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# Optimization of an Airfoil Cooling System Using a Pareto Dominance Approach

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## Abstract

The article deals with the optimization of the airfoil internal cooling system in order to find the most efficient distribution of passages to reduce thermal stress and, at the same time, retain the temperature at a permissible level. Another objective is to carry out the task with as little coolant as possible. The optimization is approached with the evolutionary algorithm, and thermo-mechanical FEM simulations are used for the assessment of the particular cooling system candidates. Since the optimization involves several criteria, the multi-objective approach needs to be applied. The optimization process may focus either on the scaled function solution or on finding the front of optimal solutions (Pareto optimal). In this article the latter approach is implemented and compared to the scaled function solution.

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**Keywords:** multi-objective optimization, evolutionary algorithm, airfoil cooling

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## Introduction

The gas turbine has now become the main source of drive in aircraft and it is an essential element of modern systems of energy conversion. The common occurrence of this type of machines has intensified the development of turbine engines to satisfy the needs of the aircraft industry. The progress has also influenced the branch of stationary turbines.

The efficiency of energy conversion in systems with a gas turbine depends on several factors, the most important of which are: the value of the gas temperature at the turbine inlet, the choice of the right compression [ratio](#) and the use of the appropriate system structure. The hottest issue at the moment, the one that engineers have been focusing on lately, is the raising of the feed temperature of the turbine, both in stationary and aircraft turbines (Shoko [et al.](#) 2005). The temperature, can significantly affect the specific unit power of the turbine. However, the raising of the inlet temperature is strongly limited by the materials used in production. That is why technical and technological works are being carried out to overcome the restrictions. Research is being made continuously in two main areas:

- materials engineering
- improvement of the [blade](#) cooling techniques

The first area is concerned with the development and production of modern materials capable of performing in high temperatures. At the same time, work is being done on the perfection of production technology and application of layers protecting airfoils from the impact of high temperatures. The second area is concerned with the use of various cooling methods of the components to make sure that their temperature is kept at the permissible levels.

The object of all these activities is to raise the Turbine Inlet Temperature (TIT), as the main parameter which determines energy conversion efficiency. It should be noted that the progress made in cooling techniques has made it possible to reach exhaust temperatures which overcome material restrictions. On the other hand, the use of more and more advanced cooling methods brings about some complications of [a](#) thermal nature. One consequence of the cooling process is the significant temperature gradients in the blades, which affects the values of thermal stress and, at the same time, the durability of the components. In this field, Bunker (2006) defines the challenges that engineers have to face to improve the performance of cooling systems of contemporary gas turbines. These are among the others:

- uniform internal cooling
- thermal stress reduction

Providing uniform inner cooling is related to the placement of cooling channels in the areas where the highest temperatures occur, relatively close to the blade edge. This can be a solution to thermo-mechanical restrictions. It also allows a more efficient use of the cooling agent through temperature reduction only in the areas which call for it.

Cooling of heat turbine components, including blades, is closely connected with loss generation which results from the flow of the cooling agent. The structure of the cooling system, and consequently the flow rate of cooling air significantly influences the total efficiency of energy conversion in circulation. On the other hand, the cooling system used should make sure that the maximum temperature of the component being cooled is kept below the allowable temperature level. The value of this temperature depends on the material and is connected with melting temperature together with thermal and creep strength.

Thus the right choice of the cooling system must be accompanied by a process of optimization leading to a system configuration which will meet the basic economy and strength criteria. These criteria are as follows:

- keeping airfoil temperature at a permissible level
- reduction of the coolant mass flow

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Another objective is to carry out the task with as little coolant as possible.¶

The optimization is approached with the evolutionary algorithm, and thermo-mechanical FEM simulations are used for the assessment of the particular cooling system candidates. Since the optimization involves several criteria, the multi-objective approach needs to be applied.

The (optimization) process may focus either on the scaled function solution or on finding the front of optimal solutions (Pareto optimal). In this paper the latter approach is implemented and compared to the scaled function solution.¶

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**Keywords:** multi-objective optimization, evolutionary algorithm, airfoil cooling¶

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- keeping stresses at a safe level
- securing the proper durability and rigidity of the component

Martin and Dulikravich (2001) put forward the following question: “Is it possible to design manufacturable turbine cooling configurations that simultaneously increase the turbine inlet temperature, decrease the coolant requirements, maintain material integrity and improve airfoil durability, so that the engine capital and operating costs are reduced?”

An application of the cooling system involves changes in the operational conditions of the component, the machine and even the whole power generation unit. The changes affect temperature distributions, stresses and, in consequence, the lifetime of components, power generation efficiency and machine complexity. In order to account for all these aspects, a formulation of the optimization task has to involve many criteria and constraints. Because the parameters of the changes are related to different physical fields, the optimization task becomes a multi-objective and multi-disciplinary one.

It is only in recent years that scientists became interested in the problems of optimization of the cooling systems of gas turbine blades. The reason for this situation was probably the high cost of optimization calculations, which results from the necessity of reliable computational models that can predict both temperature and the failure of a blade. Lack or inaccessibility of high-efficient calculating machines precluded, until a certain moment, the necessary calculations. A substantial increase in the processor capacity of PC's and the development of parallel processing techniques have made it possible to deal with the problem on a bigger scale. Optimizing calculations concerning the components of the flow system of a turbine (e.g. blades) mean having to carry out multiple flow- and thermal- analyses or conjugated tasks. Although precise solution of the processes influencing the optimization requires continuous development, the models used predict a correct trend in design changes. Since the optimization problem is a relative process, where the designs are compared to each other, the absolute values of the predictions are not always needed.

The attempts to find the optimum choice of the cooling system made recently were focused mainly on convection cooling with the use of inner cooling channels. At the moment it is probably the only cooling mechanism that can be analysed in full. Other cooling systems e.g. film cooling, impingement cooling etc., because of the complexity of the phenomena taking place, are only analysed from the point of view of the “given state” (Garg, 2002), and only some chosen aspects are the subject of optimization, e.g. the outflow of the coolant from a single passage in film cooling. Talya *et al.* (2002) took film cooling into consideration in the process of the profile shape optimization, but its structure was fixed and so were the inner channels, which were only scaled according to the size and shape of the profile. For several years now Dulikravich and his team (Martin and Dulikravich, 2001, Jeong *et al.*, 2003, Dennis *et al.*, 2003a, b) have been doing research on various problems concerning optimization of turbine blade cooling with focus on flow- thermal- and strength criteria. The outcome of that research is the Multidisciplinary Computer Automated Design Optimization (MCADO). The works are mainly concerned with optimization of the inner cooling channels (Dennis *et al.*, 2003, Martin and Dulikravich, 2001) with a possible outflow of the cooling air at the trailing edge (Dulikravich *et al.*, 1999). It can be seen in these works that, in spite of numerous decision variables used in the calculations, the optimization process is strongly restricted with relation to changes in geometry, i.e. optimization affects for example the fillet radius of the channel, the distance between the channels and the wall or the interspacing of the channels. In 2003 Dennis *et al.* (2003a) presented an article on the optimization of the location of a large number of circular cooling channels. The location of these channels, however, was restricted to a narrow area of the blade section close to the blade wall. The objective was to find a pattern for the distribution of the cooling holes so that the heat flux absorbed by the coolant could be maximally reduced, with the temperature of the blade kept at the permissible level,

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1 and so that the amount of steam necessary for the cooling could be reduced, too. Dennis *et*  
 2 *al.*, (2003b) presented another interesting work on the optimization of channels in the shape of  
 3 a serpentine with three returns of the cooling agent. The problem was dealt within 3D with the  
 4 use of thermal- and strength criteria. The results revealed a great potential in the cooling  
 5 system; the right choice of the shape and distribution of cooling channels does affect the  
 6 amount of the cooling agent and thermal stress, and, consequently, the durability of the blade.

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7 Nowak *and* Chmielniak (2005) presented the optimization of the distribution and  
 8 dimensions of circular cooling passages, with their set number. In this case the approach was  
 9 that the cooling holes could move freely in the blade cross-section with feasibility criteria  
 10 retained. In later works (Nowak *et al.*, 2007, 2008) the problem of channel optimization was  
 11 expanded to a multidisciplinary task including thermal- and strength- states with relation to  
 12 economic criteria. The latest work of Verstraete *et al.* (2008) presents attempts to connect  
 13 cooling channel optimization tasks with the thermal- flow- conjugated task. The objective of  
 14 this approach is to increase the credibility of the boundary conditions formulated at the blade  
 15 surface.

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16 The problems of the optimization of the cooling systems for the airfoils of heat turbines  
 17 usually involve a relatively large number of decision variables together with a complex, non-  
 18 overt and sensitive to even small changes in parameters response of the system being  
 19 analysed. That response, *e.g.* in temperature, stresses or other values, is not monotonic and is  
 20 characterised by a great number of (local) extremes, which is a major problem for many  
 21 optimization methods based on the gradient approach (Mueller, 2002). In these situations  
 22 stochastic methods are preferred, which in fact can be seen in all the works mentioned in this  
 23 article.

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24 The most often used stochastic method is the evolutionary algorithm, which, in spite of  
 25 its downsides, is best suited for dealing with complex technical problems. A slightly different  
 26 optimization method, also stochastic, was presented by Dennis *et al.* (2003a). The method is  
 27 called Indirect Optimization based on Self Organisation (IOSO), and is a combination of the  
 28 Response Surface Method with the evolutionary approach.

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29 In this article the problem of optimization of the spacing and diameter of cylindrical  
 30 cooling channels is dealt with in the multi-objective aspect with the use of various ways to run  
 31 the evolution algorithm (different solution quality assessment methods). The results of the  
 32 single-objective approach with objective function scaling and of the approach based on Pareto  
 33 domination are presented. Because the solutions of multi-objective tasks are often ambiguous,  
 34 the results are shown as a set of non-dominated solutions, from which the user can choose the  
 35 final solution. The optimization task is based on the evolutionary algorithm with the use of the  
 36 commercial solver FEM (ANSYS) to predict thermal and strength fields.

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### 37 The Evolutionary Optimization

38 The notion *evolutionary algorithm* encompasses a whole class of numeric methods  
 39 based on the mechanisms of natural selection and heredity. They combine the evolutionary  
 40 principle of survival of an individual which is best adapted with the systematic, though  
 41 randomised exchange of information (Goldberg, 2003). Algorithms of this type process  
 42 populations of individuals which represent potential (though of a different quality) solutions  
 43 to a problem. The essence of the genetic algorithm is to get, in subsequent generations,  
 44 individuals with increasingly higher values of the fitness function. For this kind of process to  
 45 take place, the subsequent generations have to change but the changes do not occur at random.  
 46 The transformation of the present (parental) generation into the offspring generation takes  
 47 place through a mathematical interpretation of the reproduction and mutation processes  
 48 known from biology. How individuals contribute to these processes and what chances of  
 49 survival they have is controlled by a selection procedure. The chance of survival, together

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1  
2 1 | with the chance of passing its own features on to the offspring generation, depend on the  
3 2 | quality of the individual (its fitness). Deleted: their  
4 3 | The structure of the individual plays the key role in the evolutionary optimization. So it must  
5 4 | reflect all the design parameters which take part in the optimization process. The values of the  
6 5 | parameters are most often not used directly, rather than encoded either in binary form or as a  
7 6 | floating point number. In the first case the individuals are structured as strings of bits, whereas  
8 7 | the latter requires vector form.  
9 8 | Each individual is assessed to determine to what extent it meets the criteria posed. In the  
10 9 | airfoil cooling system optimization, any relocation of channels and/or change in their size  
11 10 | affects temperature distribution, stresses, cooling agent consumption or other values. On these  
12 11 | grounds the evaluation criteria are formulated.

### 14 **Multi-objective optimization**

15 15 | As has been mentioned before, the presence of the blade cooling system, apart from  
16 16 | positive effects, causes also some negative phenomena. The objective of any optimization is  
17 17 | to strengthen the former and eliminate or reduce the latter. This calls for having to meet many  
18 18 | criteria which all contribute to the proper and efficient operation of both a single component  
19 19 | and the entire machine. Moreover, as these objectives are often contradictory, the cooling  
20 20 | system should be designed with consideration of possibly all the phenomena and parameters  
21 21 | which affect the operation of a machine. Multi-objective optimization becomes essential. Deleted: a must  
22 22 | One of the ways to deal with this type of tasks is task scaling and the use of the single-criteria  
23 23 | algorithm, which involves a multi-dimensional objective function including all the optimised  
24 24 | parameters. Owing to this solution a single configuration of design parameters (the structure  
25 25 | of the cooling system), which gives the extreme value of the objective function, is the result .  
26 26 | It turns out, however, that this approach may be ambiguous because, taking all the  
27 27 | optimization criteria into consideration, it is impossible to state beforehand which of the  
28 28 | solutions is the best. There is a group of solutions which differ only in the aspect of quality  
29 29 | with reference to specific criteria. These are the so-called non-dominated solutions, i.e. Deleted: i.e.  
30 30 | it is impossible to find better solutions that would meet all the criteria simultaneously. Deleted: et al  
31 31 | Non-dominated solutions are called *Pareto optimal* and their corresponding set of vectors Deleted: Font: Not Italic  
32 32 | containing the objectives is referred to as *Pareto front*. It can be seen that a multi-objective Deleted:  
33 33 | task usually has many solutions, from which, on the grounds of additional premises, the final  
34 34 | one is selected.

35 35 | From the mathematical point of view, we look for design parameters for which their  
36 36 | corresponding objectives are non-dominated. The domination of vector  $\mathbf{X}=\{x_1, x_2, \dots, x_n\}$   
37 37 | over vector  $\mathbf{Y}=\{y_1, y_2, \dots, y_n\}$  in the case of the minimisation task (Fig. 1) can be formulated as  
38 38 | follows (Wagner et al., 2007): Deleted: et al  
39 39 |

$$40 40 \quad \forall i \in \{1, \dots, n\}: x_i \leq y_i \quad \text{and} \quad \mathbf{X} \neq \mathbf{Y} \quad (1)$$

41 41 | Evaluation of the population can be based on the fitness function, whose value is determined  
42 42 | for each individual according to its distance from the Pareto front. For this purpose a set of  
43 43 | consecutive fronts is built. Only the first encompasses the non-dominated solutions whereas  
44 44 | the others are found one by one after the previously determined front is excluded from the  
45 45 | population. Then, the fitness value depends on the front number the individual belongs to.  
46 46 | Another way of an individual evaluation is to determine the fitness function according to the  
47 47 | number of individuals dominated by, and the ones which themselves dominate a particular  
48 48 | solution. Tests showed that in this optimization problem the latter method made the algorithm  
49 49 | perform better and it was used for the computations. Deleted: ,  
50 50 | Deleted: to

## Numerical Tests

Test calculations were made for the C3X blade mentioned above, with the use of the evolutionary algorithm.

### Evolutionary algorithm

The algorithm used for the search process was based on real-number coding of the design parameters. The operation of the change of individuals was realised by means of recombination and mutation processes. The first of these processes was used as weight averaging of selected parental individuals:

$$\begin{aligned} [a_1, a_2, \dots] &\longrightarrow [m_1 a_1 + (1 - m_1) b_1, m_2 a_2 + (1 - m_2) b_2, \dots] \\ [b_1, b_2, \dots] &\xrightarrow{\text{recombination}} [(1 - m_1) a_1 + m_1 b_1, (1 - m_2) a_2 + m_2 b_2, \dots] \\ &(m_i \rightarrow \{0,1\}), \end{aligned}$$

where the weight  $m_i$  is chosen randomly for each  $i$ .

The mutation consisted in a random disturbance introduced into the vector of design parameters. The size of the disturbance, according to Michalewicz (1999), got smaller in proportion to the number of iterations (population number):

$$\begin{aligned} [a_1, a_2, \dots] &\xrightarrow{\text{mutation}} [a_1 \pm \varepsilon_1, a_2 \pm \varepsilon_2, \dots] \\ \varepsilon_i &= \varepsilon_{\max} \left( 1 - \text{RND} \left( \left( 1 - \frac{k}{K} \right)^b \right) \right) \quad (\varepsilon_{\max} \rightarrow \{0,1\}) \end{aligned}$$

where  $\varepsilon_{\max}$  is the maximum (normalised) disturbance value,  $\text{RND}$  is the random variable with a uniform distribution,  $k/K$  is the ratio of the present iteration number to the maximum number of iterations to be carried out, and  $b$  is the constant parameter. Both operations occur with a specific probability, which is a parameter of the algorithm.

Fixed boundary conditions on the outer surface resulting from experimental research were assumed for a given profile. The values were given for 86 locations around the blade at its mid-height and between them a linear approximation was used. In the span-wise direction the boundary conditions were constant. The heat exchange on the surface of the channels was determined on the grounds of pipe-flow. Another assumption was that all the channels were fed with air of the same parameters and that in each of them the same fall in pressure is realised. Owing to this, Nusselt number was defined and then the heat transfer coefficient, which was constant in the whole channel. Following the experimental report of Hylton *et al.* (1983) the Nusselt number in a passage was found with the Dittus-Bolter equation (Hylton *et al.*, 1983):

$$Nu = 0.023 Re^{0.8} Pr^{0.4} \quad (2)$$

On the grounds of the dependence which determines how the agent is heated in a pipe with a constant temperature, the coolant local temperature was defined (Lienhard IV and Lienhard V, 2008):

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$$T(z) = (T_m - T_{wall}) \exp\left(-\frac{h\pi d}{\dot{m}c_p} z\right) + T_{wall} \quad (3)$$

Wall temperature  $T_{wall}$  was an average for the whole channel and originated from the previous iteration. To obtain the wall temperature, thermal calculations were performed twice, firstly, for assumed  $T_{wall}$  and then with the computed one. This approach made it possible to predict the wall temperature with satisfying accuracy.

It has to be stressed that a mere consideration of constant boundary conditions at the external surface taken from the experiment, together with the Dittus-Bolter relation (2) for internal passages is a simplification of the actual problem. In reality a change in the cooling system structure affects thermal boundary conditions for the external flow. To take into account changes in the external flow, a Conjugate Heat Transfer predictions need to be conducted. This, however, is time consuming and, for the time being excluded from optimization tasks. On the other hand, although a lot of improvements in the boundary condition assessment can be introduced to make them more reliable, the simplified description of the problem seems to be appropriate for the testing of optimization methods.

### Cooling system coding

Cooling system optimization is carried out for the blade C3X (Fig. 2), known from literature (Hylton *et al.*, 1983). It is originally equipped with 10 cylindrical internal cooling passages. The location of each of the passages can be described with three variables (2 coordinates of the centre and radius).

These values are the design parameters and, obviously, affect the structure of the individual. So the optimization problem is formulated and solved over a 30 dimensional design space. As Michalewicz's work (1993) shows, the real-number coding of technical problems of a continuous character is more beneficial and efficient. It has been confirmed by the author's own studies as well. Because of that, the task of optimization is dealt with the use of this type of variable representation. Earlier studies showed that the use of Cartesian coordinates is ineffective in domains as complex as the airfoil cross-section. Modification of the individuals due to the evolution within the global Cartesian system leads to a very high fraction of the population to violate the constraints. That is why the variables describing the channel are defined in a local curvilinear coordinate system, associated with the blade (Nowak G., Wróblewski W., 2007). In this case the axis of abscissa follows the airfoil's suction side and the axis of ordinates is always perpendicular to it. The coordinates are additionally normalized. In this way the individual's genotype renders the structure of the cooling system in a unit square.

### Evaluation

The choice of optimization objectives was determined on the basis of the airfoil's durability and the thermal efficiency of the whole cycle. To reduce deterioration effects (oxidation and corrosion) within the material, it was important to provide operation at the lowest component temperature possible. This temperature, on one hand, must not exceed the permissible level. On the other hand, a lower temperature results in higher material limits and extends the lifetime of the hot gas path components. The life increase can be significant for modern aero engines operated at very high gas temperature. Although in the case study the operational temperature is too low to induce the deterioration effects mentioned, it was decided to take the maximum material temperature into account to show possibilities of its reduction in parallel with other objectives. Another factor influencing the component's life is the stress level, either maximum tensile or equivalent one. The former is responsible for the

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fracture propagation, the latter - for the fatigue and creep process. Which factor prevails depends on the material behaviour. In this work it was decided to consider the stress level in the form of its relative value called the failure factor, which is a ratio of the actual stress components to the allowable (desired) ones.

$$s_{\sigma} = \frac{\sigma_{\max}^i}{\sigma_{all}^i}$$

The allowable stress components were assumed as linear functions of the blade temperature. For the normal components the allowable stress changed in the range of 350-180 MPa within the temperature interval of 400-800 K. For the shear stress the change was 200-100 MPa.

The application of a cooling system leads to a cycle efficiency drop which results from air extraction from the compressor. The air is then directed into a component being cooled, and finally the turbine flow path. The efficiency drop can be reduced by a decrease in the coolant mass flow.

The optimization calculations were made for two ways of algorithm realisation. One was based on the translation of a multi-objective optimization problem into a single-objective one by means of a weighted sum of objectives and, in consequence, it used the single-objective evolutionary algorithm. The other employed the method of Pareto domination in a multi-objective algorithm. In both cases, three values, which reflected the critical aspects of blade operation, were the subject of optimization: maximum blade temperature ( $T_{\max}$ ), maximum failure factor ( $s_{\sigma}$ ) and the total mass flow of the coolant ( $\dot{m}$ ). The stress involved in the failure factor resulted from thermal load only. The objective was to minimize all the values. In the case of scaling, the objective function was formulated as a weighted sum of the component functions.

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$$f = \sum_i w_i f_i \quad \text{where} \quad \sum_i w_i = 1 \quad (4)$$

The component functions are of the same form:

$$f_i = (y_i - y_{it})^2 \quad \text{where} \quad y_i = \{T_{\max}, s_{\sigma}, \dot{m}\} \quad (5)$$

The values marked with index  $t$  are the target values of the parameters, and for the calculations presented they are, respectively : 650K, 0.9, 50g/s.

The fitness value formulated as:

$$F = N - \frac{n_d}{(n_{ds} + 1)} \quad (6)$$

was determined for the multi-objective task on the grounds of the assessment of the objective vector of the form:

$$\mathbf{V} = \left[ \frac{T_t}{T_{\max}}, \frac{s_{\sigma t}}{s_{\sigma \max}}, \frac{\dot{m}_t}{\dot{m}} \right] \quad (7)$$

where  $N$  is the population size,  $n_d$  is the number of individuals that dominate a given solution, and  $n_{ds}$  is the number of the individuals dominated by the assessed one. This evaluation is shown in Fig. 1 for  $X_1$  solution. The individuals enclosed within the lower-left rectangle dominate over  $X_1$ , whereas those within the top-right one (in Fig.1 only  $X_5$ ) are dominated by  $X_1$ .

1 Preliminary calculations showed, however, that there is a need to complete the individual  
 2 assessment procedure with a penalty function to modify the values of the parameters being  
 3 optimised, should they exceed permissible values.

4 Since the optimization is carried out with an automatic model generation, the user's influence  
 5 on the mesh reliability is highly limited. To keep the results accuracy on an acceptable level,  
 6 it is necessary to perform the FEM structural computations with higher order elements. The  
 7 reason for this is that the high thermal gradients, which result from the cooling, produce high  
 8 stresses limited to a small area. If linear elements are used, the stresses are averaged over the  
 9 element, and in consequence the results can be substantially incorrect. The stresses (or failure  
 10 factor) can be significantly underestimated in critical locations. The application of mesh with  
 11 nonlinear elements is however much more computationally expensive both in terms of time  
 12 and computer resources.

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## 14 Results

15 The optimization calculations carried out for profile C3X provided new configurations  
 16 of the cooling system. The configurations differ from the original structure in the placement  
 17 of individual channels and their size. The results show that similar effects, from the point of  
 18 view of the optimization criteria, can be achieved with various distribution and even various  
 19 number of cooling passages. The number of channels varied in the course of the search  
 20 process usually between 7 and 10 although there were individuals with 4 or 5 passages. A  
 21 higher number of cooling channels causes a fall in the blade temperature but usually generates  
 22 higher failure factors and, in the case of larger diameters, increases coolant consumption. In  
 23 the case of configurations with fewer passages the tendencies are opposite - less cooling  
 24 results in a lower stress level and reduced coolant usage.

25 From the point of view of the optimization criteria, the new configurations of the  
 26 cooling system are more functional. All the optimised values have been improved (compare  
 27 Fig. 3 and Fig. 4). The maximum temperature of the blade fell by 8K with failure factor drop  
 28 from over 1.16 to below 1.05, and with almost 45% reduction in the total coolant mass flow  
 29 (from 90g/s in the reference case to less than 50g/s in the optimised one).

30 In the case of the method using the weighted sum of objectives, the solution of the  
 31 optimization task is obtained with a maximum value of the fitness function. This solution,  
 32 however, depends closely on how the objective function is formulated. Even a slight change  
 33 of weights in formula (4) leads to a different channel distribution. This feature of the method  
 34 usually requires some test computations in order to adjust the weights within the objective  
 35 function. It is more vivid in the cases where specific objective spaces are of a different range.  
 36 It may happen that a huge change of one objective can affect the objective function in the  
 37 same manner as slight change in another one. The algorithm will promote an objective that  
 38 requires less change to obtain a bigger effect. Different weight sets may give comparable  
 39 objective function values in which different criteria are strengthened. So the choice of weights  
 40 can be time consuming and is specific to the task in question.

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41 In the case of the method based on Pareto domination, the result is a set of *p-optimal*  
 42 solutions which best meet specific criteria. The calculations which were carried out provided  
 43 a set of 16 non-dominated solutions, some of which are presented in Fig. 5. The results are  
 44 presented also in the objective space (Fig. 6) in the form of a class scatter chart. The number  
 45 of non-dominated individuals is too small to generate a continuous 3D surface plot, so the  
 46 chart presents the Pareto front in temperature-failure factor planes for several classes of the  
 47 coolant mass flow. This figure shows that intensive cooling (low temperature) requires  
 48 increased coolant mass flow and induces higher failure factors. The opposite tendency is  
 49 visible for less cooling. If the failure factor is limited to 1, which can be treated as a value  
 50 that results in a nominal lifetime, there are three configurations which fall in this range. All of

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2 1 them are cooled with significantly lower amount of air (below 60 g/s) if compared to the  
3 2 reference case and in two of them the maximum temperature of the blade is also lower by 15-  
4 3 17K. The failure factor limitation is however not obeyed by the single-objective solution for  
5 4 which the value is 1.049 despite very low coolant consumption. Since the failure factor seems  
6 5 to be the most important from the point of view of components' lifetime further optimization  
7 6 is required, which would need weight changes within the objective function. Decrease of the  
8 7 failure factor results in an increase of lifetime. Similar tendency can be observed for the  
9 8 maximum material temperature of the hot components.

10 9 It has to be stressed that during the optimization process specific cooling candidates  
11 10 were evaluated on the basis of relationship (6) only, and no penalty method, regardless of any  
12 11 particular objective value, was used. The above selection was made for the final generation to  
13 12 present the solutions which were situated in an assumed range of the objective space. This  
14 13 space is usually limited by permissible values of temperature, the failure factor or other  
15 14 parameters. The structures of the cooling configurations obtained differ from each other  
16 15 according to their location on the Pareto front (Fig. 6). A close distance along the surface  
17 16 provides similar solutions, whereas remote solutions result in highly different cooling  
18 17 candidates. In the case of a low blade temperature the cooling system usually consists of ten  
19 18 bigger passages which provide a high coolant mass flow. Such solutions generally produce a  
20 19 higher thermal stress and in turn a higher failure factor (the actual stress growth is higher than  
21 20 the allowable stress due to temperature reduction). At the other end of the front, for low  
22 21 failure factor values, the cooling candidates are composed of a fewer number of channels (4-5  
23 22 passages), whose size is small. In consequence, the coolant mass flow is limited, which makes  
24 23 the airfoil's temperature rise.

25 24 Comparing the solution for both approaches with the same measure (weighted  
26 25 objective function in the single-objective optimization) the single-objective method provided  
27 26 a cooling configuration slightly worse (less than 1%) than the best Pareto candidate. This  
28 27 solution belongs to the Pareto front (the 2 cross-marked individuals in Fig. 6 with lower  
29 28 temperature and failure factor objectives use more coolant) and it does not dominate any  
30 29 individual within the multi-objective non-dominated set. The specific objective values and  
31 30 single-objective function values for both optimization approaches and the reference case are  
32 31 given in Table 1.

33 32 In both optimization techniques the solution time was comparable although the  
34 33 number of generations calculated was different. In the case of the single-objective approach  
35 34 the algorithm converged after about 35 generations, whereas the multi-objective one required  
36 35 about 60-70 generations. Both tasks were carried out with a population of 60 individuals.  
37 36 However in the first case the number of cooling candidates to be assessed per generation was  
38 37 essentially higher than in the other. The reason is that in the single-objective algorithm only  
39 38 5% of the best individuals were copied to the successive population, whereas in the multi-  
40 39 objective approach it was the whole Pareto front. In the initial stage of the search process the  
41 40 non-dominated set was quite small (8-10 individuals) but as the algorithm advanced, the front  
42 41 increased, until it filled up the whole population.

43 42 Numerical simulations have proved the efficiency of both approaches but the Pareto  
44 43 domination method is characterised by one specific advantage. To function properly, the  
45 44 method of objective function scaling must be duly matched to the character of the task, and  
46 45 this calls for a great number of tests to determine how individual criteria affect the process  
47 46 and direction of optimization. In the multi-objective approach it is necessary to compare the  
48 47 objectives without having to fix the impact of the individual criteria on the direction of the  
49 48 search.

## 50 49 Conclusions

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The objective of the study was to compare how two different approaches to the optimization task of the cooling system of the blades of gas turbines function when numerous criteria have to be met. The method based on objective function scaling was compared with the method of Pareto domination. The former provides a single solution which depends on the choice of the weights in the objective function; the latter offers a set of non-dominated solutions, *i.e.* solutions which best meet specific, individual criteria. The evolutionary algorithm was used for the optimization, which is a typical approach while looking for a Pareto front.

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Numerical simulations have shown that both approaches are equally efficient when it comes to the time of solution finding, but an essential advantage of the Pareto domination method is to be noticed. It does not require the knowledge of how the optimised parameters depend on each other because it is only the specific parameters themselves or the functions on which they are built that are subject to comparison. The knowledge of the dependence is essential in the weighted single-objective method. The weight values for specific criteria structure the running of the algorithm and have a direct influence on the final result of the optimization process. A proper determination of the objective function usually requires some preliminary tests which should be accepted as part of the optimization cost. The computational effort necessary for the single-objective approach is much higher and therefore the multi-objective Pareto dominance method seems to be the right choice.

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Table 1. Objective comparison

Case	$T_{max}, K$	$s_{\sigma}$	$\dot{m}, g/s$	$f$
Reference	706.7	1.163	90.0	9.06
Single-Objective	698.7	1.049	49.4	9.83
Multi-Objective	689.6	0.982	55.0	9.89

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Figure 1. Pareto domination

Figure 2. C3X airfoil

Figure 3. Temperature (left) and failure factor (right) distribution for original cooling system configuration

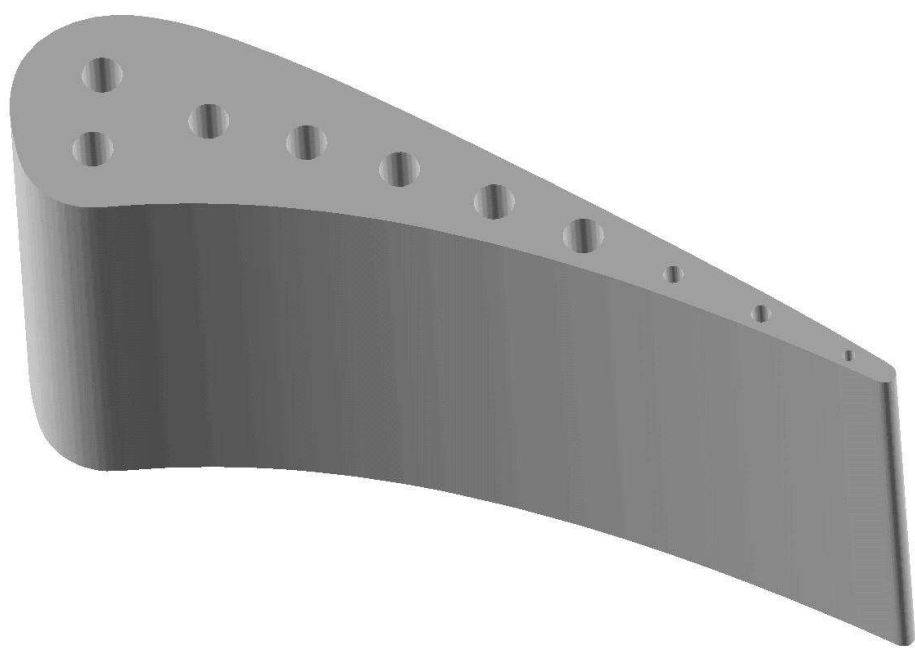
Figure 4. Temperature (left) and failure factor (right) distribution for single objective optimization

Figure 5. Selected *p-optimal* solutions (failure factor contour)

Figure 6. Pareto front

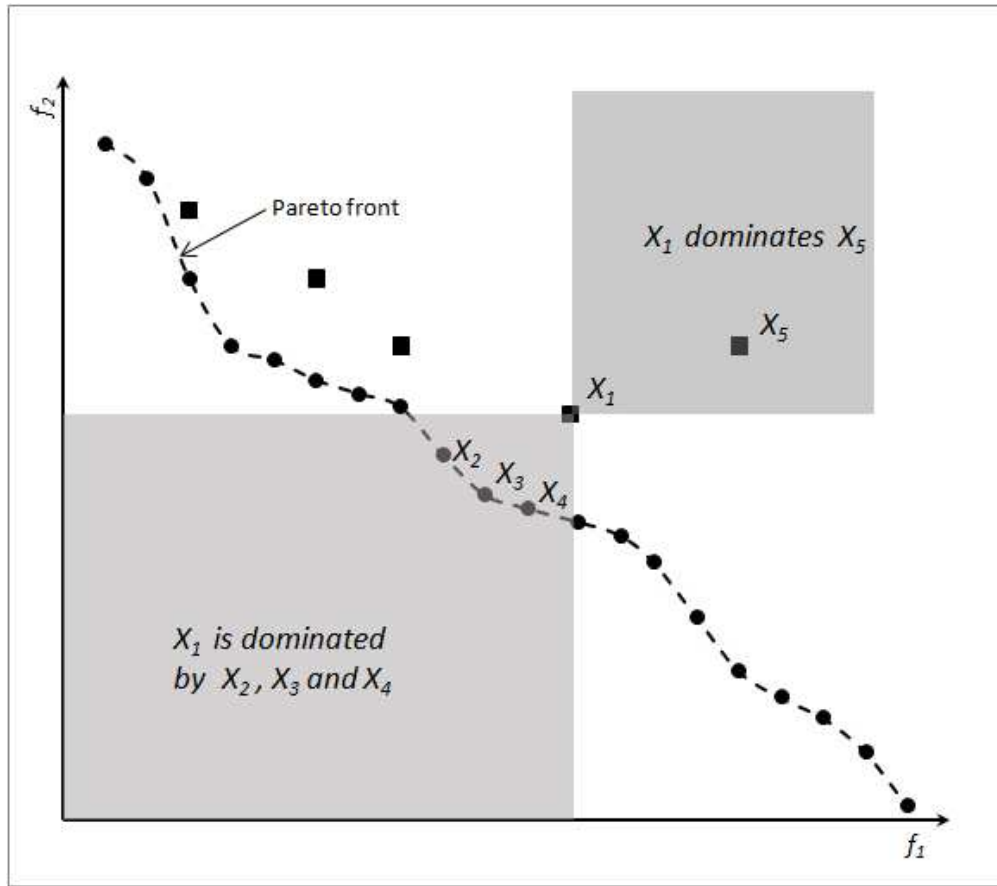
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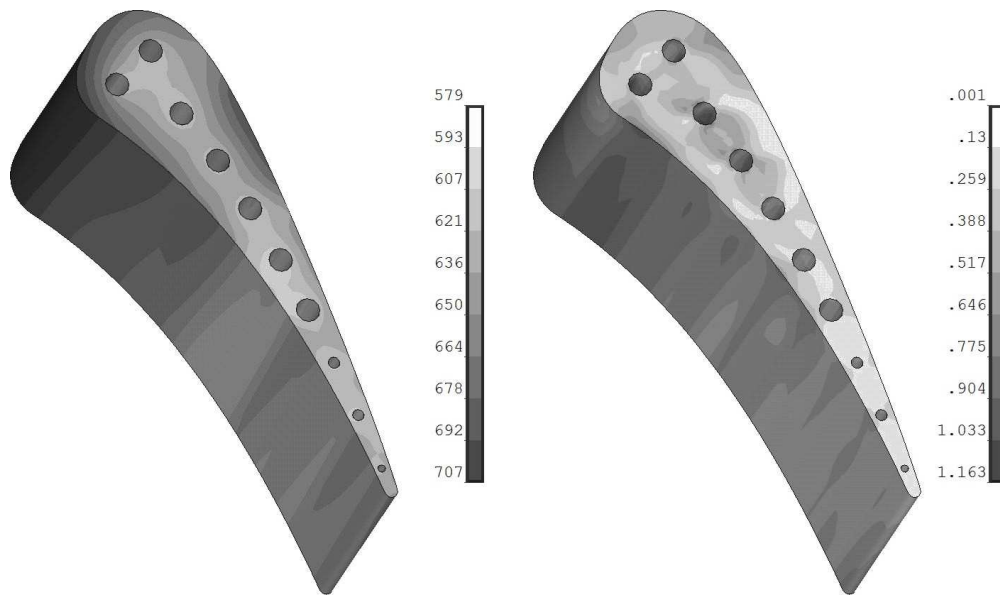
C3X airfoil  
323x248mm (96 x 96 DPI)

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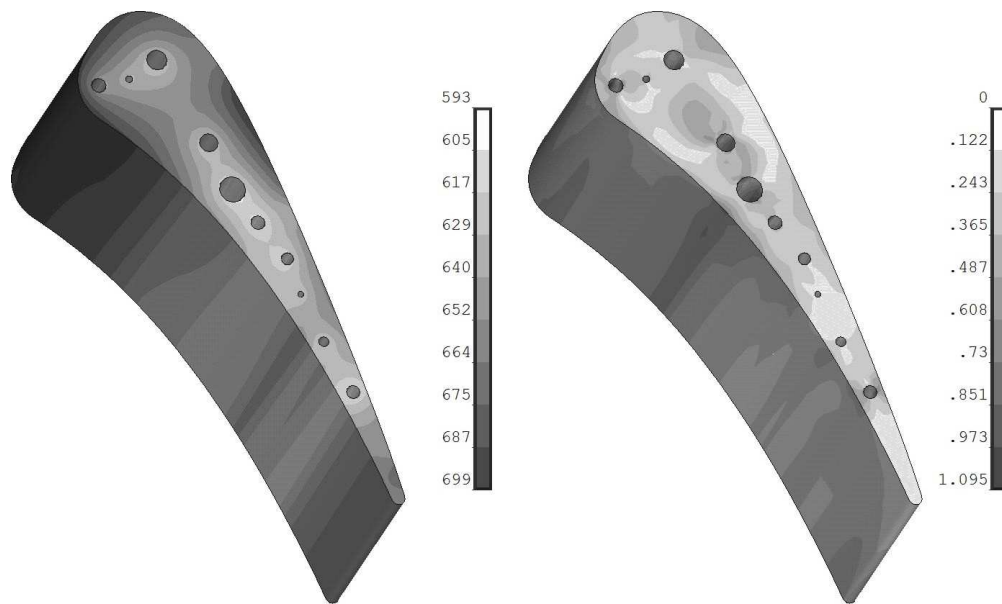
Pareto domination  
153x135mm (96 x 96 DPI)

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Temperature (left) and failure factor (right) distribution for original cooling system configuration  
238x139mm (150 x 150 DPI)

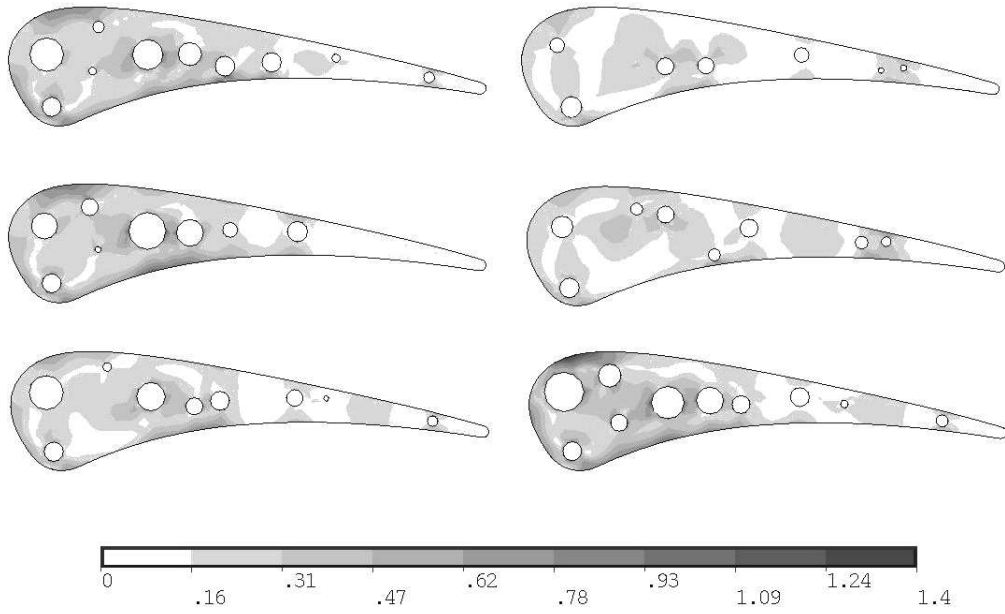
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Temperature (left) and failure factor (right) distribution for single objective optimization  
235x139mm (150 x 150 DPI)

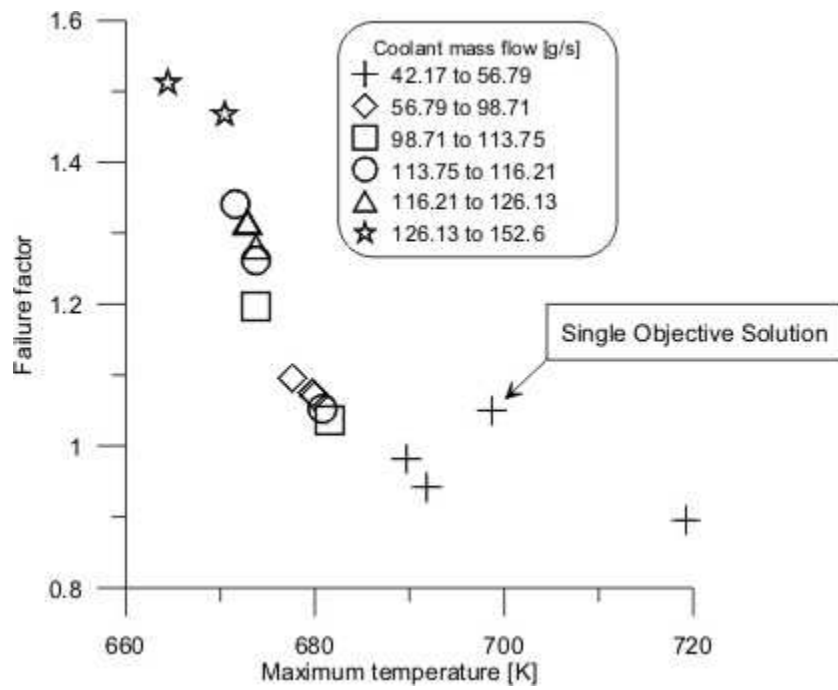


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Selected p-optimal solutions (failure factor contour)  
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Pareto front  
146x118mm (72 x 72 DPI)

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