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**Analytical and numerical modeling of mechanical properties of orthogonal 3D CFRP**

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**Abstract**

This study proposes an analytical and numerical model, for the prediction of mechanical properties of orthogonal 3D reinforcement composite materials, taking into account their structural parameters (mechanical properties of the components and geometrical architecture). This first step of this work requires the definition of the composite representative elementary volume (REV). Microscope studies made possible to visualize the architectural aspect of the internal structure. From these observations two types of REV, adapted to the types of modeling (analytical and by finite elements FE), were defined. The first one takes into account the whole of material in its thickness thus integrating the characteristics of the layers and the vertical reinforcements. The second one, strongly simplified in order to minimize the costs of calculations, is used in the FE approach. Moreover the analytical model is extended to the prediction of the ultimate properties by using the tensorial criterion 3D of Tsai. The results obtained from these modeling are compared with experimental results. This comparison highlights the interest and the limits of each approach according to the effect of the choice of the REV.

**Keywords:** 3D orthogonal; Homogenization, Modeling, Finite Elements

**Introduction**

The interest of the introduction of out of plane reinforcement into composite materials is not any more to demonstrate. In addition to increase the mechanical performances in the third

direction, this reinforcement also improves considerably inter-laminar resistance of these materials. If the weaving technique makes it possible to propose a complex 3D preforms, the current challenge consists in developing tools able to predict the mechanical behavior of these materials starting from the structural geometrical parameters. This step is belonging works initiated in eighties which currently continued and which consist in micromechanical modeling of woven composites materials. In the first models, only the undulation in the “x” direction or stuffer yarn was taken into account [1]. These models evolved by taken also into account the weaving in the “y” direction or filler yarn [2] and the relative shift of the layers. In 1994, Sankar and al [3] proposed an analytical model called Selective Avering method (SAM): this approach is based on a combination of average in stiffness and compliance to consider the thermo-elastic 3D properties.

Scida and al (98) [4] propose an analytical model of homogenization by summations of average stiffness of each discretized volume of the elementary cell, to predict the three-dimensional properties of 2D woven. Ping et al., 2000 [5] propose a modeling of the elastic behavior of 3D orthogonal by using similar analytical models, baptized XYZ, YXZ, ZXY and ZYX based on the conditions of iso-stress and iso-strain on micro-elements of representative elementary volume (REV) divided in series or parallel.

It acts in this paper of mechanical behavior modelling (elastic and failure) of 3D orthogonal carbon reinforcement composite material. Two REV or basic cells will be considered in this studies. An analytical model inspired of work of Scida et al. (98) [4] is applied to each basic cell. The results obtained are confronted with those obtained by ZYX model, with a finite elements approach based on the principle of optimization the functional calculus (Duysinx, 96) [6] and with experimental results. The effect of the choice of the basic cell is then highlighted.

The failure, in uni-axial tension, is also predicted by the model and compared with the experimental results. It uses the 3D criterion of Tsai-Wu, while having resorts to the data base established by Khellil [7].

## Material Presentation

The material of this study is an Epoxy RTM6 matrix reinforced by 3D orthogonal T300J carbon. Its thickness is 8mm. The preform is woven according to the three orthogonal directions of a Cartesian reference mark. The reinforcements in the third direction juxtapose the longitudinal and transversal yarns along the thickness. The third yarns are vertical in the heart of the preform, and take a concave weaving form at the surfaces. The whole of the reinforcement forms a structured internal architecture. Figure 1 represents a longitudinal section of the material and a schematization of the higher surface.

The yarn proportions are 46% in warp direction, 46% in weft direction and 8% in vertical reinforcement.

## Microscopic study

The observations carried out on an electronic microscope associated with images analysis, show that the material symmetry is not respected perfectly. In fact, the compaction of the preform follow-up by the resin injection creates disequilibria between geometrical dimensions of the yarns in the warp and weft direction. The analysis of the microscopic images obtained makes it possible to modeling the whole of the geometrical characteristics of the structure necessary to the next step. The sections of the wicks in the three directions are appreciably rectangular (fig. 2). Nevertheless, the horizontal section shows that the weft yarn presents, in the plane, a contracted zone due to the passage of the vertical reinforcements (fig. 2c).

Concerning the vertical reinforcement, the proportion of vertical part and the weavers part was determined. Thus the first one represents 64.14% of the total of the yarn and each curved

part 17.93%. This result allows estimating the vertical yarn average curvature radius of 0.63 mm..

### Analytical modeling

The selected representative elementary volume (REV) is decomposed according to its components (3 yarns and resin blocks). The elastic behavior of each component is then expressed by the three-dimensional stiffness matrix  $[C_{ij,k}]$ . If the component corresponds to the yarn, the stiffness matrix is expressed in the reference mark of the yarn (reference 123) then transformed in the basic cell reference mark (reference xyz) by taking account the orientation of yarn.

The knowledge of the matrices transformed for each component  $k$ , makes it possible to deduce the global stiffness matrix from of the basic cell according to the relation (1) by taking account of the volume fraction of each component. ( $V_t$  is the total volume of the REV,  $V_k$  the volume of each component)

$$C_{ij,Global} = \frac{1}{V_t} \sum_{k=1}^n V_k \cdot C'_{ij,k} \quad \text{for } i = 1 \text{ to } 6 \text{ and } j = 1 \text{ to } 6 \quad (1)$$

In addition, by preoccupation with a confrontation of this apprao with those of the literatures, a model ZYX of Ping et al., (2000) [5] based on the principle of iso-stress and iso-strain is also applied to materials of this study.

### Choice of the REV

In conventional way, the geometrical periodicity of a composite is described in term of basic cell or REV. It is defined by the condition which the entire composite can be built starting from copies of this one relocated in space. The loading response of the composite can then be calculated by analyzing the behavior of this only basic cell.

For each periodic structure, there is a multitude of possible choices. In a simplification preoccupation, symmetries in a 3D orthogonal composite impose two main roads:

*Unit cell (UC)*

This REV neglects the higher and lower undulation in the vertical reinforcement. It is then possible to propose a very simple basic cell and consequently requiring a reduced computing time. This REV consists of compartments of yarn and resin with parallelepiped form, juxtaposed the ones with the others as shown on figure 3. The volumes of each component and their proportion are given in table 1.

*Global cell (GC)*

This REV takes into account the real form of the vertical yarn, on the totality thickness of the material (fig. 4). The 16 yarns in each warp and weft direction are modeled by parallelepiped compartments form. The vertical yarns are modeled by five types of compartment chosen according to orientations of fibers: two horizontal on the surfaces, followed by two in form of curve and then the vertical one. The curve of the yarn is modeled on the basis of four tangent circle of average radius equal to  $3/2$  the width of the vertical yarn (0.67mm). This value is appreciably equal to the average value of 0.63 mm determined by the microscopic analysis. The detail of the modeling is represented in figure 5. The total volume of the REV of total cell is  $21.352 \text{ mm}^3$ . The volume of the components and their proportion is given in table 1.

The orientations of fibers in curve compartments are taking into account in the homogenization procedure.

**3D Mechanical properties**

3D elastic properties:  $E_L, E_T, E_H, G_{LT}, G_{LH}, G_{TH}, \nu_{LT}, \nu_{LH}, \nu_{TH}$  given for each type of modeling by a succession of stage are represented in the flow chart (Fig.5). Calculation is

based on the geometrical properties of the basic cells and on the mechanical properties of the components.

### Ultimate properties

The elastic modeling is extended to the case of the ultimate properties in particular in uniaxial tension. Indeed the determination of local stiffness allows determining the local states of stresses for a macroscopic level of loading. A criterion of failure 3D of Tsai-Wu type which takes into account the three-dimensional effect of the texture of material is then applied to each increment of loading.

The criterion is written by his tensorial form:

$$F_i \bar{\sigma}_i + F_{ij} \bar{\sigma}_i \bar{\sigma}_j = 1 ; \quad (2)$$

for :  $i, j = 1, \dots, 6$

The linear terms of this expression  $F_i$  et  $F_{ii}$  as well as the ultimate stress are determined by compression, tension and shearing tests on the components or, in certain cases resulting from the literature. The complexity by using the such criterion resides in fact, in the determination of the interactions' coefficients  $F_{ij}$  ; for  $i \neq j$ . In this study, the coefficients used result from Khellil works [7].

### Numerical modeling

Several digital techniques are proposed in the literature in order to homogenize an orthotropic material. We will quote in others:

- Theory of the effective modulus under two possible approaches in strain or stresses
- Method of homogenization periodic
- Energetic approach



All these techniques require a REV checking the symmetric material on the three planes of orthotropic (fig. 6).

This study uses an energetic approach based on the principle optimization of the functional [6]

$$\Pi(a) = \frac{1}{2} \iiint [\varepsilon]^T \cdot [\sigma] \cdot d\Omega - \iint a \cdot \bar{\sigma} \cdot \bar{n} \cdot dS \quad (3)$$

The first integral constitutes the deformation energy in the REV and the second represents the work applied by the load to the limits of the REV. All these variables must be expressed according to displacement 3D in elementary volume. The discretization of the REV in a whole of functional calculus is done as follows:

$$\frac{1}{2} \iiint [\varepsilon]^T \cdot [\sigma] \cdot d\Omega = \frac{1}{2} \sum_e Q_e^T \cdot K_e \cdot Q_e = \frac{1}{2} Q^T \cdot K \cdot Q \quad (4)$$

$$\iint a \cdot \bar{\sigma} \cdot \bar{n} \cdot dS = \sum_e F_e^T \cdot Q_e = Q^T \cdot F \quad (5)$$

$Q_e$  is the vector displacement on each trihedral of the REV with a meshes,  $K_e$  is the matrix of corresponding stiffness,  $F^T$  is the loading on the trihedral of the borders.

The homogenization requires the resolution of the preceding problem in six different situations of loading: three simple tensions along the longitudinal, transverse and vertical axes and three simple shear around the same axes.

On the level of the assembly of stiffness matrix, the homogenization also requires the taking into account a boundary condition expressing equality between the axial or angular strains on the requested borders in tension or torsion.

This numerical approach is realized under MatLab environment. It gives a fast convergence towards the Young modulus by a low number of trihedral meshes (592). On the other hand, convergence towards the shearing coefficient requires a high number of elements (37577).

## Results and discussion

The mechanical properties of each type of yarn warp, weft and vertical directions, given analytically (Berthelot 92) [8] different according to the selected REV. Indeed, the volume of each component is evolved and on the other hand the total fraction volume of the composite is maintained constant for both REV (table 3). The table 4 presents the elastic properties 3D obtained by each analytical model and finite elements. They are compared with some results of experimental tests carried out on this material.

This reveals that the analytical models and finite elements give results appreciably the same ones for the properties in the longitudinal directions and transversals whatever the selected of REV. On the other hand the effect of the choice of the REV feels in the determination of the properties in the thickness. Indeed, the Global cell gives results very close to the experimentation contrary to the Unit cell which over-estimates the  $E_3$  modulus. By comparing this model with ZXY model of Ping et al. [5], we note a notable difference of the shear stiffness modulus. The model using the Global cell gives best satisfaction by comparison to the experimental results. The properties predicted by the model of this study are in agreement with experimentation and rather close to the results obtained by finite elements.

The analytical results of failure strength by application of the criterion 3D of Tsai-Wu and the experimental results are given in table 5. Modeling proposes of ultimate stiffness, in tension in the longitudinal direction, is independently of the choice the REV. We also notes that the failure criterion 3D gives satisfaction in comparison with the experimental results. Indeed, the unit cell (UC) over-estimates the failure strength since it does not take account of the undulation of vertical yarn. The state of strength for the latter case is presented on figure 7. It

is noticed that the checking of the criterion of Tsai-Wu is a function of the orientation's angle of the reinforcement in this zone. As an example, in longitudinal tension, the first damages appear at 180 MPa.

## Conclusions

This study approached the modeling of the mechanical behavior of a composite material with reinforcement carbon orthogonal 3D. The determination of the elastic properties 3D homogenized was done by using an analytical and numerical model. This study reveals the importance of the choice of the REV on certain mechanical properties as observed in the case of out of plane elastic and failure properties. The models analytical and numerical suggested were validated following a comparison with the experimental tests. Moreover the use of the criterion of Tsai 3D gave satisfaction for the prediction of tension failure in the longitudinal direction. Tests in the three directions of space will have to be carried out in order to complete the validation of the model.

The numerical method which was interested to the elastic behavior tends to amplify the homogenized characteristics, unless refining more the mesh of the REV.

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Figure 1: Presentation of 3D orthogonal material

Figure 2: Microscopic studies of 3D orthogonal.

Figure 3: Schematic presentation of the VER modeling.

Figure 4 : Modeling of the curve part of the vertical yarn in the REV global cell.

Figure 5 : Flow chart for the prediction of elastic 3D properties and the failure strengths

Figure 6: Finite elements modeled of REV.

Figure 7 : Stress distribution on vertical yarn under  $\sigma_{x,t}$

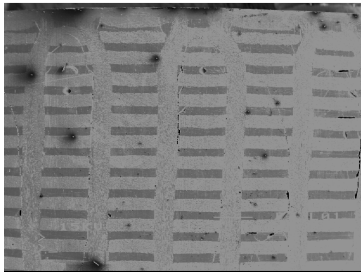
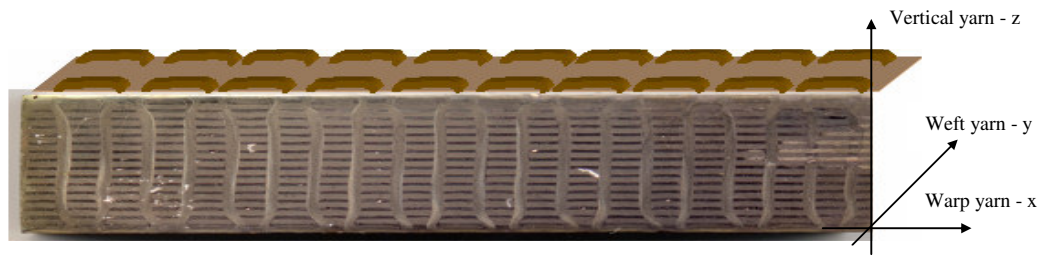
Table 1 : Volumes and proportions of the REV components

Table 2 : Elastic properties of the composite constituents

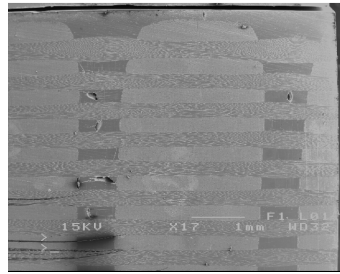
Table 3: Stiffness results of analytical, numerical and experimental studies

Table 4: Failure Strain and stress results

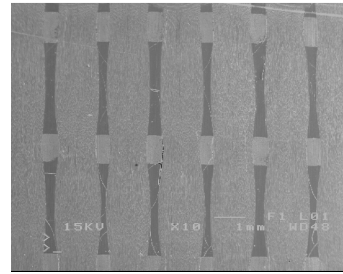
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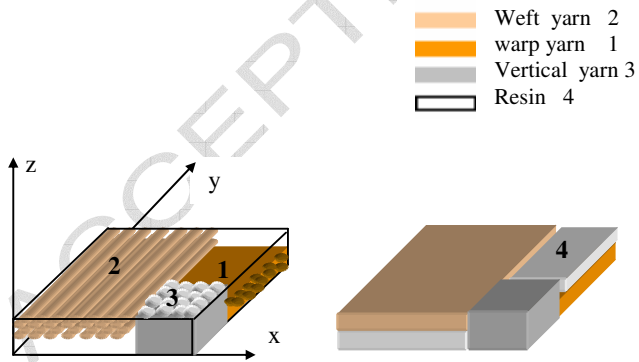
(a) Longitudinal section



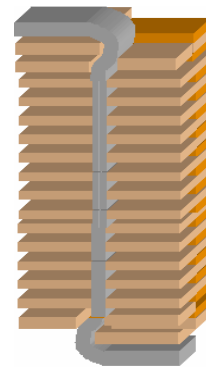
(b) Transversal section



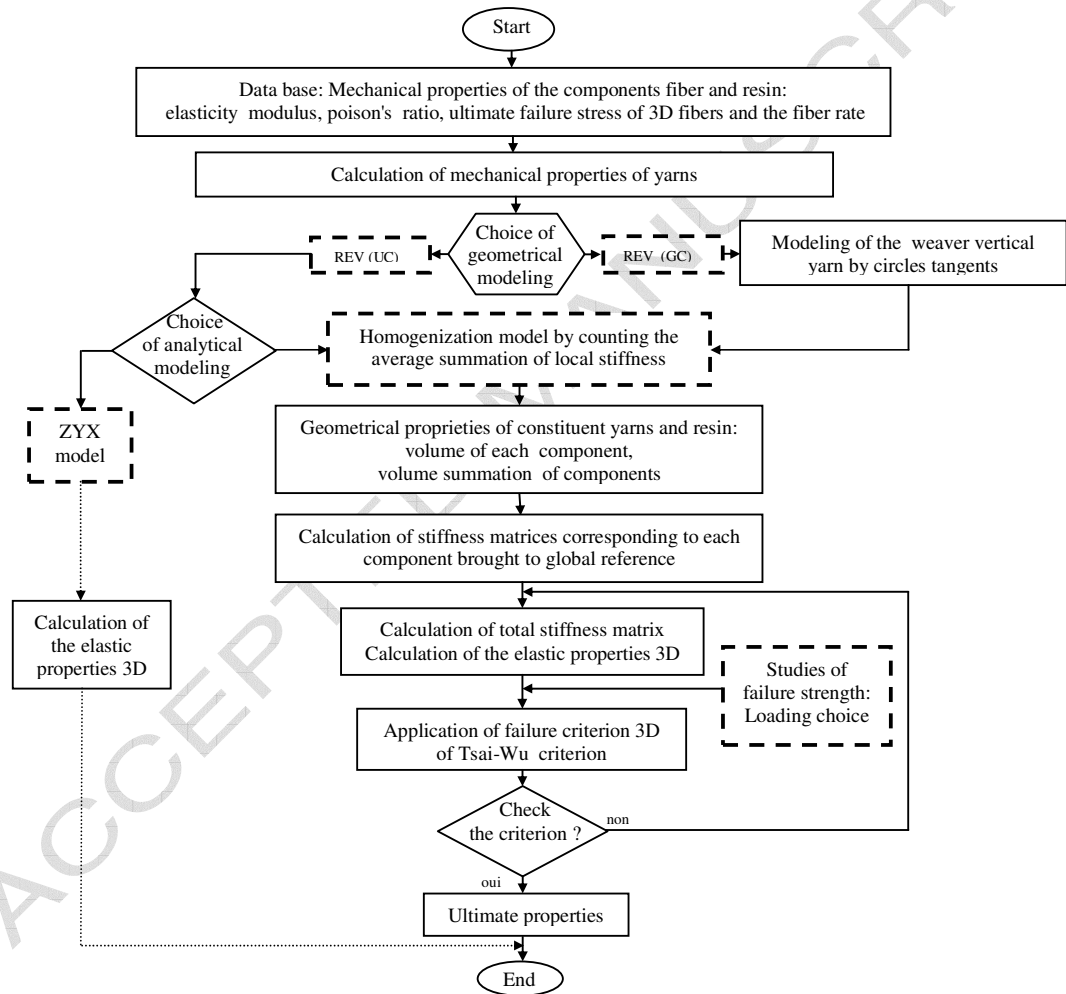
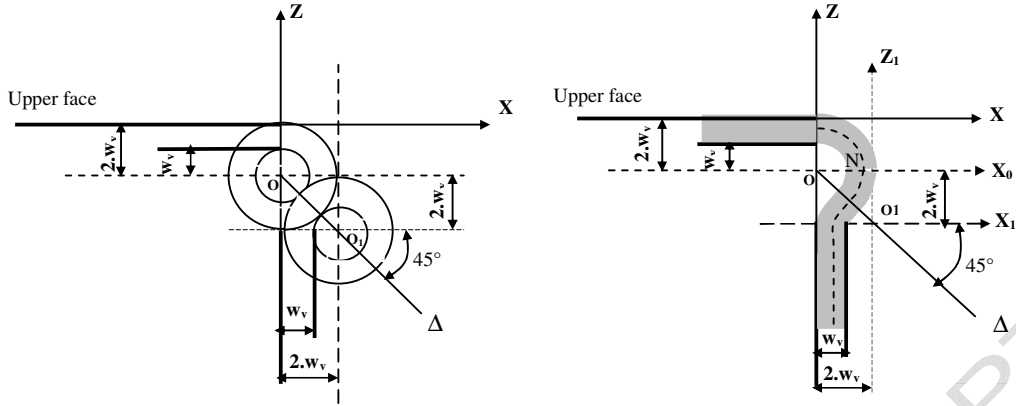
(c) Horizontal section



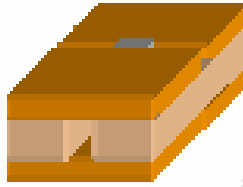
(a) Unit cell



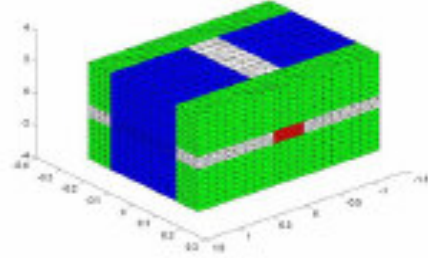
(b) Global cell



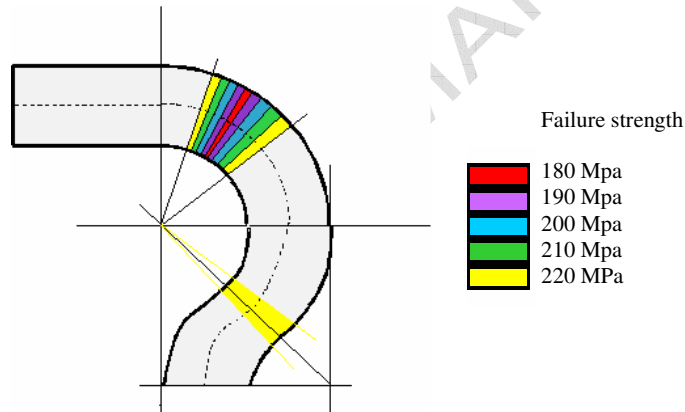




(a) Orthotropic REV of unit cell



(b) Meshing of REV, unit cell



| The components | Volume of constituents of VER<br>(mm <sup>3</sup> ) |                  | Volume proportion in the VER |                  |
|----------------|---|------------------|------------------------------|------------------|
|                | Unit cell (UC)                                      | Global cell (GC) | Unit cell (UC)               | Global cell (GC) |
| Warp yarn      | 0.1851  | 5.9233           | 30.82 %                      | 27.74 %          |
| Weft yarn      | 0.2570  | 7.7988           | 42.80 %                      | 36.53 %          |
| Vertical yarn  | 0.0395  | 1.9239           | 6.58 %                       | 9.01 %           |
| Resin          | 0.1189  | 1.9239           | 19.80 %                      | 26.72 %          |

| Analytical models | Type of yarn | Vf     | Mechanical elastic properties of yarns |                                      |  |                   |            |            |
|-------------------|--------------|--------|--|--------------------------------------|--|-------------------|------------|------------|
|                   |              |        | $E_L$<br>(Gpa)                         | $E_T$<br>(=E <sub>Z</sub> )<br>(Gpa) | $G_{LT}$<br>(=G <sub>LZ</sub> )<br>(Gpa) | $G_{TZ}$<br>(Gpa) | $\nu_{LT}$ | $\nu_{TZ}$ |
| Unit cell (UC)    | Warp         | 0.7163 | 165.58                                 | 15.11                                | 6.05                                     | 5.17              | 0.31       | 0.45       |
|                   | Weft         | 0.5159 | 120.05                                 | 8.42                                 | 3.24                                     | 2.85              | 0.32       | 0.47       |
|                   | vertical     | 0.5840 | 135.52                                 | 10.01                                | 3.91                                     | 3.52              | 0.33       | 0.49       |
| Global cell (GC)  | Warp         | 0.7959 | 183.65                                 | 21.04                                | 8.52                                     | 7.88              | 0.31       | 0.45       |
|                   | Weft         | 0.6045 | 140.18                                 | 10.59                                | 4.15                                     | 3.74              | 0.32       | 0.47       |
|                   | vertical     | 0.4262 | 99.68                                  | 6.89                                 | 2.60                                     | 2.36              | 0.33       | 0.49       |

| Propriétés mécaniques 3D du tissage orthogonal |                |                |                |            |            |            |                   |                   |                   |
|--|----------------|----------------|----------------|------------|------------|------------|-------------------|-------------------|-------------------|
|  | $E_1$<br>(Gpa) | $E_2$<br>(Gpa) | $E_3$<br>(Gpa) | $\nu_{12}$ | $\nu_{13}$ | $\nu_{23}$ | $G_{12}$<br>(Gpa) | $G_{13}$<br>(Gpa) | $G_{23}$<br>(Gpa) |
| Analytical model                               | 57.302         | 58.631         | 19.808         | 0.0723     | 0.2677     | 0.2692     | 3.687             | 3.553             | 3.451             |
| Unit cell                                      | 57.462         | 58.944         | 15.664         | 0.0676     | 0.3677     | 0.3702     | 4.307             | 4.114             | 3.960             |
| Global cell                                    |                |                |                |            |            |            |                   |                   |                   |
| ZYX model                                      | 56.570         | 56.746         | 19.304         | 0.0743     | 0.2631     | 0.2614     | 3.093             | 2.753             | 2.152             |
| Finite element (37577 trihedral)               | 59.034         | 63.313         | 34,185         | 0.1044     | 0.3334     | 0.3598     | 3.950             | 3.788             | 3.158             |
| Experimental                                   | 57.5±1.8       | -              | 15.534±1.09    | 0.028      | 0.269      | 0.268      | 4.114±0.05        | -                 | -                 |

| Strain and failure strength (Mpa) |                 |                  |             |               |
|-----------------------------------|-----------------|------------------|-------------|---------------|
| Type de sollicitation             |                 | Analytical model |             | Experimental  |
|                                   |                 | Unit cell        | Global cell |               |
| Tension in x direction            | $\sigma_{xt}$   | 770              | 770         | 780 ± 50      |
|                                   | $\epsilon_{xt}$ | 1.47 %           | 1.46 %      | 1.25 % ± 0.07 |
| Tension in y direction            | $\sigma_{yt}$   | 770              | 710         | -             |
|                                   | $\epsilon_{yt}$ | 1.46 %           | 1.45 %      | -             |