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MODIFIED DISCRETE MINDLIN HYPOTHESISES FOR LAMINATED COMPOSITE STRUCTURES

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ABSTRACT

The present work deals with a procedure of defining discrete kinematic and mechanic hypothesises for Reissner-Mindlin plate finite elements. They are associated with the transverse shear strains for which the corresponding classical approximations caused the shear locking problem. Used in the past for isotropic homogeneous bending-shear plates, these hypothesises have been locally modified in order to be used for laminated composite plates. The derived plate finite element model, labelled DDM "Displacement Discrete Mindlin" has been applied to a 3-node triangular (DMTP: Discrete Mindlin Triangle for Plates) and validated on some standard problem tests, including one or several orthotropic layers.

KEYWORDS: Plate, laminated composites, finite element, bending-shear, discrete Mindlin

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ANALYTICAL ASPECTS OF THE DDM MODEL

The formulation of simple and efficient finite elements has been always the main concern of the researchers in the field of bending-shear plates. The most interesting approach was certainly the use of discrete Kirchhoff hypothesis for formulating the well known finite element for thin plates DKT (Bathe et al, 80). The authors used first a quadratic interpolation of the normal rotations β_x and β_y. Their element had initially 6 nodes with the previous two rotations per node: the bending behaviour was thus well represented. After introducing the following hypothesis (called Discrete Kirchhoff hypothesis) on the three sides:

$$\int_0^{L_k} \gamma_s ds = 0 \quad ; \quad \gamma_s = w_{,s} + \beta_s \qquad \textit{(zero shear strains: thin plate)} \tag{1}$$

the rotational mid-side degrees of freedom (dofs) will be written in terms of the transverse displacement w (deflection) on the three corner nodes. The final element, labelled DKT (Discrete Kirchhoff Element), is very simple (3 nodes and 3 dofs per node: w,β_x,β_y) and performs well all standard tests for thin isotropic homogeneous plates. It has been implemented in the majority of finite element programs.

This popular element remained unfortunately limited for thin structures without shear effects. It became then necessary to think about new procedures leading to simple and efficient finite elements which would take into account the effects of transverse shear. The first contributions in this field (Aalto, 88) (Lardeur et al, 89) (Onate et al, 92) proposed interesting triangular elements but with some difficulties. They we will be detailed in the full paper. For isotropic plates, Katili (Katili, 93) and Ayad (Ayad et al, 01) proposed to use two discrete Mindlin hypothesises on the three sides of the element, defined initially with 6 nodes. These hypothesises are written as

$$\int_0^{L_k} \left(\gamma_s - (w_{,s} + \beta_s)\right) ds = 0 \qquad \textit{(kinematic hypothesis)} \tag{2}$$

$$\int_0^{L_k} \left(\gamma_s - \frac{T_s}{D_s}\right) ds = 0 \,;\, D_s = kGh \,;\, G = \frac{E}{2(1+\nu)} \qquad \textit{(mechanical hypothesis)} \tag{3}$$

γ_s and T_s are the shear strain and force on the side s, E the elasticity modulus, ν the Poisson coefficient, k the shear correction factor.

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The derived 3-node finite elements DKMT (displacement model proposed by Katili (Katili, 93)) and MiSP3+ (hybrid-mixed model proposed by Ayad (Ayad et al, 01)) are very simple and robust. They performed well all tests for isotropic bending-shear isotropic plates, from thin to thick situations. The second hypothesis (3) shows clearly the limitation to isotropic homogeneous plates for these elements. In order to compute laminated composite structures, we have introduced in our recent works (Ayad, 02) (Ayad et al, 03) a new modified mechanical Mindlin hypothesis, generalising then the equation (3). This new hypothesis takes the following form:

$$\int_0^{L_k} \left(\gamma_s - < H_{ck11}^{inv} \quad H_{ck12}^{inv} > \begin{Bmatrix} T_s \\ T_n \end{Bmatrix} \right) ds = 0 \tag{4}$$

H_{ck11}^{inv} and H_{ck12}^{inv} are two behaviour terms of the following shear inverse matrix $[H_{ck}]^{-1}$:

$$[H_{ck}] = [P_k][H_c][P_k]^{-1} \; ; \; [P_k] = [P_k]^{-1} = \begin{bmatrix} c_k & s_k \\ s_k & -c_k \end{bmatrix} ; \; \begin{cases} c_k = \cos\alpha_k \\ c_k = \sin\alpha_k \end{cases} \text{ (see figure 1)} \tag{5}$$

$[H_c]$ is the transverse shear behaviour matrix, defined for one or several layers (laminated composites). It will be detailed with other matrices in the full paper. Introducing the bending behaviour equation $\{M\}=[H_f]\{\chi\}$, quadratic and linear interpolations respectively for the edge rotations β_s and β_n, and the following equilibrium equations:

$$T_s = M_{s,s} + M_{sn,n} \quad ; \quad T_n = M_{sn,s} + M_{n,n} \tag{6}$$

the equation (4) becomes (ϕ_k^{lc} being the new shear influence factor for a laminated composite plate):

$$\gamma_{sk} = -\phi_k^{lc} \Delta\beta_{sk} \tag{7a}$$

$$\phi_k^{lc} = 8\left(H_{f11} \frac{H_{ck11}^{inv}}{L_k^2} + H_{f13} \frac{H_{ck12}^{inv}}{L_k^2} \right) \; ; \; \textit{(lc: laminated composite)} \tag{7b}$$

All these previous equations will contribute to final and modified expressions of curvatures, shear strains and stiffness matrices in consequence. These matrices are exactly integrated using only three hammer points. More details will be presented in the full paper.

NUMERICAL RESULTS

Simply supported under uniform loading of a sandwich plate

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A simply supported square sandwich plate (3-layers) is submitted to a uniform loading f_z. Problem data are given by figure 2. The skin material (Layers 1 and 3) is orthotropic. The material properties of layer 2 (heart) are proportional to those of layers 1 and 3 (shear modulus of the heart = shear modulus of skins divided by 10). The results of central deflection w_c are reported on the table 1. They are compared to a 3D solution given by Srinivas (Srinivas, 73) ($\overline{w}_c = w_c G_{12}(2)/hf_z$).

A good correlation between our model DMTP (Discrete Mindlin Triangular for Plates) and the 3D solution is then obtained. DMTP also performs well other multilayered plate examples. We'll present some of them in the full paper.

A Simply supported 9-layer square plate under uniform pressure load

A simply supported 9-layer unidirectional square laminate with length *L=1000* and thickness *t=250, 100* and *20,* is analysed with the proposed element using a number of mesh size (Figure 3). The plate is subjected to a distributed transverse load of intensity $q = q_0 \sin(\pi x/L) \sin(\pi y/L)$ where q_0 is its amplitude at the plate centre.

The central normalized deflections $\underline{W}_C$ and stress σ_{xz} obtained from the present elements, together with those from 3D solution given by Pagano and Hatfield[Ex5_1] and the plate element DST developed by Lardeur & Batoz [Ex5_2], are presented in Table 2. The table clearly shows that the performance of the proposed DMTS element is very good in terms of solution accuracy and the rate of convergence. It also indicates that the results obtained by PRHSDT are closer to the 3-D elasticity solution [32] compared to the PHSDT and PFSDT (Table 1) as expected.

It is obvious that the results from element DMTS are in good agreement with the results from 3D solution compared to the element DST.

CONCLUSION

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Two Mindlin hypothesises have been presented in a discrete manner. They have been adapted for being used in the simulation of laminated composite plates and shells. The corresponding DDM model (Displacement Discrete Mindlin) can be considered as a new alternative to the known Discrete Kirchooff elements DKT and DKQ (only used for thin plates). The derived finite elements Triangular DMTP or Quadrilateral DMQP (will be developed in our future works) are very simple to formulate. Thay have a minimum number of degrees of freedom.

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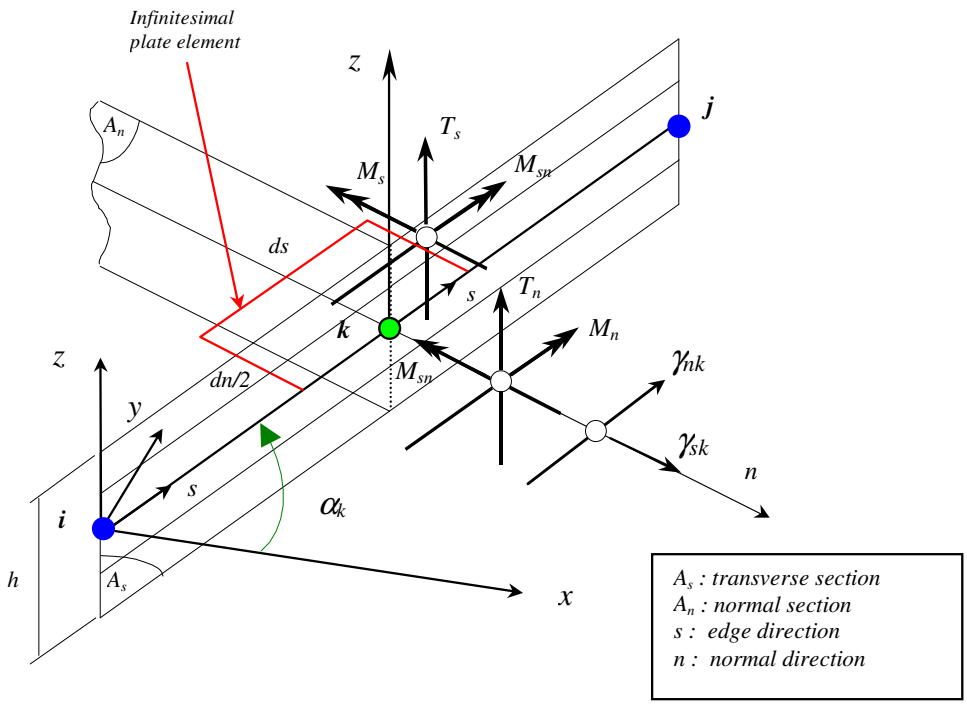
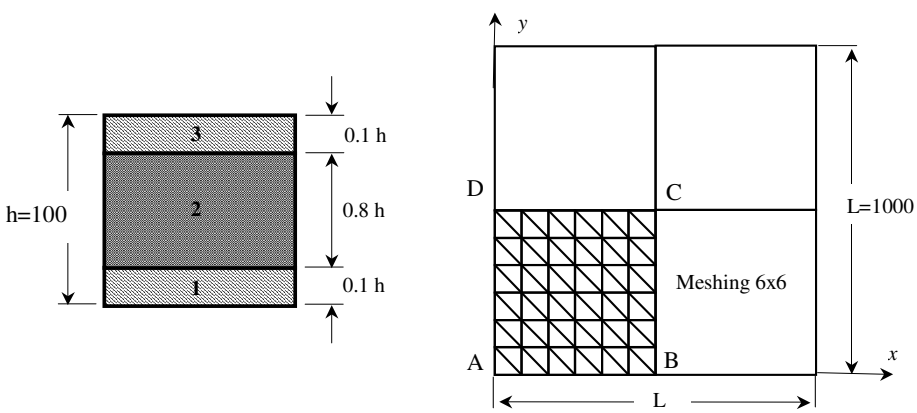


Figure 1. Modified Mindlin hypothesis on a side *ij* of a multilayered finite element



Boundary conditions:	Material: (skins & heart orthotropic)
on AB : $W=\theta_Y=0$	E_1= 3.4156, E_2= 1.7931, ν_{12}= 0.44
on AD : $W=\theta_X=0$	G_{12}= 1. , G_{13}= 0.608, G_{23}= 1.015
on BC : $U=\theta_Y=0$	Stratification: (sandwich)
on DC : $V=\theta_X=0$	3 layers 0 / 0 / 0 symmetrical
	k_{11}= k_{22}=0.3521, k_{12} =0

Figure 2 - Simply supported under uniform loading of a sandwich plate. Problem data

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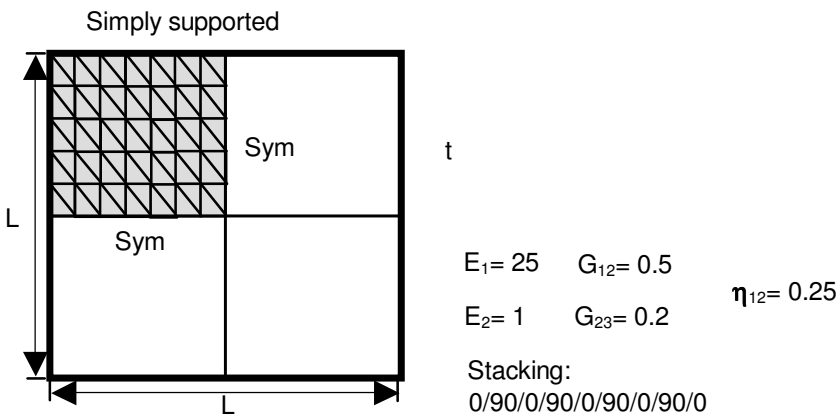

Figure 3 – Square plate geometry

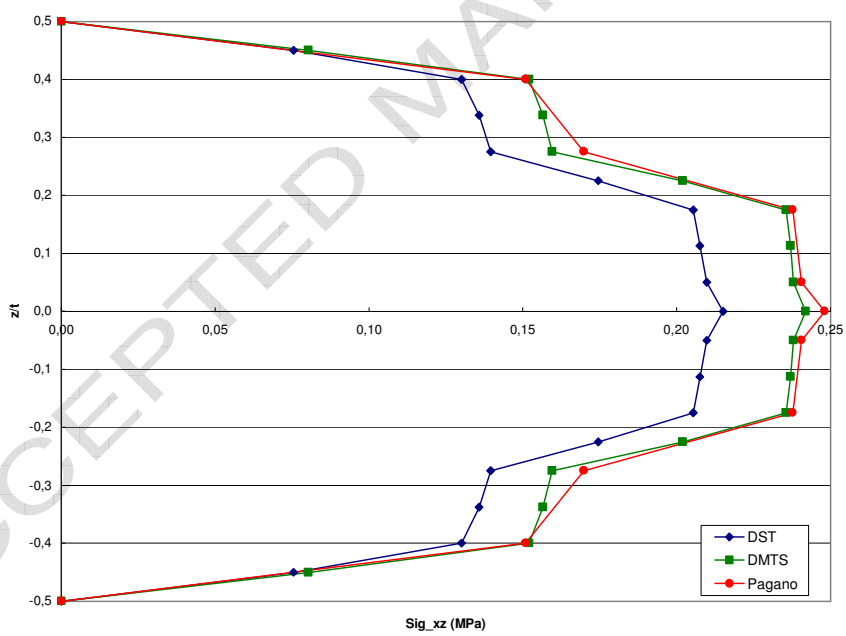

Figure 4 - Simply supported under uniform loading of a sandwich plate. Problem data

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Table 1:

Simply supported under uniform loading of a sandwich plate. Central deflection $\overline{w}_c$

	3D Sol. (Srinivas, 73)	DMTP
$k_{11}= k_{22}=0.3521$	41.91	41.31

Table 2

Convergence of the deflection and stress at point C with L/t ration

	L/t	4			10			50		
	Mesh	DMTS	DST	3D	DMTS	DST	3D	DMTS	DST	3D
W	6x6	4,419	4.227	4.079	1.612	1.513	1.512	1,091	1,007	1,021
(L/2,L/2,0	10x10	4,186	4.242			1.526			1.020	
σ_{xz}	6x6	0.221	0.221	0.223	0.213	0.213	0.247	0.185	0.185	0.259
(0,L/2,0	10x10		0.225			0.219			0.190	

$$\underline{W} = W_c 100 \frac{E_2 t^3}{q_0 a^4}$$

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