlet versus time.

$$\Theta_{wi} = [T_{wi}(L, t) - T_{\min}] / (T_{\max} - T_{\min}), \tau = t / t_{\max}, X = x / L.$$

Fig. 6

Transient temperature profiles of fluids for the sample three-fluid heat exchanger for  $t = 0.75 \text{ s}$ .

Fig. 7

Transient average temperature of walls,  $T_{wi} = 0.5(T_{wi}^a + T_{wi}^b)$ , versus co-ordinate along the exchanger for the sample three-fluid heat exchanger for  $t = 0.75 \text{ s}$ .

Fig. 8

Steady-state temperature profiles of fluids for the sample three-fluid heat exchanger (transient model for  $t \rightarrow \infty$ ). The solid lines represent the case where the heat capacity of the walls and heat conduction along and across the walls are not taken into account.

Table 1

Selected thermophysical properties of typical materials used for construction of heat exchangers

Material	$\rho$ [ $kg/m^3$ ]	$c_w$ [ $J/(kgK)$ ]	$\lambda_w$ [ $W/(mK)$ ]	$a \cdot 10^6$ [ $m^2/s$ ]
Copper alloy C24000	8530	380	111	34.20
Stainless steel AISI 410	7770	460	25	7.00
Titanium alloy Ti6Al4V	4420	610	5.80	2.15

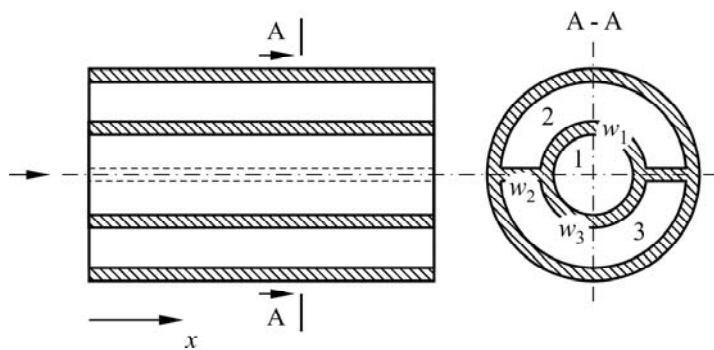


FIG. 1

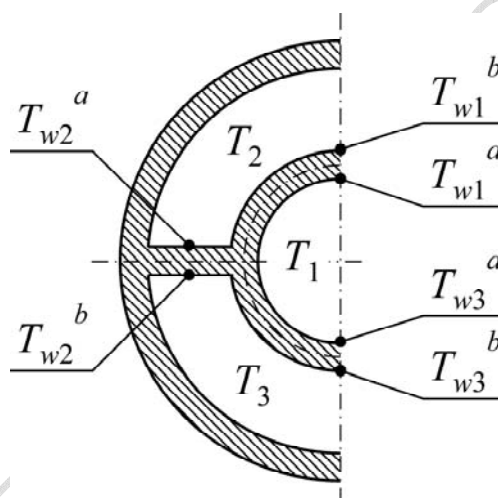


FIG. 2

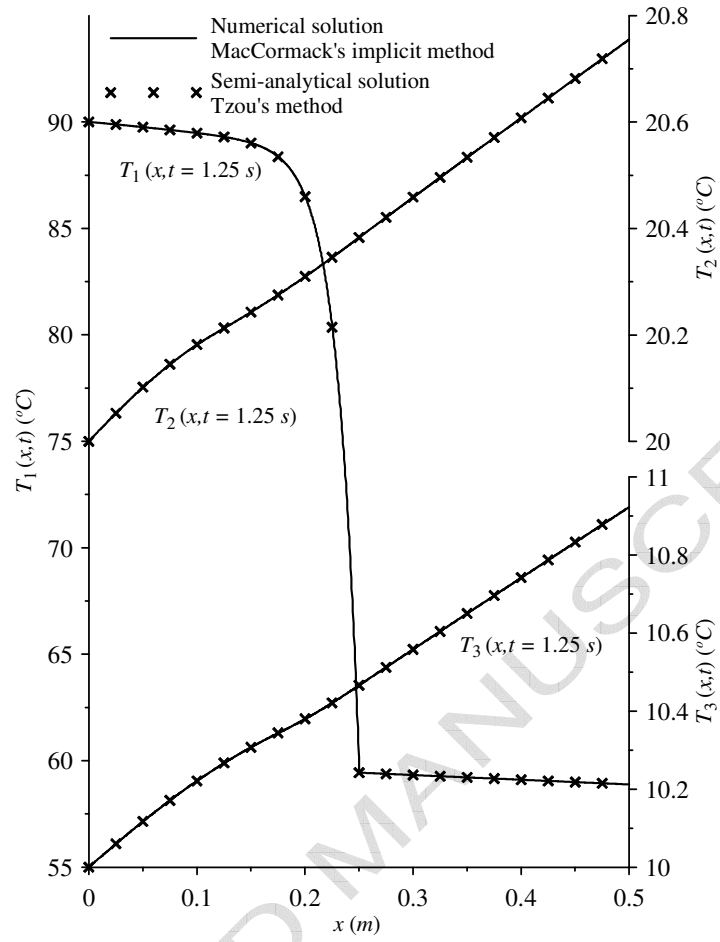


FIG. 3



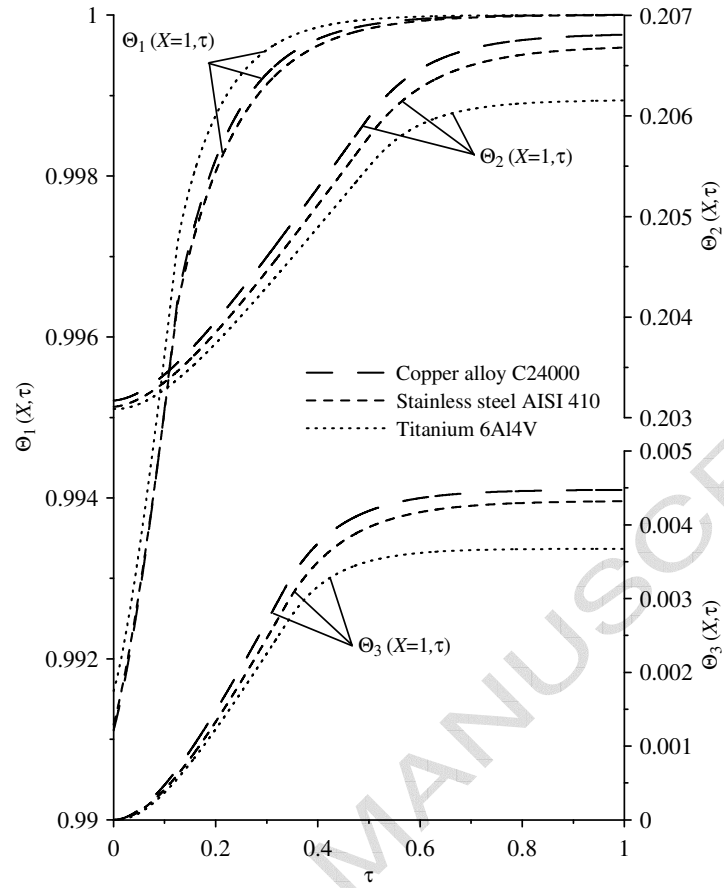


FIG. 4

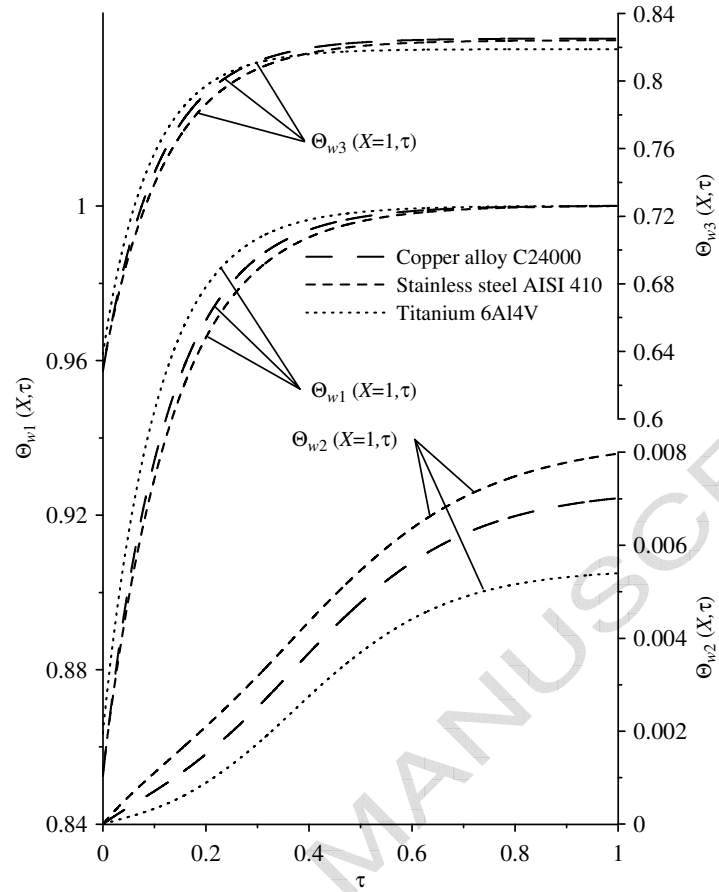


FIG. 5

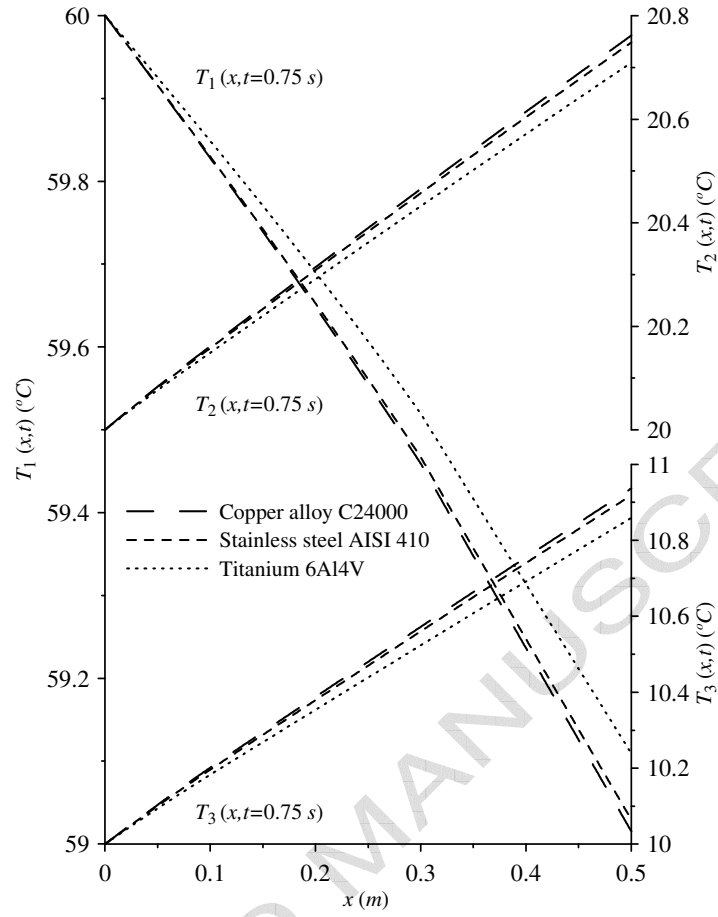


FIG. 6

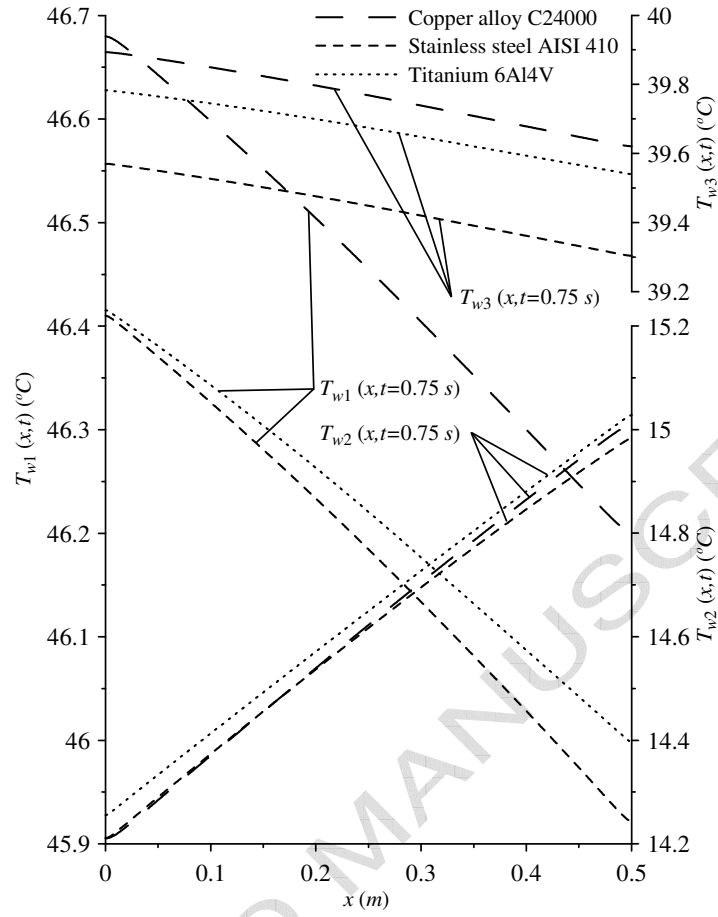


FIG. 7

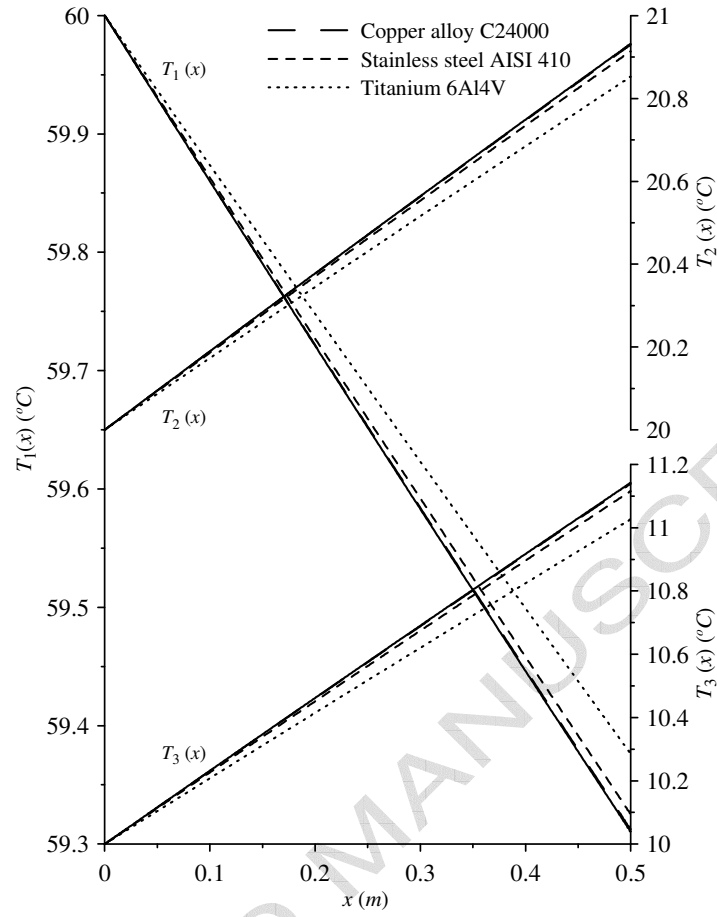


FIG. 8