

1 INTRODUCTION

Objective : Consider the *loudness* and *sustain* (duration) compromise in piano
 Influencing factor : **Input admittance** between the string and the resonator
 We propose a method of **validation of admittance measurements** on simple structures
 The method is implemented on a **simplified device : piano string coupled to a thin steel beam**
 Input admittance is derived from **vibratory measurements of the string and compared to direct measurements**

2 STRING-SOUNDBOARD COUPLING IN PIANOS

Principal sketch of the piano, with the main components. [Askenfelt, 1990]

Soundboard admittance : $Y(\omega) = \frac{V(L, \omega)}{F(L, \omega)} = G(\omega) + jB(\omega)$ (1)
 Characteristic impedance of the string : $Z_c = \frac{T}{c} = \sqrt{\mu T} = \frac{F(L, \omega)}{V(L, \omega)}$ (2)
Mean mechanical power transmitted from string to soundboard :
 $P_m(\omega) = 1/2 \operatorname{Re}\{F(L, \omega)V^*(L, \omega)\} = 1/2 G(\omega)Z_c^2|V(L, \omega)|^2$ (3)

3 SOUNDBOARD IMPEDANCE

Expected : Asymptotical value $Z(\omega) \rightarrow 8\sqrt{\rho_p h D}$ (4) [Busch-Vishniac, 1981]
 where $D \approx \sqrt{E_1 E_2} h^3 / [12(1-\nu^2)]$ is the rigidity of an orthotropic plate
Measured :

→ **Necessity of reconsidering these measurements**

4 INFLUENCE OF SOUNDBOARD ON STRING

Perturbation of string's complex eigenfrequencies :

$$\delta\omega_n = \frac{j c Z_c Y(\omega)}{L} = \frac{j \omega_n Z_c}{\pi} [G(\omega) + jB(\omega)] = j \alpha_n + 2\pi \delta f_n$$
 (5)
 Damping factors in [s⁻¹] : $\alpha_n = \frac{T}{L} G(\omega)$ (6)
 Shift of eigenfrequencies in [Hz] : $\delta f_n = \frac{-T}{2\pi L} B(\omega)$ (7)
Idea → **Why not measuring them to derive the soundboard admittance ?**

5 METHOD AND VALIDATION

- Step 1** Estimation of eigenfrequencies and decay times on an isolated string.
- Step 2** Compare with the string loaded at one end by a known admittance.
- Step 3** Derive the end admittance from these measurements.
- Step 4** Compare results of Step 3 with directly measured load admittance and asymptotical value

EXPERIMENTAL SET-UP

The prototype :

String loaded at one end by a known admittance (thin beam)

Difficulties :

- Keep same length and same tension
- Avoid exciting horizontal polarization
- The beam is prestressed by the string

RESULTS

Influence of coupling

Shift of string's eigenfrequencies + Apparition of beam's peaks

Damping factors of string partials

Precise estimation of frequencies and damping factors → **Esprit algorithm** [Roy et al., 1986 & 1989]

$$x(t) = \sum_{i=1}^L A_i e^{-\alpha_i t} \cos(2\pi f_i t + \Phi_i)$$

Amplitudes : A_i Phases : Φ_i

7 LOAD ADMITTANCE (Step 4) :

Comparison between direct measurements and calculation

Calculation derived from coupled string's spectrum (black points) Eq (6) and (7)
 Theoretical asymptotic value
 Direct measurements (Step 4)

CONCLUSION

- Frequency range **A** : $f \in [0; 7 \text{ kHz}]$
 → Calculations derived from string's spectrum are compatible with load admittance measurements
 Frequency range **B** : $f > 7 \text{ kHz}$
 → Compatibility between asymptotical value and measurements
 → The method overestimates the load admittance in high frequency
 Hypotheses : { 1. Esprit algorithm (wrong estimation)
 2. Physical model (no more acceptable)