Control of Time-Constrained Dual-Armed Cluster Tools Using (max, +) Algebra

Rachid Attia, Saïd Amari, Claude Martinez

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Rachid ATTIA[1], Saïd AMARI[1] and Claude MARTINEZ[2]

Abstract: **The problem studied in this paper is the control of discrete event systems subject to strict temporal constraints using (max, +) algebra. Initially we sought necessary and sufficient conditions for the existence of a causal control law guaranteeing the respect of the temporal constraints. Subsequently, a method for calculating the control law, if any, is proposed. The application which we are interested in is the control of a manufacturing semiconductor wafers process subject to strict temporal constraints.**

Key words: Timed Event Graphs, (max, +) algebra, temporal constraints, feedback control, cluster tools.

I. INTRODUCTION:

In this work we are interested in the control, supervision, of a class of time-constrained discrete event systems modeled by deterministic Timed Event Graphs (TEG). The problem of time constraints is encountered in many industrial applications, such processes including thermal or chemical treatments [S. Amari and *al.* 2004] , [J. Kim and *al.* 2003], the embedded systems and urban or railway transportation systems [T. van den Boom and *al*. 2004]. Let us consider semi-conductor production process, once a schedule has been determined for a cluster tool, the behavior of the production system may be described as a TEG [J. Kim and *al.* 2003]. Some tasks of the production process which are executed by a robot may need to be repeated, for alignment purpose. This would perturbate the initial schedule and consequently lead to some quality loss on a part of the production. The goal of our approach is to design a control law that would be tolerant to that perturbations.

Temporal constraints have been earlier studied has a problem of verification and validation, and many authors have addressed that point of view, see for example [B. Berthomieu and *al.* 1991]. In our work, we consider rather, a control problem. We search for a linear feedback control law determining the firing instants of the controllable transitions, source transitions, to guarantee the respect of the temporal constraints.

In our approach the time is explicitly taken into account, and that is the main difference methods [K. Yamalidou and *al.* 1996], [L. E. Holloway and *al.* 1997]. In the literature, we may also find other approaches of control using dioid algebra. In [L. Houssin and *al*. 2006] an approach based on fixed points results of antitone mappings is given, the aim of the proposed control method is to delay as less as possible the system while ensuring some given specifications. Another approach of supervision of an industrial plant is proposed in [A. M. Atto and *al.* 2008]. In both methods, the authors consider a completely controllable TEG, i.e. all transitions are controllable, which is limiting in several real applications. Other approaches are proposed in [S. Amari and *al.* 2005] and [S. Amari and *al.* 2006] in (min, +) and (max, +) algebras respectively. Some restrictive assumptions were considered for the control synthesis, e.g. existence of an empty path, with no tokens, from the control transition to the constrained place. The control laws were calculated under sufficient conditions.

We suggest to relax the assumptions taken by [S. Amari and *al.* 2006] and formulate necessary and sufficient conditions for the existence of a causal linear feedback, ensuring the respect of the temporal constraints. We consider in our study the systems that can be modeled by a linear (max, +) equations subject to temporal constraints, the temporal constraints are represented by a set of (max, +) linear inequalities. We consider first the control of systems with a single control input and extend the approach to systems with multiple control inputs, assuming that the whole set of constraints is admissible. As an application, we are interested in the control of a time-constrained dual-armed cluster tool proposed in [J. Kim and *al.* 2003].

This paper is organized as follows, the second section recalls briefly the bases and tools of (max, +) algebra and TEG modeling. In the third section, we bring to light the temporal constraints problem and their formalization in (max, +) algebra. The proposed and the established results are presented. The application of the proposed method for the control of a time-constrained dual-armed cluster tool is presented in the fourth section and then we conclude.

II. PRELIMINARIES:

1. (max, +) algebra:

A dioid, or semiring, is a set $\mathcal{D}$ equiped with two binary operations $\oplus$ and $\otimes$ called addition and multiplication, respectively. The addition is commutative, associative with identity element ε called ‘‘zero’’. The multiplication is also associative with identity element e called ‘‘identity’, if the multiplication is commutative, the dioid is commutative’. The multiplication is distributive over addition and the ‘‘zero’’ annihilates $\mathcal{D}$, with respect to multiplication.

The dioid $\overline{\mathbb{R}}_{max}$ commonly called (max, +) algebra, which we consider here, is defined over the set of complete

[1]LURPA, ENS Cachan, 61 avenue Pt Wilson 94235 Cachan, France. (said.amari@lurpa.ens-cachan.fr)
[2]IRCCyN, UMR CNRS 6597, 1 rue de la Noë, BP 92101, 44321 Nantes cedex 03, France (claude.martinez@irccyn.ec-nantes.fr)

real numbers $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty, +\infty\}$. The operations addition ($\oplus$) and multiplication ($\otimes$) correspond, respectively, to the operations maximum with ($\varepsilon = -\infty$) as a "zero" element and ordinary addition with an "identity" element ($e = 0$). The multiplication ($\otimes$) is replaced by ($\cdot$) in the rest of this paper.

2. Timed Event Graphs:

Timed Event Graphs (TEGs) define a subclass of Petri nets [T. Murata 1989] where each place has exactly one upstream transition and one downstream transition. i.e. this type of graphs allows modeling synchronization and parallelism phenomena but not resource sharing or mutual exclusion. An important property of TEGs is that for any circuit, the number of token is constant, therefore if all circuits are non-empty, the TEG is live. Several real systems, as flexible workshops, embedded systems and transportation systems are modeled by TEGs.

TEGs are temporized Petri nets; we denote p_{ij} the place relying the transition t_j to t_i, if any, the corresponding delay is denoted τ_{ij} and its initial number of tokens denoted m_{ij}. The temporization τ_{ij} corresponds to the minimal sojourn time of tokens in the place. The maximal marking arising in the graph is denoted m_{max}. The evolution mode considered for TEGs is a maximum speed mode, i.e. a transition is fired as soon as enabled.

We define a path from a given transition t_j to another transition t_i as the suite of transitions and places (t_j, $p_{k_1 j}$, t_{k_1}, $p_{k_2 k_1}$, t_{k_2}, … , t_{k_n}, $p_{i k_n}$, t_i). The marking of the path is given by the sum of the number of tokens in each place of the path. The temporization of the path is also given by the sum of the path places temporizations.

3. Linear (max, +) model and state space representation:

It is well known that the dynamical behavior of a TEG can be expressed by a system of linear inequalities in the (max, +) algebra [F. Baccelli and *al.* 1992]. For this, we associate to each transition a dater, function of the integer variable $k \in \mathbb{N}^+$, where, $x_i(k)$ corresponds to the date of the k^{th} firing of the transition t_i, such that:

$$x(k) \geq \bigoplus_{l=0}^{m_{max}} \left(A_l x(k-l) \oplus B_l u(k-l)\right) \qquad \text{(I.1)}$$

where: $x \in \overline{\mathbb{R}}_{max}^{n\times 1}$ is the state variables vector, $u \in \overline{\mathbb{R}}_{max}^{m\times 1}$ is the vector associated to the input transitions $tu_{z=1,\dots,m}$. $A_l \in \overline{\mathbb{R}}_{max}^{n\times n}$ is the matrix whose element $A_{l,ij}$ equals to the temporization τ_{ij} of the place p_{ij}, if any, and ε otherwise. $B_l \in \overline{\mathbb{R}}_{max}^{n\times m}$ is the input matrix.

Considering a maximal speed (as soon as) evolution of the TEG, the precedent inequality (I.1) can be replaced by the following equation:

$$x(k) = \bigoplus_{l=0}^{m_{max}} \left(A_l x(k-l) \oplus B_l u(k-l)\right) \qquad \text{(I.2)}$$

The implicit equation (I.2) is usually replaced by the following explicit equation:

$$x(k) = \bigoplus_{l=1}^{m_{max}} \left(A_0^* \cdot A_l x(k-l) \oplus A_0^* \cdot B_l u(k-l)\right) \qquad \text{(I.3)}$$

$A_0^* = \bigoplus_{r\geq 0} A_0^r$ where '*' is the *Kleene Star* operator. $A^r = A \cdot A \cdots \cdot A$, r times.

To get a state-space representation we must decompose each place containing more than one token to several places and transitions where each place contains at most one token. By this operation, any model (I.3) can be written in the following state-space representation:

$$x(k) = A \cdot x(k-1) \oplus B \cdot u(k) \qquad \text{(I.4)}$$

Matrices $A = (A_0^* \cdot A_l)$ and $B = (A_0^* \cdot B_l)$.

III. CONTROL SYNTHESIS:

1. Temporal constraint:

The temporizations associated with places in a TEG correspond to the minimal sojourn time. In fact, the tokens are allowed to sojourn more time. For a time-constrained place a maximal sojourn time is fixed. This limitation of the maximal allowed sojourn time appears as an additional constraint that should be verified. Let us first consider a single constraint on place p_{ij}. We associate to this place the time interval $[\tau_{ij}\ ,\ \tau_{ij}^{max}]$, τ_{ij} is the minimal sojourn time while τ_{ij}^{max} is the maximal one.

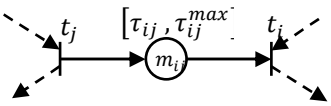

Fig 2. Place subject to strict temporal constraint.

The constraint is expressed through the following inequality:

$$x_i(k) \leq \tau_{ij}^{max} \cdot x_j(k - m_{ij}), \forall k \geq 0 \qquad \text{(II.1)}$$

where m_{ij} is the initial marking of the place p_{ij}. The inequality (II.1) is the additional constraint to be satisfied. Thus, the problem is to determine when the controllable transitions should be fired to satisfy the constraint (II.1). To consider more than a single constraint, one has to consider as many expressions (II.1) as the number of temporal constraint to satisfy.

2. Control synthesis:

We formulate the problem as follows; let a system given by its state-space representation (I.4) subject to the temporal constraint (II.1). Find a linear feedback $u(k) = G \cdot x(k-1)$, ($G \in \overline{\mathbb{R}}_{max}^{m\times n}$) such as the constraint (II.1) being always satisfied. In order to establish necessary and sufficient conditions for the existence of a control law ensuring the respect of the temporal constraints we must discard the

trivial case where $u(k) = \varepsilon$ is a solution. We distinguish the two following cases:

2.1 Trivially solvable problems:

Let the autonomous system, i.e. $u(k) = \varepsilon$, given by its state-space model:

$$x(k) = A \cdot x(k-1)$$

hence $x_i(k) = A_i \cdot x(k-1)$ and $x_j(k) = A_j \cdot x(k-1)$, where A_i and A_j are, respectively, the rows i and j of the matrix A.

Replacing x_i and x_j in the inequality constraint (II.1) we obtain:

$$A_i \cdot A^{k-1} \cdot x(0) \leq \tau_{ij}^{max} \cdot A_j \cdot A^{k-m_{ij}-1} \cdot x(0) \quad \text{(II.2)}$$

$x(0)$ being known, $k = m_{ij} + 1$. For a single constraint, it suffices to verify the inequality (II.2) to decide whether the problem is trivially solvable. Considering a set of q constraints, expression (II.2) has to be verified for each constraint, and furthermore, inequality (II.2.1) has to be satisfied:

$$H \cdot x(0) \leq Q \cdot x(0), \quad \text{(II.2.1)}$$

where $H, Q \in \overline{\mathbb{R}}_{max}^{q \times n}$, H is composed of q row matrices $A_{il} \cdot A^{k-1}$for l=1 to q, Q is composed of q row matrices $\tau_{il,jl}^{max} \cdot A_{jl} \cdot A^{k-m_{il,jl}-1}$ and $k = \max_{l \leq q}(m_{il,jl}) + 1$. In a recent work of [Allamigeon *et al*, 2010] an algorithm is given to characterize the complete set of solutions to inequalities of the form (II.2.1). In the present work, $x(0)$ is given.

2.2 Control synthesis for time-constrained systems:

Again, let us consider first a system with a single input and single temporal constraint, given by its (max, +) state-space model (I.4). Let p_{ij} being the place subject to the temporal constraint (II.1). The idea of the control is delaying the entrance of the tokens in the time-constrained place. Indeed, we block the transition t_j by blocking the input transition tu. This method of control can not operate unless the blocking of the control transition tu does not induce an "important" blocking of the transition t_i. The term "important" is informal, it is function of the marking and temporizations of the paths relying tu to t_i and t_j. Afterwards, we derive necessary and sufficient conditions for the existence of such control.

Let m_α being the smallest marking of all paths relying $\mathrm{t_u}$ to t_j, and m_β the smallest marking of all paths from $\mathrm{t_u}$ to t_i. The state-space representation (I.4) can be written, in the following form [S. Amari and *al.* 2006]:

$$x(k) = A^{\varphi} \cdot x(k-\varphi) \oplus \bigoplus_{k'=0}^{\varphi-1} \left(A^{k'} \cdot B \cdot u(k-k')\right)$$

with φ a strictly positive integer.

Hence, for $(\varphi = m_\alpha + m_{ij} + 1)$ the expressions of the component $x_i(k)$ is given by:

$$x_i(k) = (A^{m_\alpha + m_{ij}+1})_i \cdot x(k - m_\alpha - m_{ij} - 1) \oplus \bigoplus_{k'=0}^{m_\alpha + m_{ij}} \left((A^{k'} \cdot B)_i \cdot u(k-k')\right)$$

where $(A^{m_\alpha+m_{ij}+1})_i$ is the i^{th} row of the matrix $A^{m_\alpha+m_{ij}+1}$ and $(A^{k'} \cdot B)_i$ is the i^{th} element of the vector $(A^{k'} \cdot B)$. As far as, for $(\varphi = m_\alpha + 1)$ $x_j(k)$ is given by:

$$x_j(k) = (A^{m_\alpha+1})_j \cdot x(k - m_\alpha - 1) \oplus \bigoplus_{k'=0}^{m_\alpha} \left((A^{k'} \cdot B)_j \cdot u(k-k')\right)$$

Considering the definitions of m_α (resp. m_β) the terms $(A^{k'} \cdot B)_j$ (resp. $(A^{k'} \cdot B)_i$) vanish for $k' < m_\alpha$ (resp. $k' < m_\beta$). Thus, we get the following expressions for $x_i(k)$ and $x_j(k)$:

$$x_i(k) = (A^{m_\alpha+m_{ij}+1})_i \cdot x(k - m_\alpha - m_{ij} - 1) \oplus \bigoplus_{m_\alpha+m_{ij}-1} (A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k - m_\alpha - m_{ij}) \oplus \bigoplus_{k'=m_\beta} \left((A^{k'} \cdot B)_i \cdot u(k-k')\right)$$

$$x_j(k) = (A^{m_\alpha+1})_j \cdot x(k - m_\alpha - 1) \oplus (A^{m_\alpha} \cdot B)_j \cdot u(k - m_\alpha) \quad \text{(II.3)}$$

The inequality constraint (II.1) is equivalent to $x_i(k + m_{ij}) \leq \tau_{ij}^{max} \cdot x_j(k)$. Replacing $x_i(k)$ and $x_j(k)$ by their expressions in (II.3) we find:

$$(A^{m_\alpha+m_{ij}+1})_i \cdot x(k - m_\alpha - 1) \oplus (A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k - m_\alpha) \leq \tau_{ij}^{max} \cdot x_j(k) \quad \text{(II.4)}$$

$$\bigoplus_{k'=m_\beta}^{m_\alpha+m_{ij}-1} \left((A^{k'} \cdot B)_i \cdot u(k - k' + m_{ij})\right) \leq \tau_{ij}^{max} \cdot x_j(k) \quad \text{(II.5)}$$

The temporal constraint (II.1) is satisfied if and only if the two inequalities (II.4) and (II.5) are satisfied. From the two last inequalities (II.4) and (II.5) we derive necessary and sufficient conditions for the existence of a control law satisfying the temporal constraint (II.1) and calculate this law, if any. We must distinguish the two cases $(m_\beta = m_\alpha + m_{ij})$ and $(m_\beta < m_\alpha + m_{ij})$.

i) First case $(m_\beta = m_\alpha + m_{ij})$*:*

In this case, we just have to verify that inequality (II.4) is satisfied. In fact, by replacing $x_j(k)$ by its expression, we get:

$$(A^{m_\alpha+m_{ij}+1})_i \cdot x(k - m_\alpha - 1) \oplus (A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k - m_\alpha) \leq \{\tau_{ij}^{max}((A^{m_\alpha+1})_i \cdot x(k - m_\alpha - 1) \oplus (A^{m_\alpha} \cdot B)_i \cdot u(k - m_\alpha))\}$$

From which we derive the first necessary and sufficient condition, given by the following proposition:

Proposition 1*:* Consider a system given by its (max, +) state-space model (I.4), subject to temporal constraint (II.1). There is a linear feedback control law ensuring the constraint (II.1) if and only if the following condition holds:

$$(A^{m_\alpha+m_{ij}} \cdot B)_i \leq \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \quad \text{(II.6)}$$

Proof: To prove the sufficiency of the proposed condition, it suffices to prove the correctness of the following implication: $(A^{m_\alpha+m_{ij}} \cdot B)_i \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \Rightarrow x_i(k) \le \tau_{ij}^{max} \cdot x_j(k)$. We have: $(A^{m_\alpha+m_{ij}} \cdot B)_i \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j$.
By multiplying the inequality by $u(k-m_\alpha)$ we get:

$$(A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k-m_\alpha) \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha) \quad \text{(II.7)}$$

It suffices to choose: $u(k-m_\alpha) = \left(-\tau_{ij}^{max} - (A^{m_\alpha} \cdot B)_j\right) \cdot A_i \cdot A^{m_\alpha+m_{ij}} \cdot x(k-m_\alpha-1)$, to get the equality (II.8) :

$$(A^{m_\alpha+m_{ij}+1})_i \cdot x(k-m_\alpha-1) = \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha) \text{ (II.8)}$$

Now adding, in (max, +) sense, inequality (II.7) to equality (II.8) we get $(A^{m_\alpha+m_{ij}+1})_i \cdot x(k-m_\alpha-1) \oplus (A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k-m_\alpha) \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha)$. The left term, in fact, is equal to $x_i(k)$, this leads to $x_i(k+m_{ij}) \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha)$. Finally, we get: $x_i(k+m_{ij}) \le \tau_{ij}^{max} \cdot x_j(k)$
To prove the necessity of the condition (II.6) we proceed by contradiction. We suppose that the condition (II.6) doesn't hold and the constraint is satisfied and this must lead to an anomaly.

Suppose:

$$(A^{m_\alpha+m_{ij}} \cdot B)_i > \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \quad \text{(II.9)}$$

and: $x_i(k+m_{ij}) \le \tau_{ij}^{max} \cdot x_j(k)$ (II.10)

Inequality (II.10) leads to:

$$\begin{cases} (A^{m_\alpha+m_{ij}+1})_i \cdot x(k-m_\alpha-1) \le \tau_{ij}^{max} \cdot x_j(k) \\ (A^{m_\alpha+m_{ij}} \cdot B)_i \cdot u(k-m_\alpha) \le \tau_{ij}^{max} \cdot x_j(k) \end{cases} \quad \text{(II.11)}$$

Taking in account the expression of $x_j(k)$, two cases arise, the first is: $x_j(k) = (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha)$
replacing $x_j(k)$ in (II.11) by its expression, we get:

$$\begin{cases} (A^{m_\alpha+1})_i \cdot x(k-m_\alpha-1) \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha) \\ (A^{m_\alpha} \cdot B)_i \cdot u(k-m_\alpha) \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha) \end{cases} \quad \text{(II.12)}$$

The second inequality of the system (II.12) contradicts the hypothesis (II.9).

The second case is: $x_j(k) = (A^{m_\alpha+1})_j \cdot x(k-m_\alpha-1)$. Replacing $x_j(k)$ by its expression in (II.11), we find this time:

$$\begin{cases} (A^{m_\alpha+1})_i \cdot x(k-m_\alpha-1) \le \tau_{ij}^{max} \cdot (A^{m_\alpha+1})_j \cdot x(k-m_\alpha-1) \\ (A^{m_\alpha} \cdot B)_i \cdot u(k-m_\alpha) \le \tau_{ij}^{max} \cdot (A^{m_\alpha+1})_j \cdot x(k-m_\alpha-1) \end{cases} \quad \text{(II.13)}$$

In this case, the first inequality of the system (II.13) leads to an absurdity. In fact, we supposed that the autonomous system doesn't satisfy the temporal constraint, see the remark in (§III.1).■

Remark1: we proved that for the case $m_\beta = m_\alpha + m_{ij}$ the condition (II.6) is a sufficient and necessary one for the existence of a linear feedback control law satisfying the temporal constraint (II.1). We proposed also a control law given by the following equation:

$$u(k) = G \cdot x(k-1) = \bigoplus_{r=1}^{n} G_r \cdot x_r(k-1) \quad \text{(II.14)}$$

with: $G_r = \left(-\tau_{ij}^{max} - (A^{m_\alpha} \cdot B)_j\right) \cdot (A^{m_\alpha+m_{ij}+1})_{ir}$

ii) Second case $(m_\beta < m_\alpha + m_{ij})$*:*

In this case, in addition to inequality (II.4) treated in the first case, we deal with inequality (II.5) form which we derive the second necessary and sufficient condition. Thus, we get the same necessary and sufficient condition (II.6) and an additional condition driven from (II.5). The inequality to be satisfied is:
$(A^{k'} \cdot B)_i \cdot u(k-k'+m_{ij}) \le \tau_{ij}^{max} \cdot x_j(k); k' = m_\beta \; to \; (m_\alpha + m_{ij})$, which is the same as:

$$(A^{k'+m_{ij}} \cdot B)_i \cdot u(k-k') \le \tau_{ij}^{max} \cdot x_j(k); k' = m_\beta - m_{ij} \; to \; m_\alpha$$

We have $u(k-k') = G \cdot x(k-k'-1)$, from which we get $u(k-k') = G \cdot (A \oplus BG)^{m_\alpha-k'-1} \cdot (A \oplus BG) \cdot x(k-m_\alpha-1)$.
Taking into account the expression of $x_j(k)$, $x_j(k) = (A^{m_\alpha+1})_j \cdot x(k-m_\alpha-1) \oplus (A^{m_\alpha} \cdot B)_j \cdot u(k-m_\alpha)$. We derive the following condition:

$$(A^{k'+m_{ij}} \cdot B)_i \cdot G \cdot \left((A \oplus BG)^{m_\alpha-k'-1}\right) \cdot B \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \quad \text{(II.15)}$$

Proposition 2: Consider a system given by its (max, +) state-space model (I.4), subject to temporal constraint (II.1). There is a linear feedback control law guaranteeing the respect of the temporal constraint (II.1), if and only if the two following conditions hold:

$$(A^{m_\alpha+m_{ij}} \cdot B)_i \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \quad \text{(II.16)}$$

$$(A^{k'+m_{ij}} \cdot B)_i \cdot G \cdot \left((A \oplus BG)^{m_\alpha-k'-1}\right) \cdot B \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B)_j \quad \text{(II.17)}$$

Proof: The condition (II.16) arises from the case $(m_\beta = m_\alpha + m_{ij})$, while the second condition (II.6) is a consequence of the case $(m_\beta < m_\alpha + m_{ij})$. The condition (II.17) is as a limitation of the greatest feedback G. To calculate a control law we first check the condition (II.16), if it holds, we calculate a control law using the result (II.14). Then, we check condition (II.15). We don't need to solve the inequality (II.15).■

The work presented here for single control input systems subject to single temporal constraint may be applied to systems that are subject to multiple temporal constraints. After derivating a control law for each constraint, assuming (II.16) and (II.17) are satisfied for each constraint, one has to check if the resulting control laws do not influence each others.

IV. Application: Scheduling of Time-Constrained Dual-Armed Cluster Tool:

In this section we apply our control method to solve a time-constrained scheduling problem proposed in [J. Kim

and *al.* 2003]. The studied system is a semiconductor wafers manufacturing cluster tool. Processes such as some low pressure chemical vapor deposition processes require strict timing control. Unless a processed wafer leaves the chamber within a specified time limit, the wafer is subject to quality degradation due to the processing side effects. A wafer goes through different chambers and undergoes different treatments. The operations of transportation, loading and unloading to the different chambers are performed by a dual-armed handling robot. The challenge is in scheduling the operations on the handling robot to ensure the different temporal constraints of processing steps. Our approach consists in the formulation of the time-constrained scheduling problem as a control problem under strict temporal constraints. Then, we exploit the results of the proposed method to calculate a control satisfying these temporal constraints. In the following paragraphs, we give, first, a description of the manufacturing system. After, the associated TEG and (max, +) model will be given. A control law ensuring the respect of the temporal constraints is calculated, thus, a feasible schedule is found.

1. System description:

A cluster tool of type flow pattern (2,1) is compound of two load locks, two parallel chambers C1, C2 and a third chamber C3 in a serial configuration with C1 and C2. The same processing is performed in the chambers C1 and C2 while a different one in the chamber C3, that said, a new wafer unloaded from the load lock (LL) passes through one of the chambers C1 or C2 where a first treatment is completed then it passes to the chamber C3 where another treatment is offered. The following figure schematizes a cluster tool of type flow pattern (2, 1) and brings to light the robot work cycle.

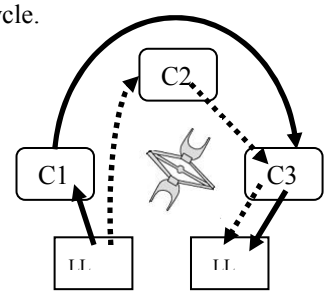

Fig 3. Wafer flow pattern (2, 1) for dual-armed cluster tool.

2. System Modeling:

The robot work cycle is defined by the following operations sequence: unload a new wafer from the load lock→ move the empty arm to C1→ swap the completed wafer with the unprocessed wafer at C1→ move to C3→ swap at C3→ move to the load lock→ return the wafer to the load lock→ unload a new wafer from the load lock→ move to C2→ swap at C2→ move to C3→ swap at C3→ move to the load lock→ return the wafer to the load lock. The TEG of the (Fig 4) traduces the robot work cycle, each place corresponds to an operation or series of operations. We denote the temporizations associated to the time-constrained places with a closed interval of the form $[\tau_i,\ \tau_i + d_i]$ where τ_i is the minimal duration of a treatment and d_i the maximal waiting time to leave chamber after treatment ends.

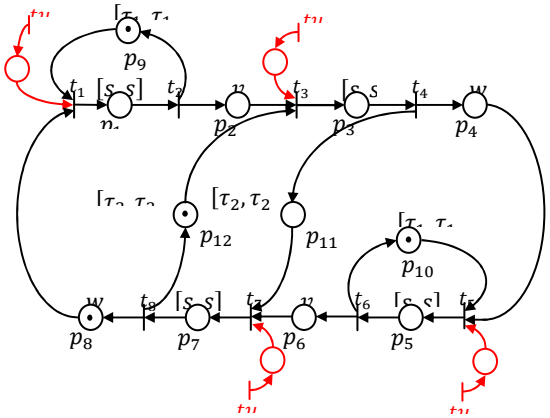

Fig 4. TEG model for the flow pattern (2,1).

On the TEG the places are denoted from p_1 to p_{10} and the transitions from t_1 to t_8. Places p_9 and p_{10} correspond to processing in chambers C1 and C2, respectively. While p_{11} and p_{12} corresponds to processing in C3. Places p_1, p_3, p_5, and p_7 correspond to swap operations on which no waiting time is allowed, thus, downstream transitions of these places are uncontrollable, i.e. transitions t_2, t_4, t_6 and t_8 are uncontrollable. The rest of transitions are controllable, we associate to each one of them a control input. The control inputs are shown on (Fig 4). To get the (max, +) model traducing the dynamical behavior of the robot work cycle, we associate to each transition t_i of the TEG (Fig 4) a dater denoted x_i and to the controllable transitions t_2, t_4, t_6 and t_8 the control inputs tu_1, tu_2, tu_3 and tu_4, respectively. The linear (max, +) model of our system is given by the following linear equations:

$$\begin{cases} x_1(k) = w \cdot x_8(k-1) \oplus \tau_1 \cdot x_2(k-1) \oplus u_1(k) \\ x_2(k) = s \cdot x_1(k) \\ x_3(k) = v \cdot x_2(k) \oplus \tau_2 \cdot x_8(k-1) \oplus u_2(k) \\ x_4(k) = s \cdot x_3(k) \\ x_5(k) = w \cdot x_4(k) + \tau_1 \cdot x_6(k-1) \oplus u_3(k) \\ x_6(k) = s \cdot x_5(k) \\ x_7(k) = v \cdot x_6(k) + \tau_2 \cdot x_4(k) \oplus u_4(k) \\ x_8(k) = s \cdot x_7(k) \end{cases}$$

with: $w = 4; v = 1; s = 2;\ \tau_1 = 22;\ \tau_2 = 9; d_1 = 1; d_2 = 1.$
The above system of equations can be written in a state-space representation as follows (see §II.3), $x(k) = A \cdot x(k-1) \oplus B \cdot u(k)$. Matrices *A* and *B* are given below.

2. *Control of the time-constrained system:*

We mentioned above the problem of the strict time processing in the chambers. On the TEG modeling the robot work cycle, we denoted $[\tau_1, \tau_1 + d_1]$ the temporization associated to processing in chambers C1 and C2 while $[\tau_2, \tau_2 + d_2]$ for the temporization associated to the once in chamber C3.

$$A = \begin{bmatrix} \varepsilon & 22 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 4 \\ \varepsilon & 24 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 6 \\ \varepsilon & 25 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 9 \\ \varepsilon & 27 & \varepsilon & \varepsilon & \varepsilon & \varepsilon & \varepsilon & 11 \\ \varepsilon & 31 & \varepsilon & \varepsilon & \varepsilon & 22 & \varepsilon & 15 \\ \varepsilon & 33 & \varepsilon & \varepsilon & \varepsilon & 24 & \varepsilon & 17 \\ \varepsilon & 36 & \varepsilon & \varepsilon & \varepsilon & 25 & \varepsilon & 20 \\ \varepsilon & 38 & \varepsilon & \varepsilon & \varepsilon & 27 & \varepsilon & 22 \end{bmatrix}; B = \begin{bmatrix} e & \varepsilon & \varepsilon & \varepsilon \\ 2 & \varepsilon & \varepsilon & \varepsilon \\ 3 & e & \varepsilon & \varepsilon \\ 5 & 2 & \varepsilon & \varepsilon \\ 9 & 6 & e & \varepsilon \\ 11 & 8 & 2 & \varepsilon \\ 14 & 11 & 3 & e \\ 16 & 13 & 5 & 2 \end{bmatrix}$$

The temporal constraints are written in (max, +) algebra as follows:

$$\begin{cases} x_1(k) \le (\tau_1 + d_1) \cdot x_2(k-1) \\ x_5(k) \le (\tau_1 + d_1) \cdot x_6(k-1) \\ x_3(k) \le (\tau_2 + d_2) \cdot x_8(k-1) \\ x_7(k) \le (\tau_2 + d_2) \cdot x_4(k) \end{cases} \quad \text{(III.1)}$$

The analysis is performed for each input towards all of the temporal constraints by checking the conditions (II.6) for each input regarding the temporal constraints (III.1). Let B_i being the i^{th} column of the matrix B, so we check the following condition:

$$(A^{m_\alpha + m_{ij}} \cdot B_l)_i \le \tau_{ij}^{max} \cdot (A^{m_\alpha} \cdot B_l)_j \;\; ; \;\; l = 1 \; to \; 4 \quad \text{(III.2)}$$

We find that the condition does not hold for $l = 1$ and $l = 3$, while it holds for $l = 2$ and $l = 4$. From this result we decide to keep just u_2 and u_4 for control synthesis.

To calculate u_2 and u_4 control laws we apply (II.14), $u_2(k) = \begin{bmatrix} \varepsilon & 24 & \varepsilon & \varepsilon & \varepsilon & 13 & \varepsilon & 8 \end{bmatrix} \cdot x(k-1)$, and $u_4(k) = \begin{bmatrix} \varepsilon & 37 & \varepsilon & \varepsilon & \varepsilon & 24 & \varepsilon & 19 \end{bmatrix} \cdot x(k-1)$, u_1 and u_3 are not used for control, this mean that they are vanished. To control the system we connect the TEGs resulting from u_1 and u_3 to the TEG of the system, thus we get the TEG of the controlled system.

3. Results analysis:

In this application, we have shown that the (max, +) method proposed in this paper can bring solutions to real-world problems encountered in industrial applications. About comparing our results to those proposed by [J. Kim and *al.* 2003], it remains not easy and less evident since the authors in [J. Kim and *al.* 2003] are interested in scheduling of dual-armed cluster tools, while we are interested in developing formal methods for control and supervision of time-constrained discrete event systems. For us, the scheduling of the dual-armed cluster tool studied here is an application between many other applications where the proposed approach can be applied.

V. CONCLUSION

In this paper we proposed a (max, +) formal method for the control of time-constrained discrete event systems. The principal contribution of this work consists in formulating necessary and sufficient conditions for the existence of linear feedback control law ensuring the respect of the strict temporal constraints. Originality lies also in the application of the proposed method for solving a time-constrained scheduling problem. We wish to generalize the method to multivariable systems, multi-inputs and several temporal constraints. Working on the application presented in this paper and many other examples, we notice that the control law found is always optimal in cycle time sense, i.e. it delays as less as possible the system while keeping the temporal constraints satisfied.

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