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Information provision in financial markets

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Abstract

We set up a rational expectations model in which investors trade a risky asset based on a private signal they receive about the quality of the asset, and a public signal that represents a noisy aggregation of the private signals of all investors. Our model allows us to examine what happens to market performance (market depth, price efficiency, volume of trade, and expected welfare) when regulators can induce improved information provision in one of two ways. Regulations can be designed that either provide investors with more accurate prior information by improving the quality of prior information, or that enhance the transparency of the market by improving the quality of the public signal. In our rational expectations equilibrium, improving the quality of the public signal can be interpreted as a way of providing information about the anticipations and trading motives of all market participants. We find that both alternatives improve market depth. However, in the limit, we show that improving the precision of prior information is a more efficient way to do so. More accurate prior information decreases asymmetric information problems and consequently reduces the informativeness of prices, while a more accurate public signal increases price informativeness. The volume of trade is independent of the quality of prior information and is increasing in the quality of the public signal. Finally, expected welfare can sometimes fall as prior information or the public signal become more precise.

1 Introduction

In recent years, regulations have been implemented in financial markets throughout the world designed to induce improved information provision in order to reduce existing uncertainty about the fundamentals of publicly traded firms. There are two alternative means available to regulators trying to stimulate better information provision. The first involves tightening the standards of information dissemination of publicly traded firms. This is achieved by improving the precision of the prior information available to all investors about asset value. The Sarbanes-Oxley Act of 2002 is an example of a regulation designed to provide investors with more accurate accounting information.

The second way to improve information provision is to enhance the transparency of the market. The main argument for this alternative is that uncertainty is more about the actions of competing investors than about fundamentals, and so enhancing transparency can be achieved by providing all market participants better access to information about the information held by their competitors. This can involve giving access to more information about the order book, order imbalances, or about all the transactions that have taken place in the market. For example, in April of 1990 the Toronto Stock Exchange (TSE) established a computerized system, Market by Price, to disseminate real-time detailed information on the limit order book to the public. In particular it provided information on the depth and quotes for the current inside market and also depth and limit order prices for up to four price levels above and below the current market, and it automatically displayed all depth (see Madhavan et al., 2005 for further details). The London Stock Exchange has experimented with different regimes for the publication of last-traded prices (see Gemmill, 1996).

The objective of this paper is to determine the ways in which these two means of improving information provision affect market performance. We set up a rational expectations model in which investors trade a risky asset based on a private signal they receive about the quality of the asset, and a public signal that represents a noisy aggregation of the private signals of all investors. Our model allows us to examine what happens to market performance when regulators can decrease uncertainty about fundamentals by improving information provision in one of the two ways described above. In our setup, regulators can provide investors with more accurate prior information about the asset's value by improving the precision of all private signals, and/or can enhance the transparency of the

market by improving the quality of the public signal. In our model improving the quality of the public signal can be interpreted as a way of providing information about the information of others. Moreover, because expectations are rational in our set up, pre-trade information, for instance from the order book or order imbalances, contains no additional information than would be revealed in equilibrium. We therefore focus on the provision of post-trade information about past transactions such as last-traded prices.

We consider the effect of these alternatives on different measures of market performance. We focus on market depth, price efficiency, volume of trade, and ex-ante expected welfare. Improving the precision of prior information or increasing the level of transparency in the market always increases market depth. However, the effect of increasing transparency is less important than the effect of increasing the precision of prior information. In the limit, while very precise prior information results in an infinitely deep market, increasing market transparency results in a level of market depth that is bounded from above. The reason is that the informational content of each of the alternatives is different. Improving information provision by increasing the precision of prior information directly affects the trading strategies of agents, while enhancing market transparency does so only indirectly by affecting the precision of their information about their competitors signals and the information they infer from the equilibrium price.

Turning to the effect of improved information provision on price efficiency we find that price efficiency is decreasing in improvements in the accuracy of prior information. More accurate prior information reduces price efficiency since the availability of higher quality information implies that the information transmitted by prices is less important. The opposite effect occurs when the market is made more transparent. The provision of the public signal compensates for the reduction of information asymmetries.

Next we examine the effect of improving the precision of prior information or increasing the level of transparency on the volume of trade that takes place in equilibrium. We find that the volume of trade is independent of the quality of prior information and is increasing in the quality of the public signal.

Finally, to study the effect on expected welfare we turn to numerical analysis since analytical

analysis is intractable. Our simulations show that improving information provision is not always welfare increasing. In particular, we show that starting with very imprecise prior information and/or a very opaque market, improving the precision of prior information, and/or enhancing the transparency of the market will cause expected welfare to fall up to a point, after which expected welfare will increase. The reason is that in our framework expectations are rational and investors are imperfectly competitive and so improving information can eliminate potential informational rents.

We are not the first to study these issues. In the market microstructure literature, researchers have examined the impact of increasing financial market transparency on market performance. Theoretical (Pagano and Röell 1996, Madhavan 1995 and 1996, and Baruch 2005), empirical (Gemmill 1996, Porter and Weaver 1998, and Madhavan et al. 2005) and experimental (Bloomfield and O'Hara 1999, and Flood et al. 1999) studies reached mixed conclusions about the impact of transparency on financial markets. In all of these studies transparency is defined as the information that the market or some participants should disclose in order to increase the information shared between agents. This transparency can be either pre-trade (for instance information about the book) or post-trade (for instance information on past trades). All of this information will affect the trading behavior of agents because it allows information about the trading strategies of competitors to be shared. We are not interested in modeling transparency, but simply in determining its ultimate impact on market performance. Therefore, in our model, we capture the impact of transparency by simply allowing for the existence of a public signal that is a noisy aggregation of the signals held by all the players in the market and which, given our rational expectations equilibrium, provides information about the anticipations of all market participants. In contrast with the market microstructure literature on market transparency, since the precision of the public signal can be varied continuously, our model allows us to study the marginal impact of decreasing uncertainty by increasing transparency.

There have also been a number of papers that study the impact of reducing uncertainty directly by announcing a public signal that is independent of the private signals held by investors. Morris and Shin (2002) show that public information may be harmful for expected social welfare in a beauty-contest set up where the payoff of an agent decreases in the distance between his action and the actions of the others. The main intuition for their result is that in the beauty-contest set up, public information

helps investors achieve coordination, but greater coordination between agents is assumed to be socially irrelevant.¹ Clark and Polborn (2006) focus on situations where individuals care about making the correct choice, but also about the fraction of the population that make the same choice that they do and in which the preferred level of coordination lies between 1/2 and the whole population. They focus on two types of pure strategy equilibria—one in which agents ignore their private information in favor of the public information, and one in which every individual chooses the action that they believe is most likely to be correct. Like Morris and Shin they find that better public information need not increase players' expected utility, but, to the contrary, may even decrease it. This can occur when in the initial situation both types of equilibria exist, and the one with the higher expected utility is the one in which people heed their private information; in this situation, increases in public signal quality can eliminate this equilibrium type, and the remaining equilibrium in which all players follow the public signal may yield a lower expected utility for all players. The same is true for the private signal, as it may cause overcrowding in situations where agents choose the action they believe is most likely to be correct.

There are two important differences between our model and Morris and Shin (2002) and Clark and Polborn (2006). First, there is no direct coordination effect in our model. As a result, the positive effect of decreasing uncertainty would be related to a decrease in the adverse selection problem which would affect their trading behavior. Second, we consider a rational expectations equilibrium set up in which a small number of investors engage in imperfect competition. We model the strategic behavior of investors and explore the way it is affected by the existence of a more valuable information.

The rest of the paper proceeds as follows. In the next section we describe the model. Section 3 characterizes the equilibrium. In Section 4 we analyse the equilibrium effects of improving information provision on market performance. We conclude in Section 5. All proofs are in the Appendix.

¹Svensson (2006) shows that this result holds only for special cases and that when the parameters of the Morris and Shin (2002) model take reasonable values the condition for public information to be welfare decreasing is violated. Other papers have challenged Morris and Shin's result by showing that the increase in public information announcements would increase the expected social welfare if sent only to a proportion of agents (Cornand and Heinemann, 2008), in economies with investment complementarity, or in economies featuring monopolistic competition among heterogeneously informed firms (Angeletos and Pavan, 2004 and 2007).

2 The model

Consider a market where Q units of a risky asset are traded. There are n investors who participate in this market. The value of the risky asset is denoted by θ and is unobserved by the participants. However, each investor has some information about θ . Based on the information available to them, each submits a demand function and in equilibrium total demand will equal the quantity supplied, Q .

As in Kyle (1989), we consider a rational expectations equilibrium with imperfect competition among agents in which investors submit downward sloping demand curves. The main difference between our model and Kyle's is that, in our model, investors not only observe a private signal about the value of θ , but also observe a common public signal, which is itself an imperfect signal of the average of individual private signals.

2.1 Information structure

We assume that prior to trading each investor i receives two imperfect signals about the value of the risky asset, θ . The first signal is private for investor i , and is denoted by s_i . The second signal, denoted by S , is a public signal observed by all investors. Both signals are noisy and all random variables are assumed to be normally distributed. So, we suppose that

$$s_i = \theta + t_i \quad \text{where} \quad \theta \sim N(\bar{\theta}, \sigma_\theta^2) \quad \text{and} \quad t_i \sim N(0, \sigma_t^2), \quad (1)$$

and

$$S = \sum_i \frac{s_i}{n} + \mu = \theta + \sum_j \frac{t_j}{n} + \mu \quad \text{where} \quad \mu \sim N(0, \sigma_\mu^2). \quad (2)$$

From these expressions we can see the two means available to regulators for reducing uncertainty about the value of a financial asset.² One option is for standards on information dissemination of publicly traded assets to be tightened. In our setup this is captured by reducing σ_θ^2 . The second alternative is to enhance the transparency of financial markets by giving access to more information about the average private signal in the market. This can be done by providing information on past transactions in the market. In our model this is captured by a decrease in σ_μ^2 .³

²Note that we do not focus on a trade-off between these two types of signals, but rather on their respective impacts on market performance. See Morris and Shin (2002), Clark and Polborn (2006) for models in which agents weigh the relative benefits of the two types of information.

³Modeling transparency in this way allows us to study the marginal impact of decreasing uncertainty. In the market-microstructure literature transparency is always defined as the information that the market or some participants should

Since S is a noisy signal about the average of all private signals received by investors, if $\sigma_\mu^2 \rightarrow 0$, it is as if participants can observe the average of the signals received by all investors. This would correspond to the case where investors have access to all of the existing valuable information. Alternatively, if $\sigma_\mu^2 \rightarrow \infty$, we have $E(\theta|S, s_i) = E(\theta|s_i)$, so that investors can infer no additional information about the final value of the asset from observing S . By changing the value of σ_μ^2 we will explore the impact of information aggregation and market transparency on the market equilibrium and the trading strategies of agents.

In practice, even complete access to information about past trades may not provide investors with all existing information. This is because information on past trades is a compound of private signals and idiosyncratic liquidity shocks. Investors can be motivated to trade either to capitalize on valuable trading opportunities they perceive given their fundamental information or for liquidity needs. As a result it is probably most appropriate to think of μ as the sum of two variables: one that captures information-based trading and one that captures liquidity-based trading. We are of course interested in the incentive to trade that is marginal relative to the liquidity-based motivation.

2.2 Agents

We suppose that agents have CARA preferences with r denoting their risk aversion coefficient. Given the assumption of normally distributed random variables, this implies that preferences can be denoted by the mean-variance representation. An agent who trades a quantity x_i has expected utility of:

$$E_\theta [W + \theta(w_i + x_i + \epsilon_i) - px_i|I] - \frac{r}{2} \text{Var} [\theta|I] (w_i + x_i + \epsilon_i)^2, \quad (3)$$

where W is some initial wealth, w_i is agent i 's initial endowment of the stock, and ϵ_i corresponds to an idiosyncratic liquidity shock that the agent receives before participating in the market. I denotes all information available to the investor. The initial endowment w_i allows us to model the supply of shares in the market. We assume that $\sum_i w_i = Q$. We assume that ϵ_i is private information for the agent and that it follows distribution $N(0, \sigma_\epsilon^2)$. The introduction of ϵ_i adds a second motivation for trading as in Glosten (1989). Without these shocks, the rational expectations equilibrium would be fully revealing and so traders would have no incentive to gather information. The result would be a

disclose in order to increase the information shared between agents. Essentially this boils down to providing a public signal about the signal of others.

no-trade equilibrium as in Grossman and Stiglitz (1980).⁴ With the inclusion of ϵ_i , higher demand by a particular agent may be interpreted as being the result either of good information or a large negative liquidity shock.

The precision of the liquidity shock will affect the existence of the linear equilibrium. We prove in what follows that unless this shock is sufficiently noisy, agents may refuse to take part in the market.

3 Equilibria with linear downward sloping demand

As in Kyle (1989) we focus on linear equilibria by assuming that investors submit linear demand functions. Under simplifying assumptions of normal and independent idiosyncratic shocks and a CARA utility function, we are able to compute a symmetric rational expectations equilibrium with downward sloping demand curves. The equilibrium is derived by maximizing each agent's expected utility against the residual demand curve. Indeed, in the spirit of rational expectations equilibria, after making conjectures about the optimal demand functions of his competitors, each agent will choose his optimal strategy by acting as a monopsonist with respect to a residual demand curve conditional on these conjectures.

Let us suppose that agent i conjectures that each agent $j \neq i$ has the following inverse demand function :

$$P(s_j, \epsilon_j, S, w_j, x_j) = \beta_0 + \beta_1 s_j - \beta_2 \epsilon_j + \beta_3 S - \beta_4 w_j - \delta x_j \quad (4)$$

Since, the equilibrium price is the same for all agents, summing the demand functions for all $j \neq i$, dividing by $(n - 1)$, and then using the market clearing condition, $\sum_{j \neq i} x_j = -x_i$, yields the inverse residual demand function for agent i :

$$P(x_i, w_i, S, \epsilon_{-i}, s_{-i}) = \left[\beta_0 + \beta_1 \frac{\sum_{j \neq i} s_j}{(n - 1)} - \beta_2 \frac{\sum_{j \neq i} \epsilon_j}{(n - 1)} + \beta_3 S - \beta_4 \frac{(Q - w_i)}{(n - 1)} + \frac{\delta x_i}{(n - 1)} \right], \quad (5)$$

where ϵ_{-i} and s_{-i} represent the liquidity shocks and private signals for all agents other than i respectively. This can be rewritten as

$$P(x_i, w_i, S, y_i) = \left[\beta_0 + \beta_1 y_i + \beta_3 S - \beta_4 \frac{(Q - w_i)}{(n - 1)} + \frac{\delta x_i}{(n - 1)} \right], \quad (6)$$

⁴The alternative is to introduce noise traders into the model.

where $y_i \equiv \left[\frac{\sum_{j \neq i} s_j}{(n-1)} - \frac{\beta_2}{\beta_1} \frac{\sum_{j \neq i} \epsilon_j}{(n-1)} \right]$. y_i represents the indirect information that agent i can infer about the other investors' strategies given his demand function $x_i(\cdot)$ and the equilibrium price p . When the variance of ϵ_j , σ_ϵ^2 , goes to zero, $y_i = \left[\frac{1}{(n-1)} \sum_{j \neq i} s_j \right]$ and the market equilibrium is fully informative. y_i is a sufficient statistic for the trading motivations of agents. Its two components represent the different trading motivations of agents alluded to above. The first term captures the information-based trading motive, while the second represents the liquidity-based motive. If an agent only observes y_i , he has no way to distinguish between these two motives. The role of the public signal, S , as we have defined it here is to provide a noisy signal about the first term of y_i which allows agents to better distinguish between the two trading motives.⁵

Under the rational expectation equilibrium, we will assume that investor i selects his optimal demand function as if he were observing the y_i from the residual demand function. This is illustrated in Figure 1. Suppose that agent i faces the residual demand curve associated with some y_{i0} . He can select the pair (p_0^*, x_0^*) lying along this curve which maximizes his expected utility given s_i, S , and y_{i0} . Now suppose instead that $y_i = y_{i1}$. The agent can select a price and quantity pair (p_1^*, x_1^*) that lies along the new residual demand curve associated with $y_i = y_{i1}$ and that maximizes his expected utility given s_i, S , and y_{i1} . The optimal demand curve for agent i can, therefore, be constructed by connecting the optimal price-quantity pairs for each value of y_i . The optimal demand curve provides an optimal response for all values of s_i, S , and y_i even if y_i is not directly observed by the investor i .

Formally, for some S, y_i , and s_i , agent i 's problem consists of selecting a demand function $x_i(p)$ that solves the following problem

$$\max_{x_i(p)} W + E_\theta [\theta | y_i, S, s_i] (w_i + x_i + \epsilon_i) - px_i - \frac{r}{2} \text{Var} [\theta | y_i, S, s_i] (w_i + x_i + \epsilon_i)^2$$

subject to

$$p = \left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\beta_4 (Q - w_i)}{(n-1)} + \frac{\delta x_i}{(n-1)} \right].$$

The first-order condition is

$$E_\theta [\theta | y_i, S, s_i] - \left[\beta_0 + \beta_1 y_i + \beta_3 S - \frac{\beta_4 (Q - w_i)}{(n-1)} + \frac{2\delta x_i}{(n-1)} \right] - r \text{Var} [\theta | y_i, S, s_i] (w_i + x_i + \epsilon_i) = 0, \quad (7)$$

⁵Alternatively, we could model the public signal as being a noisy signal about the liquidity-based trading motives. That is, a noisy signal about the sum of the ϵ_j . In this case, enhancing transparency can be achieved by providing market participants better access to information about liquidity-based trading motives of their competitors. This could be achieved for instance by implementing sunshine trading which identifies informationless trading (see for instance Admati and Pfleiderer 1991).

and the second-order condition is

$$\frac{2\delta}{(n-1)} + r\text{Var}(\theta|y_i, S, s_i) > 0. \quad (8)$$

In order to further characterize the structure of the demand function, we state the following lemma:

Lemma 1 *Given the definitions of y_i, S and s_i , and the assumption that all random variables are distributed normally, we have the following*

$$E[\theta|y_i, S, s_i] = \left(\frac{k_0}{k}\right)\bar{\theta} + \left(\frac{k_1}{k}\right)y_i + \left(\frac{k_2}{k}\right)\frac{(nS - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right)s_i \quad (9)$$

$$\text{and } \text{Var}(\theta|y_i, S, s_i) = \sigma_t^2 k_3/k, \quad (10)$$

with

$$k_0 = \sigma_t^2 \quad (11)$$

$$k_1 = \frac{(n-1)\sigma_\theta^2\sigma_t^2\sigma_\mu^2}{[X^2\sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + \frac{(n-1)}{n^2}X^2\sigma_t^2]} \quad (12)$$

$$k_2 = \frac{\frac{(n-1)^2}{n^2}X^2\sigma_\theta^2\sigma_t^2}{[X^2\sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + \frac{(n-1)}{n^2}X^2\sigma_t^2]} \quad (13)$$

$$k_3 = \sigma_\theta^2 \quad (14)$$

$$\text{and } k = [k_0 + k_1 + k_2 + k_3] \quad (15)$$

where $X = \frac{\beta_2}{\beta_1}\sigma_\epsilon$.

Solving the first-order condition yields the optimal demand function which is of the form conjectured in (4) and which we characterize in the following proposition:

Lemma 2 *For strictly positive $r, \sigma_\epsilon^2, \sigma_\theta^2, \sigma_\mu^2$, and σ_t^2 , and $n > 2$, if a (linear) rational expectations equilibrium exists, then each agent must submit a strictly decreasing inverse demand curve of the form:*

$$P(x_i, S, \epsilon_i, s_i) = \beta_0 + \beta_1 s_i - \beta_2 \epsilon_i + \beta_3 S - \beta_4 w_i - \delta x_i \quad (16)$$

where:

$$\delta = \frac{(n-1) \left(k_3 - \frac{k_2}{(n-1)} + k_1 \right) r \sigma_t^2 k_3 / k}{\left((n-2) k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1 \right)} \quad (17)$$

$$\beta_0 = \left(\frac{k_0}{k} \right) \bar{\theta} + \frac{k_1 r \sigma_t^2 \frac{k_3}{k}}{\left((n-1)k_3 - k_2 - k_1 \right)} Q \quad (18)$$

$$\beta_1 = \frac{k_3}{k} - \frac{k_2}{k(n-1)} + \frac{k_1}{k} \quad (19)$$

$$\beta_2 = r \sigma_t^2 \frac{k_3}{k} \left[\frac{\left(k_3 - \frac{k_2}{(n-1)} + k_1 \right)}{\left(k_3 - \frac{k_2}{(n-1)} \right)} \right] \quad (20)$$

$$\beta_3 = \frac{nk_2}{k(n-1)} \quad (21)$$

$$\beta_4 = \left[r \sigma_t^2 \frac{k_3}{k} \right] \left[\frac{(n-1)k_3 - k_2 + (n-1)k_1}{(n-1)k_3 - k_2 - k_1} \right] \quad (22)$$

and k, k_0, k_1, k_2 and k_3 are defined above.

In order to show that the equilibrium inverse demand function is well-defined, we must write the unknown values of the equilibrium inverse demand function ($\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$, and δ) in terms of the parameters of the model ($n, r, \sigma_\varepsilon^2, \sigma_\theta^2, \sigma_\mu^2$, and σ_t^2) and confirm that the conjectures on prices are satisfied. Since $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$, and δ depend on k, k_0, k_1, k_2 , and k_3 which in turn depend on β_1 and β_2 through the value of $X = \frac{\beta_2}{\beta_1} \sigma_\varepsilon$, the problem is somewhat complicated. Using (19) and (20), we can write $X = \frac{\beta_2}{\beta_1} \sigma_\varepsilon$ in terms of the parameters of the model ($n, r, \sigma_\varepsilon^2, \sigma_\theta^2, \sigma_\mu^2$, and σ_t^2). We have:

$$X = \frac{\beta_2}{\beta_1} \sigma_\varepsilon = r \sigma_t^2 \sigma_\varepsilon \left[\frac{[X^2 \sigma_\mu^2 + \sigma_t^2 \sigma_\mu^2 + \frac{(n-1)}{n^2} X^2 \sigma_t^2]}{\sigma_\mu^2 [X^2 + \sigma_t^2]} \right], \quad (23)$$

or alternatively,

$$[X - r \sigma_t^2 \sigma_\varepsilon] = r \sigma_\varepsilon (n-1) \frac{X^2}{[X^2 + \sigma_t^2]} \frac{\sigma_t^4}{n^2 \sigma_\mu^2}. \quad (24)$$

So if we can show that there exists a (unique) X that solves the above equation and that satisfies the second-order condition (8), then we can characterize the (unique) equilibrium with linear demand. The following lemma describes when such an X exists and is unique.

Lemma 3 *When $r^2 \sigma_\varepsilon^2 \geq \left(\frac{n}{(n-2) \sigma_t^2} \right)$, the equilibrium defined in Lemma 2 is unique and is fully characterized by $X = \frac{\beta_2}{\beta_1} \sigma_\varepsilon$. There is a unique positive X that solves (23) and that satisfies the second-order condition given by (8). When $r^2 \sigma_\varepsilon^2 < \left(\frac{n}{(n-2) \sigma_t^2} \right)$, there are values of σ_μ^2 for which no equilibrium exists.*

The equilibrium existence condition is sufficient but not necessary. When this condition is violated, we can only show that an equilibrium does not exist for certain values of σ_μ^2 . Note that the condition does not depend on σ_μ^2 or on σ_θ^2 which are the variables defining the alternatives available to regulators for reducing uncertainty in our model. This fact allows us to perform comparative statics without being constrained by bounded values for σ_μ^2 and σ_θ^2 .

The equilibrium existence condition stems from the second-order condition of the optimality of the trading strategies of agents. Roughly speaking, it imposes that the lower bound on X given by (24) (which is equal to $r\sigma_t^2\sigma_\varepsilon$) is higher than the lower bound given by the second-order condition (which is equal to $\left(\frac{n}{(n-2)}\right)^{1/2}\sigma_t$). Therefore changing the values of different parameters does not violate the second-order condition.

Intuitively, the equilibrium existence condition states that a linear equilibrium exists when (i) there is enough noise caused by liquidity shocks, which would make private signals more valuable and increase the likelihood of trading; (ii) agents are sufficiently risk averse and therefore willing to trade in the market in order to share risk; and/or (iii) the variance of private signals is sufficiently high that adverse selection problems are minimized.

In the following, we summarize the results about existence of equilibrium that will be helpful when we turn to equilibrium analysis in the following section.

Proposition 1 *For all values of σ_μ^2 and σ_θ^2 , a rational expectations equilibrium with linear demand functions exists if and only if $r^2\sigma_\varepsilon^2 \geq \left(\frac{n}{(n-2)}\sigma_t^2\right)$.*

4 Equilibrium analysis

As mentioned above, we are interested in determining the effect of improved information provision on market performance. In particular, we would like to investigate the two alternatives available to regulators for reducing uncertainty about the value of financial assets. In this section we consider what happens when standards on information dissemination of publicly traded assets are tightened (ie. when σ_θ^2 is reduced), and what happens when more information about past transactions in the market are provided (ie. when σ_μ^2 is reduced).

Our main focus is on the impact of changes in these variables on different measures of market performances. We concentrate our analysis on the effect of reducing σ_θ^2 and/or σ_μ^2 on market depth, market efficiency, and expected welfare. As in Kyle (1985) we define market depth as the inverse of sensitivity of prices to changes in quantities; that is, the size of an order flow innovation required to change prices a given amount. So market depth provides a measure of the liquidity of the market. By market efficiency we mean the way prices aggregate the available information in the market. We study the impact on these two measures since the objective of regulators is to ensure high quality –fair, orderly, and efficient⁶ markets, and the microstructure literature studying the impact of transparency on market quality generally uses liquidity as a measure of market quality (see for instance Pagano and Röell 1996, Madhavan 1995 and 1996, and Madhavan et al. 2005).

4.1 Market depth and price efficiency

In order to study the effect of reducing σ_θ^2 and σ_μ^2 on market depth and price efficiency we start by characterizing the way in which these variables affect X and the ex-post variance of θ . Using (24), we can state the following:

Lemma 4 *X has the following properties:*

- (i) X does not depend on σ_θ^2
- (ii) X is a decreasing function of σ_μ^2 . X converges to $r\sigma_\varepsilon^2$ as σ_μ^2 goes to infinity and to infinity as σ_μ^2 converges to 0.

Using (10) and (23), we obtain an expression for the ex-post variance of θ .

$$\begin{aligned}
\text{Var}(\theta|y_i, S, s_i) &= \frac{\sigma_t^2 k_3}{k_0 + k_1 + k_2 + k_3} \\
&= \frac{\sigma_t^2 \sigma_\theta^2}{\sigma_t^2 + n\sigma_\theta^2 - \frac{\sigma_\theta^2(n-1)X^2\sigma_\mu^2}{[X^2\sigma_\mu^2 + \sigma_t^2\sigma_\mu^2 + \frac{(n-1)}{n^2}X^2\sigma_t^2]}} \\
&= \frac{\sigma_t^2 \sigma_\theta^2}{(\sigma_t^2 + n\sigma_\theta^2) - \frac{\sigma_\theta^2 r \sigma_\varepsilon^2 (n-1)X}{[X^2 + \sigma_t^2]}} \tag{25}
\end{aligned}$$

Note that the expression for $\text{Var}(\theta|y_i, S, s_i)$ is written so that it varies with σ_μ^2 only indirectly through its dependence on X , and that X does not depend at all on σ_θ^2 . Hence, we can use (25) to compute the impact of changes in σ_μ^2 and σ_θ^2 . $\text{Var}(\theta|y_i, S, s_i)$ is increasing in $\frac{X}{[X^2 + \sigma_t^2]}$, which in turn is decreasing

⁶See, for example, <http://www.sec.gov/about/whatwedo.shtml>.

in X whenever $X^2 \geq \sigma_t^2$. But this is always true under the assumption of Lemma (3), indeed we have $X \geq r\sigma_t^2\sigma_\varepsilon$ and $r^2\sigma_\varepsilon^2 \geq \left(\frac{n}{(n-2)\sigma_t^2}\right)$ which imply that $X^2 \geq \left(\frac{n\sigma_t^2}{(n-2)}\right)$. Hence, $Var(\theta|y_i, S, s_i)$ is decreasing in X and increasing in σ_μ^2 . Also $Var(\theta|y_i, S, s_i)$ is increasing in σ_θ^2 . We summarize these results with the following lemma:

Lemma 5 *The ex-post variance of θ , $Var(\theta|y_i, S, s_i)$, is*

- (i) *decreasing in X and increasing in σ_μ^2*
- (ii) *increasing in σ_θ^2 .*

4.1.1 Market depth analysis

In our rational expectations equilibrium set-up market depth is measure by $1/\delta$. The value δ measures the way prices change in response to quantity changes and is defined in Proposition 2. The lower is δ , the deeper is the market.⁷

Using (17), we have:

$$\begin{aligned}
\delta &= rVar(\theta|y_i, S, s_i) \frac{(n-1) \left(k_3 - \frac{k_2}{(n-1)} + k_1 \right)}{\left((n-2) k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1 \right)} \\
&= rVar(\theta|y_i, S, s_i) \left[\frac{(n-1) (X^2 + n\sigma_t^2)}{(n-2)X^2 - n\sigma_t^2} \right] \\
&= \left[\frac{\sigma_t^2 \sigma_\theta^2}{(\sigma_t^2 + n\sigma_\theta^2) - \frac{\sigma_\theta^2 r \sigma_\varepsilon \sigma_t^2 (n-1) X}{[X^2 + \sigma_t^2]}} \right] \left[\frac{(n-1) (X^2 + n\sigma_t^2)}{(n-2)X^2 - n\sigma_t^2} \right] \tag{26}
\end{aligned}$$

Note that δ is increasing in σ_θ^2 , and that both its terms are decreasing in X . Hence, δ is also increasing in σ_μ^2 .

We summarize the results for market depth with the following proposition:

Proposition 2 (i) *Market depth, is decreasing in both σ_θ^2 and σ_μ^2 .*

(ii) *As σ_θ^2 converges to zero, market depth converges to infinity. However, as σ_μ^2 converges to zero, market depth converges to a finite value. In the limit, we have:*

$$\lim_{\sigma_\mu^2 \rightarrow 0} \frac{1}{\delta} = \left(\frac{(n-2)n}{(n-1)r\sigma_t^2} \right).$$

(iii) *The marginal effect of σ_θ^2 on market depth is increasing in σ_μ^2 , i.e. $\frac{\partial^2(\frac{1}{\delta})}{\partial \sigma_\theta^2 \partial \sigma_\mu^2} > 0$.*

⁷Note also that in our set-up δ is a measure of the effective bid-ask spread.

Making the market more transparent increases market depth. Increasing the precision of the prior information available to all investors about asset value also increases market depth, and the marginal effect is increasing in market transparency. This suggests that regulators should somehow link market transparency to an increase in the quality of prior information.

More interestingly, (ii) states that in the limit, as prior information becomes extremely precise, the market becomes infinitely deep, while an extremely transparent market would yield a level of market depth bounded from above. This demonstrates the difference between the two alternative ways of improving information provision. While, both generate an increase in market depth, they do so with different levels of efficiency. Increasing the precision of prior information is more efficient since doing so would have a direct impact on the trading behavior of agents by affecting the accuracy of all available information. On the other hand, enhancing transparency only influences agents' strategies by affecting the precision of their information about their competitors signals and the information they infer from the equilibrium price. Therefore, we argue that a further increase in the quality of prior information or of private signals has a more important impact on market depth than does a marginal increase in market transparency. Henceforth, working on rules that allow for an improvement in the quality of the information about firms (whether by increasing prior information or even the private signals) would be a more efficient way of increasing market depth than would rules that increase the quality of the information of the traders' behavior reflecting a higher market transparency.

Note, however, that (iii) implies that the two effects reinforce one another. This finding is particularly interesting since almost all of the literature focuses on one or the other of these types of information, whereas our setup considers both. It suggests that a regulator should encourage both types of informational improvements if it wants to achieve greater market depth.

4.1.2 Price efficiency analysis

We now consider the effect of changes of σ_θ^2 and σ_μ^2 on price efficiency. By price efficiency we mean the quality of information about the asset value transmitted by equilibrium price. We consider the

following measure of price efficiency:

$$\begin{aligned}
e &= 1 - \frac{\text{Var}(\theta|p)}{\text{var}(\theta)} = 1 - \frac{\text{Var}(\theta|y_i, s_i, S)}{\sigma_\theta^2} = 1 - \frac{\sigma_t^2}{(\sigma_t^2 + n\sigma_\theta^2) - \frac{\sigma_\theta^2 r \sigma_\varepsilon \sigma_t^2 (n-1)X}{[X^2 + \sigma_t^2]}} \\
&= \frac{n[X^2 + \sigma_t^2] - r\sigma_\varepsilon \sigma_t^2 (n-1)X}{\left(\frac{\sigma_t^2}{\sigma_\theta^2} + n\right) [X^2 + \sigma_t^2] - r\sigma_\varepsilon \sigma_t^2 (n-1)X}, \tag{27}
\end{aligned}$$

which lies in $[0, 1]$.⁸ When e is zero (one) p is completely uninformative (perfectly informative) about the final value θ . Note that e varies with σ_μ^2 only indirectly through its dependence on $\text{Var}(\theta|y_i, s_i, S)$. Hence, we can use (27) to compute the impact of σ_μ^2 and σ_θ^2 on e .

Proposition 3 (i) Price efficiency is increasing in σ_θ^2 . e converges to 0 as σ_θ^2 goes to 0, and to 1 as σ_θ^2 goes to ∞ .

(ii) Price efficiency is decreasing in σ_μ^2 . As σ_μ^2 goes to 0, X goes to infinity and e converges to $\frac{n\sigma_\theta^2}{\sigma_t^2 + n\sigma_\theta^2}$. As σ_μ^2 goes to infinity, X goes to $r\sigma_t^2\sigma_\varepsilon$ and e converges to

$$\lim_{X \rightarrow r\sigma_t^2\sigma_\varepsilon} e = \frac{\left([(r\sigma_t^2\sigma_\varepsilon)^2 + n\sigma_t^2] \right)}{\frac{\sigma_t^2}{\sigma_\theta^2} [(r\sigma_t^2\sigma_\varepsilon)^2 + \sigma_t^2] + \left([(r\sigma_t^2\sigma_\varepsilon)^2 + n\sigma_t^2] \right)}.$$

The first part of (i) may seem counter-intuitive in the sense that it states that increasing the quality of prior information reduces price efficiency. However, since price efficiency measures the ability of prices to transmit signals about the information available in the market, it makes sense that if the available information is of higher quality, prices are less efficient as information aggregators. Furthermore, a marginal increase in the quality of information would negatively affect the informativeness of prices. On the other hand, (ii) states that enhancing market transparency by increasing the quality of the aggregate signal (decreasing σ_μ^2) positively affects the informativeness of the equilibrium price. However, the positive effect of transparency on market efficiency is bounded since, in the limit, as σ_μ^2 converges to zero or to infinity, price efficiency converges to finite values. As σ_μ^2 is lowered, it becomes easier to distinguish the informational and liquidity motivations for trading. This allows traders to better assess the information-motivations for trading and so the resulting price is more informative.

⁸See Brown and Zhang (1997) for a discussion of the properties of this measure of price efficiency.

4.2 Equilibrium trade

From the derived equilibrium we have that

$$x_i = \frac{\left(E_\theta [\theta|y_i, S, s_i] - \left[\beta_0 + \beta_1 \frac{\sum_{j \neq i} s_j}{(n-1)} - \beta_2 \frac{\sum_{j \neq i} \epsilon_j}{(n-1)} + \beta_3 S - \frac{\beta_4(Q-w_i)}{(n-1)} \right] \right) - rVar [\theta|y_i, S, s_i] (w_i + \epsilon_i)}{2 \left(\frac{\delta}{(n-1)} + \frac{r}{2} Var [\theta|y_i, S, s_i] \right)}.$$

If we sum the above expression for all x_i , then since $\sum_j x_j = 0$, we have that $x_i - \sum_j x_j = x_i$, which can be written as

$$x_i = \frac{\frac{\beta_1}{(n-1)} \left(s_i - \frac{\sum_j s_j}{n} \right) - \left(\frac{\beta_4}{(n-1)} + rVar [\theta|y_i, S, s_i] \right) \left(w_i - \frac{Q}{n} \right) + \left(\frac{\beta_2}{(n-1)} + rVar [\theta|y_i, S, s_i] \right) \left(\frac{\sum_j \epsilon_j}{n} - \epsilon_i \right)}{\left(\frac{2\delta}{(n-1)} + rVar [\theta|y_i, S, s_i] \right)}.$$

Substituting for the equilibrium values of $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, Var [\theta|y_i, S, s_i]$, and δ and using (23), we obtain:

$$x_i = \frac{[(n-2)X^2 - n\sigma_t^2]}{(n-1)[X^2 + \sigma_t^2]} \left[\frac{\sigma_\varepsilon}{X} \left(t_i - \frac{\sum_j t_j}{n} \right) + \left(\frac{[X^2 + \sigma_t^2]}{[X^2]} \right) \left(\frac{Q}{n} - w_i \right) + \left(\frac{\sum_j \epsilon_j}{n} - \epsilon_i \right) \right]. \quad (28)$$

Note here that, interestingly, x_i does not depend on S . So, the trading quantity of the agent is not affected directly by the public signal. The agent's net trade will depend on the difference between his private signal the average signal in the market, the difference between his endowment and the average endowment in the market, and the difference between his liquidity shock and the average liquidity shock. Furthermore, recall that X^2 is independent of σ_θ^2 and therefore so is an agent's net trade.

Now, consider the impact of σ_μ^2 on net trade. The variance σ_μ^2 affects net trade indirectly through its impact on X , which is decreasing in σ_μ^2 . Because $\left(t_i - \frac{\sum_j t_j}{n} \right)$, $\left(\frac{Q}{n} - w_i \right)$, and $\left(\frac{\sum_j \epsilon_j}{n} - \epsilon_i \right)$ can be negative or positive, the impact of σ_μ^2 is ambiguous.

However, we can derive unambiguous results on the dispersion of net trades among agents, $E(x_i^2)$. Doing so allows us to say something about the impact of σ_μ^2 on the volume of trade (since $E(x_i^2)$ is related to $\sum |x_i|$). For tractability, we assume that for i , we have $w_i = \frac{Q}{n}$.

Proposition 4 *The dispersion of net trades among agents, $E(x_i^2)$, is given by*

$$E(x_i^2) = \frac{[(n-2)X^2 - n\sigma_t^2]^2}{(n-1)[X^2 + \sigma_t^2]} \frac{\sigma_\varepsilon^2}{nX^2},$$

and is increasing in X^2 and decreasing in σ_μ^2 .

The above result may seem paradoxical. An increase in σ_μ^2 lowers the value of the public signal which reduces the relative importance of shared information. It follows that individual demand will

rely more on the idiosyncratic information available. This effect tends to increase the amount of trade. However, despite this, there is in fact less trade since there is another more dominant effect. An increase in σ_μ^2 increases the adverse selection effect and therefore increases the absolute value of the slope of the demand curve (or equivalently decreases market depth). This reduces trade in the market.

4.3 Expected welfare

Finally we analyse the effect of σ_μ^2 and σ_θ^2 on expected welfare. We start by defining an agent's welfare conditional on all of the available information

$$V(s_i, y_i, S, \epsilon_i) = (E(\theta|s_i, y_i, S, \epsilon_i) (w_i + x + \epsilon_i) - px - \frac{r}{2} \text{var}(\theta|s_i, y_i, S, \epsilon_i)(w_i + x + \epsilon_i)^2.$$

By substituting the first order condition (7) we obtain

$$\begin{aligned} V(s_i, y_i, S, \epsilon_i) &= (E(\theta|s_i, y_i, S, \epsilon_i) (w_i + \epsilon_i) - \frac{r}{2} \text{Var}(\theta|s_i, y_i, S, \epsilon_i)(w_i + \epsilon_i)^2 \\ &\quad + \left(\frac{r}{2} \text{Var}(\theta|s_i, y_i, S, \epsilon_i) + \frac{\delta}{(n-1)} \right) x_i^2, \end{aligned} \quad (29)$$

where x_i is given by (28).

When there is no uncertainty (ie. $\text{var}(\theta|s_i, y_i, S, \epsilon_i) = 0$), $V(s_i, y_i, S, \epsilon_i) = E(\theta|s_i, y_i, S, \epsilon_i) (w_i + \epsilon_i) = \theta (w_i + \epsilon_i)$. That is, when there is no uncertainty the equilibrium price is equal to the value of the asset; also because there is no risk there are no gains from trade. When there is uncertainty, the second and third terms of (29) are non-zero. The second term is negative and it reflects the disutility associated with risk. The third term is positive and it measures to what extent trade can mitigate risk. The impact of an increase in variance depends on an agent's initial endowment and liquidity shock ($w_i + \epsilon_i$). For instance if $w_i + \epsilon_i = 0$, then agent i is initially not exposed to any risk and so can trade purely in order to take advantage of the uncertainty faced by rival agents.

Using $V(s_i, y_i, S, \epsilon_i)$ we can define the ex-ante expected welfare of an agent as

$$EW = -e^{-rW} E_{(s_i, y_i, S, \epsilon_i)} [-\exp[-rV(s_i, y_i, S, \epsilon_i)]] \quad (30)$$

where the variables, s_i , y_i , S , and ϵ_i have means equal to $\bar{\theta}$, $\bar{\theta}$, $\bar{\theta}$, and 0 respectively, and have variance-

covariance matrix given by

$$\Omega = \begin{pmatrix} \sigma_\theta^2 + \sigma_t^2 & \sigma_\theta^2 & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & 0 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{X^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & 0 \\ \sigma_\theta^2 + \frac{\sigma_t^2}{n} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} + \sigma_\mu^2 & 0 \\ 0 & 0 & 0 & \sigma_\epsilon^2 \end{pmatrix}. \quad (31)$$

Note that $V(s_i, y_i, S, \epsilon_i)$ is a quadratic function and so by applying moment generating techniques we can rewrite (30). For tractability we assume that $w_i = \frac{Q}{n}$. We summarize the result in the following proposition.

Proposition 5 *Assuming that $w_i = \frac{Q}{n}$, expected welfare can be written in the following form*

$$\begin{aligned} EW &= -\frac{|\Lambda|^{1/2}}{|\Omega|^{1/2}} \exp \left\{ -r \left(\left(W + \frac{Q\bar{\theta}}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n} \right)^2 \right) - \frac{r}{2} t \Lambda t \right) \right\} \\ &= -\exp \left\{ -r \left(\left(W + \frac{Q\bar{\theta}}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n} \right)^2 - \frac{\log(|\Lambda|)}{2r} \right) - \frac{r}{2} t \Lambda t \right) \right\} \end{aligned} \quad (32)$$

where Λ and t are defined in the appendix.

Solving for the impact on expected welfare of reducing σ_μ^2 and σ_θ^2 is analytically intractable and so we turn to simulation analysis. Figure 2 plots expected welfare as a function of σ_θ^2 and/or σ_μ^2 . From the picture it can be seen that starting at high values of σ_θ^2 and/or σ_μ^2 , expected welfare is initially decreasing as σ_θ^2 and/or σ_μ^2 are reduced, before finally increasing as σ_θ^2 and/or σ_μ^2 become very small. Figure 3a plots ex-ante expected welfare as a function of σ_θ^2 for different values of σ_μ^2 , while figure 3b plots ex-ante expected welfare as a function of σ_μ^2 for different values of σ_θ^2 . In each we see the same pattern as in the the three-dimensional plot. Expected welfare is initially decreasing as σ_θ^2 or σ_μ^2 are reduced, before finally increasing as σ_θ^2 or σ_μ^2 become very small.

To explain what is going on in these figures, we turn back to equation (32) from which it can be seen that expected welfare is made up of mean and variance components. The second term in the exponent of (32), $\frac{r}{2} t \Lambda t$, represents the variance component. Expected welfare is decreasing in this component. The first term, $W + \frac{Q\bar{\theta}}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n} \right)^2 - \frac{\log(|\Lambda|)}{2r}$, therefore represents the mean component, which is positively related to expected welfare.

The variance component is always increasing in σ_θ^2 and/or σ_μ^2 since more precise information (more precise prior information and/or greater transparency) will always reduce uncertainty (see Figure 3).

The relationship between the mean component and $\sigma_\theta^2/\sigma_\mu^2$ is more complicated (see Figure 4). Starting from perfect information ($\sigma_\theta^2 = \sigma_\mu^2 = 0$) where there is no trading, we consider a small increase in σ_θ^2 and/or σ_μ^2 . This reduces the mean component since for these values trading will occur only for liquidity purposes. This is only true up to a certain point, after which increasing σ_θ^2 and/or σ_μ^2 increases the mean component since less precise information provides the opportunity for informational rents.

So what is the overall effect of more precise information on expected welfare? Starting from high σ_θ^2 and σ_μ^2 we consider the impact of a decrease in σ_θ^2 and/or σ_μ^2 on expected welfare. As σ_θ^2 and/or σ_μ^2 falls both the variance component and the mean component decrease leading to conflicting effects on welfare since welfare is decreasing in the former and increasing in the latter. Because of the greater effect of informational rents, the mean-component effect will dominate leading to an overall reduction in expected welfare. This is true up until information becomes relatively precise (low σ_θ^2 and/or σ_μ^2). At this point trading becomes more liquidity motivated and the uncertainty effect comes to dominate causing expected welfare to increase as information becomes more precise.⁹

As mentioned in the Introduction, other articles have also found that improved information may lower welfare, but these mostly focus on a class of economies that feature externalities, strategic complementarity or substitutability (see for instance Morris and Shin 2002, and Clark and Polborn 2006, and see Angeletos and Pavan 2007 for a summary). We focus instead on a rational expectations set up with imperfect competition and improving information can lower welfare in this case since doing so eliminates potential informational rents.

5 Concluding remarks

In this paper we have studied the impact on market performance of regulations that would improve the level of information provision in financial markets. In our rational expectations model we are able to examine what happens to market performance when regulators can provide investors with more accurate prior information by improving the precision of all private signals, and/or can enhance the transparency of the market. We show that the two alternative ways of decreasing uncertainty have the same effect on market depth, but opposite effects on market efficiency. Providing more precise

⁹Note that the extent to which expected welfare decreases and the point at which expected welfare begins to increase as information becomes more precise depend on W , Q , $\bar{\theta}$, and r .

prior information increases market depth and decreases market efficiency since adverse selection effects are less relevant. Enhancing market transparency will both increase the market depth and market efficiency. The effect of increasing transparency is less important for market depth than the effect of increasing the precision of prior information since in the limit, while very precise prior information results in an infinitely deep market, increasing market transparency results in a level of market depth that is bounded from above. The volume of trade is independent of the quality of prior information and is increasing in the quality of the public signal. Finally, we show that starting with very imprecise private signals and/or a very opaque market, improving the precision of all private signals, and/or enhancing the transparency of the market will cause expected welfare to fall up to a point, after which expected welfare will increase.

Appendix

Proof of Lemma 1. To simplify notation we define the following:

$$\begin{aligned}
Z_4 &= \theta \sim N(\bar{\theta}, \sigma_\theta^2), \\
Z_3 &= s_i = \theta + t_i \sim N(\bar{\theta}, \sigma_\theta^2 + \sigma_t^2), \\
Z_2 &= \frac{nS - s_i}{(n-1)} = \frac{n\left(\theta + \frac{\sum_i t_i}{n} + \mu\right) - s_i}{(n-1)} = \theta + \frac{\sum_{j \neq i} t_j}{n-1} + \frac{n}{n-1}\mu \sim N\left(\bar{\theta}, \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{n^2\sigma_\mu^2}{(n-1)^2}\right), \\
Z_1 &= y_i = \frac{1}{(n-1)} \sum_{j \neq i} s_j - \frac{\beta_2}{\beta_1(n-1)} \sum_{j \neq i} \epsilon_j \sim N\left(\bar{\theta}, \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{\beta_2^2}{\beta_1^2(n-1)}\sigma_\varepsilon^2\right).
\end{aligned}$$

We then let

$$X = \frac{\beta_2}{\beta_1}\sigma_\varepsilon,$$

so that $Z_1 \sim N\left(\bar{\theta}, \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{X^2}{(n-1)}\right)$. Our objective is to compute $E(Z_4|Z_1, Z_2, Z_3)$ and $Var(Z_4|Z_1, Z_2, Z_3)$.

Note that all variables are normally distributed, so the joint distribution is

$$(Z_1, Z_2, Z_3, Z_4) \sim N(m; \Sigma),$$

where $m = (E(Z_1), E(Z_2), E(Z_3), E(Z_4)) = (\bar{\theta}, \bar{\theta}, \bar{\theta}, \bar{\theta})$ and Σ is the variance-covariance matrix defined

as follows

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{X^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} + \frac{n^2\sigma_\mu^2}{(n-1)^2} & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 + \sigma_t^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \end{pmatrix}.$$

For notation purposes, let $\Sigma = (\eta_{ij})$ for $i, j = 1, \dots, 4$ and $\Sigma^{-1} = (\eta^{ij})$ for $i, j = 1, \dots, 4$. Moreover, let

$\Sigma_{(-4)}$ be the upper left 3 by 3 corner of Σ , so

$$\Sigma_{(-4)} = \begin{pmatrix} \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{X^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} & \sigma_\theta^2 \\ \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{(n-1)} + \frac{n^2\sigma_\mu^2}{(n-1)^2} & \sigma_\theta^2 \\ \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 + \sigma_t^2 \end{pmatrix}.$$

Finally, let $\Sigma_{(-4)}^{-1} = (\phi^{pq})$ for $p, q = 1, \dots, 3$. Since all variables are normally distributed, we have

$(Z_4|Z_1 = z_1, Z_2 = z_2, Z_3 = z_3) \sim N(m^*; \sigma^2)$, where $\sigma^2 = \frac{\det(\Sigma)}{\det(\Sigma_{(-4)})}$,¹⁰ and

$$m^* = E(Z_4) + \sum_{l=1}^3 \tau_l(z_l - E(Z_l)), \quad (33)$$

with

$$\tau_l = \sum_{q=1}^3 \phi^{lq} \eta_{q,4}, \text{ pour } l = 1, 2, \text{ et } 3. \quad (34)$$

¹⁰See Hoel (1984).

For the conditional variance, σ^2 , we have

$$\det \Sigma_{(-4)} = \frac{\sigma_t^2 [(n-1)X^2\sigma_t^2 + n^2X^2\sigma_\mu^2 + n^2\sigma_t^2\sigma_\mu^2] + \sigma_\theta^2 [n^2X^2\sigma_\mu^2 + n^3\sigma_\mu^2\sigma_t^2 + n(n-1)X^2\sigma_t^2]}{(n-1)^3}$$

and

$$\det(\Sigma) = \sigma_\theta^2\sigma_t^2 \left[\frac{n^2\sigma_\mu^2\sigma_t^2 + (n-1)X^2\sigma_t^2 + n^2X^2\sigma_\mu^2}{(n-1)^3} \right].$$

Then,

$$\begin{aligned} \text{Var}(Z_4|Z_1, Z_2, Z_3) &= \sigma^2 = \frac{\det(\Sigma)}{\det(\Sigma_{(-4)})} \\ &= \frac{\sigma_\theta^2\sigma_t^2}{k} \end{aligned} \quad (35)$$

with

$$k = \sigma_t^2 + \sigma_\theta^2 + (n-1) \frac{\sigma_\theta^2 [n^2\sigma_\mu^2\sigma_t^2 + (n-1)\sigma_t^2X^2]}{[n^2\sigma_\mu^2\sigma_t^2 + n^2\sigma_\mu^2X^2 + (n-1)\sigma_t^2X^2]}. \quad (36)$$

In order to derive the conditional expectation of Z_4 , we need to derive the inverse matrix of $\Sigma_{(-4)}$.

We present $\Sigma_{(-4)}^{-1}$ as follows:

$$\Sigma_{(-4)}^{-1} = \frac{(n-1)}{k[n^2\sigma_\mu^2\sigma_t^2 + n^2\sigma_\mu^2X^2 + (n-1)\sigma_t^2X^2]} \times (\psi^1, \psi^2, \psi^3)$$

where

$$\psi^1 = \begin{pmatrix} \psi_1^1 \\ \psi_2^1 \\ \psi_3^1 \end{pmatrix} = \begin{pmatrix} [n(n-1)\sigma_t^2\sigma_\theta^2 + (n-1)\sigma_t^4 + n^2\sigma_\mu^2\sigma_\theta^2 + n^2\sigma_\mu^2\sigma_t^2] \\ -(n-1)\sigma_t^2(n\sigma_\theta^2 + \sigma_t^2) \\ -n^2\sigma_\mu^2\sigma_\theta^2 \end{pmatrix},$$

$$\psi^2 = \begin{pmatrix} \psi_1^2 \\ \psi_2^2 \\ \psi_3^2 \end{pmatrix} = \begin{pmatrix} -(n-1)\sigma_t^2(n\sigma_\theta^2 + \sigma_t^2) \\ (n-1)(n\sigma_t^2\sigma_\theta^2 + \sigma_t^4 + X^2\sigma_\theta^2 + X^2\sigma_t^2) \\ -(n-1)X^2\sigma_\theta^2 \end{pmatrix},$$

and

$$\psi^3 = \begin{pmatrix} \psi_1^3 \\ \psi_2^3 \\ \psi_3^3 \end{pmatrix} = \begin{pmatrix} -n^2\sigma_\mu^2\sigma_\theta^2 \\ -(n-1)X^2\sigma_\theta^2 \\ (n-1)X^2\sigma_\theta^2 + n^2\sigma_\mu^2\sigma_\theta^2 + X^2\sigma_t^2 + \frac{n^2\sigma_\mu^2(\sigma_t^2 + X^2)}{(n-1)} \end{pmatrix}.$$

From (34) and since $(\eta_{\cdot 4}) = (\sigma_\theta^2, \sigma_\theta^2, \sigma_\theta^2)$ we have

$$\tau_l = \frac{(n-1)\sigma_\theta^2 (\psi_1^l + \psi_2^l + \psi_3^l)}{k[n^2\sigma_\mu^2\sigma_t^2 + n^2\sigma_\mu^2X^2 + (n-1)\sigma_t^2X^2]}, \quad (37)$$

for $l = 1, 2, 3$. For τ_1 , substitution of the values of ψ_i^1 (for $i = 1, 2, 3$) in (37) and simplifications give

$$\tau_1 = \frac{(n-1)n^2\sigma_\mu^2\sigma_t^2\sigma_\theta^2}{k[n^2\sigma_\mu^2\sigma_t^2 + n^2\sigma_\mu^2X^2 + (n-1)\sigma_t^2X^2]}.$$

Similar manipulations allow us to write

$$\tau_2 = \frac{(n-1)^2 X^2 \sigma_\theta^2 \sigma_t^2}{k[n^2 \sigma_\mu^2 \sigma_t^2 + n^2 \sigma_\mu^2 X^2 + (n-1) \sigma_t^2 X^2]},$$

and

$$\tau_3 = \frac{\sigma_\theta^2}{k}.$$

Substitution of the values of z_i in (33) gives

$$\begin{aligned} m^* &= \bar{\theta} + \sum_{l=1}^3 \tau_l (z_l - \bar{\theta}) \\ &= \left(1 - \sum_{l=1}^3 \tau_l\right) \bar{\theta} + \tau_1 y + \tau_2 \frac{nS - s_i}{(n-1)} + \tau_3 s_i. \end{aligned}$$

Note that

$$1 - \sum_{l=1}^3 \tau_l = \frac{\sigma_t^2}{k},$$

then substitution of τ_l (for $l = 1, 2, 3$) gives

$$\begin{aligned} m^* &= E[\theta | y_i, S, s_i] \\ &= \left(\frac{\sigma_t^2}{k}\right) \bar{\theta} + \frac{n^2(n-1)\sigma_\mu^2 \sigma_t^2 \sigma_\theta^2}{[n^2 \sigma_\mu^2 \sigma_t^2 + n^2 \sigma_\mu^2 X^2 + (n-1) \sigma_t^2 X^2]} y_i \\ &\quad + \frac{(n-1)^2 X^2 \sigma_\theta^2 \sigma_t^2}{[n^2 \sigma_\mu^2 \sigma_t^2 + n^2 \sigma_\mu^2 X^2 + (n-1) \sigma_t^2 X^2]} \frac{nS - s_i}{(n-1)} + \frac{\sigma_\theta^2}{k} s_i. \end{aligned}$$

Finally let: $k_0 = \sigma_t^2$, $k_1 = \frac{n^2(n-1)\sigma_\mu^2 \sigma_t^2 \sigma_\theta^2}{[n^2 \sigma_\mu^2 \sigma_t^2 + n^2 \sigma_\mu^2 X^2 + (n-1) \sigma_t^2 X^2]}$, $k_2 = \frac{(n-1)^2 X^2 \sigma_\theta^2 \sigma_t^2}{[n^2 \sigma_\mu^2 \sigma_t^2 + n^2 \sigma_\mu^2 X^2 + (n-1) \sigma_t^2 X^2]}$ and $k_3 = \sigma_\theta^2$.

Note that from these definitions and from (36) that $k = [k_0 + k_1 + k_2 + k_3]$. This establishes the results of Lemma 1. ■

Proof of Lemma 2. Recall that we have:

$$y_i = \frac{p - \beta_3 S + \frac{\beta_4(Q-w_i)}{(n-1)} - \frac{\delta x}{(n-1)} - \beta_0}{\beta_1} \quad \text{and} \quad (38)$$

$$E[\theta | y_i, S, s_i] = \left(\frac{k_0}{k}\right) \bar{\theta} + \left(\frac{k_1}{k}\right) y_i + \left(\frac{k_2}{k}\right) \frac{(nS - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right) s_i \quad (39)$$

Hence, we can rewrite the first-order condition in (7) in order to find a relationship between x , a demand quantity, and, p , the equilibrium price which should not depend on y_i . Since $Var[\theta | y_i, S, s_i] = \sigma_t^2 \frac{k_3}{k}$, we obtain the following relationship:

$$\begin{aligned} 0 &= \left[\left(\frac{k_0}{k}\right) \bar{\theta} + \left(\frac{k_2}{k}\right) \frac{(nS - s_i)}{(n-1)} + \left(\frac{k_3}{k}\right) s_i + \left(\frac{k_1}{k}\right) \left(\frac{p - \beta_0 - \beta_3 S + \frac{\beta_4(Q-w_i)}{(n-1)} - \frac{\delta x}{(n-1)}}{\beta_1} \right) \right] \\ &\quad - p - \left(\frac{\delta}{(n-1)} + r \sigma_t^2 \frac{k_3}{k} \right) x - r \sigma_t^2 \frac{k_3}{k} (w_i + \epsilon_i) \end{aligned} \quad (40)$$

Isolating p , we obtain the following inverse demand function:

$$p \left[1 - \left(\frac{k_1}{\beta_1 k} \right) \right] = \left[\left(\frac{k_0}{k} \right) \bar{\theta} + \left(\frac{k_1}{\beta_1 k} \right) \frac{\beta_4 Q}{(n-1)} - \left(\frac{k_1 \beta_0}{k \beta_1} \right) \right] + \left(\frac{k_3}{k} - \left(\frac{k_2}{(n-1)k} \right) \right) s_i - \left(r \sigma_t^2 \frac{k_3}{k} \right) \epsilon_i$$

$$+ \left[\frac{nk_2}{(n-1)k} - \left(\frac{k_1}{k} \frac{\beta_3}{\beta_1} \right) \right] S - \left[\left(\frac{k_1}{\beta_1 k} \right) \frac{\beta_4}{(n-1)} + r \sigma_t^2 \frac{k_3}{k} \right] w_i \quad (41)$$

$$- \left[\frac{\delta}{(n-1)} + r \sigma_t^2 \frac{k_3}{k} + \left(\frac{k_1}{\beta_1 k} \right) \frac{\delta}{(n-1)} \right] x \quad (42)$$

Since by assumption:

$$P(x_i, S, \epsilon_i, s_i) = \beta_0 + \beta_1 s_i - \beta_2 \epsilon_i + \beta_3 S + \beta_4 w_i - \delta x_i$$

matching the arguments of the two above equations, we get

$$\beta_0 = \left[\frac{\left(\frac{k_0}{k} \right) \bar{\theta} + \left(\frac{k_1}{\beta_1 k} \right) \frac{\beta_4 Q}{(n-1)} - \left(\frac{k_1 \beta_0}{k \beta_1} \right)}{1 - \left(\frac{k_1}{\beta_1 k} \right)} \right]$$

$$\beta_1 = \left[\frac{\left(\frac{k_3}{k} - \frac{k_2}{(n-1)k} \right)}{\left[1 - \left(\frac{k_1}{\beta_1 k} \right) \right]} \right] = \left(\frac{k_3}{k} - \frac{k_2}{(n-1)k} + \frac{k_1}{k} \right)$$

$$\beta_2 = \frac{\left(r \sigma_t^2 \frac{k_3}{k} \right)}{\left[1 - \left(\frac{k_1}{\beta_1 k} \right) \right]}$$

$$\beta_3 = \frac{\left[\frac{nk_2}{(n-1)k} - \left(\frac{k_1}{k} \frac{\beta_3}{\beta_1} \right) \right]}{\left[1 + \left(\frac{k_1}{\beta_1 k} \right) \right]}$$

$$\beta_4 = \frac{\left[\left(\frac{k_1}{\beta_1 k} \right) \frac{\beta_4}{(n-1)} + r \sigma_t^2 \frac{k_3}{k} \right]}{\left[1 - \left(\frac{k_1}{\beta_1 k} \right) \right]}$$

$$\delta = \frac{\left[\frac{\delta}{(n-1)} + r \sigma_t^2 \frac{k_3}{k} + \left(\frac{k_1}{\beta_1 k} \right) \frac{\delta}{(n-1)} \right]}{\left[1 - \left(\frac{k_1}{\beta_1 k} \right) \right]}$$

The expression for β_1 is as in the Lemma. Subbing this into the other expressions, solving, and simplifying, we obtain the expressions in Lemma 2. ■

Proof of Lemma 3. From (24), the right side of this equation is positive. So, the left side must also be positive and consequently X must be strictly bigger than $r \sigma_t^2 \sigma_\varepsilon$. Now in order to show the existence of a solution to (24), we only need to show that the left- and right- hand sides defined as functions of X cross at least once. To do this note that at $X = r \sigma_t^2 \sigma_\varepsilon$, the left-hand side of (24) is 0 and is lower than the right-hand side which is strictly positive. On the other hand, when X goes to infinity, the left-hand side goes to infinity and is bigger than the right side which is finite.

Since both sides are continuous functions in X , there exists at least one solution to this equation. Note also that the left-hand side is linear and strictly increasing in X while the right-hand side is strictly increasing and concave in X for all $X > r\sigma_t^2\sigma_\varepsilon$, so they must cross only once which guarantees equilibrium uniqueness. Also, we must show that the obtained X satisfies the second-order condition. This condition is represented in (8),

$$\frac{2\delta}{(n-1)} + r\text{Var}(\theta|y, s_i, S) > 0.$$

Substitution of δ from (17) and using the fact that $\text{Var}(\theta|y_i, S, s_i) = \sigma_t^2 k_3/k$ yields

$$r\sigma_t^2 \frac{k_3}{k} \left[\left(\frac{2k_3 - \frac{2k_2}{(n-1)} + 2k_1}{(n-2)k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1} \right) + 1 \right] > 0$$

or, since $r\sigma_t^2 \frac{k_3}{k} > 0$,

$$\frac{n \left(k_3 - \frac{k_2}{(n-1)} \right)}{(n-2)k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1} > 0.$$

Then, substitution of the values of k_1 , k_2 and k_3 from equations (12), (13) and (14) and simplification gives the following condition

$$\frac{n(X^2 + \sigma_t^2)}{(n-2)X^2 - n\sigma_t^2} > 0.$$

Hence, the second-order condition is verified for all $X^2 > \frac{n\sigma_t^2}{(n-2)}$. So, a sufficient condition for the equilibrium existence is that the lowest value of X satisfies the second order condition, i.e.,

$$(r\sigma_t^2\sigma_\varepsilon)^2 > \frac{n\sigma_t^2}{(n-2)}$$

or, after simplification

$$r^2\sigma_\varepsilon^2 > \frac{n}{(n-2)\sigma_t^2}.$$

Under this condition, the equilibrium exists and is unique. Finally, if this condition does not hold, i.e., $r^2\sigma_\varepsilon^2 \leq \frac{n}{(n-2)\sigma_t^2}$, then we can show that the equilibrium may not exist. In order to see this, note that the right side of (24) converges to 0 when σ_μ^2 goes to infinity. So, from the left side, if X exists it must converge to $r\sigma_t^2\sigma_\varepsilon$. Thus, there exists a σ_μ^2 sufficiently large so that the solution to (24) is such that $X^2 < \frac{n\sigma_t^2}{(n-2)}$, and, consequently, the second-order condition is not verified. ■

Proof of Lemma 4. The result in (i) follows immediately from (24) which does not depend on σ_θ^2 .

In order to prove the results in (ii), let us rewrite (24) as follows

$$\frac{[X - r\sigma_t^2\sigma_\varepsilon][X^2 + \sigma_t^2]}{X^2} = \frac{r\sigma_\varepsilon\sigma_t^4(n-1)}{n^2} \frac{1}{\sigma_\mu^2}, \quad (43)$$

where the right hand side is decreasing in σ_μ^2 . The left hand side must be consequently decreasing in σ_μ^2 . So, in terms of differentiation with respect to σ_μ^2 , this gives

$$\frac{\partial}{\partial \sigma_\mu^2} \left(\frac{[X - r\sigma_t^2\sigma_\varepsilon][X^2 + \sigma_t^2]}{X^2} \right) = \frac{\partial X}{\partial \sigma_\mu^2} \left(\frac{X^3 - \sigma_t^2 X + 2r\sigma_t^4\sigma_\varepsilon}{X^3} \right) \leq 0.$$

We can easily show that the polynomial $(X^3 - \sigma_t^2 X + 2r\sigma_t^4\sigma_\varepsilon)$ is positive for all $X \geq r\sigma_t^2\sigma_\varepsilon$ when the equilibrium existence condition is satisfied. So, X must be decreasing in σ_μ^2 . Limit computations establish that when σ_μ^2 goes to infinity, $[X - r\sigma_t^2\sigma_\varepsilon]$ must converge to zero and, when σ_μ^2 goes to zero, $[X - r\sigma_t^2\sigma_\varepsilon]$ must converge to infinity. ■

Proof of Proposition 2. Part (i) is proven in the text. For part (ii), if we take the limit as σ_θ^2 converges to 0, $\frac{1}{\delta}$ goes to ∞ . From Lemma 4, when σ_μ^2 converges to 0, X goes to $r\sigma_t^2\sigma_\varepsilon$. Hence, we have:

$$\begin{aligned} \lim_{\sigma_\mu^2 \rightarrow \infty} \frac{1}{\delta} &= \lim_{X \rightarrow r\sigma_t^2\sigma_\varepsilon} \frac{1}{\delta} \\ &= \frac{[(n-2)(r^2\sigma_t^2\sigma_\varepsilon^2) - n]}{[(r^2\sigma_t^2\sigma_\varepsilon^2) + n]} \left(\frac{(\sigma_t^2 + n\sigma_\theta^2)}{(n-1)r\sigma_t^2\sigma_\theta^2} - \frac{r\sigma_\varepsilon^2(n-1)}{[(r^2\sigma_t^2\sigma_\varepsilon^2) + 1]} \right) \end{aligned}$$

This establishes (ii). Using (26) in order to write $\frac{1}{\delta}$ and differentiating with respect to σ_θ^2 and X gives

$$\frac{\partial^2 \left(\frac{1}{\delta} \right)}{\partial \sigma_\theta^2 \partial X} = \frac{\partial}{\partial X} \left(\frac{[(n-2)X^2 - n\sigma_t^2]}{[X^2 + n\sigma_t^2]} \right) \frac{\partial}{\partial \sigma_\theta^2} \left(\frac{\left(\frac{\sigma_t^2}{\sigma_\theta^2} + n \right)}{(n-1)r\sigma_t^2} \right) < 0.$$

From Lemma 4 we have $\frac{\partial X}{\partial \sigma_\mu^2} < 0$, so

$$\frac{\partial^2 \left(\frac{1}{\delta} \right)}{\partial \sigma_\theta^2 \partial \sigma_\mu^2} > 0$$

which shows (iii). ■

Proof of Proposition 3. We can easily see from (27) that e is increasing in σ_θ^2 . When σ_θ^2 goes to 0, e goes to 0. Conversely, if σ_θ^2 goes to ∞ , then $\frac{1}{e}$ and e go to 1. This establishes (i).

Note that e is decreasing in $\frac{X}{[X^2 + \sigma_t^2]}$, and that $\frac{X}{[X^2 + \sigma_t^2]}$ is decreasing in X since we have $X^2 > \sigma_t^2$. It follows that e is increasing in X . Since X decreases in σ_μ^2 , market efficiency, e , must decrease in σ_μ^2 . When σ_μ^2 goes to 0, X goes to infinity. Hence, we have:

$$\lim_{\sigma_\mu^2 \rightarrow 0} e = \lim_{X \rightarrow \infty} e = \frac{n}{\frac{\sigma_t^2}{\sigma_\theta^2} + n} = \frac{\sigma_\theta^2 n}{\sigma_t^2 + \sigma_\theta^2 n}$$

Also when σ_μ^2 goes to ∞ , X goes to $r\sigma_\varepsilon\sigma_t^2$, hence the result presented in the Proposition. ■

Proof of Proposition 4. Since all variables are independently and normally distributed. We have:

$$\begin{aligned}
E(x_i^2) &= \left(\frac{[(n-2)X^2 - n\sigma_t^2]}{(n-1)[X^2 + \sigma_t^2]} \right)^2 \left(\frac{\sigma_\varepsilon^2}{X^2} E \left[\left(t_i - \frac{\sum_j t_j}{n} \right)^2 \right] + E \left[\left(\frac{\sum_j \varepsilon_j}{n} - \varepsilon_i \right)^2 \right] \right) \\
&= \left(\frac{[(n-2)X^2 - n\sigma_t^2]}{(n-1)[X^2 + \sigma_t^2]} \right)^2 \left(\frac{\sigma_\varepsilon^2}{X^2} \frac{(n-1)\sigma_t^2}{n} + \frac{(n-1)\sigma_\varepsilon^2}{n} \right) \\
&= \frac{[(n-2)X^2 - n\sigma_t^2]}{(n-1)[X^2 + \sigma_t^2]} \frac{[(n-2)X^2 - n\sigma_t^2]}{nX^2} \sigma_\varepsilon^2
\end{aligned}$$

which is increasing in X^2 , hence from Lemma (4) is decreasing in σ_μ^2 . ■

Analysis of ex ante expected welfare. We need to calculate

$$\begin{aligned}
EW &= E_{(\theta, s_i, y_i, S, \varepsilon_i)} [-\exp[-r(W + \theta(w_i + x + \varepsilon_i) - px)]] \\
&= -e^{-rW} E_{(s_i, y_i, S, \varepsilon_i)} [E_\theta [\exp[-r(\theta(w_i + x + \varepsilon_i) - px) | s_i, y_i, S, \varepsilon_i]]] \\
&= -e^{-rW} E_{(s_i, y_i, S, \varepsilon_i)} [-\exp[-rV(s_i, y_i, S, \varepsilon_i)]]
\end{aligned}$$

where $V(s_i, y_i, S, \varepsilon_i)$ is defined in (29).

Substitution of the expression for x_i in (29) as well as $(E(\theta | s_i, y_i, S, \varepsilon_i), \text{var}(\theta | s_i, y_i, S, \varepsilon_i), \text{and } \delta,$ and assuming a symmetric initial portfolio position, i.e., $w_i = Q/n$ for all i , gives

$$\begin{aligned}
V &= \left[\left(\frac{k_0}{k} \right) \bar{\theta} + \left(\frac{k_1}{k} \right) y_i + \left(\frac{k_2}{k} \right) \frac{nS}{(n-1)} + \left(\frac{k_3}{k} - \frac{k_2}{k(n-1)} \right) s_i \right] \left(\frac{Q}{n} + \varepsilon_i \right) - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n} + \varepsilon_i \right)^2 \\
&\quad + \left(r\sigma_t^2 \frac{k_3}{2nk} \left(k_3 - \frac{k_2}{(n-1)} \right) \left((n-2)k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1 \right) \right) \left[\left(\frac{(s_i - y_i)}{r\sigma_t^2 k_3} - \frac{\varepsilon_i}{\left(k_3 - \frac{k_2}{(n-1)} \right)} \right) \right]^2.
\end{aligned}$$

In the following, we rewrite V so as to bring out a quadratic expression in the vector of variables

$Z = (s_i, y_i, S, \varepsilon_i)$. In order to simplify notation, let

$$\begin{aligned}
A &= \frac{\left(k_3 - \frac{k_2}{(n-1)} \right) \left((n-2)k_3 - \frac{(n-2)k_2}{(n-1)} - 2k_1 \right)}{2nk(r\sigma_t^2 k_3)} \\
B &= -r\sigma_t^2 \frac{k_3}{k} \left(\frac{\left(k_3 - \frac{k_2}{(n-1)} + k_1 \right)}{n \left(k_3 - \frac{k_2}{(n-1)} \right)} \right) \\
C &= \frac{(n-2)}{nk} \left(k_3 - \frac{k_2}{(n-1)} + k_1 \right) \\
D &= \left(\frac{nk_2}{(n-1)k} \right).
\end{aligned}$$

So we can write V as follows

$$\begin{aligned}
V &= \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 \\
&+ \left(\frac{k_3}{k} - \frac{k_2}{k(n-1)}\right)\frac{Q}{n}s_i + \left(\frac{k_1}{k}\right)\frac{Q}{n}y_i + \left(\frac{k_2}{k}\right)\frac{QS}{(n-1)} + \left(\frac{k_0}{k}\bar{\theta} - \frac{r\sigma_t^2 k_3}{k}\frac{Q}{n}\right)\epsilon_i \\
&+ A(s_i^2 + y_i^2) + B\epsilon_i^2 - 2As_i y_i + Cy_i \epsilon_i + DS\epsilon_i + \frac{2C}{n-2}s_i \epsilon_i.
\end{aligned}$$

Since we have $E(Z) = (\bar{\theta}, \bar{\theta}, \bar{\theta}, 0)$, we can write

$$\begin{aligned}
V &= \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 \\
&+ \left(\frac{k_3}{k} - \frac{k_2}{k(n-1)}\right)\frac{Q}{n}s_i + \left(\frac{k_1}{k}\frac{Q}{n}y_i\right) + \left(\frac{k_2}{k}\frac{QS}{(n-1)}\right) + \left(\bar{\theta} - \frac{r\sigma_t^2 k_3}{k}\frac{Q}{n}\right)\epsilon_i \\
&+ A\left((s_i - \bar{\theta})^2 + (y_i - \bar{\theta})^2\right) + B\epsilon_i^2 - 2A(s_i - \bar{\theta})(y_i - \bar{\theta}) + C(y_i - \bar{\theta})\epsilon_i + D(S - \bar{\theta})\epsilon_i + \frac{2C}{n-2}(s_i - \bar{\theta})\epsilon_i
\end{aligned}$$

which can be written as follows

$$V = \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 + \langle t, Z \rangle + (Z - E(Z))' \Delta (Z - E(Z)) \quad (44)$$

with the vector t is such that

$$t' = \left(\left(\frac{k_3}{k} - \frac{k_2}{k(n-1)}\right)\frac{Q}{n}, \frac{k_1}{k}\frac{Q}{n}, \frac{k_2}{k}\frac{Q}{(n-1)}, \left(\bar{\theta} - \frac{r\sigma_t^2 k_3}{k}\frac{Q}{n}\right) \right) \quad (45)$$

$\langle \cdot, \cdot \rangle$ is the dot product and Δ is a 4×4 matrix defined as follows

$$\Delta = \begin{pmatrix} A & -A & 0 & \frac{C}{(n-2)} \\ -A & A & 0 & \frac{C}{2} \\ 0 & 0 & 0 & \frac{D}{2} \\ \frac{C}{(n-2)} & \frac{C}{2} & \frac{D}{2} & B \end{pmatrix}. \quad (46)$$

Substitution of the value of V in the expression of Expected welfare gives

$$\begin{aligned}
EW &= -e^{-rW} E_{(s_i, y_i, S, \epsilon_i)} \left[\exp \left[-r \left(\left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 + \langle t, Z \rangle + (Z - E(Z))' \Delta (Z - E(Z)) \right) \right] \right] \\
&= -e^{-r \left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 \right)} E_{(s_i, y_i, S, \epsilon_i)} \left[\exp \left[-r \left(\langle t, Z \rangle + (Z - E(Z))' \Delta (Z - E(Z)) \right) \right] \right] \\
&= \frac{-e^{-r \left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 \right)}}{(2\pi)^2 |\Omega|^{1/2}} * \\
&\quad \int_{(s_i, y_i, S, \epsilon_i)} \exp \left\{ -r \left(\langle t, Z \rangle + (Z - E(Z))' \Delta (Z - E(Z)) \right) - \frac{1}{2} (Z - E(Z))' \Omega^{-1} (Z - E(Z)) \right\} dZ \\
&= \frac{-e^{-r \left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k} \left(\frac{Q}{n}\right)^2 \right)}}{(2\pi)^2 |\Omega|^{1/2}} * \int_{(s_i, y_i, S, \epsilon_i)} \exp \left\{ -r \langle t, Z \rangle - \frac{1}{2} (Z - E(Z))' (\Omega^{-1} + 2r\Delta) (Z - E(Z)) \right\} dZ
\end{aligned}$$

where Ω is the variance covariance matrix of Z and it is equal to

$$\Omega = \begin{pmatrix} \sigma_\theta^2 + \sigma_t^2 & \sigma_\theta^2 & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & 0 \\ \sigma_\theta^2 & \sigma_\theta^2 + \frac{1}{(n-1)}\sigma_t^2 + \frac{X^2}{(n-1)} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & 0 \\ \sigma_\theta^2 + \frac{\sigma_t^2}{n} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} & \sigma_\theta^2 + \frac{\sigma_t^2}{n} + \sigma_\mu^2 & 0 \\ 0 & 0 & 0 & \sigma_\epsilon^2 \end{pmatrix}.$$

Let Λ such that

$$\Lambda^{-1} = [\Omega^{-1} + 2r\Delta]$$

then

$$\begin{aligned} EW &= \frac{-e^{-r\left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k}\left(\frac{Q}{n}\right)^2\right)} |\Lambda|^{1/2}}{(2\pi)^2 |\Omega|^{1/2}} \\ & * \int_{(s_i, y_i, S, \epsilon_i)} \frac{1}{(2\pi)^2 |\Lambda|^{1/2}} \exp(-r\langle t, Z \rangle) \exp\left\{-\frac{1}{2}(Z - E(Z))' \Lambda^{-1} (Z - E(Z))\right\} dZ \end{aligned}$$

If the matrix Λ is positive definite then the integral part in this equation is written as the moment generating function of a multi-normal distribution with mean $E(Z)$ and variance-covariance matrix Λ . It is such that

$$\int_{(s_i, y_i, \epsilon_i)} \frac{1}{(2\pi)^2 |\Lambda|^{1/2}} \exp(\langle -rt, Z \rangle) \exp\left\{-\frac{1}{2}\left((Z - \mu)^T \Lambda^{-1} (Z - \mu)\right)\right\} dZ = \exp\left\{\left(-rE(Z)'t + \frac{r^2}{2}t\Lambda t\right)\right\}.$$

Thus we have

$$\begin{aligned} EW &= \frac{-e^{-r\left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k}\left(\frac{Q}{n}\right)^2\right)} |\Lambda|^{1/2}}{|\Omega|^{1/2}} \exp\left\{-rE(Z)'t + \frac{r^2}{2}t\Lambda t\right\} \\ &= -\frac{|\Lambda|^{1/2}}{|\Omega|^{1/2}} \exp\left\{-r\left(W + \left(\frac{k_0}{k}\right)\bar{\theta}\frac{Q}{n} - \frac{r\sigma_t^2 k_3}{2k}\left(\frac{Q}{n}\right)^2 + E(Z)'t - \frac{r}{2}t\Lambda t\right)\right\}. \end{aligned}$$

Yet we have

$$E(Z)'t = Q\bar{\theta}\left(\frac{k_1 + k_2 + k_3}{nk}\right)$$

so that

$$EW = -\frac{|\Lambda|^{1/2}}{|\Omega|^{1/2}} \exp\left\{-r\left(\left(W + \frac{Q\bar{\theta}}{n} - \frac{r\sigma_t^2 k_3}{2k}\left(\frac{Q}{n}\right)^2\right) - \frac{r}{2}t\Lambda t\right)\right\}$$

This gives the expression of expected welfare that will be used in the numerical analysis. ■

Figures

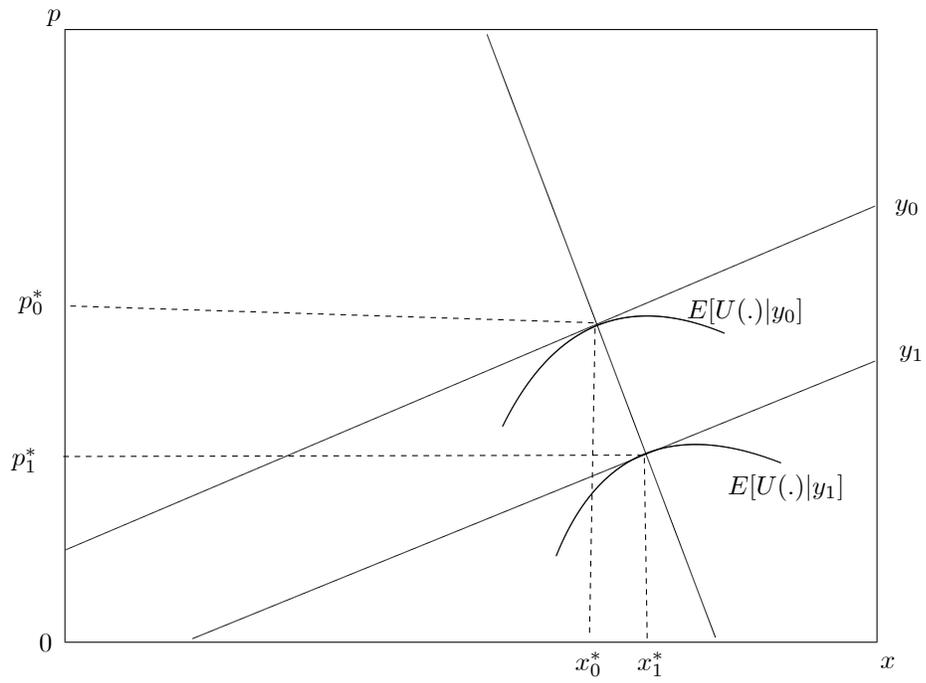


Figure 1: construction of bid functions

Note: For the following figures, the parameters values considered are: $n = 20$, $r = 2$, $Q = 15$, $W = 10$, $\sigma_t = \sigma_\varepsilon = 2$ and $\bar{\theta} = 1$.

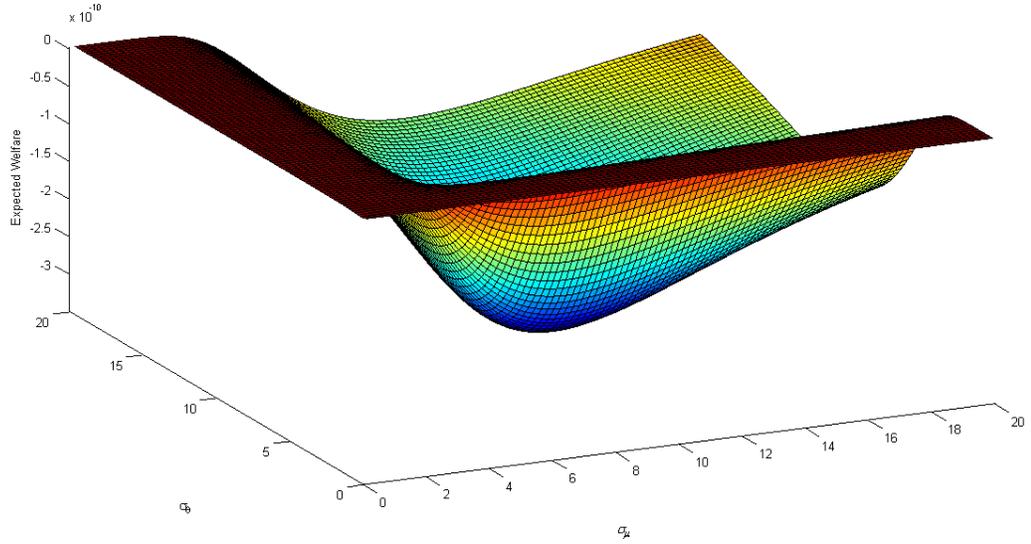


Figure 2: Expected welfare as function of σ_μ and σ_θ

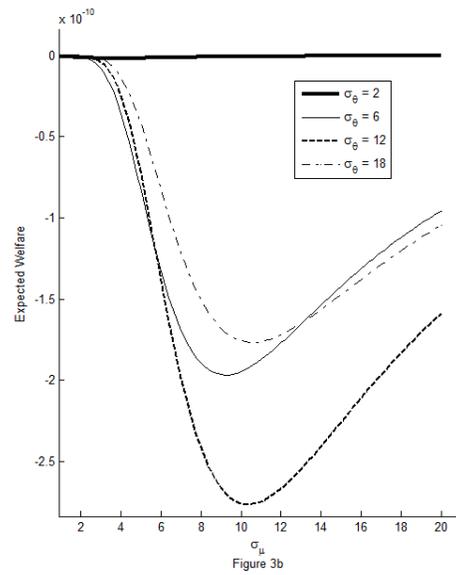
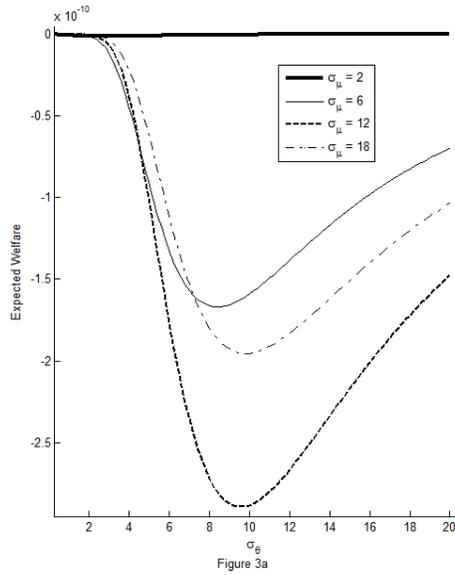


Figure 3: Expected welfare as function of σ_θ (Figure 3a) and σ_μ (Figure 3b)

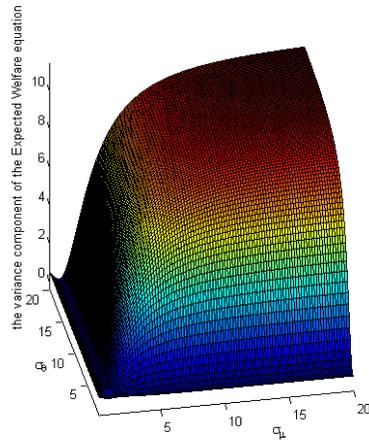


Figure 4: The variance component of the Expected Welfare equation as a function of σ_μ and σ_θ

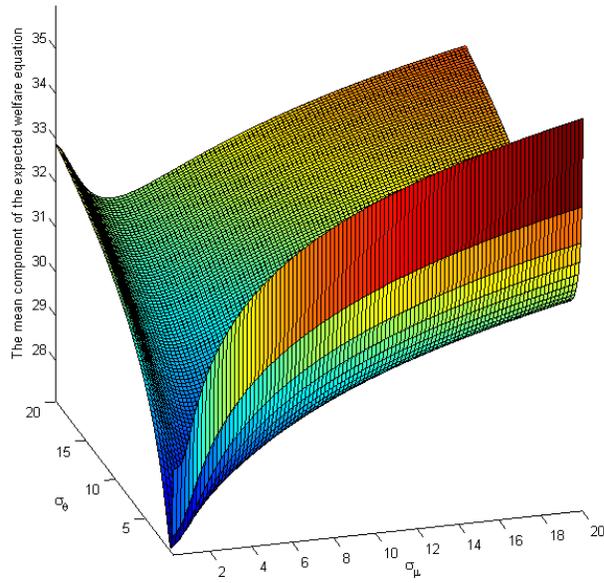


Figure 5: The mean component of the expected welfare equation as a function of σ_μ and σ_θ

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