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Optimization of Aeroelastic Composite Structures using Evolutionary Algorithms

Abdul. Manan, Gareth.A. Vio, M.Yazdi. Harmin and Jonathan.E. Cooper
Department of Engineering, University of Liverpool, Liverpool, L69 7EF, UK

Corresponding Author

Professor Jonathan Cooper

Tel: +44 151 794 5232

Fax: + 44 151 794 4848

Email: j.e.cooper@liverpool.ac.uk

Optimization of Aeroelastic Composite Structures using Evolutionary Algorithms

A. Manan, G.A. Vio, M.Y. Harmin and J.E. Cooper

Department of Engineering, University of Liverpool, Liverpool, L69 7EF, UK

The flutter / divergence speed of a simple rectangular composite wing is maximised through the use of different ply orientations. Four different biologically inspired optimization algorithms (binary genetic algorithm, continuous genetic algorithm, particle swarm optimization and ant colony optimization) and a simple meta-modelling approach are employed statistically on the same problem set. It was found in terms of the best flutter speed, that similar results were obtained using all of the methods, although the continuous methods gave better answers than the discrete methods. When the results were considered in terms of the statistical variation between different solutions, Ant Colony Optimization gave estimates with much less scatter.

Keywords: Aeroelastic Tailoring, Composite Lay-up, Flutter, Evolutionary Optimization

I. Introduction

Aeroelasticity (Wright & Cooper 2007) is the science that incorporates the interactions between a flexible structure and surrounding aerodynamic forces. In general, aeroelastic phenomena are undesirable and can either reduce aircraft performance or, in extreme cases, cause structural failure either statically (divergence) or dynamically (flutter). Aircraft designers are interested in the speeds at which any instabilities occur, the dynamic response due to gusts and manoeuvres, and also the shape that the aircraft wings take in-flight as this has a significant effect on the drag and resulting fuel efficiency and performance.

Traditional aircraft design has always tended to avoid aeroelastic problems through stiffening the structure with extra material and accepting the resulting weight penalty. In recent years, there has been a move towards trying to use aeroelastic deflections in a positive manner, and this has resulted in research programmes such as the Active Flexible Wing (Perry et al. 1995), the Active Aeroelastic Wing (Pendleton et al. 2000), the Morphing Program (Wlezin et al. 1998) and Active Aeroelastic Aircraft Structures project (Schweiger & Suleman 2003) that have used made used to several novel active technologies. However, a more traditional passive approach to making use of aeroelastic deflections in a positive way is to use carbon fibre composite structures.

Despite the benefits of composite structures, it is only recently that the main load bearing structures in large aircraft such as the Boeing 787 and Airbus A350 have started to be manufactured using carbon fibre composites. Even then, the unique directionality properties of composite laminates have yet to be exploited fully in order to improve the aircraft performance. Examples of work that has optimized the stacking sequence of composite structures include (Pagano et.al. 1971) who studied the effect of 15,45 ply lay-up on the laminate strength. Genetic Algorithms have been widely used couple in multi-level optimisation routines to optimise wing stiffeners elements (Herencia et.al. 2007, Liu et al. 2008. The effect of stacking sequence was investigated for the creation of composite bi-stable laminates (Mattioni et al. 2009) using 0,90 and 45 degrees ply directions. (Autio 2000) used a lamination parameter in order to optimise buckling/frequency of a composite plate with a ply angle search.

The idea of using the directional property of composites for aeroelastic tailoring has been around since the 1970s (Shirk et al. 1986, Weisshaar 1981). However, since tailoring was demonstrated on the X-29 in the late 1970s and early 1980s, very few aircraft have used these directional properties to achieve beneficial aeroelastic effects. The

original application was to reduce the likelihood of divergence occurring on forward-swept wings (Shirk et al. 1986, Weisshaar 1981); recent applications have included weight reduction (Arizono & Isogai 2005, Kim et al. 2007, Kameyama & Fukunaga 2007, Eastep et al. 1999, Guo 2007), drag reduction (Weisshaar and Duke 2006) and passive gust alleviation (Petit & Grandhi 2003, Kim & Hwang 2005) of composite wings. Although the new generation of commercial civil aircraft has started to use composites, they have only exploited the superior strength/weight ratio of composite materials rather than employing aeroelastic tailoring, in effect using the composite as a “black metal”.

There are a wide range of different optimization approaches that can be used for design problems. The two main categories are Hill-Climbing and Evolutionary Methods. Evolutionary algorithms tend to be based upon the mimicry of some biological or physical process and have been proven to be effective for large parameter space solutions and do not suffer from the local optima problems that can occur in the Hill-Climbing approaches; however, Evolutionary Algorithms do not guarantee to achieve the global optimum solution. In the aeroelastic tailoring field, Genetic Algorithms have been used to minimise the structural weight whilst satisfying a number of aeroelastic parameters such as flutter and divergence (Shirk et al. 1986, Weisshaar 1981, Kim et al. 2007). However there have been few known aeroelastic applications of Particle Swarm Optimization or Ant Colony Optimization.

In this study, a simple assumed modes mathematical model of a rectangular composite wing with unsteady aerodynamics was developed in order to assess the ability of tailoring the composite structure to maximise the flutter / divergence speed. Four different biologically inspired optimization techniques, including discrete and continuous approaches, were applied 100 times each to this problem in order to evaluate their performance in a statistical manner. A further set of analysis was performed using a simple meta-modelling approach.

II. Mathematical Model

A. Structural Modelling

The composite lifting surface was idealised as a rectangular plate, as shown in Figure 1, using the Rayleigh-Ritz assumed modes method (Wright & Cooper 2007, Al-Obeid & Cooper 1995), which addresses the minimization of energy functional composed of strain and kinetic energy. The problem considered here is a rectangular composite plate wing with a semi-span to chord ratio of four (i.e. on a full aircraft modeling both wings this would give an aspect ratio of eight) and a six layer fibre angle lay-up of $(\theta_1, \theta_2, \theta_3)_s$.

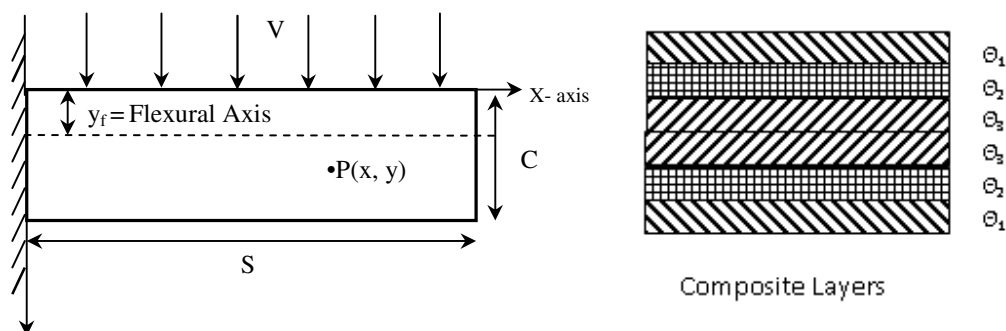


Figure 1. Rectangular Wing with Six Composite Layers

The strain energy over the entire plate domain Ω can be expressed as

$$U = \frac{1}{2} \iiint_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \sigma_{xz} \varepsilon_{xz} + \sigma_{yz} \varepsilon_{yz} + \sigma_{xy} \varepsilon_{xy}) dx dy dz \quad (1)$$

which reduces, by taking into account of assumptions that transverse shear and normal stresses are negligible, to

$$U = \frac{1}{2} \iiint_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_{xy} \varepsilon_{xy}) dx dy dz \quad (2)$$

in which σ_i and ε_i are stress and strain (where $i = x, y, xy$) that are related to each other via transformed reduced stiffness matrix, \bar{Q}_{ij} , by the following relation for each ply k

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \quad (3)$$

Replacing stress relations defined in (3) into (2) we get following equation

$$U = \frac{1}{2} \iiint_{\Omega} (\bar{Q}_{11} \varepsilon_x^2 + \bar{Q}_{12} \varepsilon_x \varepsilon_y + 2\bar{Q}_{16} \varepsilon_x \varepsilon_{xy} + 2\bar{Q}_{26} \varepsilon_y \varepsilon_{xy} + \bar{Q}_{22} \varepsilon_y^2 + \bar{Q}_{66} \varepsilon_{xy}^2) dx dy dz \quad (4)$$

Then, utilising the strain-displacement relations for this pure bending problem with symmetric lay-up configurations, expression (4) reduces to

$$U_{\max} = \frac{1}{2} \iint_{\Omega} \left\{ \begin{aligned} & D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2D_{12} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + \dots \\ & 4D_{16} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial x \partial y} \right) + 4D_{26} \left(\frac{\partial^2 w}{\partial y^2} \right) \left(\frac{\partial^2 w}{\partial x \partial y} \right) + 4D_{66} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \end{aligned} \right\} dx dy \quad (5)$$

where $D_{ij} = \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$ in which z_k is k th layer distance along z -axis from mid-plane of the plate and w is the out of plane deflection of the plate. Out of plane deflections (expressed at some point $P(x,y)$ on the surface of the wing) of the composite lifting surface are expressed as

$$w = \sum_{i=1}^n \gamma_i(x, y) q_i(t) \quad (6)$$

where $q_i(t)$ is the generalised displacement of the i^{th} mode represented with $\gamma_i(x, y)$ (Taken here as polynomial series of $x^2, x^3, x^4, x^2(y - y_f), x^3(y - y_f), x^4(y - y_f), x^2(y - y_f)^2, x^3(y - y_f)^2$ and $x^4(y - y_f)^2 \dots$, then strain energy can be calculated by utilizing equation (5). Similarly, the maximum kinetic energy of the entire plate domain Ω can be formulated as

$$T_{\max} = \frac{1}{2} \rho h \omega^2 \iint_{\Omega} w^2(x, y) dx dy \quad (7)$$

where ρ is the mass per unit area of plate, h is plate thickness and ω is the frequency of vibration. Minimization of the functional ($U_{\max} - T_{\max}$) with respect to each coefficient q_i results the following stiffness and mass matrices respectively

$$E_{ij} = \iint_{\Omega} \left\{ \begin{array}{l} D_{11} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial x^2} + D_{12} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial x^2} + 2D_{16} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial x^2} + \dots \\ D_{12} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial y^2} + D_{22} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial y^2} + 2D_{26} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial y^2} + \dots \\ 2D_{16} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial x \partial y} + 2D_{26} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial x \partial y} + 2D_{66} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial x \partial y} \end{array} \right\} dx dy \quad (8)$$

$$A_{ij} = \iint_{\Omega} \rho \gamma_i \gamma_j dx dy \quad (9)$$

Comparison with Finite Element analysis showed that a very good representation of the dynamic behavior of the composite wing could be achieved using the above model.

B. Aerodynamic Modelling

In this work a modified strip theory approach, in which unsteady effects are introduced via the torsional velocity term (Wright & Cooper 2007), was employed to develop the aerodynamic model. Strip theory divides the wing into infinitesimal strips on which lift acting on the quarter chord is assumed to be proportional to the dynamic pressure, local angle of attack, lift curve slope and the downwash due to the vertical motion. Although strip theory would not be used for the design of commercial jet aircraft by the aerospace industry, the approach does give a representative conservative model of the static and dynamic aeroelastic behavior of high aspect ratio wings at low speeds, and remains as a possible analysis approach in the airworthiness regulations. Previous unpublished studies have shown that similar results for the type of wing model considered here are obtained using the modified strip theory compared to the Doublet Lattice approach used in commercial software packages. The aim of this paper is to investigate the behavior of several optimization methods and not to produce a perfect aeroelastic model, thus any variations in the aerodynamic modeling will not affect the optimization methods, however, further investigation is required in the transonic flight regime where the aerodynamic behavior becomes nonlinear. For ease of computational effort it was decided to use the modified strip theory approach throughout this work.

C. Aeroelastic Modelling

Considering the incremental work done over entire wing due to the lift and pitching moment (about the flexural axis), application of Lagrange's equations eventually leads to the formulation of the B (aerodynamic damping) and C (aerodynamic stiffness) matrices (Wright & Cooper 2007) which can be coupled with the above structural terms to give the aeroelastic equation of motions in the classical form

$$(A) \ddot{\underline{q}} + (\rho V B + D) \dot{\underline{q}} + (\rho V^2 C + E) \underline{q} = \underline{0} \quad (10)$$

D is the structural damping matrix which cannot be predicted and requires test measurements to get accurate estimates. As with most aeroelastic modeling, the structural damping can be set to zero as the aerodynamic damping terms have a far greater effect (Wright & Cooper 2007).

Rewriting into first order matrix form, equation (10) becomes

$$\begin{pmatrix} \dot{q} \\ \ddot{q} \end{pmatrix} - \begin{pmatrix} 0 \\ (\rho V^2 C + E) \end{pmatrix} \begin{pmatrix} I \\ (\rho V B + D) \end{pmatrix} \begin{pmatrix} q \\ \dot{q} \end{pmatrix} = \dot{x} - Qx = \underline{Q} \quad (11)$$

and the eigenvalues of matrix Q lead to the system frequency and damping ratios at any given flight condition. The flutter speed at a given altitude can be determined by finding the air speed where one of the damping values becomes negative. In some cases divergence might occur before flutter, and this is characterised by positive real eigenvalues occurring.

A total of nine assumed modes were assumed for the composite wing model for which material properties used are given in Table 1. The frequency and damping ratio trends of the lowest three modes for a typical case are shown in Figure 2, and it can be seen that a classical flutter mechanism results through the coupling between the first two modes and the flutter speed occurs when the Mode 2 damping curve becomes zero.

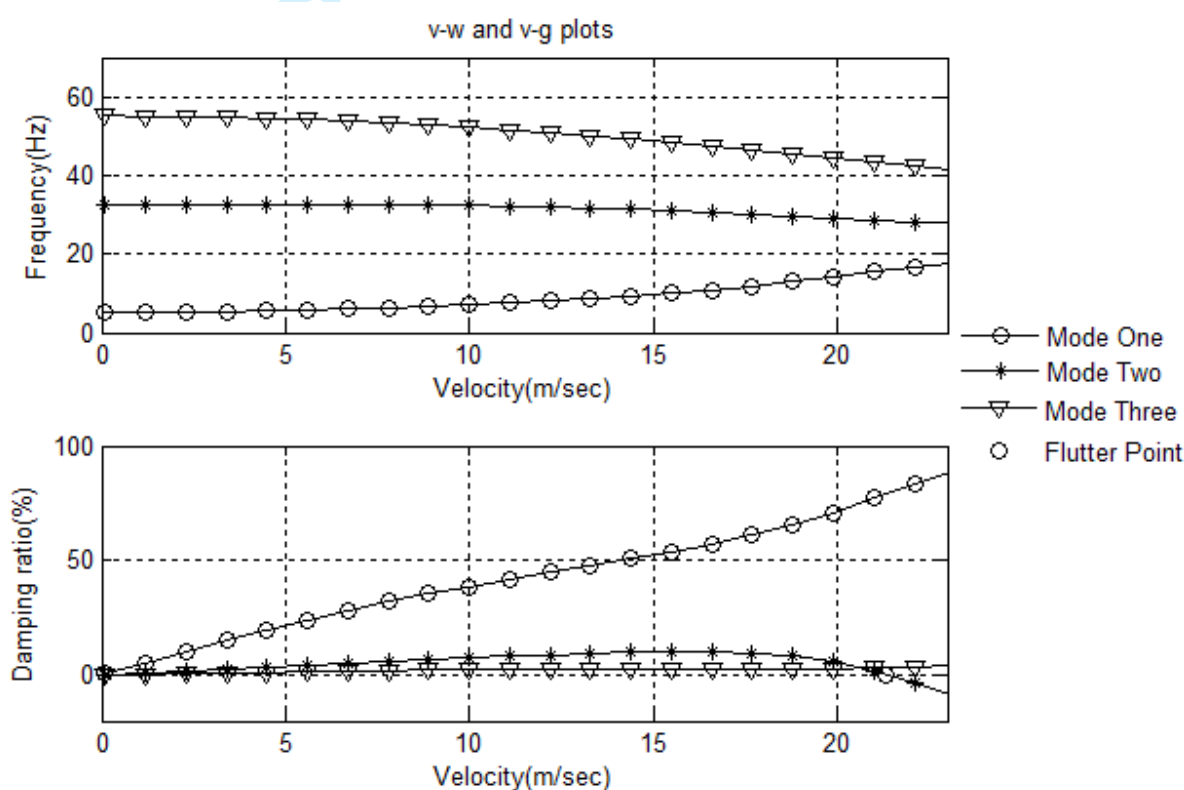


Figure 2. Typical Frequency and Damping Ratio Trends vs. Speed

III. Optimization Methods and Application

Four different optimization methods were considered for this study, all of which are based upon some form of evolutionary search based upon the mimicry of various types of biological system. Two of the approaches, Binary Genetic Algorithm (BGA) and Ant Colony Optimization (ACO), are discrete in the sense that the set of possible solutions are defined a-priori; here the number of possible orientations for each layer is defined by the number of bits in each gene (BGA) or the number of possible paths between each waypoint (ACO). The other two methods, Continuous Genetic Algorithm (CGA) and Particle Swarm Optimization (PSO) allow any orientation within the defined search space ($-90^\circ \leq \theta \leq 90^\circ$ for each layer) to be obtained.

In all cases, the objective was to find the combinations of ply orientations that maximised the air speed at which aeroelastic instability occurred, due to either flutter or divergence. Although this might seem to be a trivial problem

with only three parameters (θ_1 , θ_2 , θ_3) needing to be determined, due to the highly coupled nature of the aeroelastic system the optimized lay-up is not easy to find. Figure 3 shows a section of the solution space for constant $\theta_3 = 50^\circ$, calculated by enumeration, and it can be seen that there are a number of local optima and also regions of rather low instability speeds corresponding to solutions where the divergence speed is the critical case.

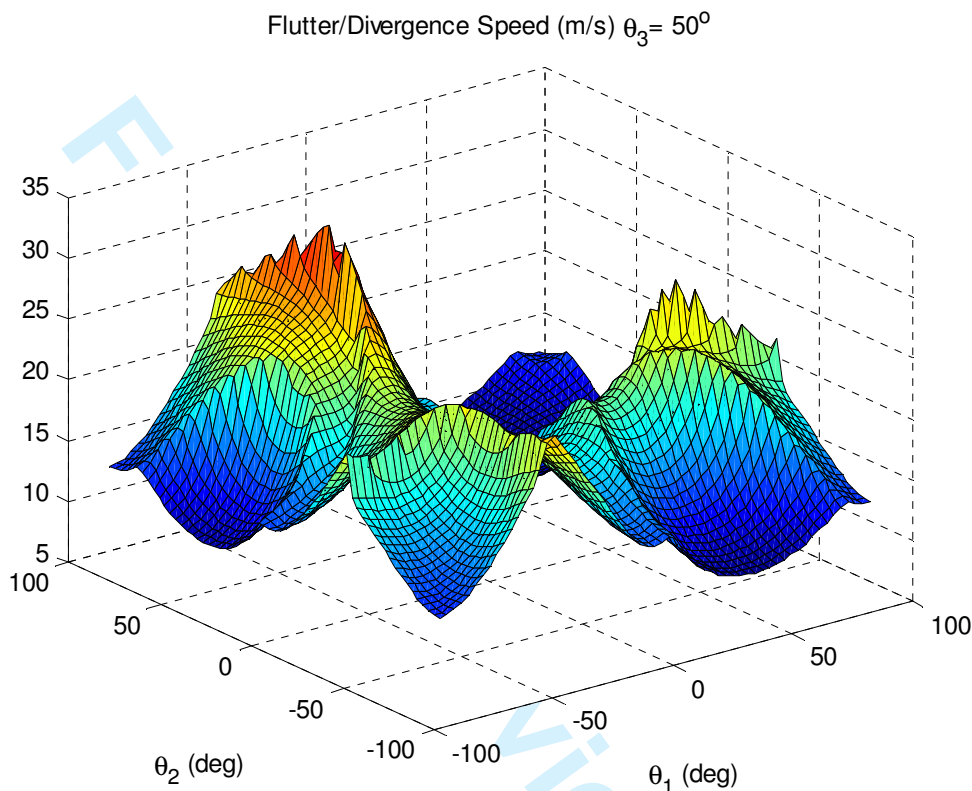


Figure 3. Flutter Speed Solutions for θ_1 vs. θ_2 for constant θ_3

A total of 20 solutions (genes, particles or ants) were used for each approach. For each solution case the methods were run for a maximum of 100 generations / iterations, with convergence considered to occur if the best solution did not vary for 20 iterations. A very brief description of the application of each optimization algorithm follows.

A. Binary Genetic Algorithm

Genetic Algorithms attempt to mimic Darwinian theory of natural selection which is based upon the traits of the most successful animals being passed onto future generations. In an optimization setting (Haupt & Haupt 2004), the characteristics of the best solutions from a range of initial estimates ("genes") are passed onto subsequent iterations via a series of mathematical operators, which is repeated until convergence is achieved. Randomness is added via application of a mutation function and also, possibly, the inclusion of "new-blood" solutions. The most common approach is to use a binary representation (BGA) of the system parameters.

Here, the binary representation of the composite lay-up assigned 5 bits (32 possible orientations – i.e. 5.625° between each possible orientation) to each layer, giving a gene length of 15 bits. A classical implementation of the binary GA was employed, with 20 genes being included in the gene pool, the 4 best genes saved after each iteration, a 90% probability of crossover, 5% probability of mutation and a 10% likelihood of translation.

B. Continuous Genetic Algorithm

Continuous or real number genetic algorithms (CGA) work (Haupt & Haupt 2004) in a similar way to the binary genetic algorithm described above. However, as the name suggests, the primary difference is in the variable representation of each gene. In CGA, the genes are represented using real numbers and consequently a re-definition of the mutation and crossover operators must be employed.

1. Mutation

The mutation operator for CGA requires the selection of a number of variables based on a mutation rate to be replaced by a new random variable. The best gene is left untouched in order to give an element of elitism to the generation.

2. Crossover

As for the binary BGA, a pair of genes is selected to create any offspring. For the BGA, if two points are selected and swapped, i.e.

$$\begin{aligned} \text{parent}_1 &= [p_{11} \ p_{12} \ p_{13} \ p_{14} \ p_{15} \ p_{16} \ \dots \ p_{1N}] \\ \text{parent}_2 &= [p_{21} \ p_{22} \ p_{23} \ p_{24} \ p_{25} \ p_{26} \ \dots \ p_{2N}] \end{aligned} \quad (12)$$

where N is the number of genes. By applying crossover (randomly chosen to occur after the 2nd cell), the following offspring are obtained

$$\begin{aligned} \text{offspring}_1 &= [p_{11} \ p_{12} \ p_{23} \ p_{24} \ p_{25} \ p_{16} \ \dots \ p_{1N}] \\ \text{offspring}_2 &= [p_{21} \ p_{22} \ p_{13} \ p_{14} \ p_{15} \ p_{26} \ \dots \ p_{2N}] \end{aligned} \quad (13)$$

It can be seen that no new information is passed to the offspring. However, for the CGA, new genetic material is introduced into the cross over process via the use of a blending function β such that

$$\text{offspring}_1 = \text{parent}_1 - \beta(\text{parent}_1 - \text{parent}_2) \quad (14)$$

where β is a random number between 0 and 1.

In this application, a population of 20 genes was chosen, with the mutation rate set at 0.2 and the crossover rate at 0.5.

C Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a heuristic search method which is based on a simplified social model that is closely tied to swarming theory and intelligence in which each particle of the swarm has memory and can also communicate with each other (Clerc 2006). The position and velocity of particle is updated by knowing the previous best values of each particle and overall swarm such that for the k th iteration

$$v_{id}(k+1) = wv_{id}(k) + c_1\phi_{1d}(p_{id}(k) - x_{id}(k)) + c_2\phi_{2d}(g_d(k) - x_{id}(k)) \quad (15)$$

$$x_{id}(k+1) = x_{id}(k) + v_{id}(k) \quad (16)$$

where v_i and x_i are the velocity and position of particle i , p_i and g_i are the best positions found by each particle and the entire population; ϕ_{1d} and ϕ_{2d} are independent uniformly distributed random numbers and are generated

independently. w , c_1 and c_2 are the user defined inertia factor ($w=1$), particle belief factor ($c_1=2$) and swarm belief ($c_2=2$) factors respectively.

In this application, 20 particles were selected and the process was continued for 100 iterations or until convergence occurred.

D Ant Colony Optimization

Ant Colony Optimization (Dorigo & Stutzle 2004) attempts to mimic mathematically the process by which ant colony sends out scouts to search for food and a pheromone is laid upon the trail depending upon the success of that route. The probability of ants following a particular trail is increased by the amount of pheromone that it contains. ACO has primarily been applied for scheduling / routing problems, however, in this application a different approach has to be employed.

In figure 4 it can be seen composite layer optimization problem is formulated as a series of way-points that each ant must pass through, representing each of the composite layer, however, there are many paths that can be taken between each way-point representing the possible orientations for each layer, in this case the same discrete orientations as used for the BGA were used, i.e. 32 possible orientations, leading to increments of 5.625° in the range -90° to 90° .

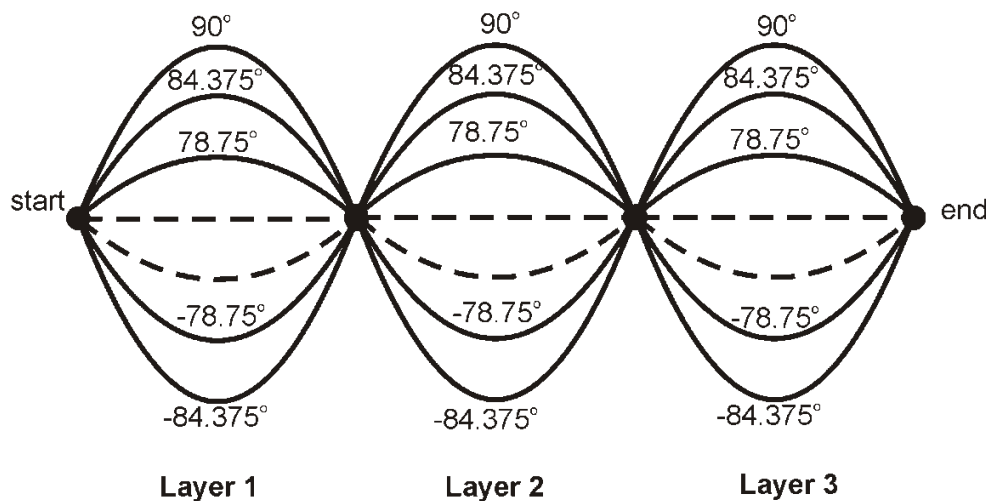


Figure 4 ACO Solution of Composite Layer Optimization Problem (dashed line indicates all paths between 78.75° and -78.75°)

Suppose there are n_{ant} ants that initially take a random set of routes. The single ant with the best route found at each iteration deposits pheromone, which will also evaporate at some predefined rate. Further sets of routes are chosen, with those sections containing more pheromone being more likely to be chosen. This process is repeated either for a set number of times, or until convergence to a solution is found.

Mathematically, the route that each ant takes depends upon the probabilities P_{ij} (ith layer and jth orientation) assigned to each path via pheromone intensity (τ_{ij}) information contained in the $m * N_{\theta}$ pheromone matrix, where m is the number of layers that need to be defined ($m = 3$) and N_{θ} is the number of possible orientations ($N_{\theta} = 32$).

The probability of choosing a particular composite lay-up sequence for the each ant was set as

$$P_{ij} = \frac{\tau_{ij} + z}{\sum_{k=1}^{N_{\theta}} (\tau_{ij} + z)}, \quad i = 1 \dots m, j = 1 \dots N_{\theta} \quad (17)$$

with all pheromone intensities set as zero for the first iteration

The updating process consists of adding and evaporating the pheromone intensity is represented as

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (18)$$

where ρ is the amount of percentage evaporation ($\rho = 0.02$) and $\Delta\tau_{ij}$ is an additional pheromone deposited on each best route, defined here as

$$\Delta\tau_{ij} = \begin{cases} \frac{J(X)_n}{Q} & \text{if arc (i,j) belongs to iteration best solution} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

where Q is a constant and $\{J(X)_n\}$ is the best solution (cost function) of a colony at the nth iteration (best iteration ant). In this work, the constant Q was taken as a maximum possible of cost function (35 m/s) so that the maximum value of $\Delta\tau_{ij}$ was of order unity.

In order to optimize the ACO solution, it is essential to have a proper parameter setup of the evaporation constant ρ , $\Delta\tau_{ij}$ and pheromone constant z so as to ensure there is an appropriate balance of inclusion of random material, avoiding premature convergence whilst still ensuring that the solution converges. The inclusion of the pheromone constant term in equation (17), defined as

$$z = \left(\frac{1}{\rho}\right) \left(\frac{1 - P_{max}}{N_{\theta} P_{max} - 1}\right) \quad (20)$$

where P_{max} is the maximum allowable size of probability (taken as 50%) that correspond to the maximum allowable pheromone intensity in the pheromone matrix. This approach has been found by the authors to address these issues, preventing some of the pheromone intensities becoming either too high or too low and leading to more variation on the pheromone matrix.

E Meta-Modelling Approach

A further approach that was employed was to take 20 emulations of the system with parameters that were chosen using a random Latin Hypercube (Sacks et al. 1989) to ensure that a broad distribution of the parameters was considered. A cubic model of the form

$$V_f = A + B_1\theta_1 + B_2\theta_2 + B_3\theta_3 + C_1\theta_1^2 + \dots + D_1\theta_1\theta_2 + \dots + E_1\theta_1^3 + \dots + F_1\theta_1^2\theta_2 + \dots + G\theta_1\theta_2\theta_3 \quad (21)$$

was chosen where the unknown A,B,...G parameters are found from a simple regression analysis using the test data sets. The maximum value of the resulting reduced order model was then determined. It was found that in order to ensure that a concave solution was found it was necessary to include a further set of 26 sample points around the edge of the solution space. This process was repeated 100 times using a different set of Latin Hypercube solutions each time with a resolution of 1° .

IV. Results

Figures 5 – 9 show the best flutter speed solutions and corresponding ply angles from all 100 solutions for each of the methods and figure 10 shows the number of iterations required to achieve convergence of each solution along with the corresponding optimized instability speed. Table 2 shows the best solutions and corresponding composite layer orientations achieved by all the methods over the 100 runs, whereas Table 3 shows the statistical behavior of the 100 solution set, showing both the mean and also the standard deviations of the instability speed, flutter frequency, lay-up orientations and number of iterations to achieve convergence.

In terms of the overall best solution from the 100 runs, the PSO method gave the best answer with the CGA approach giving a very similar result. Both these continuous solutions give better solutions than the two discrete methods (which both found the same optimum solution) as they have an infinite possible number of possible solutions. All of the four optimization methods found very similar orientations for θ_1 and θ_2 however, there is a marked difference in the θ_3 solution found by the PSO and CGA methods compared to the BGA and ACO approaches. The performance of the meta-modelling approach was much worse than the optimization methods, highlighting that the problem requires a significantly higher order model than the cubic one that was employed.

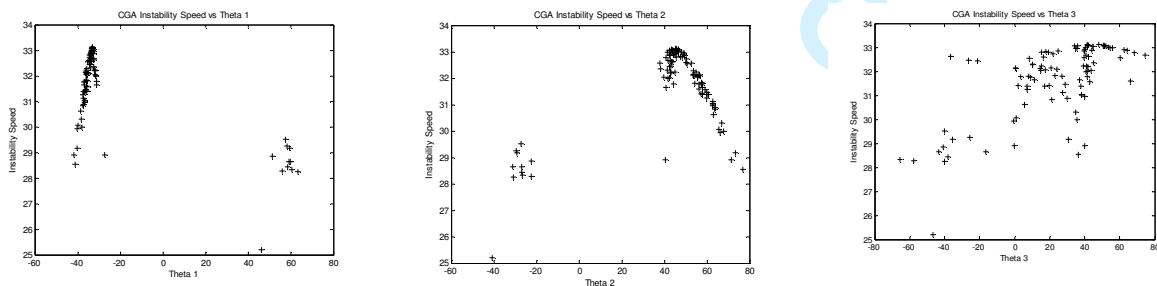


Figure 5. CGA Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

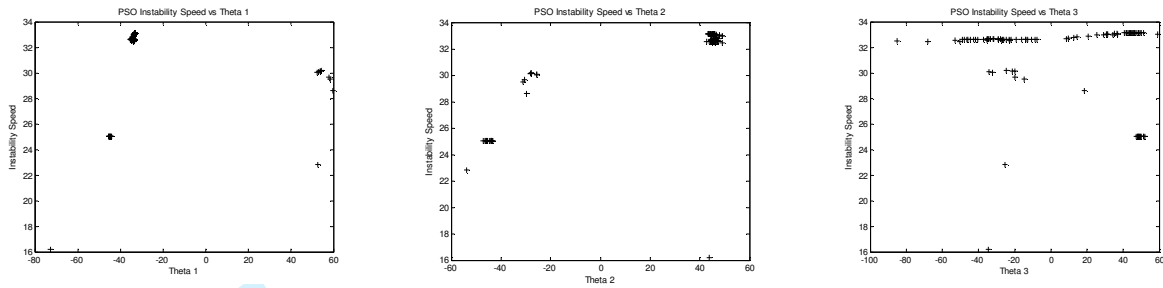


Figure 6. PSO Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

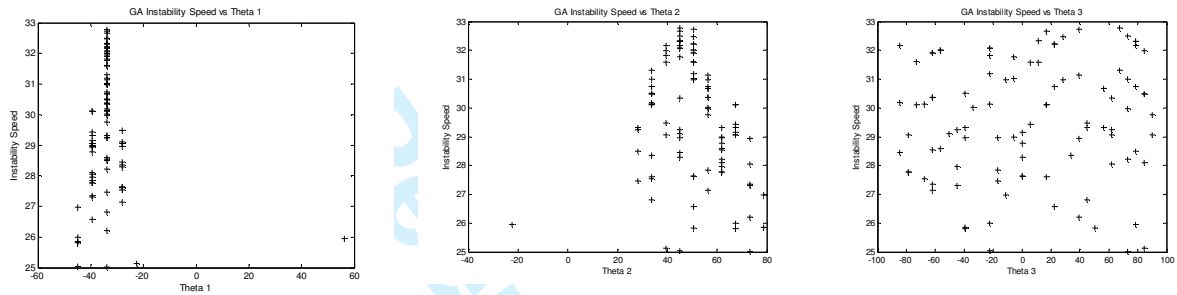


Figure 7. BGA Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

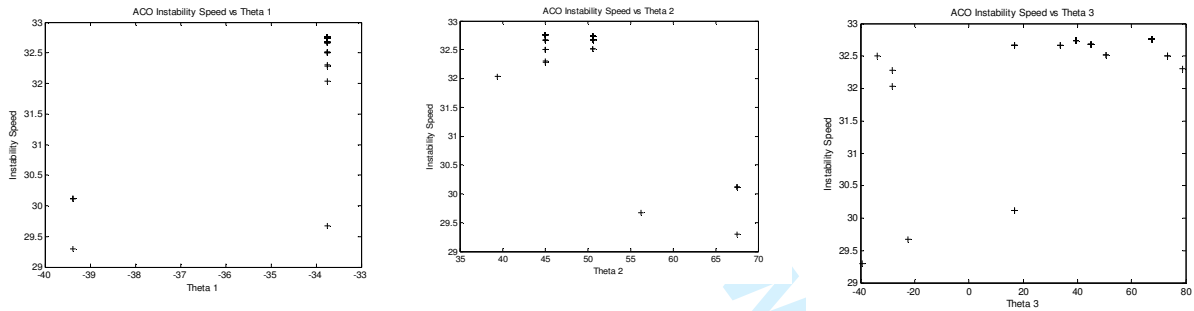


Figure 8. ACO Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

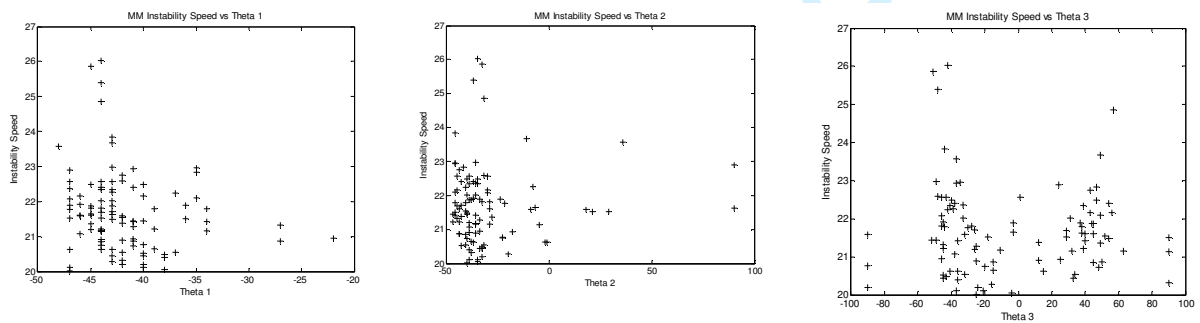


Figure 9. Meta-Model Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

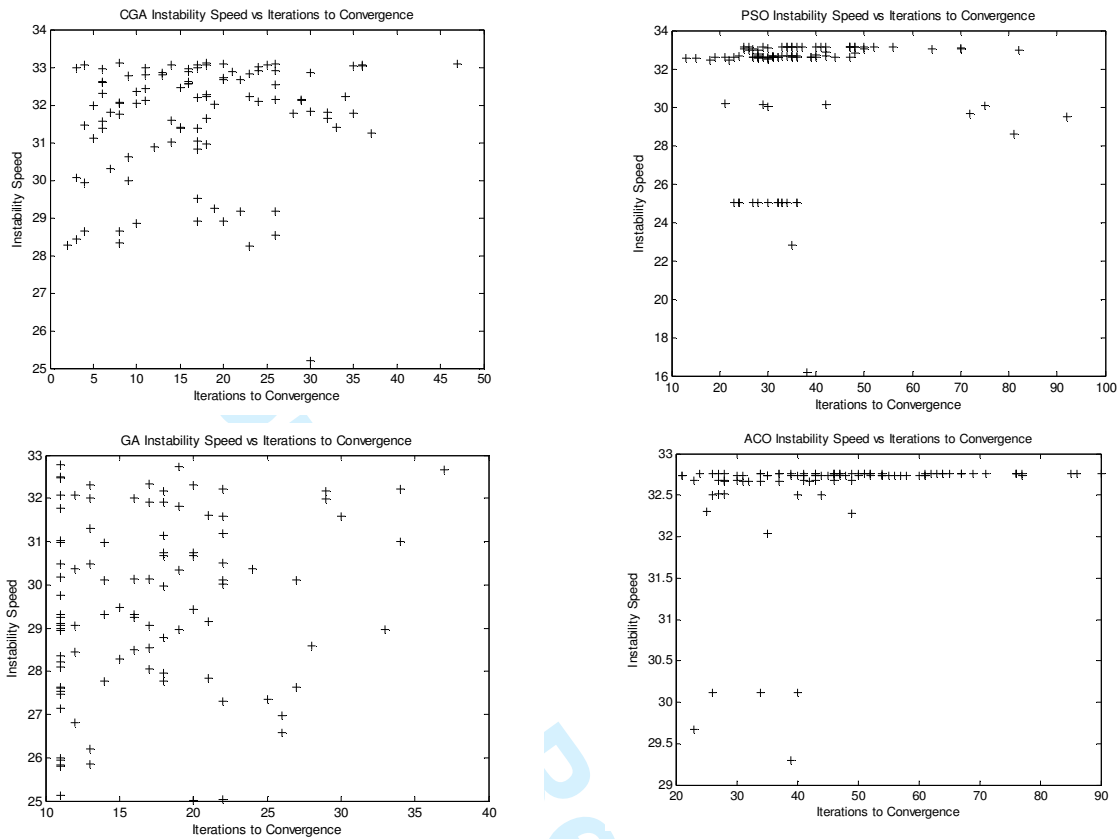


Figure 10. Iterations Required for Convergence for the Four Different Methods

Property	Value
E_1 (GPa)	98.0
E_2 (GPa)	7.9
V_{12}	0.28
G_{12}	5.6
G_{13}	5.6
G_{23}	5.6
Ply thickness	0.134 (mm)
Density	1520(Kg/m ³)

Table 1. Composite Material Properties

Best Speed	Best Speed m/s	Θ_1 (deg)	Θ_2 (deg)	Θ_3 (deg)
CGA	33.12	-33.16	45.16	48.29
PSO	33.13	-33.08	44.26	48.34
BGA	32.77	-33.75	45.00	67.50
ACO	32.77	-33.75	45.00	67.50
MM	26.03	-44.00	-35.00	-42.00

Table 2. Best Speeds and Orientations from all 100 solution cases.

Mean	Best Speed m/s	Θ_1 (deg)	Θ_2 (deg)	Θ_3 (deg)	Iterations to convergence
CGA	31.63	-24.63	42.52	22.13	17.23
PSO	31.14	27.82	25.10	6.24	36.49
BGA	29.57	-33.86	50.23	5.79	17.11
ACO	32.57	-33.98	48.38	47.76	46.51
MM	21.70	-41.88	-30.17	-4.32	
Standard Deviations					
CGA	1.56	29.07	26.56	29.98	9.68
PSO	3.31	26.79	36.87	38.83	14.39
BGA	2.05	10.05	14.81	53.72	6.29
ACO	0.64	1.11	4.93	24.34	15.95
MM	1.16	4.44	23.41	43.42	

Table 3. Mean and Standard Deviations of Speeds, Orientations and Required Iterations from all 100 solution cases.

	D_{11} (N.m)	D_{16} (N.m)	D_{66} (N.m)	D_{16}/D_{11}	D_{66}/D_{11}
CGA	2.0866	-0.5704	1.0074	-0.27	0.48
PSO	2.1078	-0.5638	1.0059	-0.27	0.48
BGA	2.0269	-0.5888	1.0008	-0.29	0.49
ACO	2.0269	-0.5888	1.0008	-0.29	0.49
MM	1.7019	-1.0622	1.0784	-0.62	0.63

Table 4. Bending, Bend-Torsion and Torsion stiffness terms for the Optimal Solutions for each Method

The statistical investigation provides a rather different picture. When all 100 solutions are considered, the ACO approach gives the best average result and has a much lower standard deviation, however it does take around 2.5 times as many iterations than the GA method and 30% more computation than PSO. There is very little scatter in the ACO results and the mean values are close to the optimal answers, many of the estimates are found repeatedly. There is a large variance in both the PSO and CGA solutions, however it can be seen that the PSO results for the ply orientations are in distinct closely formed clusters whereas there is a much more scattered appearance for the CGA results. The BGA scatter is between that of the ACO and the other two continuous methods for θ_1 and θ_2 however the variation for θ_3 is larger. The variance is very large in most cases as its calculation included all possible solutions which includes some significantly different answers. To use this information in practice, the worst solutions should be discarded and some form of clustering algorithm used to determine groups about which meaningful information on the solution distributions.

Figure 10 highlights how much better the ACO and to some extent the PSO methods are in consistently producing good estimates, however, the key observation is that for all methods there is no correlation between the number of iterations used and the optimality of the solution.

The results give optimal orientations between $\pm 45^\circ$ for a composite wing as predicted by Weisshaar, however, it should be noted that due to coupling between the bending and stiffness behavior, optimal results are not simply found from $\pm 45^\circ$ lay-ups. The θ_3 layer has the least effect due to it being placed closest to the neutral axis, resulting in a greater scatter in its results. Table 4 shows the Bending (D_{11}), Bend-Torsion (D_{16}) and Torsion (D_{66}) stiffness terms for the best results obtained by all the methods. It can be seen that the ratio between the torsion stiffness and the other two terms results remains almost constant for all results showing that the same stiffness ratio pattern is found by all methods.

The above discussion is enhanced by comparison with exhaustive searches for the more usual industry lay-up using any possible combination of $(0^\circ, \pm 45^\circ, 90^\circ)$ and $(0^\circ, \pm 30^\circ, \pm 45^\circ, \pm 60^\circ, 90^\circ)$, leading to optimum lay-ups and maximum instability speeds of $[-45 \ 45 \ 45]_s$, 25.43m/s and $[-30 \ 45 \ 45]_s$, 30.73m/s respectively. These results

demonstrate the sensitivity of the flutter process to the orientation angles and show how relatively small changes in the lay-up can make a big difference.

V. Conclusions

Four biologically inspired optimization methods (Genetic Algorithms (binary and continuous), Particle Swarm Optimization and Ant Colony Optimization) were used to determine the optimal lay-up for a simple composite wing in order to maximize the flutter and divergence speeds. A statistical investigation was performed in order to investigate the variation of the parameters that were optimized. The best single results were found using the Particle Swarm and Continuous Genetic Algorithm however, the statistical investigation showed that Ant Colony Optimization gave results with much less scatter than the other methods. A polynomial based meta-modelling approach gave much worse answers than the other methods. It was also shown that for all methods there was no correlation between the accuracy of the optimization and the number of iterations required for convergence.

Obviously these results only refer to the optimization of a single aeroelastic system, however, it is conjectured that similar findings would be found if the methods were applied to larger more realistic models. Further work is currently investigating the application of these evolutionary approaches in combination with gradient based methods to industrial type wing Finite Element models combined with potential flow aerodynamics.

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Optimization of Aeroelastic Composite Structures using Evolutionary Algorithms

Abdul. Manan, Gareth.A. Vio, M.Yazdi. Harmin and Jonathan.E. Cooper
Department of Engineering, University of Liverpool , Liverpool, L69 7EF, UK

Corresponding Author

Professor Jonathan Cooper

Tel: +44 151 794 5232

Fax: + 44 151 794 4848

Email: j.e.cooper@liverpool.ac.uk

Optimization of Aeroelastic Composite Structures using Evolutionary Algorithms

A. Manan, G.A. Vio, M.Y. Harmin and J.E. Cooper

Department of Engineering, University of Liverpool, Liverpool, L69 7EF, UK

The flutter / divergence speed of a simple rectangular composite wing is maximised through the use of different ply orientations. Four different biologically inspired optimization algorithms (binary genetic algorithm, continuous genetic algorithm, particle swarm optimization and ant colony optimization) and a simple meta-modelling approach are employed statistically on the same problem set. It was found in terms of the best flutter speed, that similar results were obtained using all of the methods, although the continuous methods gave better answers than the discrete methods. When the results were considered in terms of the statistical variation between different solutions, Ant Colony Optimization gave estimates with much less scatter.

Keywords: Aeroelastic Tailoring, Composite Lay-up, Flutter, Evolutionary Optimization

I. Introduction

Aeroelasticity (Wright & Cooper 2007) is the science that incorporates the interactions between a flexible structure and surrounding aerodynamic forces. In general, aeroelastic phenomena are undesirable and can either reduce aircraft performance or, in extreme cases, cause structural failure either statically (divergence) or dynamically (flutter). Aircraft designers are interested in the speeds at which any instabilities occur, the dynamic response due to gusts and manoeuvres, and also the shape that the aircraft wings take in-flight as this has a significant effect on the drag and resulting fuel efficiency and performance.

Traditional aircraft design has always tended to avoid aeroelastic problems through stiffening the structure with extra material and accepting the resulting weight penalty. In recent years, there has been a move towards trying to use aeroelastic deflections in a positive manner, and this has resulted in research programmes such as the Active Flexible Wing (Perry et al. 1995), the Active Aeroelastic Wing (Pendleton et al. 2000), the Morphing Program (Wlezin et al. 1998) and Active Aeroelastic Aircraft Structures project (Schweiger & Suleman 2003) that have used made use to several novel active technologies. However, a more traditional passive approach to making use of aeroelastic deflections in a positive way is to use carbon fibre composite structures.

Despite the benefits of composite structures, it is only recently that the main load bearing structures in large aircraft such as the Boeing 787 and Airbus A350 have started to be manufactured using carbon fibre composites. Even then, the unique directionality properties of composite laminates have yet to be exploited fully in order to improve the aircraft performance. Examples of work that has optimized the stacking sequence of composite structures include (Pagano et al. 1971) who studied the effect of 15,45 ply lay-up on the laminate strength. Genetic Algorithms have been widely used couple in multi-level optimisation routines to optimise wing stiffeners elements (Herencia et al. 2007, Liu et al. 2008). The effect of stacking sequence was investigated for the creation of composite bi-stable laminates (Mattioni et al. 2009) using 0,90 and 45 degrees ply directions. (Autio 2000) used a lamination parameter in order to optimise buckling/frequency of a composite plate with a ply angle search.

The idea of using the directional property of composites for aeroelastic tailoring has been around since the 1970s (Shirk et al. 1986, Weisshaar 1981). However, since tailoring was demonstrated on the X-29 in the late 1970s and early 1980s, very few aircraft have used these directional properties to achieve beneficial aeroelastic effects. The

original application was to reduce the likelihood of divergence occurring on forward-swept wings (Shirk et al. 1986, Weisshaar 1981); recent applications have included weight reduction (Arizono & Isogai 2005, Kim et al. 2007, Kameyama & Fukunaga 2007, Eastep et al. 1999, Guo 2007), drag reduction (Weisshaar and Duke 2006) and passive gust alleviation (Petit & Grandhi 2003, Kim & Hwang 2005) of composite wings. Although the new generation of commercial civil aircraft has started to use composites, they have only exploited the superior strength/weight ratio of composite materials rather than employing aeroelastic tailoring, in effect using the composite as a “black metal”.

There are a wide range of different optimization approaches that can be used for design problems. The two main categories are Hill-Climbing and Evolutionary Methods. Evolutionary algorithms tend to be based upon the mimicry of some biological or physical process and have been proven to be effective for large parameter space solutions and do not suffer from the local optima problems that can occur in the Hill-Climbing approaches; however, Evolutionary Algorithms do not guarantee to achieve the global optimum solution. In the aeroelastic tailoring field, Genetic Algorithms have been used to minimise the structural weight whilst satisfying a number of aeroelastic parameters such as flutter and divergence (Shirk et al. 1986, Weisshaar 1981, Kim et al. 2007). However there have been few known aeroelastic applications of Particle Swarm Optimization or Ant Colony Optimization.

In this study, a simple assumed modes mathematical model of a rectangular composite wing with unsteady aerodynamics was developed in order to assess the ability of tailoring the composite structure to maximise the flutter / divergence speed. Four different biologically inspired optimization techniques, including discrete and continuous approaches, were applied 100 times each to this problem in order to evaluate their performance in a statistical manner. A further set of analysis was performed using a simple meta-modelling approach.

II. Mathematical Model

A. Structural Modelling

The composite lifting surface was idealised as a rectangular plate, as shown in Figure 1, using the Rayleigh-Ritz assumed modes method (Wright & Cooper 2007, Al-Obeid & Cooper 1995), which addresses the minimization of energy functional composed of strain and kinetic energy. The problem considered here is a rectangular composite plate wing with a semi-span to chord ratio of four (i.e. on a full aircraft modeling both wings this would give an aspect ratio of eight) and a six layer fibre angle lay-up of $(\theta_1, \theta_2, \theta_3)_s$.

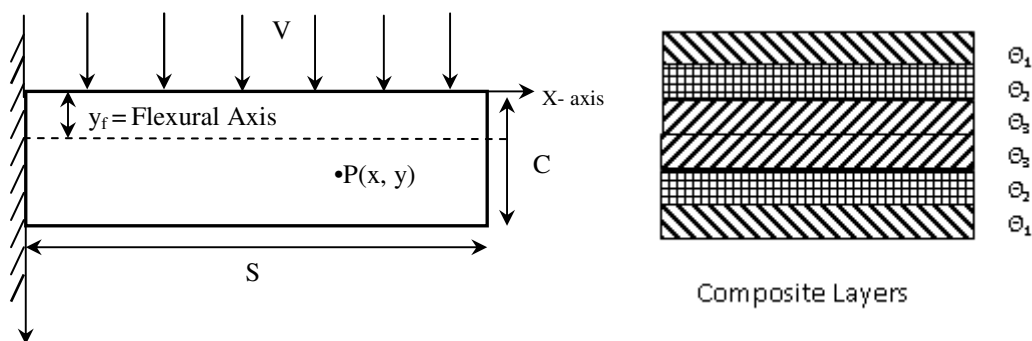


Figure 1. Rectangular Wing with Six Composite Layers

The strain energy over the entire plate domain Ω can be expressed as

$$U = \frac{1}{2} \iiint_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \sigma_{xz} \varepsilon_{xz} + \sigma_{yz} \varepsilon_{yz} + \sigma_{xy} \varepsilon_{xy}) dx dy dz \quad (1)$$

which reduces, by taking into account of assumptions that transverse shear and normal stresses are negligible, to

$$U = \frac{1}{2} \iiint_{\Omega} (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_{xy} \varepsilon_{xy}) dx dy dz \quad (2)$$

in which σ_i and ε_i are stress and strain (where $i = x, y, xy$) that are related to each other via transformed reduced stiffness matrix, \bar{Q}_{ij} , by the following relation for each ply k

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix} \quad (3)$$

Replacing stress relations defined in (3) into (2) we get following equation

$$U = \frac{1}{2} \iiint_{\Omega} (\bar{Q}_{11} \varepsilon_x^2 + \bar{Q}_{12} \varepsilon_x \varepsilon_y + 2\bar{Q}_{16} \varepsilon_x \varepsilon_{xy} + 2\bar{Q}_{26} \varepsilon_y \varepsilon_{xy} + \bar{Q}_{22} \varepsilon_y^2 + \bar{Q}_{66} \varepsilon_{xy}^2) dx dy dz \quad (4)$$

Then, utilising the strain-displacement relations for this pure bending problem with symmetric lay-up configurations, expression (4) reduces to

$$U_{\max} = \frac{1}{2} \iint_{\Omega} \left\{ \begin{aligned} & D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 + 2D_{12} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial y^2} \right) + D_{22} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 + \dots \\ & 4D_{16} \left(\frac{\partial^2 w}{\partial x^2} \right) \left(\frac{\partial^2 w}{\partial x \partial y} \right) + 4D_{26} \left(\frac{\partial^2 w}{\partial y^2} \right) \left(\frac{\partial^2 w}{\partial x \partial y} \right) + 4D_{66} \left(\frac{\partial^2 w}{\partial y^2} \right)^2 \end{aligned} \right\} dx dy \quad (5)$$

where $D_{ij} = \frac{1}{3} \sum_{k=1}^n (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3)$ in which z_k is k th layer distance along z -axis from mid-plane of the plate and

w is the out of plane deflection of the plate. Out of plane deflections (expressed at some point $P(x,y)$ on the surface of the wing) of the composite lifting surface are expressed as

$$w = \sum_{i=1}^n \gamma_i(x, y) q_i(t) \quad (6)$$

where $q_i(t)$ is the generalised displacement of the i^{th} mode represented with $\gamma_i(x, y)$ (Taken here as polynomial series of $x^2, x^3, x^4, x^2(y - y_f), x^3(y - y_f), x^4(y - y_f), x^2(y - y_f)^2, x^3(y - y_f)^2$ and $x^4(y - y_f)^2 \dots$, then strain energy can be calculated by utilizing equation (5). Similarly, the maximum kinetic energy of the entire plate domain Ω can be formulated as

$$T_{\max} = \frac{1}{2} \rho h \omega^2 \iint_{\Omega} w^2(x, y) dx dy \quad (7)$$

where ρ is the mass per unit area of plate, h is plate thickness and ω is the frequency of vibration. Minimization of the functional $(U_{\max} - T_{\max})$ with respect to each coefficient q_i results the following stiffness and mass matrices respectively

$$E_{ij} = \iint_{\Omega} \left\{ \begin{array}{l} D_{11} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial x^2} + D_{12} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial x^2} + 2D_{16} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial x^2} + \dots \\ D_{12} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial y^2} + D_{22} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial y^2} + 2D_{26} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial y^2} + \dots \\ 2D_{16} \frac{\partial^2 \gamma_i}{\partial x^2} \frac{\partial^2 \gamma_j}{\partial x \partial y} + 2D_{26} \frac{\partial^2 \gamma_i}{\partial y^2} \frac{\partial^2 \gamma_j}{\partial x \partial y} + 2D_{66} \frac{\partial^2 \gamma_i}{\partial x \partial y} \frac{\partial^2 \gamma_j}{\partial x \partial y} \end{array} \right\} dx dy \quad (8)$$

$$A_{ij} = \iint_{\Omega} \rho \gamma_i \gamma_j dx dy \quad (9)$$

Comparison with Finite Element analysis showed that a very good representation of the dynamic behavior of the composite wing could be achieved using the above model.

B. Aerodynamic Modelling

In this work a modified strip theory approach, in which unsteady effects are introduced via the torsional velocity term (Wright & Cooper 2007), was employed to develop the aerodynamic model. Strip theory divides the wing into infinitesimal strips on which lift acting on the quarter chord is assumed to be proportional to the dynamic pressure, local angle of attack, lift curve slope and the downwash due to the vertical motion. Although strip theory would not be used for the design of commercial jet aircraft by the aerospace industry, the approach does give a representative conservative model of the static and dynamic aeroelastic behavior of high aspect ratio wings at low speeds, and remains as a possible analysis approach in the airworthiness regulations. Previous unpublished studies have shown that similar results for the type of wing model considered here are obtained using the modified strip theory compared to the Doublet Lattice approach used in commercial software packages. The aim of this paper is to investigate the behavior of several optimization methods and not to produce a perfect aeroelastic model, thus any variations in the aerodynamic modeling will not affect the optimization methods, **however, further investigation is required in the transonic flight regime where the aerodynamic behavior becomes nonlinear.** For ease of computational effort it was decided to use the modified strip theory approach throughout this work.

C. Aeroelastic Modelling

Considering the incremental work done over entire wing due to the lift and pitching moment (about the flexural axis), application of Lagrange's equations eventually leads to the formulation of the B (aerodynamic damping) and C (aerodynamic stiffness) matrices (Wright & Cooper 2007) which can be coupled with the above structural terms to give the aeroelastic equation of motions in the classical form

$$(A) \ddot{\underline{q}} + (\rho V B + D) \dot{\underline{q}} + (\rho V^2 C + E) \underline{q} = \underline{0} \quad (10)$$

D is the structural damping matrix which cannot be predicted and requires test measurements to get accurate estimates. As with most aeroelastic modeling, the structural damping can be set to zero as the aerodynamic damping terms have a far greater effect (Wright & Cooper 2007).

Rewriting into first order matrix form, equation (10) becomes

$$\begin{pmatrix} \dot{\underline{q}} \\ \ddot{\underline{q}} \end{pmatrix} - \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ (\rho V^2 \mathbf{C} + \mathbf{E}) & (\rho \mathbf{V} \mathbf{B} + \mathbf{D}) \end{pmatrix} \begin{pmatrix} \underline{q} \\ \dot{\underline{q}} \end{pmatrix} = \underline{\dot{x}} - \mathbf{Q} \underline{x} = \underline{Q} \quad (11)$$

and the eigenvalues of matrix Q lead to the system frequency and damping ratios at any given flight condition. The flutter speed at a given altitude can be determined by finding the air speed where one of the damping values becomes negative. In some cases divergence might occur before flutter, and this is characterised by positive real eigenvalues occurring.

A total of nine assumed modes were assumed for the composite wing model for which material properties used are given in Table 1. The frequency and damping ratio trends of the lowest three modes for a typical case are shown in Figure 2, and it can be seen that a classical flutter mechanism results through the coupling between the first two modes and the flutter speed occurs when the Mode 2 damping curve becomes zero.

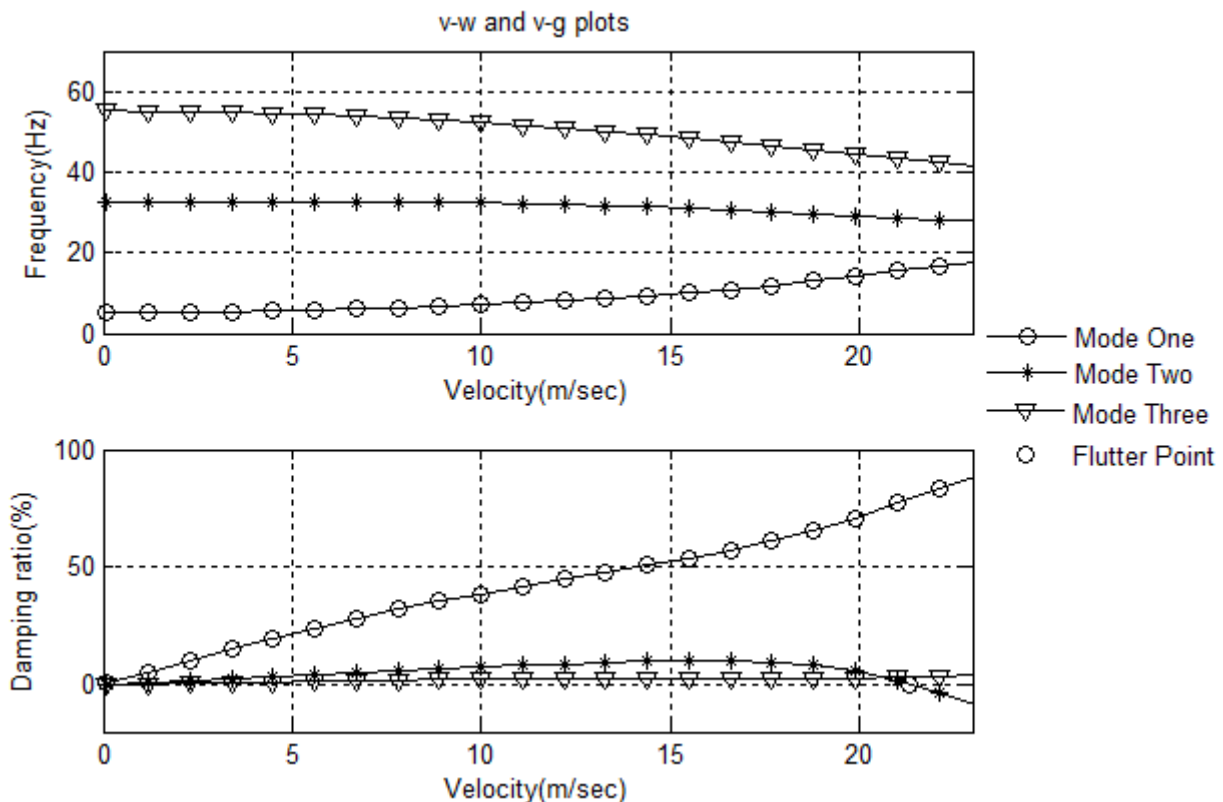


Figure 2. Typical Frequency and Damping Ratio Trends vs. Speed

III. Optimization Methods and Application

Four different optimization methods were considered for this study, all of which are based upon some form of evolutionary search based upon the mimicry of various types of biological system. Two of the approaches, Binary Genetic Algorithm (BGA) and Ant Colony Optimization (ACO), are discrete in the sense that the set of possible solutions are defined a-priori; here the number of possible orientations for each layer is defined by the number of bits in each gene (BGA) or the number of possible paths between each waypoint (ACO). The other two methods, Continuous Genetic Algorithm (CGA) and Particle Swarm Optimization (PSO) allow any orientation within the defined search space ($-90^\circ \leq \theta \leq 90^\circ$ for each layer) to be obtained.

In all cases, the objective was to find the combinations of ply orientations that maximised the air speed at which aeroelastic instability occurred, due to either flutter or divergence. Although this might seem to be a trivial problem with only three parameters ($\theta_1, \theta_2, \theta_3$) needing to be determined, due to the highly coupled nature of the aeroelastic system the optimized lay-up is not easy to find. Figure 3 shows a section of the solution space for constant $\theta_3 = 50^\circ$, calculated by enumeration, and it can be seen that there are a number of local optima and also regions of rather low instability speeds corresponding to solutions where the divergence speed is the critical case.

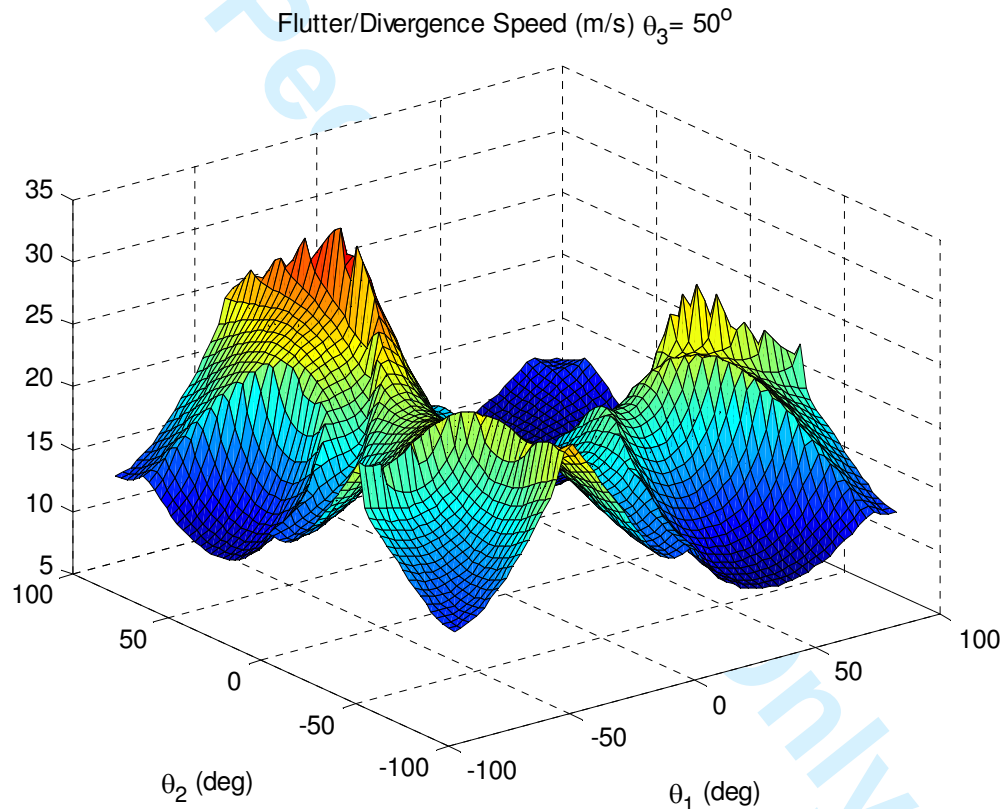


Figure 3. Flutter Speed Solutions for θ_1 vs. θ_2 for constant θ_3

A total of 20 solutions (genes, particles or ants) were used for each approach. For each solution case the methods were run for a maximum of 100 generations / iterations, with convergence considered to occur if the best solution did not vary for 20 iterations. A very brief description of the application of each optimization algorithm follows.

A. Binary Genetic Algorithm

Genetic Algorithms attempt to mimic Darwinian theory of natural selection which is based upon the traits of the most successful animals being passed onto future generations. In an optimization setting (Haupt & Haupt 2004), the

characteristics of the best solutions from a range of initial estimates (“genes”) are passed onto subsequent iterations via a series of mathematical operators, which is repeated until convergence is achieved. Randomness is added via application of a mutation function and also, possibly, the inclusion of “new-blood” solutions. The most common approach is to use a binary representation (BGA) of the system parameters.

Here, the binary representation of the composite lay-up assigned 5 bits (32 possible orientations – i.e. 5.625° between each possible orientation) to each layer, giving a gene length of 15 bits. A classical implementation of the binary GA was employed, with 20 genes being included in the gene pool, the 4 best genes saved after each iteration, a 90% probability of crossover, 5% probability of mutation and a 10% likelihood of translation.

B. Continuous Genetic Algorithm

Continuous or real number genetic algorithms (CGA) work (Haupt & Haupt 2004) in a similar way to the binary genetic algorithm described above. However, as the name suggests, the primary difference is in the variable representation of each gene. In CGA, the genes are represented using real numbers and consequently a re-definition of the mutation and crossover operators must be employed.

1. Mutation

The mutation operator for CGA requires the selection of a number of variables based on a mutation rate to be replaced by a new random variable. The best gene is left untouched in order to give an element of elitism to the generation.

2. Crossover

As for the binary BGA, a pair of genes is selected to create any offspring. For the BGA, if two points are selected and swapped, i.e.

$$\begin{aligned} parent_1 &= [p_{11} \ p_{12} \ p_{13} \ p_{14} \ p_{15} \ p_{16} \ \dots \ p_{1N}] \\ parent_2 &= [p_{21} \ p_{22} \ p_{23} \ p_{24} \ p_{25} \ p_{26} \ \dots \ p_{2N}] \end{aligned} \quad (12)$$

where N is the number of genes. By applying crossover (randomly chosen to occur after the 2^{nd} cell), the following offspring are obtained

$$\begin{aligned} offspring_1 &= [p_{11} \ p_{12} \ p_{23} \ p_{24} \ p_{25} \ p_{16} \ \dots \ p_{1N}] \\ offspring_2 &= [p_{21} \ p_{22} \ p_{13} \ p_{14} \ p_{15} \ p_{26} \ \dots \ p_{2N}] \end{aligned} \quad (13)$$

It can be seen that no new information is passed to the offspring. However, for the CGA, new genetic material is introduced into the cross over process via the use of a blending function β such that

$$offspring_1 = parent_1 - \beta(parent_1 - parent_2) \quad (14)$$

where β is a random number between 0 and 1.

In this application, a population of 20 genes was chosen, with the mutation rate set at 0.2 and the crossover rate at 0.5.

C Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a heuristic search method which is based on a simplified social model that is closely tied to swarming theory and intelligence in which each particle of the swarm has memory and can also communicate with each other (Clerc 2006). The position and velocity of particle is updated by knowing the previous best values of each particle and overall swarm such that for the k th iteration

$$v_{id}(k+1) = wv_{id}(k) + c_1\phi_{1d}(p_{id}(k) - x_{id}(k)) + c_2\phi_{2d}(g_d(k) - x_{id}(k)) \quad (15)$$

$$x_{id}(k+1) = x_{id}(k) + v_{id}(k) \quad (16)$$

where v_i and x_i are the velocity and position of particle i , p_i and g_i are the best positions found by each particle and the entire population; ϕ_{1d} and ϕ_{2d} are independent uniformly distributed random numbers and are generated independently. w , c_1 and c_2 are the user defined inertia factor ($w=1$), particle belief factor ($c_1=2$) and swarm belief ($c_2=2$) factors respectively.

In this application, 20 particles were selected and the process was continued for 100 iterations or until convergence occurred.

D Ant Colony Optimization

Ant Colony Optimization (Dorigo & Stutzle 2004) attempts to mimic mathematically the process by which ant colony sends out scouts to search for food and a pheromone is laid upon the trail depending upon the success of that route. The probability of ants following a particular trail is increased by the amount of pheromone that it contains. ACO has primarily been applied for scheduling / routing problems, however, in this application a different approach has to be employed.

In figure 4 it can be seen composite layer optimization problem is formulated as a series of way-points that each ant must pass through, representing each of the composite layer, however, there are many paths that can be taken between each way-point representing the possible orientations for each layer, in this case the same discrete orientations as used for the BGA were used, i.e. 32 possible orientations, leading to increments of 5.625° in the range -90° to 90° .

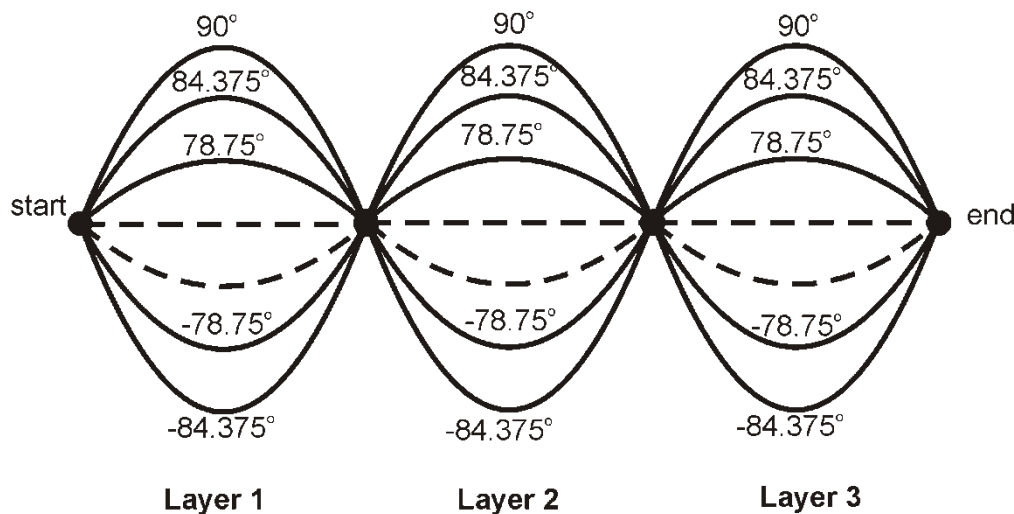


Figure 4 ACO Solution of Composite Layer Optimization Problem (dashed line indicates all paths between 78.75° and -78.75°)

Suppose there are n_{ant} ants that initially take a random set of routes. The single ant with the best route found at each iteration deposits pheromone, which will also evaporate at some predefined rate. Further sets of routes are chosen, with those sections containing more pheromone being more likely to be chosen. This process is repeated either for a set number of times, or until convergence to a solution is found.

Mathematically, the route that each ant takes depends upon the probabilities P_{ij} (ith layer and jth orientation) assigned to each path via pheromone intensity (τ_{ij}) information contained in the $m * N_{\theta}$ pheromone matrix, where m is the number of layers that need to be defined ($m = 3$) and N_{θ} is the number of possible orientations ($N_{\theta} = 32$).

The probability of choosing a particular composite lay-up sequence for the each ant was set as

$$P_{ij} = \frac{\tau_{ij} + z}{\sum_{k=1}^{N_{\theta}} (\tau_{ij} + z)}, \quad i = 1 \dots m, j = 1 \dots N_{\theta} \quad (17)$$

with all pheromone intensities set as zero for the first iteration

The updating process consists of adding and evaporating the pheromone intensity is represented as

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \Delta\tau_{ij}(t) \quad (18)$$

where ρ is the amount of percentage evaporation ($\rho = 0.02$) and $\Delta\tau_{ij}$ is an additional pheromone deposited on each best route, defined here as

$$\Delta\tau_{ij} = \begin{cases} \frac{J(X)_n}{Q} & \text{if arc (i,j) belongs to iteration best solution} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

where Q is a constant and $\{J(X)\}_n$ is the best solution (cost function) of a colony at the nth iteration (best iteration ant). In this work, the constant Q was taken as a maximum possible of cost function (35 m/s) so that the maximum value of $\Delta\tau_{ij}$ was of order unity.

In order to optimize the ACO solution, it is essential to have a proper parameter setup of the evaporation constant ρ , $\Delta\tau_{ij}$ and pheromone constant z so as to ensure there is an appropriate balance of inclusion of random material, avoiding premature convergence whilst still ensuring that the solution converges. The inclusion of the pheromone constant term in equation (17), defined as

$$z = \left(\frac{1}{\rho}\right) \left(\frac{1 - P_{max}}{N_{\theta} P_{max} - 1}\right) \quad (20)$$

where P_{max} is the maximum allowable size of probability (taken as 50%) that correspond to the maximum allowable pheromone intensity in the pheromone matrix. This approach has been found by the authors to address these issues, preventing some of the pheromone intensities becoming either too high or too low and leading to more variation on the pheromone matrix.

E Meta-Modelling Approach

A further approach that was employed was to take 20 emulations of the system with parameters that were chosen using a random Latin Hypercube (Sacks et al. 1989) to ensure that a broad distribution of the parameters was considered. A cubic model of the form

$$V_f = A + B_1\theta_1 + B_2\theta_2 + B_3\theta_3 + C_1\theta_1^2 + \dots + D_1\theta_1\theta_2 + \dots + E_1\theta_1^3 + \dots + F_1\theta_1^2\theta_2 + \dots + G\theta_1\theta_2\theta_3 \quad (21)$$

was chosen where the unknown A,B,...G parameters are found from a simple regression analysis using the test data sets. The maximum value of the resulting reduced order model was then determined. It was found that in order to ensure that a concave solution was found it was necessary to include a further set of 26 sample points around the edge of the solution space. This process was repeated 100 times using a different set of Latin Hypercube solutions each time with a resolution of 1° .

IV. Results

Figures 5 – 9 show the best flutter speed solutions and corresponding ply angles from all 100 solutions for each of the methods and figure 10 shows the number of iterations required to achieve convergence of each solution along with the corresponding optimized instability speed. Table 2 shows the best solutions and corresponding composite layer orientations achieved by all the methods over the 100 runs, whereas Table 3 shows the statistical behavior of the 100 solution set, showing both the mean and also the standard deviations of the instability speed, flutter frequency, lay-up orientations and number of iterations to achieve convergence.

In terms of the overall best solution from the 100 runs, the PSO method gave the best answer with the CGA approach giving a very similar result. Both these continuous solutions give better solutions than the two discrete methods (which both found the same optimum solution) as they have an infinite possible number of possible solutions. All of the four optimization methods found very similar orientations for θ_1 and θ_2 however, there is a marked difference in the θ_3 solution found by the PSO and CGA methods compared to the BGA and ACO approaches. The performance of the meta-modelling approach was much worse than the optimization methods, highlighting that the problem requires a significantly higher order model than the cubic one that was employed.

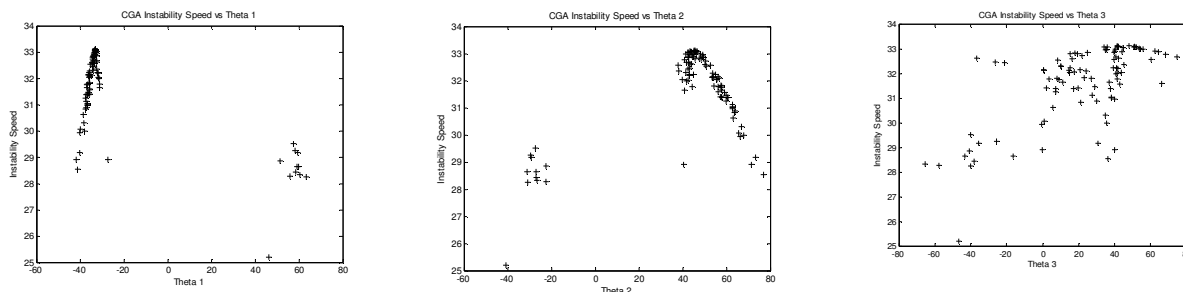


Figure 5. CGA Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

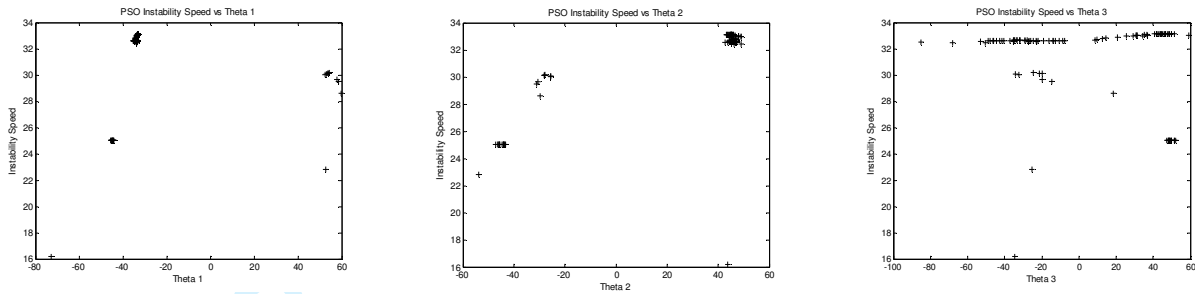


Figure 6. PSO Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

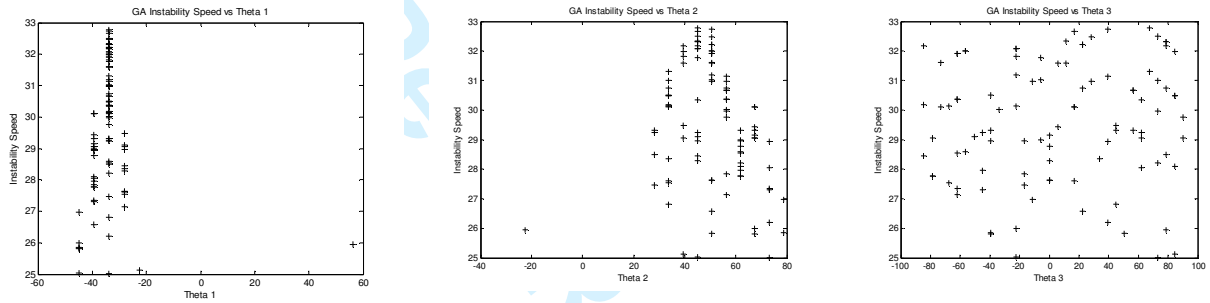


Figure 7. BGA Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

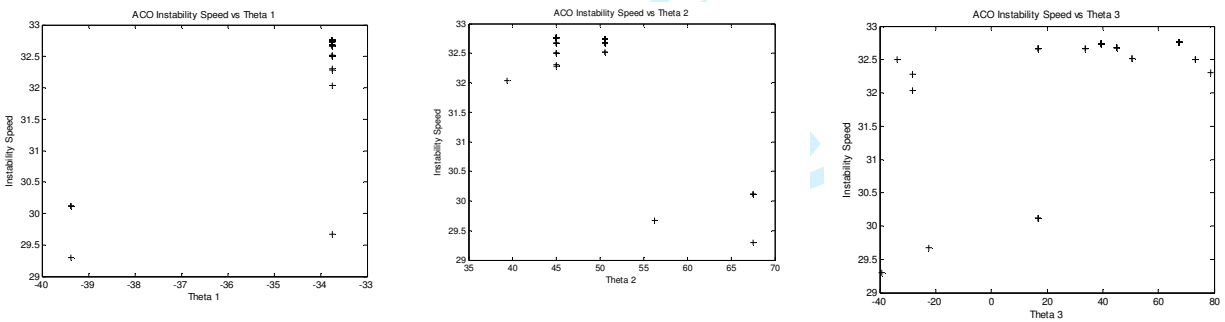


Figure 8. ACO Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

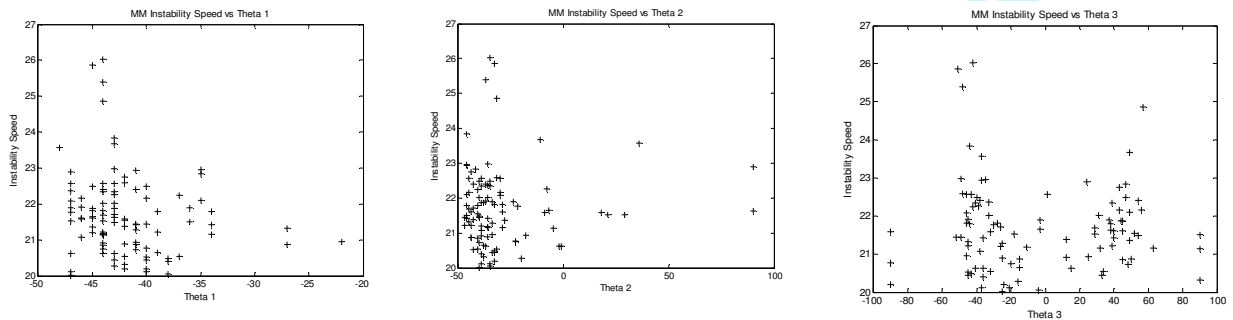


Figure 9. Meta-Model Maximum Flutter Speeds for θ_1 , θ_2 and θ_3

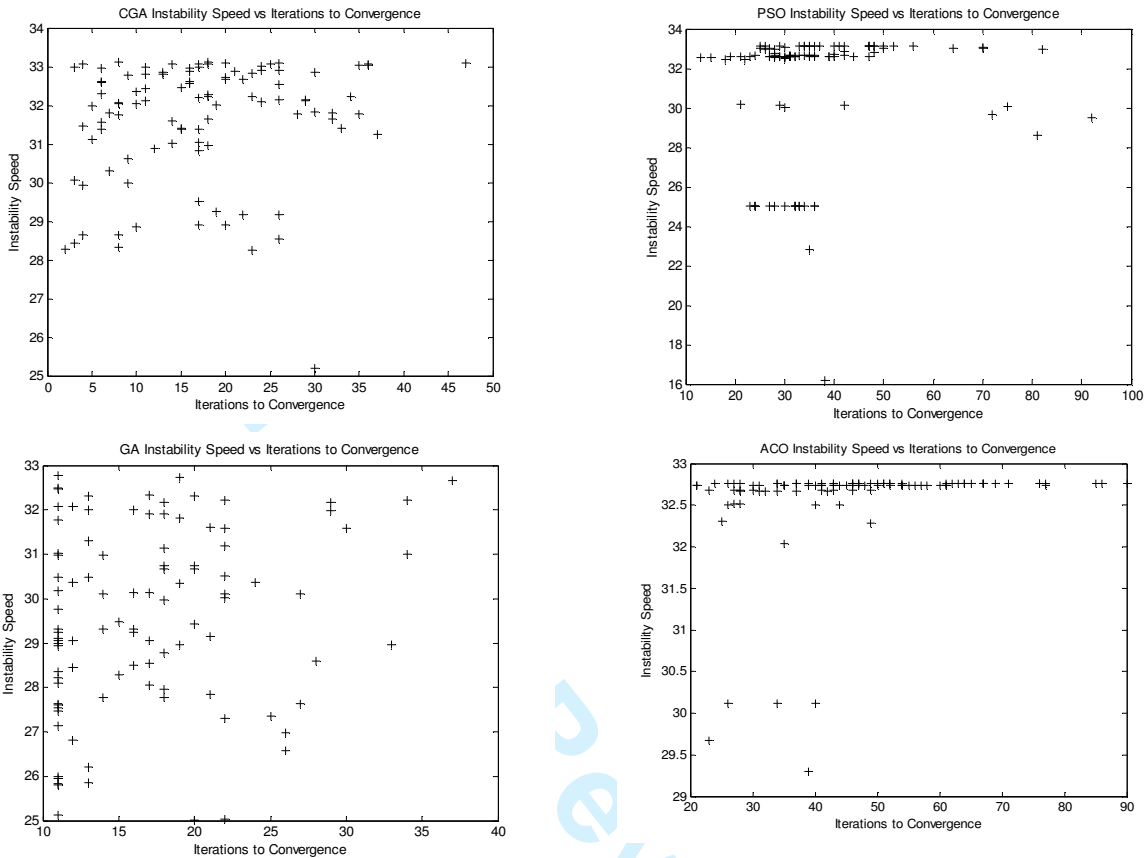


Figure 10. Iterations Required for Convergence for the Four Different Methods

Property	Value
E_1 (GPa)	98.0
E_2 (GPa)	7.9
V_{12}	0.28
G_{12}	5.6
G_{13}	5.6
G_{23}	5.6
Ply thickness	0.134 (mm)
Density	1520(Kg/m ³)

Table 1. Composite Material Properties

Best Speed	Best Speed m/s	Θ_1 (deg)	Θ_2 (deg)	Θ_3 (deg)
CGA	33.12	-33.16	45.16	48.29
PSO	33.13	-33.08	44.26	48.34
BGA	32.77	-33.75	45.00	67.50
ACO	32.77	-33.75	45.00	67.50
MM	26.03	-44.00	-35.00	-42.00

Table 2. Best Speeds and Orientations from all 100 solution cases.

Mean	Best Speed m/s	Θ_1 (deg)	Θ_2 (deg)	Θ_3 (deg)	Iterations to convergence
CGA	31.63	-24.63	42.52	22.13	17.23
PSO	31.14	27.82	25.10	6.24	36.49
BGA	29.57	-33.86	50.23	5.79	17.11
ACO	32.57	-33.98	48.38	47.76	46.51
MM	21.70	-41.88	-30.17	-4.32	
Standard Deviations					
CGA	1.56	29.07	26.56	29.98	9.68
PSO	3.31	26.79	36.87	38.83	14.39
BGA	2.05	10.05	14.81	53.72	6.29
ACO	0.64	1.11	4.93	24.34	15.95
MM	1.16	4.44	23.41	43.42	

Table 3. Mean and Standard Deviations of Speeds, Orientations and Required Iterations from all 100 solution cases.

	D_{11} (N.m)	D_{16} (N.m)	D_{66} (N.m)	D_{16}/D_{11}	D_{66}/D_{11}
CGA	2.0866	-0.5704	1.0074	-0.27	0.48
PSO	2.1078	-0.5638	1.0059	-0.27	0.48
BGA	2.0269	-0.5888	1.0008	-0.29	0.49
ACO	2.0269	-0.5888	1.0008	-0.29	0.49
MM	1.7019	-1.0622	1.0784	-0.62	0.63

Table 4. Bending, Bend-Torsion and Torsion stiffness terms for the Optimal Solutions for each Method

The statistical investigation provides a rather different picture. When all 100 solutions are considered, the ACO approach gives the best average result and has a much lower standard deviation, however it does take around 2.5 times as many iterations than the GA method and 30% more computation than PSO. There is very little scatter in the ACO results and the mean values are close to the optimal answers, many of the estimates are found repeatedly. There is a large variance in both the PSO and CGA solutions, however it can be seen that the PSO results for the ply orientations are in distinct closely formed clusters whereas there is a much more scattered appearance for the CGA results. The BGA scatter is between that of the ACO and the other two continuous methods for θ_1 and θ_2 however the variation for θ_3 is larger. The variance is very large in most cases as its calculation included all possible solutions which includes some significantly different answers. To use this information in practice, the worst solutions should be discarded and some form of clustering algorithm used to determine groups about which meaningful information on the solution distributions.

Figure 10 highlights how much better the ACO and to some extent the PSO methods are in consistently producing good estimates, however, the key observation is that for all methods there is no correlation between the number of iterations used and the optimality of the solution.

The results give optimal orientations between $\pm 45^\circ$ for a composite wing as predicted by Weisshaar, however, it should be noted that due to coupling between the bending and stiffness behavior, optimal results are not simply found from $\pm 45^\circ$ lay-ups. The θ_3 layer has the least effect due to it being placed closest to the neutral axis, resulting in a greater scatter in its results. Table 4 shows the Bending (D_{11}), Bend-Torsion (D_{16}) and Torsion (D_{66}) stiffness terms for the best results obtained by all the methods. It can be seen that the ratio between the torsion stiffness and the other two terms results remains almost constant for all results showing that the same stiffness ratio pattern is found by all methods.

The above discussion is enhanced by comparison with exhaustive searches for the more usual industry lay-up using any possible combination of $(0^\circ, \pm 45^\circ, 90^\circ)$ and $(0^\circ, \pm 30^\circ, \pm 45^\circ, \pm 60^\circ, 90^\circ)$, leading to optimum lay-ups and

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2
3 maximum instability speeds of $[-45 \ 45 \ 45]_s$, 25.43m/s and $[-30 \ 45 \ 45]_s$, 30.73m/s respectively. These results
4 demonstrate the sensitivity of the flutter process to the orientation angles and show how relatively small changes in
5 the lay-up can make a big difference.
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9 V. Conclusions

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11 Four biologically inspired optimization methods (Genetic Algorithms (binary and continuous), Particle Swarm
12 Optimization and Ant Colony Optimization) were used to determine the optimal lay-up for a simple composite wing
13 in order to maximize the flutter and divergence speeds. A statistical investigation was performed in order to
14 investigate the variation of the parameters that were optimized. The best single results were found using the Particle
15 Swarm and Continuous Genetic Algorithm however, the statistical investigation showed that Ant Colony
16 Optimization gave results with much less scatter than the other methods. A polynomial based meta-modelling
17 approach gave much worse answers than the other methods. It was also shown that for all methods there was no
18 correlation between the accuracy of the optimization and the number of iterations required for convergence.
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21 Obviously these results only refer to the optimization of a single aeroelastic system, however, it is conjectured that
22 similar findings would be found if the methods were applied to larger more realistic models. Further work is
23 currently investigating the application of these evolutionary approaches in combination with gradient based methods
24 to industrial type wing Finite Element models combined with potential flow aerodynamics.
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