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Performance Measurement in Supply Chain: New Network Analysis and Entropic Indexes

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Abstract

Industrial organisations must supply a variety of products and services, meet the needs of fragmented customer expectations, and cope with the consequences of the globalisation of world markets, all of which are producing significant levels of complexity. This study develops a new quantitative measurement of complexity for supply network based on Network Analysis, which is often used to study natural ecosystems, focussing in particular on the concept of entropy of information. The research reports advances in both theory on Supply Network Analysis problem and on its application to industrial contexts. Eight indexes based on entropy are presented. These measures provide a meaningful analysis of the level of complexity in the whole supply network mapping the exchanges of goods between the different actors in the network. The impact of possible modifications of the structure can simply be evaluated using these tools, providing a simple evaluation of the different scenarios. The proposed method takes an holistic point of view to tackle the problem of supply network optimisation. A real world application of the developed new methodology is presented.

1 Introduction

A modern supply network provides very complex inter-correlations between its various actors (i.e. the suppliers, manufacturers, distributors, customers, …) based on material, data and financial flows (Harland, 1998). This work focuses on complex supply network (which is the 4th level supply chain, as defined by Harland, 1996), concentrating not only on the materials and information flow analysis, but also on the network structure development and re-design.

We can generally distinguish two types of Supply Network Optimization problems:

1) Network flows optimization: in this case we consider a pre-designed or existing distribution network, and we want to optimize the flows of goods-information-money through the network, without changing its structure (number of layers, depots, warehouses, ...).

2) Network design or re-design: in this case we want to find the best configuration of facilities and relationships in order to satisfy the goals of the company and reduce the complexity of the network structure.

This research addresses in particular this last point. The links and the constraints on actors are numerous and mutually interdependent, with the traditional approach providing research into optimal local work conditions: each actor aims at obtaining best performance for his own local system. Consequently, optimal effectiveness in a global logistic network is not usually reached. One of the fundamental challenges for Supply Chain Management (Harland, 1998; Tapscott, 2000) is to reach optimal performance for the whole network from an holistic point of view. For example, this idea is stressed by Manzoni and Islam (2006). Companies cannot afford to remain isolated as their survival depends on their ability to organise an efficient "supply web" that brings value for all participants.

This work tackles supply chain complexity analysis, proposing a new methodology for supply network structure optimization and monitoring based on an entropic model using eight different performance measures.

Monitoring supply chain complexity is very important for two reasons. First, the information obtained results in good knowledge of the global system, and so a clear definition of the causes and effects of problems. Second, it supports the research into the best solutions for a network very effectively by comparing the various possible alternatives to provide objective and quantitative analysis.

Specifically this study looks deeply into natural ecosystems, following the idea that there is a great morphological analogy between ecosystem networks and industrial supply networks, and because

of these analogies, several innovative concepts and methodologies successfully applied in natural systems can be adapted to industrial supply networks.

This work represents a second phase of the research initiated by Battini et al. (2006). The fundamental development is a new set of quantitative entropic measures that can be used to analyse supply networks and to assess their performances in terms of complexity and development in the network structure. These are introduced, discussed theoretically, and then applied to real world applications.

The next section presents a brief review of the literature focusing on supply chain management and on the measurement of network complexity. Following this, Section 3 presents the proposed new methodology, after which a real world application and its results are discussed in section 4. Finally, conclusions and limitations concerning the potential application of entropy measurement in real supply chains are discussed in section 5 and 6.

2 Review of the literature

In recent years several authors have proposed different approaches to Supply Chain Management techniques and to Supply Network Complexity Computation.

Figure 1 summarises published studies that analyse Supply Chain Management and production systems complexity considering both internal chain and external chain aspects (Harland, 1996).

The literature divides complexity into three types – first, "static complexity" i.e. linked to system structure, second "dynamic complexity" i.e. related to the material and data flows between different actors, and third "decisional complexity" created by the managerial choices required.

Four types of approaches can be identified:

•Introductions and/or general studies. The whole problem of Supply Chain Performance Management and Control is presented, and the complex features of modern supply network are underlined, with a large set of supply chain performance indexes and software packages being introduced to support the decision making and mapping of the supply network (i.e. Huan et al., 2004; Tan and Platts, 2004).

•Statistical approaches. Analysis of the correlation between qualitative measurements of complexity and general supply chain performance indexes (i.e. Perona and Miragliotta, 2004; Milgate, 2001).

•Entropic models. Mathematical models derived by the "entropy of information" (developed during WWII for measuring and separating information codes, Shannon and Weaver, 1948) that quantify the complexity of supply chain and manufacturing systems (i.e. Frizelle and Woodcock, 1995, Calinescu 1998, and Sividasan 2002).

•Surveys: Mills et al. (2004) propose and discuss several methods that provide support to companies in a complex supply chain, populated by a great many actors. The Authors, however, do not discuss the problem of complexity in the supply chain but rather emphasise the need for studies on this important topic, justifying and validating the aims and objectives of the present research.

Place Figure 1

A great many authors have examined the relationship between performance/flexibility and complexity (Calinescu et al., 1998; Milgate 2001; Perona and Miragliotta, 2004; Arteta and Giachetti, 2004), the results are however insufficient to validate a robust relationship.

Nevertheless, it is extremely important to find the best trade-off between these parameters since poor control of complexity can produce poor performance and poor quality, generating significant additional costs. This is a substantial challenge, and Helo et al. (2006) even assert that a supply chain may be too complex and too difficult to analyse, exceeding human information-processing capabilities.

After applying this principle, Meijer (2002) proposed an organisation design methodology based on the development of different alternative solutions and then reducing the number of alternatives until only the best alternative is left. Similarly Tan and Platts (2004) developed software to support managerial decisions. Several authors (Huan et al., 2004; Bulliger et al., 2002) trust to the SCOR model (Supply Chain Operation Reference model) developed by the Supply Chain Council (SCC) (http://www.supply-chain.org/cs/root/home).

The recent literature shows that companies are taking a growing interest in the global supply network, from raw materials to final products. So it becomes intrinsically more important to consider company performance by correlating it strictly to its global supply chain performance that covers suppliers, production system, and distribution network. From this reason, from an holistic point of view, system management is confronted by great complexity, and optimal measurement and management of complexity are strategic advantages.

Perona and Miragliotta (2004) define the skill of managing supply chain complexity as strategically fundamental to modern organisations. Complexity is always transferred between actors in a Supply Chain (Sivadasan and Efstathiou, 2002). The empirical evidence shows that companies usually manage complexity in four ways (Sivadasan et al., 2003):

- by exporting operational complexity to other actors in their own supply chain;
- by charging for the service of coping with imported complexity;
- by investing in precautionary systems that work to avoid complexity generation;

- by investing in resources to absorb complexity.

The most important activities in confronting complexity are to understand it, and above all, to measure it. People generally have an intuitive understanding of complexity, but experience great difficulty confronting it rigorously according to Arteta and Giachetti, (2004), or according to Bullinger et al. (2002) "only something that can or has been measured improves and only an holistic approach prevents the adoption of sub-optimal decisions". And in literature it is possible to find several methodologies to measure and reduce complexity. Some are either models based on graph theory (Seese and Schlottmann, 2001), or statistical models (Milgate, 2001; Beamon et al., 2001; Blackhurst et al., 2004), others exploit entropic measurements (information entropy).

The concept of entropy was introduced by Shannon and Weaver (1948) to measures the level of uncertainty (or the information level) found in an unclear signal. Since complexity produces uncertainty in flows (materials and information), increases lead times, and results in aleatory operations, entropy of information is a valid system for the measurement of complexity in an industrial system, and can specifically be used to measure the complexity of a global supply chain (Frizelle and Woodcock, 1995). Karp and Ronen (1992) propose entropic indexes to demonstrate that decreasing batch dimensions and the use of just in time (JIT) solutions require less information, which means that the level of uncertainty is less critical.

Frizelle and Woodcock (1995) defined a measurement of the first type of complexity (static) and introduced a definition for the second type of complexity (dynamic). This complexity deals with the uncertainty found in material and data flows, which mainly evidences itself in supply chains in queue formation in input from and/or output to different participants (Sivadasan and Efstathiou, 2002). While Deshmukh et al. (1998) enlarged this approach by considering the relationship between resources, the concept is further developed by Shih and Efstathiou (2002), who proposed an algorithm to analyse the effects of different manufacturing network configurations, and Calinescu et al. (1998) put forward two complementary methodologies to estimate the complexity of a production system: the entropic procedure introduced by Frizelle and Woodcock (1995), and a similar method named MFC proposed by Mayer and Foley Curley (1995). Efstathiou et al. (2002) proposed a web-based expert system that mainly focuses on a third kind of complexity (organisational), and is based on measurement of the entropy generated by information transfers. They defined "decision making entropy" as the level of entropy (organisational entropy) required for decisions to be taken correctly, and Fujimoto et al. (2003) published a very practical application of complexity measurement for which they use an entropic approach to evaluate the complexity of an assembly line.

A new measurement of complexity at the business process level of an organisation was developed by Arteta and Giachetti (2004) by creating a Petri net model of the system in order to obtain a probabilistic analysis of the system. They argued that less complex processes are easier to change and thus more agile, but much more extensive validation and exploration of the link between agility and complexity is required.

Deshmukh et al. (1998) tried to take the fundamental step of introducing a potential link between complexity and performance of a production system. Sivadasan et al. (2003) applied this approach to different real world cases so as to check its robustness, while Wu et al. (2001) used simulation to carry out a similar validation, while Battini et al. (2006) demonstrated the potential of using the Average Mutual Information (AMI) index to classify the level of organisation (the opposite of complexity) in a supply chain by applying an entropic parameter in measuring material flows to a real supply network.

In the belief that information entropy is a very promising measurement of supply chain complexity, this work presents the theory and then applies a new approach that can be viewed as a logical extension of the above mentioned studies of manufacturing system complexity.

3 New proposed method

As stressed by Calinescu et al. (1998), opinions on what complexity is, why it should be measured, and how this can be done vary widely. The literature proposing entropic methods to analyse complexity have focused the attention on the information entropy as explained by Shannon, aiming to measure the uncertainty characteristics associated with material and information flows of a manufacturing network (i.e. Frizelle and Woodcock, 1995; Karp and Ronen ,1992). Sivadasan and Efstathiou (2002) extended this concept to a single-vendor and single-buyer system for the first time, but what we propose is a quantitative method to consider an entire supply network in which many different partners, positioned in different levels of the chain, are involved.

Since this study investigates complexity and organization levels in supply networks, the following two definitions are reported to help the reader's comprehension of the procedure presented below:

a. Network Complexity: a measure of the uncertainty level associated to all flows (i.e. material and data flows) exchanged between partners in the network (derived from Frizelle and Woodcock, 1995).
b. Network Organization: a measure of the specialization (or certainty) level associated to all flows (i.e. material and data flows) exchanged between partners in the network (derived from Ulanowicz, 2003).

As mentioned previously, the proposed procedure is derived from ecological theory, and quantifies not a single, but a set of parameters to better understand network structure. Both Food Webs and Ecological Networks use the graph-theory (Bang-Jensen and Gutin, 2000) to describe ecosystems by examining compartments (animal or other species) and their links and exchanges (flows of energy and matter). Ecological Network Analysis (ENA) consists of a set of tools used by researchers to test the degree of organisation in an ecosystem, analyse the pathways occurring in the system, evaluate the number of levels, estimate indirect effects, and more. Ecologists have worked to identify which ecosystem indexes quantify global attributes of a natural ecosystem (Ulanowicz and Kay, 1991). The analysis of ecosystem network relationships and transfers (Ulanowicz, 2003) and ecological network indicators (Ulanowicz, 2004) are also useful tools with which to study the performance of industrial supply chain networks. They help understand and quantify the complexity of process interactions, measure the organisation level and identify dynamic bottlenecks in the system. Ecosystems are collections of plant and animal species organised in complicated web-like structures by which energy and matter are transferred and transformed (Battini et al, 2006). Similarly, the supply chain of a company consists of different departments ranging from material procurement to customer services. A number of socio-economic activities take place along the supply chain that transfer and transform energy, information, and goods or services. These processes create functional connections that link the activities to one another in a web-like structure. In both supply chains and ecological systems this web-like structure is described by a network.

In both cases the performance of the whole system is strongly dependent on the uncertainty of flows, on the number of partners involved ("network nodes") and on the number and kind of exchanges between them ("network edges"). Furthermore, it is important to understand the trade-off between complexity and organisation of the network structure. In fact, maximum efficiency (minimum complexity) for the network often means maximum vulnerability and less flexibility in the management of sudden changes. On the other hand, high redundancy and complexity of nodes and edge links increases total costs and reduces system performance. Battini et al. (2006) proposed a new morphology analogy between these two types of networks. In order to find common study methodologies, this work applies network analysis paradigms and ecological indicators to modern supply chains so that the complexity and organisation levels of an industrial network can be studied. The method is applied in two main steps: 1.Network mapping and quantification of flows 2.Network analysis and computation of indexes.

Ecological network analysis provides a set of tools that can be used to picture the structure of the supply network under investigation. These tools are divided into: 1. Input/Output analysis; 2. Trophic level analysis; 3. Analysis of cycles; 4. Performance indexes computation. This last one is

the focus of our work, which presents how to compute performance network indexes and how they can help to understand and analyse a supply network.

3.1 Phase 1: Network mapping and quantification of flows

Ecological networks can be summarised using vectors and matrices. A network is composed of a triplet G (V; E; W), where V represents the nodes (compartments) and E the edges (arcs, arrows) associated with weights W. To help understand and illustrate the methodology, a simple industrial example involving a small supply network composed of 7 different partners (compartments) is presented in Figure 2. The network consists of 3 suppliers of manufacturing components or subassemblies, 1 manufacturer (where the final product is assembled), and 3 customers.

Place Figure 2

First of all, a unit of measurement needs to be selected in order to express flows between partners in the network, i.e. goods exchanged in tons/year. Then the inputs from outside the system into each compartment in the given period need to be measured, that means for example raw materials coming from the environment outside the network. This will form the *Import* vector, called "I". The flows exiting the system can be divided into reusable material, *Exports*, called "E", and *Dissipations*, called "D" (that means for example production losses).

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$$I = \begin{bmatrix} 103 \\ 154 \\ 52 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad E = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 178 \\ 19 \\ 88 \end{bmatrix} \qquad D = \begin{bmatrix} 3 \\ 4 \\ 2 \\ 7 \\ 5 \\ 1 \\ 2 \end{bmatrix}$$

Following a matrix representing the *transfers* of goods between partners, inside the system, called the *Transfers Matrix T*, can be set up, and an *Extended Transfers Matrix T** can be associated with the oriented graph. This matrix reports all information about exchanges occurring in the network (Figure 3). The Extended Transfer Matrix T* in Figure 2 includes all the flows occurring inside the system and all the exchanges with the external environment, as shown in Figure 4.

Place Figure 3

Place Figure 4

If mass balance is met for the system, then:

$$\sum t_{0,i} + \sum t_{j,i} = \sum t_{i,j} + \sum t_{i,n+2} + \sum t_{i,n+1} \quad (1)$$

The same formula written in compact notation for each compartment *i* is:

$$T_{.i} + I_i = T_{i.} + E_i + D_i \quad (2)$$

where the 'dot' stands for summation across the whole row/column. Because of the complex procedure of network construction, the rough data is unlikely to be balanced (steady state). If steady state is achieved, then mass balance exists around every node (incoming edges perfectly balance outgoing ones). In order to achieve steady state condition in ecological networks, researchers change the coefficients as seldom as possible. It is worth noting that, while mass balance plays a fundamental role in some of the procedures sketched above, it is not as important when dealing with information indexes (Ulanowicz, 2004). We can therefore assume without loss of generality that the networks are in steady state. The results will also extend to the non-stationary case.

3.2 Phase 2: Network analysis and calculation of entropic indexes

Ulanowicz assembled the primary methods used in Network Analysis into a single software package, NETWRK (Ulanowicz and Kay, 1991), used in the present study. Among the various types of analysis performed by the software, this study focuses on the calculation of entropic indexes that characterise the entire system: Total System Throughput (TST), Average Mutual Information (AMI), Ascendancy (ASC), Development Capacity (DC), and Overhead and its four components, the first of which is Redundancy (RED). Having identified the indexes requiring calculation, they need to be computed and used in understanding and analysing a supply network.

Total System Throughput - TST

The sum of all coefficients i.e. the “size” of the system or the total amount of the medium (goods, product pieces, product tons, money, etc..) flowing through the network is called Total System Throughput, and can be expressed with the following formula:

$$TST = \sum_{i=0}^{N+2} \sum_{j=0}^{N+2} t_{ij} = t_{..} \quad (3)$$

Where $t_{..}$ means sum across all rows (first dot) and columns (second dot). Similarly, $t_{i.}$ is the sum of the i^{th} row, and $t_{.j}$ the sum of the j^{th} column. Consequently, the TST for the network in Figure 2 can be computed as the sum of all flows:

TST = 1211 tons/year

This index quantifies the growth of the network because it is based on the number of nodes and the quantities transferred in the system. However, this quantity does not provide information about the distribution of the flows inside the system.

Average Mutual Information AMI

The Extended Transfer Matrix T* includes all flows inside the system and all exchanges with the external environment.

The probability of a product (unit of load, work piece, truck, ton of materials, etc..) moving from compartment *i* to compartment *j* is assumed to be proportional to the flow from *i* to *j*:

$$p_{O,I}(i,j) = \frac{T^{ij}}{T_{..}} \quad (4)$$

Shannon and Weaver (1948) introduced a measurement of the entropy associated with a process, and his published theory (1948) explains the computation of the analytical entropy measurement. The entropy is the sum of the probabilities of each possible outcome *i* times the logarithm of the associated probability:

$$H_X = -\sum_{i \in X} p(i) \log p(i) \quad (5)$$

A supply chain network can be depicted as a collection of transition probabilities (i.e. the probability of finding a "quantum" of the exchanged goods or product pieces moves from a certain box to another at any time), and the entropy of the system can be computed ($p \cdot \log(p)$) by considering inputs and outputs to and from any node.

In particular, in this study the network is represented as a matrix (T*) and the entropy associated with row sums (probabilities of leaving the boxes) and column sums (probabilities of entering the boxes) is computed. If, at a given time, a product travelling in the system is marked at random, the probability associated with the event "the product moving from compartment *i* to compartment *j*" will be found, and this quantity is the probability associated with the arrow from i to j.

The entropy associated with events such as "a product leaving compartment *i* and entering compartment *j*" is usually called the *joint entropy* $H_{O,I}$:

$$H_{I,O} = -\sum_{i=0}^{N+2}\sum_{j=0}^{N+2} p_{I,O}(i,j)\log p_{I,O}(i,j) = -\sum_{i=0}^{N+2}\sum_{j=0}^{N+2} \frac{t_{ij}}{t_{..}}\log\frac{t_{ij}}{t_{..}} \quad (6)$$

The entropy associated with outputs from compartments will therefore be:

$$H_O = -\sum_{i=0}^{N+2} p_O(i)\log p_O(i) = -\sum_{i=0}^{N+2} \frac{t_{i.}}{t_{..}}\log\frac{t_{i.}}{t_{..}} \quad (7)$$

and the entropy associated with inputs into compartments:

$$H_I = -\sum_{j=0}^{N+2} p_I(j)\log p_I(j) = -\sum_{j=0}^{N+2} \frac{t_{.j}}{t_{..}}\log\frac{t_{.j}}{t_{..}} \quad (8)$$

These quantities will be positive or null, and will possess all the properties of entropies. In the example network, the contribution of each coefficient to the joint entropy is $-\frac{t_{ij}}{t_{..}}\cdot\log\left(\frac{t_{ij}}{t_{..}}\right)$

The joint entropy is obtained by summing all contributions (Figure 5): $H_{I,O}$= 3.463 bits.

Place Figure 5

In the same way, the contribution of each compartment to the entropy associated with inputs H_I can be found by computing the column sum $t_{.j}$ for each compartment: the contribution of compartment j will be $-\frac{t_{.j}}{t_{..}}\log\left(\frac{t_{.j}}{t_{..}}\right)$ and the entropy associated with output *Ho* will be: $-\frac{t_{i.}}{t_{..}}\log\left(\frac{t_{i.}}{t_{..}}\right)$.

Conditional probabilities and entropies associated with events of the form "a product that is now in compartment i moves to compartment j" can be defined. In this case it is known that the product is currently in compartment i, but the uncertainty associated with the next destination needs to be measured. The associated entropy is:

$$_{I|O}(j\,|\,i)\frac{p_{I|O}(i\,|\,j)}{p_O(i)}\frac{t_{ij}}{t_{i.}} \quad (9)$$

In the same way, conditional probabilities and entropies associated with events of the form "a product that is now in compartment j moves to compartment i" can be defined as:

$$_{O|I}(i\,|\,j)\frac{p_{I|O}(i\,|\,j)}{p_I(j)}\frac{t_{ij}}{t_{.j}} \quad (10)$$

The associated total entropy is:

$$H_{I|O} = -\sum_{i=0}^{N+2} p_{I|O}(j \mid i) \log p_{I|O}(j \mid i) = -\sum_{i=0}^{N+2} \sum_{j=0}^{N+2} \frac{t_{ij}}{t_{..}} \log \frac{t_{ij}}{t_{i.}} \quad (11)$$

$$H_{O|I} = -\sum_{i=0}^{N+2} p_{O|I}(i \mid j) \log p_{O|I}(i \mid j) = -\sum_{i=0}^{N+2} \sum_{j=0}^{N+2} \frac{t_{ij}}{t_{..}} \log \frac{t_{ij}}{t_{.j}} \quad (12)$$

The following important identity will be used to define *Average Mutual Information or AMI*:

$$H_{I,O} = H_I + H_{O|I} = H_O + H_{I|O} \quad (13)$$

This identity shows that the joint entropy is equal to the sum of the entropy associated with Inputs (Outputs) plus the conditional entropy on Outputs given the Inputs (Inputs given the Outputs).

$$H_{I,O} \leq H_I + H_O \quad (14)$$

The "Average Mutual Information" (AMI) is defined as:

$$AMI = H_O - H_{O|I} = H_I - H_{I|O} = H_O + H_I - H_{I,O} \quad (15)$$

This formula explicitly states that the information is equal to the decrease in entropy associated with inflows once the outflows are known (or the decrease in outflow entropies once the inflows are known), and that AMI possesses symmetry. The AMI index of the network model in Figure 2 is:

$$AMI = H_O + H_I - H_{I,O} = 2.666 + 2.766 - 3.463 = 1.969$$

In a network of exchanges many configurations are compatible with the same *throughput level* (TST). More constrained topologies are those in which a restricted number of flows exists so that the medium is forced to move along a limited number of pathways. This occurs when compartments in the system are more functionally specialised. The AMI index measures this degree of specialisation or the amount of constraints on the medium.

Place Figure 6

The two entropies are represented as areas (Figure 6): their joint entropy H(x; y) is represented by the area in the bottom left-hand corner. The AMI is the overlap of the two areas. AMI is a measure of how constrained the material flows are. When each compartment is connected with every other compartment and the flows are all the same, the AMI is 0 (the two areas are disjoint). In other words, the fact that a product exits in a certain compartment provides no information on the next destination. The opposite case is represented by the complete overlap of the two areas. In this situation knowing that a product is in a certain compartment implies that it will enter another known compartment. The flows are completely constrained.

Ascendancy, Capacity, and Overhead

Because AMI is a-dimensional, Ulanowicz and Kay (1991) proposed to scale it for the sum of all TST flows. This would combine the size of the system (TST) with its degree of organisation/development (AMI), creating a combined measurement called *Ascendancy*.

$$A = TST \times AMI = \sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij}\log\left(\frac{t_{ij}t_{..}}{t_{i.}t_{.j}}\right) \qquad (16)$$

Ascendancy is defined as the product of the AMI and the TST, and measures the level of development within a system considering both the size of the flows (the TST) and their organisation (the Average Mutual Information Index (AMI).

The sum of all the flows in a network yields the total amount of goods, money, information that flows through the industrial system. This quantity estimates the level of activity pertaining to the supply network, in other words the level of activity that quantifies the size of the network. The process that is directly linked with size is growth. Therefore, the growth of a supply network could be quantified by measuring TST, which depends on both magnitude of flows and number of partners involved. Growth pertains to the "extension" of a system, but does not provide detail about how material and money are distributed within the network. It is possible for supply chains with the same TST to be characterised by totally different flow configurations. As shown above, higher values of AMI pertain to flow structures maximally constrained in terms of goods movement. Highly organised supply chains are those where the distribution of goods takes place along a few efficient routes, which decreases the managing costs of the whole system. From this we can deduct that highly redundant flow networks are considered to be less organised and they possess lower AMI values.

Just as in ecological ecosystems, Supply Chains should develop in the direction of a more organised structure of exchanges, where development is identified by any increase in the mutual information of the exchange configuration as quantified by the AMI. Ascendancy measures the fraction of goods, money, and information that a supply network distributes in an efficient way. In combining system activity and organisation, it provides a unique measurement of growth and development.

"In ecology, high values for ascendancy represent a mature food web where species are specialised, exchanges are structured, and internal cycle and transfer are efficient. Should an ecosystem be developed and organised to its fullest potential, the ascendancy equals the Development Capacity, which forms the upper boundary of the ascendancy" (Allesina, 2004). If Supply Network life could be subdivided into four stages of a) introduction b) growth c) maturation d) decline, such as in the life cycle of a product, it is likely that increase in activity will dominate the first two stages and will decline as the ecosystem becomes more organised. "In this latter phase, the throughput accumulated

at the beginning is redistributed and organised so that the mutual information of flows increases", (Allesina, 2004). By scaling the joint entropy using TST, the maximum development capacity of the system is obtained:

$$C = TST \times H_{I,O} = \sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij} \log\left(\frac{t_{ij}}{t_{..}}\right) \quad (17)$$

The development capacity is calculated by multiplying the TST by the entropy generated by the flows (i.e. how different compartments are used as inputs by other living compartments). The total Capacity C represents the maximum potential at the disposal of a system and as this is what can be used to achieve further development, it is the upper limit for ecosystem organisation. The capacity is then partitioned into organisation of flows (Ascendancy A) and redundant, non-organised flows (Overhead Φ). The amount of the Development Capacity remaining non-organised is called Overhead and this is equal to the differences between C and A:

$$\Phi = C - A = TST \times (H_{I,O} - AMI) = TST \times (H_{O/I} + H_{I/O}) = -\sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij} \log\left(\frac{t_{ij}^2}{t_{i.}t_{.j}}\right) \quad (18)$$

The overheads can be partitioned into 4 different contributions: Overhead on Imports, Exports, Dissipations, and Redundancy. The first three components are based on the exchanges with outside the system, while the latter pertains to the functional overlap of the pathways in the system. All four contributions are usually expressed by ecologists as a percentage on the Capacity of the system: this aspect is useful as it allows different networks to be compared one with another.

High values of Redundancy reflect a high proportion of parallel pathways in the system.

Φ_I represents the Overhead in Input, Φ_E the Overhead in Export, Φ_D the Overhead in Dissipation, and R the Redundancy. Figure 7 reports all principal system entropic indexes introduced in this paragraph and numerical values for the example network in figure 2. It is interesting to note that these four contributions are usually expressed by ecologists as a percentage on the Capacity of the system: this aspect is useful as it allows different networks to be compared one with another.

Place Figure 7

As Bullinger et al. (2002) stated in their study, to achieve logistic excellence in such complex and highly dynamic supply chains requires continuous in-depth analysis of the entire network reality, supported by measurements and a holistic point of view. In agreement with this point of view, the quantitative measurements presented in this work provide a picture of the complexity and the organisation level of the whole supply network.

4 Methodology application

Computer simulation is widely used in manufacturing systems to validate the effectiveness of tentative decisions, such as a new plan or a new schedule, and to study supply chain behaviour and performance. Wu et al. (2001) applied simulation to study aspects of complexity in the supply chain and through a case study demonstrated and validated that the complexity index as a measure of uncertainty (Frizelle and Woodcock, 1995) is generic and stable. However, simulation is often difficult and time consuming when applied to very articulated supply chain and so this study aims to demonstrate that even a set of entropic parameters like the one proposed is easy to compute and can support evaluation of the potential for structural changes.

This study investigates the supply network of an Italian company selected to test the research methodology. The company produces industrial catering equipment in sheet stainless steel for both the professional market and domestic use. It comprises three manufacturing units, with widespread Italian sales coverage and an international distribution network. Initially 9 classes of nodes/partners by which we can map the supply network of the company need to be identified:

1.Raw Material supplier ("RM supplier")

2.Semi-finished Components Supplier ("SFC supplier")

3.Sub-contractor

4.Production Plant

5.Distributor

6.Direct Sales Agency ("DS Agency")

7.Standard Customers ("SC")

8.Directional Customers ("DC", i.e. big supermarkets)

9.Hotel Chains Customers ("HC").

Material Flows between partners could be measured in different units of value, such as tons of steel per year, Europallets per year, money value per year, etc. In order to ease the ecological method application, values of goods flows are measured in one unit only: tons/year of steel, which is the raw material of all equipment produced by the company studied. An industrial network with a large number of nodes and edges produces uncertainty in the medium that flows through the network, as demonstrated for information by Shannon and Weaver (1948). The uncertainty is linked with the nature of the network structure (graph). In the industrial example reported, the supply chain is depicted in its real configuration (as is) and in its future (foreseen, after improvement) configuration (to be). For this reason historical data have been collected to depict the supply chain "as is" and they are certain and deterministic (tons of goods per last year) and predictions of future data have

been made to depict the supply chain "to be". The uncertainty is due to the complexity of the graph structure.

Figure 8 represents the whole complex network of the industrial group and depicts the present situation of the supply network ("as is"). The company is planning five different management strategies to improve network organisation and increase global efficiency:

1) To reduce steel scraps and increase the productivity of the production plant by purchasing new pre-cut sheet steel in different sizes. This choice will reduce Dissipation values inside the production plant b) by approximately 50%, reducing unorganised flows (Overhead Φ) and Total System Throughput (TST), which is simply the sum of all coefficients, that is to say, the "size" of the system or the total amount of medium flowing through the network.
2) To cut redundant connections and the recycling of goods via the Sub-contractor (second level components supplier). Consequently, the 4th component of the Overhead, the Redundancy, which reflects parallelisms in trophic pathways, is reduced and network organisation will consequently increase. Total System Throughput (TST) will decrease, cutting redundant connections in the web.
3) To reduce the number of Raw Materials suppliers from 5 to 3, which means simplifying the in-bound net. The presence of positive feedback partnership with Raw Material Suppliers forces the supply network to develop towards less redundant and more efficient configurations. The new network will have two nodes less and the Overhead in Input (Φ_I) will consequently decrease.
4) To provide direct shipments of finished products from Production Plant c) and its direct customers. By performing direct shipments the Redundancy of flows and TST will be reduced, but at the same time the Overhead in Export (Φ_E) will increase.
5) To manage and provide direct shipments from Production Plant b) and all foreign directional customers. This will decrease the Redundancy in out-bound and TST value.

Place Figure 8

To increase network efficiency, possible management choices were represented: the future configuration ("to be") of the resulting network is shown in Figure 9 where the dashed lines indicate the material flows subject to changes according to the strategies explained above.

The supply chain network has been translated in the Extended Transfer Matrix T^*, which as discussed in §3, reports all information about the network exchanges, and the computations

explained above have been applied to quantify the supply chain organisation level and complexity level before and after the improvements.

Complexity makes it difficult to make decisions and understand the consequences and result of the modifications. From this point of view, Bullinger et al. (2002) stress the idea that to benefit from supply chain management an essential precondition is a structured analysis of the network-specific optimisation opportunities. Furthermore, Shih and Efstathiou (2002) applied information entropy to indicate the effect of modification in the manufacturing network.

The case study in this section also aims to demonstrate whether or not network analysis is a useful tool, able to compare alternative supply chain configurations arising out of different choices and strategies. Figure 10 reports the two matrixes [T*] in which flows have been quantified in steel tons/year for both present and future configuration.

Place Figure 9

Figure 11 reports the values of system network indexes and the percentage improvement made. Measuring TST quantifies the size of total supply chain, which depends on both magnitude of flows and number of compartments. In this figure it is clear that the future configuration is reduced by approximately 9% due to the reduction in dissipations and redundant flows. At the same time, the total Capacity C, which represents the maximum potential that a system has at its disposal to achieve further development, decreases by 12.8% as a result of the reduction in TST and Joint Entropy. As shown in section § 3, higher values of AMI are obtained for flow structures in which movement of goods and energy within the system are maximally constrained.

These systems are also highly organised. Only a small increase in AMI index is obtained in this industrial case, so the other performance indexes must be computed in order to understand whether or not the supply network might develop a more organised structure of exchanges. In other words, identifying this organisation degree only as any increase in the mutual information of the exchange configuration is not enough, so the other six system indexes need to be computed and expressed as a percentage of system Capacity. This will be useful when one network needs to be compared with another (Figure 12).

Place Figure 10

The calculations in Figure 11 show an increase in Ascendancy of 5.9% within the new network, while Input Overhead (Φ_I) decreased by 21.7%, Dissipation Overhead (Φ_D) by 8.9%, and

Redundancy (R) by 6%. These values were all achieved by reducing dissipations, pruning redundant connections, limiting partner duplication in the supply web, depending on innovative management strategies. Otherwise, the reduction in dissipation would provide an increase in the productivity of finished products with an obvious increase in Export Overhead (Φ_E) of 19.9%. An increase in Export Overhead is often a direct result of both improved system productivity and of a higher degree of complexity, which arises from new direct shipments between company and the customers. On the one hand, this will provide an increase in sales, as desired by the company, while on the other hand, it will result in an increase in management shipments and sales costs for the company.

Figure 13 shows that the capacity of the present "as is" network configuration is divided into 46.9% in flows organisation (Ascendancy A) and 53% in redundant unorganised flows (Overhead Φ). The capacity in the future configuration is then divided into 49.7% Ascendancy and 50.2% Overhead.

To conclude, the management choices of the company will increase the Ascendancy of the network from 46.9% to 49.7%, but there is still the question of what is the best trade-off between organisation and disorganisation of a web. In other words, what is the best trade off between simplification and complexity in a supply network? 50.2% of the complexity is retained in this network, fundamentally due to logistic and economic constraints and the rigour of the environment. In fact, "dissipation may never equal zero, and pruning away redundant connections is only convenient when the risk of disrupting the remaining connections is low, that is, when the "external environment" is more benign" (Battini et al, 2006).

Place Figure 11

Place Figure 12

Place Figure 13

The aim of this research is to test a new application of the methodology developed and successfully used in other branches of science, such as ecology and information systems. The case study reported in this section demonstrates that a real application is feasible, even if the authors are well aware of the preliminary nature of the results. Moreover, the research could very well spin off into useful applications.

5 Conclusions

This work emphasises the new idea that Network Analysis is a promising method to study supply chain as a complex web, and can be understood from a systemic point of view. It provides a simple and fast method providing an idea of the complexity level of a supply network. The methodology proposed is in direct agreement with both Shannon's model and the entropic measurements introduced by ecologists (especially Ulanowicz and colleagues) to study ecosystem network structure and organisation. The research in this work aims to apply these measures to a new environment: the supply chain network. As far as the authors are aware, this is the first time these indexes have been applied to the supply chain measurements.

This work introduces eight entropic performance indexes: Total System Throughput, Average Mutual Information, Ascendancy, Development Capacity, Overhead in Input, Overhead in Export, Overhead in Dissipation, and Redundancy. The ecological entropic indexes introduced in this study provide the analyst with an immediate comparison of various alternative complex industrial webs in terms of network organisation and network complexity. The most representative index seems to be Ascendancy, expressed as a percentage on the system Capacity. Nevertheless, it is important to evaluate all performance indexes because each one of them communicates different information about the graphic structure of the supply network. The results of the applications carried out by the authors are coherent with modern supply chain management paradigms. The reduction in goods rejection (dissipations), elimination of redundant connections, limitation of partner duplication, re-cycle in the supply web, and simplification of in-bound and out-bound partnerships work to reduce the network disorder and increase the Ascendancy value. Furthermore, ascendancy is a measure of how developed a system is. It considers both the size of the flows (Total System Throughput, TST) and their organisation (the Average Mutual Information Index, AMI). A company could apply this set of performance measurements to quantifying the potential of structural changes in the supply network, to understanding the impact that strategic choices will have on the whole system, to comparing the actual structure of the network with the future structure, and identifying critical parts of the network structure.

6 Limitations and future research

As previously discussed at length, the results of this study are largely in accord with our theoretical exceptions. However, like the earlier studies, the present one has its limitations that must be addressed in future researches. The analysis of a supply network in which goods flows are measured in different units of value (kg, m^3, units of load, containers, trucks, pallets, money value, etc.) examines the system different points of view and consequently different entropic performance

index values. By itself using only one unit of measurement may not be enough to explain a complex supply network structure and might be seen as a limitation of the methodology developed.

Future research in this field should develop guidelines to support the choice of the best set of measurement units used to depict network flows. Moreover, the usability of these sets of measurements in practice needs to be investigated because of the existing limitations, mainly due to the analytical model comprehension difficulty for industrial practitioners. For this reason, further studies into the practical application of this multi-unit scenario are required. Nonetheless, just as Efstathiou et al. (2002) developed a computer program to calculate manufacturing complexity under different system layouts and operating characteristics, one future direction of this research should be to program a software tool capable of analysing the network structure and of quickly computing the proposed new entropic performance indexes overcoming the limitations explained above. Finally, future tests of the procedure on new industrial supply networks might lead to consider the "time" factor and propose a comparison between "Dynamic Network Analysis" and "Static Network Analysis".

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		Targets										
		Complexity analysis internal Supply Chain			Complexity analysis external Supply Chain			Supply Chain Management				
Year	Authors	static complexity	dynamic complexity	decisional complexity	static complexity	dynamic complexity	decisional complexity	performance indexes	operative planning	tactical planning	strategical planning	Methodologies
1992	Karp et al.		×									entropic model development
1995	Frizelle et al.	#	#									entropic model development
1998	Deshmukh et al.	V										entropic model development
1998	Calinescu et al.	#	#									entropic model development
2001	Seese et al.				V							graph theory
2001	Shih et al.	×										entropic model development
2001	Milgate				×	×		×				entropic model development
2001	Beamon et al.							×				sattistical model and simulation
2002	Sivadasan et al.		×			×			×			entropic model development
2002	Meijer	V		V								teorical model
2002	Wu et al.				#	#						simulation and entropic parameters
2002	Efstathiou et al.	V	V	V								software development and entropic parameters
2002	Makui et al.	V										entropic model development
2002	Bulliger et al.						V	V			V	decision making model
2003	Sivadasan et al.								V			teorical model
2003	Albino et al.				#	#			#	#		linear progamming model
2004	Huan et al.						V	V			V	decision making model
2004	Arteta et al.				#			#				Petri net model
2004	Mills et al.				–	–						survey
2004	Perona et al.	#	#		#	#						complexity index development
2004	Tan et al.			#								software development and entropic parameters
2004	Blackhurst et al.				#	#			#	#		statistical model and Petri net model
2004	Blecker et al.				V	V					V	teorical model
2006	Manzoni et al.							×				perfomance index model
2006	Helo et al.				×			×	×	×		software development
2006	Laumanns et al.				×	×			×			numerical model
2006	Battini et al.				#	#		#	#	#	#	entropic model development

LEGEND

V teorical approach

– survey

× numerical application

case study

Figure 1 – Literature review matrix.

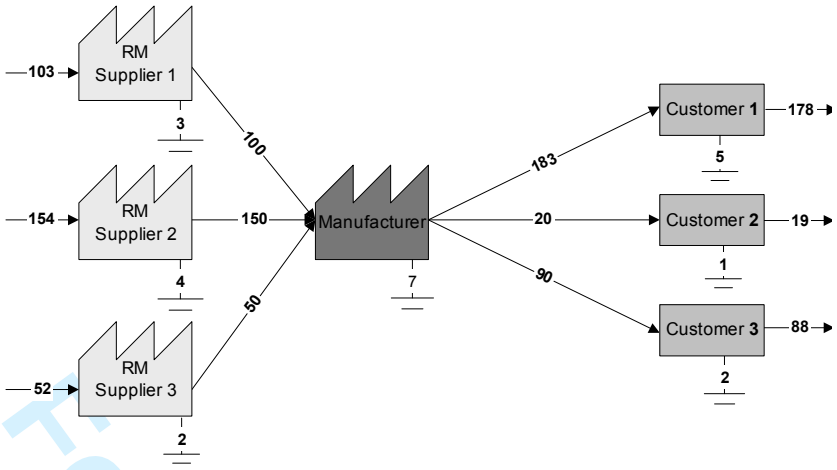

Figure 2 – Example of a typical industrial supply chain.

	0	1	2	…	N	N+1	N+2
0	0	Input [I]				0	0
1	0	Transfers between compartments [T]				Export [E]	Dissipation [D]
2	0						
…	0						
N	0						
N+1	0	0	0	0	0	0	0
N+2	0	0	0	0	0	0	0

Figure 3- Extended transfer matrix T*.

	Imp	S1	S2	S3	M	C1	C2	C3	Exp	Diss
Imp		103	154	52						
S1					100					3
S2					150					4
S3					50					2
M						183	20	90		7
C1									178	5
C2									19	1
C3									88	2
Exp										
Diss										

Figure 4 – Extended transfer matrix T* of network in Figure 2.

	Imp	S1	S2	S3	M	C1	C2	C3	Exp	Diss
Imp	0	0,302	0,378	0,195	0	0	0	0	0	0
S1	0	0	0	0	0,297	0	0	0	0	0,021
S2	0	0	0	0	0,373	0	0	0	0	0,027
S3	0	0	0	0	0,190	0	0	0	0	0,015
M	0	0	0	0	0	0,412	0,098	0,279	0	0,043
C1	0	0	0	0	0	0	0	0	0,407	0,033
C2	0	0	0	0	0	0	0	0	0,094	0,009
C3	0	0	0	0	0	0	0	0	0,275	0,015
Exp	0	0	0	0	0	0	0	0	0	0
Diss	0	0	0	0	0	0	0	0	0	0

Figure 5 - Matrix of joint entropy contributions.

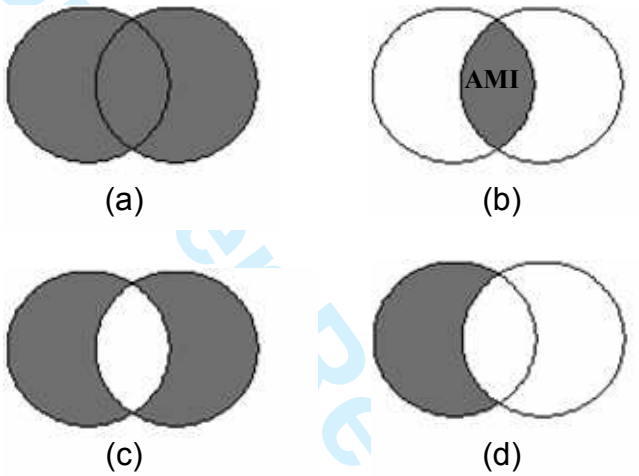

Figure 6 - Venn diagrams expressing the relations between entropies and information: H_I and H_O are sketched as circles that intersect. (a) The joint entropy expressed as the union of the two circles; (b) Average Mutual Information expressed as the intersection between the two circles; (c) the sum of the conditional entropies expressed as the union minus the intersection of the two entropies; (d) the conditional entropy $H_{I/O}$ expressed as the union minus the output entropy.

Indexes	Value	Percentage
$TST = \sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij} = t_{..}$	1211.0	
$C = TST \times H_{I,O} = \sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij} \log\left(\frac{t_{ij}}{t_{..}}\right)$	4194.0	100.00%
$A = TST \times AMI = \sum_{i=0}^{N+2}\sum_{j=0}^{N+2} t_{ij} \log\left(\frac{t_{ij}t_{..}}{t_{i.}t_{.j}}\right)$	2384.2	56.85%
$\Phi_I = -\sum_{j=1}^{N} t_{0,j} \log\left(\frac{t_{0,j}^2}{\sum_{i=1}^{N} t_{ij}\sum_{j=1}^{N} t_{0,j}}\right)$	451.7	10.77%
$\Phi_E = -\sum_{j=1}^{N} t_{i,N+1} \log\left(\frac{t_{i,N+1}^2}{\sum_{j=1}^{N} t_{ij}\sum_{i=1}^{N} t_{i,N+1}}\right)$	355.7	8.48%
$\Phi_D = -\sum_{j=1}^{N} t_{i,N+2} \log\left(\frac{t_{i,N+2}^2}{\sum_{j=1}^{N} t_{ij}\sum_{i=1}^{N} t_{i,N+2}}\right)$	187.0	4.46%
$R = -\sum_{i=1}^{N}\sum_{j=1}^{N} t_{ij} \log\left(\frac{t_{ij}^2}{\sum_{j=1}^{N} t_{ij}\sum_{i=1}^{N} t_{ij}}\right)$	815.5	19.44%

Figure 7 - Entropic Indexes (Ascendancy, Capacity, and Overhead) for the supply chain analysed.

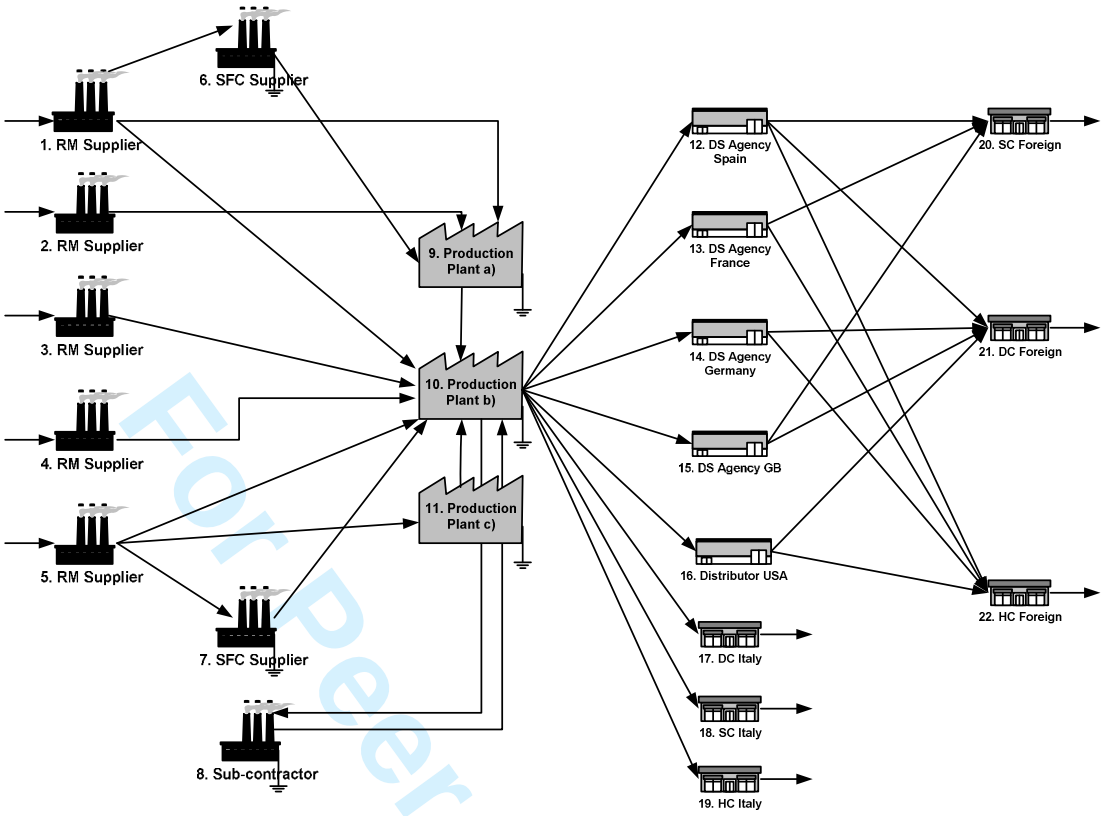

Figure 8- Goods exchanges in the industrial group analysed: the present "as is" configuration.

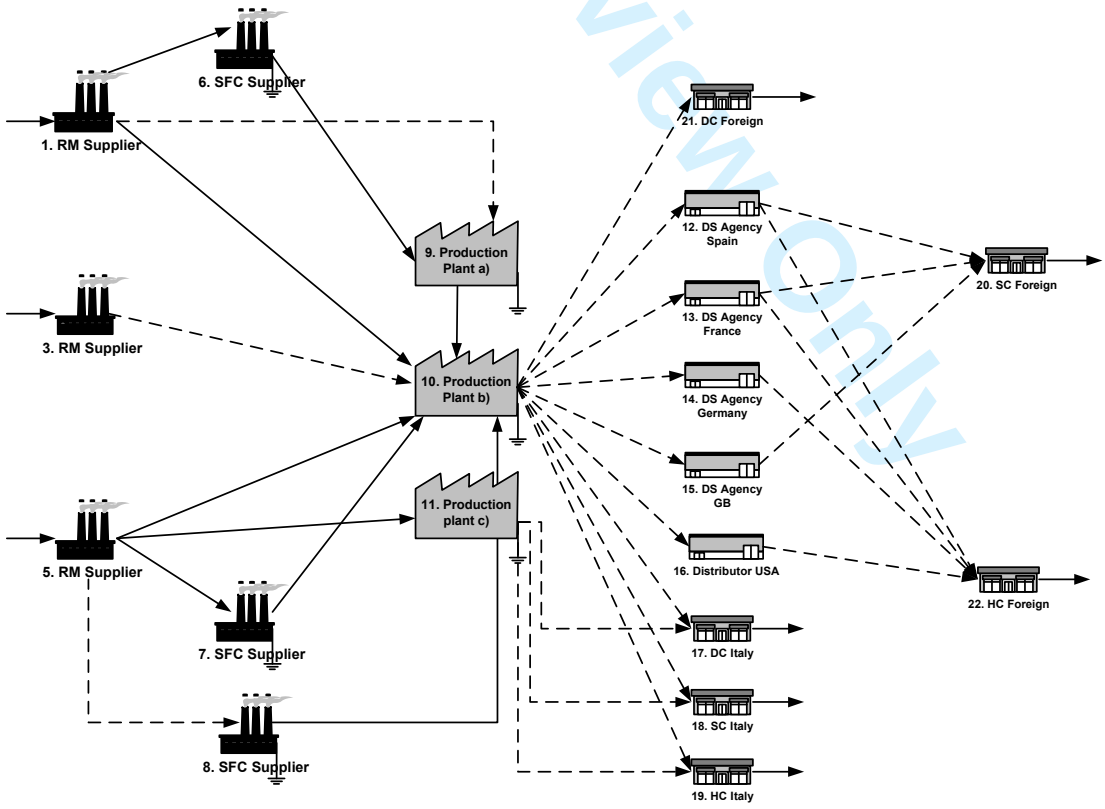

Figure 9 - Goods exchanges in the industrial group analysed: the future "to be" configuration.

i)

	Imp.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Exp.	Diss.
Imp.		521	85	580	340	620																			
1							110			125	286														
2										85															
3											580														
4											340														
5								336			130	154													
6										94															16
7											280														56
8											278														62
9											243														61
10									340				191	85	65	110	120	480	400	150					323
11											127														27
12																					46	55	90		
13																					50		35		
14																						45	20		
15																					52	58			
16																						90	30		
17																								480	
18																								400	
19																								150	
20																								148	
21																								248	
22																								175	
Exp.																									
Diss.																									

ii)

	Imp.	1	3	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	Exp.	Diss.
Imp.		606	480	960																			
1					110			210	286														
3									480														
5						336	340		130	154													
6								94															16
7									280														56
8									278														62
9									243														61
10											145	85	38	52	64	447	345	110		248			163
11																33	54	40					27
12																			45		100		
13																			45		40		
14																					38		
15																			52				
16																					64		
17																						480	
18																						399	
19																						150	
20																						142	
21																						248	
22																						242	
Exp.																							
Diss.																							

Figure 10- Extended Transfer Matrix *T** of network in Figure 8 ("as is" – i)) and network in Figure 9 ("to be" – ii)).

	TST	C	AMI	A	Φ_I	Φ_E	Φ_D	R
Network								
"AS IS"	9972	51734	2,435	24281,5	4569,1	3881,5	2471,3	16532,3
"TO BE"	9078	45083	2,469	22411,5	3115,8	4057,7	1961,9	13537,3
Difference	-894	-6651	0,034	-1870	-1453	176	-509	-2995
Difference %	-8,97%	-12,86%	1,39%	-7,70%	-31,81%	4,54%	-20,61%	-18,12%

Figure 11- Results of network analysis: values of system indexes for the two networks in Figure 8 and Figure 9.

	%A	%Φ	%ΦI	%ΦE	%ΦD	%R
Network						
"AS IS"	46,94%	53,06%	8,83%	7,50%	4,78%	31,96%
"TO BE"	49,71%	50,29%	6,91%	9,00%	4,35%	30,03%
Difference %	5,92%	-5,23%	-21,75%	19,96%	-8,90%	-6,04%

Figure 12- Percentage values of Ascendancy and Overhead in the two supply network configurations.

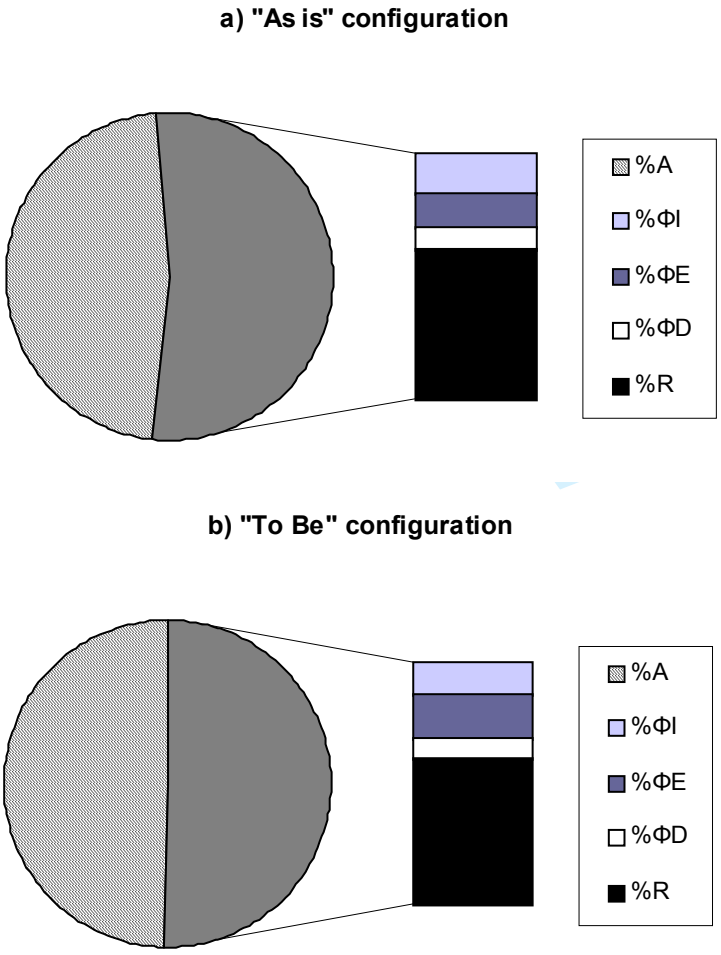

Figure 13- Graph representation of Performance System indexes of the supply network analysed.