

**TOWARDS A UNIFICATION OF SOME
LINGUISTIC REPRESENTATION MODELS:
A VECTORIAL APPROACH**

ISIS TRUCK

*LIASD – EA 4383, Université Paris 8,
2 rue de la Liberté, Saint-Denis, F-93526 Cedex, France
E-mail: truck@ai.univ-paris8.fr
www.univ-paris8.fr*

JACQUES MALENFANT

*LIP6 – UMR 7606, Université Pierre et Marie Curie-Paris 6,
104 av. du Président Kennedy, Paris, F-75016, France
E-mail: Jacques.Malenfant@lip6.fr
www.upmc.fr*

This paper takes place in the computing with words framework where a unified model of several linguistic representation models is proposed. We consider (i) the *2-tuple fuzzy linguistic representation model*, (ii) the *proportional 2-tuple fuzzy linguistic representation model*, (iii) the *linguistic degrees* and their associated *generalized symbolic modifiers*. Our approach is based on a vectorial representation. Using vectors and scalar multiplications, we translate and then unify these three models into a single one. In this paper, the scope of our research is limited to the definitions of a linguistic value, the attached aggregation operators remaining the next logical step.

Keywords: Linguistic representation models; Unification; Vectorial approach.

1. Introduction

An important part of artificial intelligence research has been and is still dedicated to qualitative information and its representation. Zadeh has introduced the computing with words (CW) paradigm¹ where several linguistic representation models take place.²⁻⁴ A key issue in CW is to relate all the operations and the intermediate results to the original set of linguistic terms defined by the user, as a way to convey an understandable semantics to these results in the terms used by the humans. Since Herrera and Martínez have proposed their 2-tuple models for CW without loss of pre-

cision,² other models have emerged towards the same goal. The key idea behind all of these models is to maintain a representation of fuzzy subsets¹ that relates to the original linguistic terms while retaining full precision over intermediate results during all the operations of CW. Although different, all of these models²⁻⁴ appear to share a common conceptual basis. In this paper, we propose that linear algebra, and its concepts of basis, can capture the relationships between these models and provide for means not only to relate the models to each other but also allows CW to use the simplest possible model for its computation, without loss of precision, while being able to come back to any of these models as soon as human-related interventions require results to be expressed in terms of the original linguistic term set. Models could then be used interchangeably, depending on the properties of the linguistic terms set that may simplify the expression using one 2-tuple model rather than another.

2. Conceptual Framework and Related Work

2.1. Linguistic Representation Models for CW

To avoid lack of precision and linguistic approximation, Herrera & Martínez consider an ordered linguistic term set $\{s_0, \dots, s_g\}$ and a symbolic translation α attached to each term that will support the difference between the term and the value to express. They thus obtain 2-tuples noted (s_i, α) , $\alpha \in [-.5, .5)$. Wang & Hao's have proposed proportional 2-tuples where the lack of precision is supported by a proportion $\alpha \in [0, 1]$ attached to the linguistic term l_i : they define a proportional 2-tuple as: $(\alpha l_i, (1 - \alpha)l_{i+1})$. It is to notice that these models imply the use of a continuous domain. Truck & Akdag, as for them, have considered the case where the domain is discrete. To avoid the loss of information they change the scale granularity in adding or subtracting terms in the original term set thanks to what they call *generalized symbolic modifiers* (GSM). The model they propose is quite simple: the linguistic term set is represented by a scale of b ordered symbolic values. A symbolic value is noted by a and is specified by its position in the scale: $p(a)$, with $p(a) \in \mathbb{N}$.

To combine such linguistic terms,⁵ the authors propose GSMs as mappings from an initial 2-tuple (a, b) to a new 2-tuple (a', b') :

Definition 1. Let \mathcal{L}_b be a set of b linguistic terms, with $b \in \mathbb{N}^* \setminus \{1\}$. A GSM m_ρ is defined as:
$$m_\rho : \mathcal{L}_b \rightarrow \mathcal{L}_{b'} \quad \begin{array}{l} \text{with } p(a) < b, \quad p(a') < b' \text{ and } \rho \in \mathbb{N}^*. \\ a \mapsto a' \end{array}$$

For example the DW GSM is defined as $DW(\rho): p(a') = p(a)$, $b' = b + \rho$

2.2. Basic Linear Algebra Concepts

The cornerstone of our proposal is based on the idea that each 2-tuple model defines a vector space, and strives to express all values in this space using a set of independent vectors, chosen to relate to the original linguistic terms set. To be more precise, key definitions from linear algebra are summarized:

Definition 2 (Basis). *For a given vector space V , a basis is a (finite or infinite) set $B = \{\vec{v}_i \mid i \in I\}$ of vectors \vec{v}_i indexed by some index set I that spans the whole vector space, and is minimal with this property.*

Given a finite basis, any vector \vec{v} can be expressed as a linear combination of the basis elements: $\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 + \dots + a_{\#I}\vec{v}_{\#I}$

Basis are unfortunately a too strong concept to deal with 2-tuple models. The need for 2-tuple models to retain a link with the original linguistic terms set, comes at odd with the minimality required for a basis. We therefore define the concept of constrained basis as follows:

Definition 3 (Constrained basis). *For a given vector space V , a constrained basis is a set $B = \{\vec{v}_i \mid i \in I\}$ of vectors \vec{v}_i indexed by some index set I iff for all $\vec{v} \in V$, there exists $a_1, \dots, a_{\#I}$ such that $\vec{v} = a_1\vec{v}_1 + a_2\vec{v}_2 + \dots + a_{\#I}\vec{v}_{\#I}$, and where the a_i are subject to constraints such that $a_i \in A_i \subset \mathbb{R}$.*

The key concept is that a constrained basis may require more vectors to span the whole vector space, given the constraints on the scalars that can be used to combine them linearly. Hence, they are no longer minimal.

3. Proposition

We now show how all three models are actually constrained bases, such as the one of Figure 1.

Conjecture 3.1. *Herrera & Martínez 2-tuple model forms a constrained basis for the space $[0, g]$, where the vectors $\vec{v}_0, \vec{v}_1, \dots, \vec{v}_g$ represent the values $\{0, 1, \dots, g\}$ respectively, where the vector \vec{u} represents a unit vector such that $\vec{v}_i + \vec{u} = \vec{v}_{i+1}, i \in \{0, g-1\}$, and where all values can be expressed as a linear combination: $\vec{v}_i + \alpha_i \vec{u}$ subject to the constraints $\alpha_0 \in [0, .5), \alpha_i \in [-.5, .5), i \in \{1, g-1\}$ and $\alpha_g \in [-.5, 0]$.*

Conjecture 3.2. *The set (\vec{v}_0, \vec{u}) also forms a constrained basis for the space $[0, g]$, where all values can be expressed as a linear combination: $\vec{v}_0 + \alpha \vec{u}$ subject to the constraints $\alpha \in [0, g]$.*

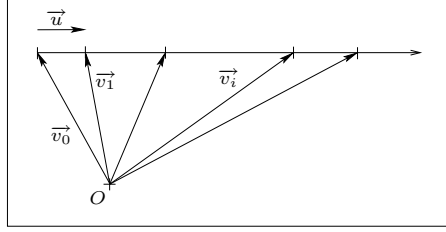


Fig. 1. Vectorial plane.

The interest of this latter basis is that it makes computations very simple. As all values are expressed with the same base vector \vec{v}_0 , any aggregator can simply be computed over the α of each unit vector.

Conjecture 3.3. *Wang & Hao 2-tuple model forms a constrained basis for the space $[0, g]$, where the vectors $\vec{v}_0, \vec{v}_1, \dots, \vec{v}_g$ represent the values $\{0, 1, \dots, g\}$ respectively, and where all values can be expressed as a linear combination:*

$$\alpha \vec{v}_i + \beta \vec{v}_{i+1}, i \in \{0, \dots, g\}$$

subject to the constraints $\alpha, \beta \geq 0$ and $\alpha + \beta = 1$.

Wang & Hao show that there are always two ways to write a value x : either through a term and its predecessor or through the same term and its successor: $(\alpha l_i, (1 - \alpha) l_{i+1}) = (1 - \alpha l_{i-1}, \alpha l_i)$. This equality is easily provable in our model thanks to the parallelogram property: in our formalism, $(\alpha l_i, (1 - \alpha) l_{i+1})$ corresponds to $\alpha \vec{v}_i + \beta \vec{v}_{i+1}$ and $(1 - \alpha l_{i-1}, \alpha l_i)$ to $\alpha \vec{v}_{i-1} + \beta \vec{v}_i = \beta \vec{v}_i + \alpha \vec{v}_{i-1}$. Consider that \vec{v}_i has point O as origin, these four vectors form a parallelogram whose opposite points are O and x .

Conjecture 3.4. *Truck & Akdag model without the recourse of a GSM forms a constrained basis for the space $[0, b - 1]$, where the vectors $\vec{v}_0, \vec{v}_1, \dots, \vec{v}_{b-1}$ represent the values $\{0, 1, \dots, b - 1\}$ respectively, and where all values can be expressed as a linear combination:*

$$\vec{v}_i + \beta \vec{u}, i \in \{0, \dots, b - 1\}$$

subject to the constraints $i = p(a), \beta = 0$.

Applying a GSM permits to obtain a pair (a', b') from a pair (a, b) .

Conjecture 3.5. *Truck & Akdag model with the recourse of a GSM $m(\rho)$ forms a constrained basis for the space $[0, b' - 1]$, where the vectors*

$\vec{v}_0, \vec{v}_1, \dots, \vec{v}_{b'-1}$ represent the values $\{0, 1, \dots, b'-1\}$ respectively, and where all values can be expressed as a linear combination:

$$\vec{v}_{i'} + \beta' \vec{u}, i' \in \{0, \dots, b'-1\}$$

subject to the constraints $i' = i = p(a), \beta' = (p(a')(b-1)/(b'-1)) - p(a)$.

NB: \vec{v}_i and \vec{u} are unchanged because we intend to express the *modified* pair (a', b') in the same vectorial notation, changing only β .

However, the values obtained for $\vec{v}_{i'}$ and β' don't permit to write the canonical form of the vectorial notation. Indeed β' is bounded by $(1-b)$ and by $(b-1)$ (knowing that $0 \leq p(a') \leq (b'-1)$ and $(1-b) \leq -p(a) \leq 0$) and for the canonical form it is required that $\beta' \in [-.5, .5]$ as β' is the factor of the unit vector.

Let us take an example. When transforming (a, b) into (a', b') using DW(10) with $p(a) = 3$ and $b = 5$, we obtain $p(a') = p(a) = 3$ and $b' = b + 10 = 15$. Thus $\beta' = (3 * 4/14) - 3 = -2.143$ and the vectorial form is $\vec{v}_3 - 2.143 \vec{u}$. Adding (subtracting in this case) to \vec{v}_3 a vector greater than the unit vector is equivalent to increment (resp. decrement) the subscript of \vec{v} . Hence $\vec{v}_3 - 2 \vec{u}$ is equivalent to exactly \vec{v}_1 .

Conjecture 3.6. We denote by $\widehat{v}_{i'} + \widehat{\beta}' \vec{u}$ the canonical vectorial form of (a', b') and $[x]$ is the integer part of x .

if $\beta' < 0$

then if $\beta' - [\beta'] < -.5$

then $\widehat{\beta}' = 1 - \beta' - [\beta']$; $\widehat{v}_{i'} = \vec{v}_{i'+[\beta']-1}$

else $\widehat{\beta}' = \beta' - [\beta']$; $\widehat{v}_{i'} = \vec{v}_{i'+[\beta']}$

else if $\beta' - [\beta'] \geq .5$

then $\widehat{\beta}' = -1 + \beta' - [\beta']$; $\widehat{v}_{i'} = \vec{v}_{i'+[\beta']+1}$

else $\widehat{\beta}' = \beta' - [\beta']$; $\widehat{v}_{i'} = \vec{v}_{i'+[\beta']}$

This implies that $\widehat{\beta}' \in [-.5, .5]$ as required.

In our example, we obtain $\widehat{\beta}' = -.143$ and $\widehat{v}_{i'} = \vec{v}_1$. The canonical form is thus $\vec{v}_1 - .143 \vec{u}$.

Figure 2 sums up our proposition for all three 2-tuple models: Herrera & Martínez, Wang & Hao and Truck & Akdag respectively.

4. Conclusion

In this paper we have shown how different linguistic representation models (even over discrete *and* continuous domains) can be unified within a

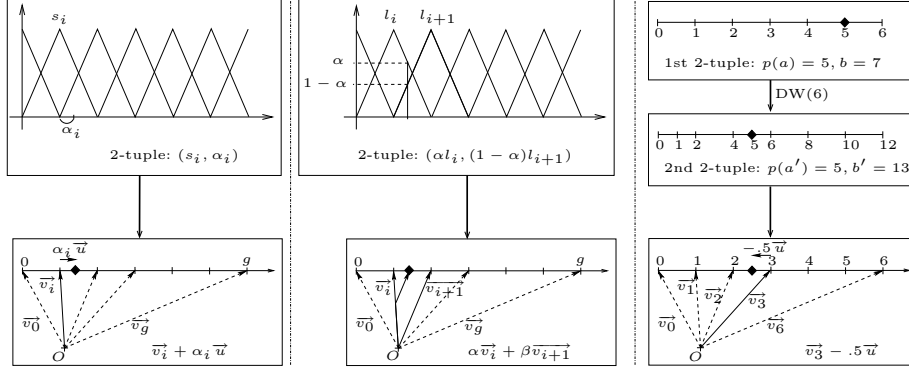


Fig. 2. Vectorial representation of the three considered cases.

single formalism. Fuzzy, proportional and discrete 2-tuples are considered as bases from a vectorial space. Generally speaking, all three models have their values that can be expressed as follows: $\alpha \vec{v}_i + \beta \vec{v}_j$ with constraints over i, j, α and β . This approach is advantageous because it is very simple to switch from one model to another, depending on which is preferred. Moreover, and this is future works, for each model it will be of great interest to reconsider the aggregation operators under our “vectorial vision”. This will allow us to change the model *even during the calculation process* to match the computational requirements of the operator with the best-suited model.

References

1. L. Zadeh, Fuzzy logic = computing with words, *IEEE Trans. Fuzzy Systems* **4**, 103 (1996).
2. F. Herrera and L. Martínez, A 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Trans. Fuzzy Systems* **8**, 746 (2000).
3. J. Wang and J. Hao, A new version of 2-tuple fuzzy linguistic representation model for computing with words, *IEEE Trans. Fuzzy Systems* **14**, 435 (2006).
4. I. Truck and H. Akdag, Manipulation of Qualitative Degrees to Handle Uncertainty : Formal Methods and Applications, *Knowledge and Information Systems (KAIS)* **9**, 385 (2006).
5. I. Truck and H. Akdag, A tool for aggregation with words, *Information Sciences* **179**, 2317 (2009).