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# Kinetics of cubic to tetragonal transformation in Ni-V-X alloys. 

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#### Abstract

Computer simulations based on the Khachaturyan's Onsager diffusion equation have been performed to model the microstructure evolution during the cubic-to-tetragonal transformation in Ni-based alloys. A 2D model was employed. The fcc- $\mathrm{DO}_{22}$ ordering has been studied as a model case of this type of transformations. A particular emphasis is on the formation mechanism of $\mathrm{DO}_{22}$ monovariant structures. Computer simulations demonstrate that strain-induced interactions between coherent $\mathrm{DO}_{22}$ precipitates lead to the formation of intermediate two-variant chessboard-like structures, which are found to be unstable and coalesce into subsequent single-variant maze structures. It is shown that the stability of the chessboard-like structures is very sensitive to lattice misfits. These simulation results are in good agreement with TEM observations.


Keywords: phase transformation kinetics, microstructure, precipitation, modelling

## 1. Introduction

Characteristic microstructures, so-called tweed or twinned structures, have been observed frequently in various alloys which undergo cubic to tetragonal phase transformations, such as $\mathrm{NiV}, \mathrm{NiCoV}, \mathrm{NiFeV}, \mathrm{FePd}$ and CoPt [1-5]. In these transformations, the new phase is tetragonal and can be formed in the three different orientations with respect to the cubic mother phase. One might expect that the three variants would be formed with equal probability. The formation of tetragonal phase precipitates in cubic matrix, however, inevitably introduces elastic strain and random arrangements of variants would result in a large increase in the elastic energy of the system. Consequently, it is well understood that formation of tweed or twinned structures is driven by the elastic strain energy reduction.

The precipitation of $\mathrm{DO}_{22}$ tetragonal phase in Ni-based alloys is one of the examples of a cubic to tetragonal transition. In Ni-V-X systems, a low symmetry ordered phase (tetragonal $\mathrm{Ni}_{3} \mathrm{~V}$ phase with the $\mathrm{DO}_{22}$ ordered structure) coexists with the high symmetry fcc parent Ni -$\mathrm{V}-\mathrm{X}$ solid solution ("A1" phase). In such systems, the formation of the $\mathrm{DO}_{22}$ tetragonal phase produces an anisotropic strain field due to the contraction and expansion of the tetragonal $a$ and $c$ axes against the cubic matrix. Lattice misfit strain along a- and c-axes of $\mathrm{DO}_{22}$ phase, $\delta_{11}$ and $\delta_{33}$, can be determined using following equations: $\delta_{11}=\left(a_{D O 22}-a_{A 1}\right) / a_{A 1}$ and $\delta_{33}=\left(c_{D O 22} / 2-a_{A 1}\right) / a_{A 1}$, respectively. The tetragonal phase has three [100] orientation variants, The resulting microstructure exhibits a fine brick-like "multi-variant structure (MVS)", and the variants are found to be twin-related to each other across the $\{110\}$ planes.

There have been a number of attempts to model microstructure evolution in systems with coexisting tetragonal and cubic phases. Recently, Y. Ni and A. G. Khachaturyan [6,7] studied using 3D modelling based on the phase field microelasticity (PFM) approach the formation of chessboard-like microstructure in cubic matrix with different initial concentrations. It was remarked that there are two thermodynamically distinct regions within the two-phase field of the phase diagram that described the equilibrium coexistence of the cubic and tetragonal phases. These regions are determined by the relative values of the free energies of the cubic and tetragonal phases as a function of their composition. It was shown that the transformation pathways to the chessboard-like microstructure are very different in these two thermodynamically distinct regions. Y. Le Bouar et al. [8-10] investigated the effect of elastic energy on the morphology of chessboard-like microstructures in Co-Pt and $(\mathrm{CuAu})_{1-\mathrm{x}}-\mathrm{Pt}_{\mathrm{x}}$ alloys. It was shown that the parent cubic phase $\left(\mathrm{L}_{2}\right.$ for $\mathrm{Co}-\mathrm{Pt}$ and disordered fcc for $\left.(\mathrm{CuAu})_{1-\mathrm{x}}-\mathrm{Pt}_{\mathrm{x}}\right)$ coexist with the three different orientation variants of the tetragonal $\mathrm{L1}_{0}$ ordered phase. In this case, the final microstructure is multi-variant and strongly depends on the misfits between the two coexisting phases. Wen et al predicted the similar chessboardlike and other complex mulitivariant precipitate structures during precipitation of an orthorhombic phase in a hexagonal Ti-Al-Nb alloy [11,12]. All these simulations show that strain energy is the dominant factor that determines the stability of CB microstructures.

In contrast to Co-Pt and other alloys, experiments revealed that long time aging of binary Ni-V alloys leads to monovariant microstructures [13-15]. It was shown that during coarsening, the major variants in initial MVS grow at the expense of minor variants and disordered A1 phase. To understand the influence of misfit on stability of MVS A. Suzuki and al. [13] have investigated the evolution of microstructure in ternary Ni-V-X (where $\mathrm{X}=\mathrm{Co}, \mathrm{Fe}$ and Nb ) alloys. It was shown that the morphology of ordered precipitates as well as the habit plane between A 1 and $\mathrm{DO}_{22}$ structure can be changed by alloying the binary $\mathrm{Ni}-\mathrm{V}$ system with $\mathrm{Co}, \mathrm{Fe}$ and Nb atoms. It was concluded that the misfit strain ratio $\mathrm{d}=\delta_{11} / \delta_{33}$ and the magnitude of $\delta_{11}$ are important in determining the habit plane and the shape of $\mathrm{DO}_{22}$ particles.

As the magnitude of $\left|\delta_{11}\right|$ becomes smaller than 0.002 , the shape of the $\mathrm{DO}_{22}$ variant changes from plate to prism elongated along a-axis on the habit plane, resulting in chessboard microstructure. Therefore, the main objective of this work is to understand the underlying thermodynamic driving forces and kinetic mechanisms leading to the formation of monovariant microstructures in binary Ni-V systems as well as the stability of MVS during cubic to tetragonal transformation in ternary Ni-V-X alloys.

To predict the microstructure evolution and precipitation kinetics of binary $\mathrm{Ni}-\mathrm{V}$ and ternary Ni-V-X alloys, 2D computer simulations are performed based on the Khachaturyan's microscopic diffusion equation. Simulation results were compared with our TEM observations for $\mathrm{Ni}-19.5 \% \mathrm{~V}$ binary system and with experimental data for ternary $\mathrm{Ni} \mathrm{-V}-\mathrm{X}$ ( $\mathrm{X}=\mathrm{Co}, \mathrm{Fe}$ and Nb ) extracted from literature [13].

## 2. Model

To investigate the morphological evolution in the Ni-V system, we employed a computer simulation model based on the Onsager-type microscopic diffusion equations first proposed by Khachaturyan [16]. For a binary alloy this equation can be written as:

$$
\begin{equation*}
\frac{\mathrm{dP}(\mathbf{r}, \mathrm{t})}{\mathrm{dt}}=\sum_{\mathrm{r}^{\prime}} \mathrm{L}\left(\mathbf{r}-\mathbf{r}^{\prime}\right)\left[\frac{\delta \mathrm{F}}{\delta \mathrm{P}\left(\mathbf{r}^{\prime}, \mathrm{t}\right)}\right] \tag{1}
\end{equation*}
$$

where $\mathrm{P}(\mathbf{r}, \mathrm{t})$ is the probability of finding of a solute atom (Vanadium) at a given lattice site $\mathbf{r}$ and at a given time $t, L\left(\mathbf{r}-\mathbf{r}^{\prime}\right)$ are the exchange probabilities between the atoms at lattice sites $\mathbf{r}$ and $\mathbf{r}^{\prime}$ per unit time and $F$ is the free energy of system. In the case of a cubic-totetragonal transformation the free energy of system contains two terms:

$$
\begin{equation*}
\mathrm{F}=\mathrm{F}_{\text {chem }}+\mathrm{E}_{\text {elas }} \tag{2}
\end{equation*}
$$

where the first term $\mathrm{F}_{\text {chem }}$ corresponds to the chemical energy and $\mathrm{E}_{\text {elas }}$ is the total elastic strain energy.

In this paper, the mean-field approach is used to calculate the chemical free energy of system. For the binary alloys the chemical energy is given by:

$$
\begin{equation*}
\mathrm{F}_{\text {chem }}=\frac{1}{2} \sum_{\mathrm{r}, \mathrm{r}^{\prime}} \mathrm{P}(\mathbf{r}) \mathrm{P}\left(\mathbf{r}^{\prime}\right) \mathrm{V}\left(\mathbf{r}-\mathbf{r}^{\prime}\right)+\mathrm{k}_{\mathrm{B}} \mathrm{~T} \sum_{\overrightarrow{\mathrm{r}}}[\mathrm{P}(\mathbf{r}) \cdot \ln \mathrm{P}(\mathbf{r})+(1-\mathrm{P}(\mathbf{r})) \cdot \ln (1-\mathrm{P}(\mathbf{r}))] \tag{3}
\end{equation*}
$$

where $\mathrm{V}\left(\mathbf{r}-\mathbf{r}^{\prime}\right)$ is exchange energy, T is the temperature and $\mathrm{k}_{\mathrm{B}}$ the Boltzmann constant.
A thermodynamic model is developed using the static concentration wave (SCW) formalism applied to the $\mathrm{DO}_{22}$ structure. This structure is generated by the wave vectors: $\mathbf{k}_{1}=\frac{2 \pi}{a_{A 1}}(100)$ and $\mathbf{k}_{2}=\frac{2 \pi}{a_{A 1}}(1 / 210)$ where $\mathbf{k}=\left(k_{x} k_{y} k_{z}\right)$ and $k_{x}, k_{y}$ and $k_{z}$ are components of the vector $\mathbf{k}$ along the $\mathrm{x}, \mathrm{y}$ and z axes parallel to the [100], [010] and [001] directions in the reciprocal space of f.c.c. lattice and $\mathrm{a}_{\mathrm{A} 1}$ is the lattice parameter of the f.c.c. lattice. Using the SCW formalism, the probability distribution function $\mathrm{P}(\mathbf{r}, \mathrm{t})$ for the $\mathrm{DO}_{22}$ structure can be written as:

$$
\begin{equation*}
\mathrm{P}(\mathbf{r}(\mathrm{x}, \mathrm{y}, \mathrm{z}))=\mathrm{c}+\gamma_{1} \eta_{1} \mathrm{e}^{2 \mathrm{i} \pi \mathrm{x}}+\gamma_{2} \eta_{2} \cos \left(2 \pi\left(\frac{\mathrm{x}}{2}+\mathrm{y}\right)\right) \tag{4}
\end{equation*}
$$

where $c$ is the nominal vanadium concentration of the alloy, $\eta_{1}$ and $\eta_{2}$ are the order parameters, they vary from 0 (disordered state) to 1 (fully ordered state). In the case of $\mathrm{DO}_{22}$ structure the coefficients of symmetry are $\gamma_{1}=1 / 4$ and $\gamma_{2}=1 / 2$. For the $\mathrm{DO}_{22}$ structure, there are three orientation and fourth translation variants. Substituting (4) into (3) gives the expression for the chemical free energy for a $\mathrm{DO}_{22}$ structure:

$$
\begin{equation*}
\mathrm{F}_{\mathrm{DO}_{22}}=\frac{1}{2} \mathrm{Nc}^{2} \mathrm{~V}(\mathbf{k}=\mathbf{0})+\frac{1}{2} \mathrm{~N}\left(\frac{\eta_{1}}{4}\right)^{2} \mathrm{~V}\left(\mathbf{k}_{1}\right)+\mathrm{N}\left(\frac{\eta_{2}}{4}\right)^{2} \mathrm{~V}\left(\mathbf{k}_{2}\right)-\mathrm{TS}_{\mathrm{DO}_{22}} \tag{5}
\end{equation*}
$$

where entropy $S$ is:

$$
\mathrm{S}_{\mathrm{DO}_{22}}=-\mathrm{N}_{\mathrm{B}} \cdot\left\{\begin{array}{l}
\frac{1}{2} \cdot\left[\left(\mathrm{c}-\frac{1}{4} \cdot \eta_{1}\right) \cdot \ln \left(\mathrm{c}-\frac{1}{4} \cdot \eta_{1}\right)+\left(1-\mathrm{c}+\frac{1}{4} \cdot \eta_{1}\right) \cdot \ln \left(1-\mathrm{c}+\frac{1}{4} \cdot \eta_{1}\right)\right]+ \\
\frac{1}{4}\left[\left(\mathrm{c}+\frac{1}{4} \cdot \eta_{1}+\frac{1}{2} \cdot \eta_{2}\right) \cdot \ln \left(\mathrm{c}+\frac{1}{4} \cdot \eta_{1}+\frac{1}{2} \cdot \eta_{2}\right)+\right. \\
\left(1-\mathrm{c}-\frac{1}{4} \cdot \eta_{1} \cdot \frac{1}{2} \cdot \eta_{2}\right) \cdot \ln \left(1-\mathrm{c}-\frac{1}{4} \cdot \eta_{1}-\frac{1}{2} \cdot \eta_{2}\right)+ \\
\left(\mathrm{c}+\frac{1}{4} \cdot \eta_{1}-\frac{1}{2} \cdot \eta_{2}\right) \cdot \ln \left(\mathrm{c}+\frac{1}{4} \cdot \eta_{1}-\frac{1}{2} \cdot \eta_{2}\right)+ \\
\left.\left(1-\mathrm{c}-\frac{1}{4} \cdot \eta_{1}+\frac{1}{2} \cdot \eta_{2}\right) \cdot \ln \left(1-\mathrm{c}-\frac{1}{4} \cdot \eta_{1}+\frac{1}{2} \cdot \eta_{2}\right)\right]
\end{array}\right\}
$$

where N is the total number of sites. $\mathrm{V}(\mathbf{k})=\sum_{\overrightarrow{\mathrm{F}}} \mathrm{V}(\mathbf{r}) \cdot \exp (\mathrm{i} \mathbf{k} . \mathbf{r})$ is the Fourier transform of exchange energies. For a f.c.c. lattice and for the $\mathbf{k}$ vectors which generate the $\mathrm{DO}_{22}$ ordered structure, we can write:

$$
\left\{\begin{array}{l}
\mathrm{V}(\mathbf{k}=\mathbf{0})=12 \omega_{1}+6 \omega_{2}+24 \omega_{3}+12 \omega_{4} \ldots  \tag{6}\\
\mathrm{~V}\left(\mathbf{k}_{1}\right)=-4 \omega_{1}+6 \omega_{2}-8 \omega_{3}+12 \omega_{4} \\
\mathrm{~V}\left(\mathbf{k}_{2}\right)=-4 \omega_{1}+2 \omega_{2}+8 \omega_{3}-4 \omega_{4}
\end{array} \ldots\right.
$$

where $\omega_{1}, \omega_{2}, \omega_{3}$ and $\omega_{4}$ are first-, second-, third- and fourth-nearest neighbour effective exchange interaction energies.
The elastic energy term was calculated using the elastic theory of multiphase coherent solids with homogeneous modulus approximation proposed in [16]. If the strain effect is predominantly caused by the long range order parameter heterogeneity, then the stress-free strain can be expanded with respect to the long range order fields. As the $\eta->-\eta$ transition does not affect the macroscopic stress-free strain, the first non-vanishing term of the expansion is of second order in $\eta_{\mathrm{p}}(\mathrm{r})$ :

$$
\varepsilon_{i j}^{0}(\mathbf{r})=\sum_{p} \eta_{p}^{2}(\mathbf{r}) \varepsilon_{i j}^{00}(p)
$$

where the tensor $\varepsilon_{i j}^{00}(p)$ describes the stress-free transformation strain from parent cubic phase to the pth orientation variant of the tetragonal phase. In this case and under stress free boundary conditions, the elastic strain energy is:

$$
\begin{equation*}
\mathrm{E}_{\text {elas }}=\frac{1}{2} \sum_{\mathrm{p}, \mathrm{q}} \int^{\prime} \frac{\mathrm{d}^{3} k}{(2 \pi)^{3}} \mathrm{~B}_{\mathrm{pq}}(\mathbf{n})\left\{\eta_{p}^{2}\right\}_{k}\left\{\eta_{q}^{2}\right\}_{k}^{*} \tag{7}
\end{equation*}
$$

where $\left\{\eta_{p}^{2}(\mathbf{r})\right\}_{k}$ is the Fourier transform of $\eta_{p}^{2}(\mathbf{r})$ and $\left\{\eta_{q}^{2}(\mathbf{r})\right\}_{k}$ is the Fourier transform of $\eta_{p}^{2}(\mathbf{r}), B_{p q}(\mathbf{n})=c_{i j k l} \varepsilon_{i j}^{00}(p) \varepsilon_{q l}^{00}(q)-n_{i} \sigma_{i j}^{0}(p) \boldsymbol{\Omega}_{j k}(\mathbf{n}) \sigma_{k l}^{0}(q) n_{l}, \mathbf{n}=\frac{\mathbf{k}}{k}$ is a unit vector and the function, $\mathrm{c}_{\mathrm{ijkl}}$ is the elastic moduli tensor, $\sigma_{i j}^{0}(p)=c_{i j k l} \varepsilon_{k l}^{00}(p)$, and $\Omega_{i j}(\mathbf{n})$ is a Green function tensor which is inverse to the tensor $\boldsymbol{\Omega}_{i j}^{-1}(n)=c_{i j k l} n_{k} n_{l} .\left\{\eta_{p}^{2}(\mathbf{r})\right\}_{k}$ which is the Fourier transform of $\eta_{p}^{2}(\mathbf{r})$. The sign ' near integral has the meaning that $\boldsymbol{k}=\mathbf{0}$ is to be excluded from the integration. According to [8], for a cubic-tetragonal transformation with isotropic elasticity, the function $\mathrm{B}_{\mathrm{pq}}(\mathbf{n})$ can be written in 2 D as:

$$
\left\{\begin{array}{l}
\mathrm{B}_{\mathrm{xx}}(\overrightarrow{\mathrm{n}})=\frac{2 \mu \delta_{33}^{2}}{1-v}\left(\left[(1-\mathrm{d}) \mathrm{n}_{1}^{2}-(1+v \mathrm{~d})\right]^{2}+\left(1-v^{2}\right) \mathrm{d}^{2}\right)  \tag{8}\\
\mathrm{B}_{\mathrm{yy}}(\overrightarrow{\mathrm{n}})=\frac{2 \mu \delta_{33}^{2}}{1-v}\left(\left[(1-\mathrm{d}) \mathrm{n}_{2}^{2}-(1+v \mathrm{~d})\right]^{2}+\left(1-v^{2}\right) \mathrm{d}^{2}\right) \\
\mathrm{B}_{\mathrm{xy}}(\overrightarrow{\mathrm{n}}) \\
\quad=\frac{2 \mu \delta_{33}^{2}}{1-v}\left((1-\mathrm{d})^{2} \mathrm{n}_{1}^{2} \mathrm{n}_{2}^{2}+(1+v) \mathrm{d}(1+\mathrm{d})\right)
\end{array}\right.
$$

where $\mathrm{d}=\delta_{11} / \delta_{33}, v$ is the Poisson ratio and $\mu$ the shear modulus.
The local order parameters $\eta_{p}$ in equation (7) can be defined as follows. As we remark previously, each orientation variant of the $\mathrm{DO}_{22}$ unit cell is generated by two vectors. The x variant is generated by $\mathbf{k}_{1 x}=\frac{2 \pi}{a_{A 1}}(100)$ and $\mathbf{k}_{2 x}=\frac{2 \pi}{a_{A 1}}(1 / 210)$ vectors, and y -variant by $\mathbf{k}_{1 y}=\frac{2 \pi}{a_{A 1}}(010)$ and $\mathbf{k}_{2 y}=\frac{2 \pi}{a_{A 1}}\left(1^{1} / 20\right)$ vectors. In our simulations, we distinguish the $\mathrm{DO}_{22}$ phase using only two order parameters $\eta_{1 x}$ and $\eta_{1 y}$. These two local order parameters are calculated using a small simulation box $\mathrm{S}_{\mathrm{B}}(5 \times 5$ sites) around a given lattice site. In this case the order parameters can be defined as:

$$
\left\{\begin{array}{l}
\eta_{1 \mathrm{x}}(\mathbf{r})=\exp \left(-\mathrm{i} \mathbf{k}_{1 \mathrm{x}} \mathbf{r}\right) \cdot\left(\operatorname{coeff}_{1 \mathrm{x}}(\mathbf{r}) \otimes \mathrm{P}(\mathbf{r})\right)  \tag{9}\\
\eta_{1 \mathrm{y}}(\mathbf{r})=\exp \left(-\mathrm{i} \mathbf{k}_{1 \mathrm{y}} \mathbf{r}\right) \cdot\left(\operatorname{coeff}_{1 \mathrm{y}}(\mathbf{r}) \otimes \mathrm{P}(\mathbf{r})\right)
\end{array}\right.
$$

where symbol $\otimes$ represent the product of convolution and

$$
\operatorname{coeff}_{p}(\mathbf{r})=\left\{\begin{array}{ll}
\frac{1}{\mathrm{~N}_{\mathrm{B}} \gamma_{\mathrm{p}}} \cdot \exp \left(\mathrm{ik}_{\mathrm{p}} \mathbf{r}\right) & \text { if } \mathbf{r} \in \mathrm{S}_{\mathrm{B}} \\
0 & \text { otherwise }
\end{array} \quad \mathrm{p}=1 \mathrm{x} \text { or } 1 \mathrm{y}\right.
$$

Here $N_{B}$ is the number of sites in $S_{B}$ and $\gamma_{p}$ is the symmetry coefficients in eq. (4). $\gamma_{p}=1 / 4$ for $\eta_{1 \mathrm{x}}$ and $\eta_{1 \mathrm{y}}$. In the case of the order parameters associated to wave vectors $\mathbf{k}_{1 x}$ and $\mathbf{k}_{1 y}$ the coefficients coeff ${ }_{1 x}$ and coeff $f_{1 y}$ can be represented in the matrix form:

$$
\operatorname{coeff}_{1 \mathrm{x}}=\frac{1}{4}\left(\begin{array}{ccccc}
1 / 4 & -1 / 2 & 1 / 2 & -1 / 2 & 1 / 4 \\
1 / 2 & -1 & 1 & -1 & 1 / 2 \\
1 / 2 & -1 & 1 & -1 & 1 / 2 \\
1 / 2 & -1 & 1 & -1 & 1 / 2 \\
1 / 4 & -1 / 2 & 1 / 2 & -1 / 2 & 1 / 4
\end{array}\right) \quad \text { and } \quad \operatorname{coeff}_{1 \mathrm{y}}=\frac{1}{4}\left(\begin{array}{ccccc}
1 / 4 & 1 / 2 & 1 / 2 & 1 / 2 & 1 / 4 \\
-1 / 2 & -1 & -1 & -1 & -1 / 2 \\
1 / 2 & 1 & 1 & 1 & 1 / 2 \\
-1 / 2 & -1 & -1 & -1 & -1 / 2 \\
1 / 4 & 1 / 2 & 1 / 2 & 1 / 2 & 1 / 4
\end{array}\right)
$$

where each element is calculated at site $r=a(m, n)$ where $m$ and $n$ are integer values and vary from -2 to 2 and $a=a_{A 1} / 2$ is the lattice parameter of disordered f.c.c. phase in 2 D .

Due to the periodic boundary conditions applied to the simulations box, the equation (7) can be rewritten in the discrete form as follows:

$$
\begin{equation*}
E_{\text {elas }}=\frac{v_{0}}{2 N} \sum_{p, q} \sum_{k \in B_{1}}^{\prime} B_{p q}(\mathbf{n})\left\{\eta_{1 p}^{2}(\mathbf{r}, \mathrm{t})\right\}_{\mathbf{k}}\left\{\eta_{1 q}^{2}(\mathbf{r}, \mathrm{t})\right\}_{\mathbf{k}}^{*} \tag{10}
\end{equation*}
$$

where $v_{0}$ is the fcc unit cell volume, $N$ is the number of sites in the simulation box, and the summation is carried over the first Brillouin zone $B_{1}$. With definitions (9), equation (10) gives explicitly the elastic energy for a given probability distribution function $\mathrm{P}(\mathbf{r}, \mathrm{t})$. In this form the elastic energy can be directly incorporated in the Onsager equation.

## 3. Experimental results

The investigated alloy had a nominal composition of 19.5 at. $\% \mathrm{~V}$ (Ni balance). A $<001>$ oriented single crystal rod was cast by the withdrawal process at ONERA (Office National d'Etudes et de Recherches Aérospatiales). This alloy was then annealed at $1300^{\circ} \mathrm{C}$ for 3 h and quenched in iced water. Finally, aging treatments were performed at $800^{\circ} \mathrm{C}$ for 15 min, $1 \mathrm{~h}, 3 \mathrm{~h}$ and 10 h . Samples were cut with a drilling machine from the single crystal and mechanically thinned down to $100 \mu \mathrm{~m}$. The electron transparency of the specimens was achieved with a twin-jet electropolisher, using $2 \%$ perchloric acid in 2-butoxyethanol solution at $20^{\circ} \mathrm{C}$ with a voltage of 30 V . Observations were made with a JEOL 2000FX microscope operating at 200 kV .

## Figure 1

Figure 2

Selected area electron diffraction (SAED) patterns were recorded along the [001] fcc axis in samples aged $15 \mathrm{~min}, 1 \mathrm{~h}, 3 \mathrm{~h}$ and 10 h at $800^{\circ} \mathrm{C}$. In the early stage of the precipitation and coarsening, the three variants of the $\mathrm{DO}_{22}$ phase are clearly seen (Figure 1a). At intermediate time, only two variants remain in the alloy (Figure 1b and c). Finally, for longer ageing treatment, only one variant of the ordered $\mathrm{DO}_{22}$ phase is present (Figure 1d).

In the early stage of ageing $(15 \mathrm{~min})$ at $800^{\circ} \mathrm{C}, \mathrm{DO}_{22}$ ordered precipitates are smaller than 10 nm , which makes them very difficult to image. Figure 2(a) shows a dark field image of the microstructure after 3 hour of ageing at $800^{\circ} \mathrm{C}$. This image was obtained from superlattice reflections of the two variants of the $\mathrm{DO}_{22}$ phase ([100] and [010]) and thus highlights all $\mathrm{DO}_{22}$ precipitates. These precipitates are parallelogram-shaped and this image shows a typical "chessboard" microstructure with a wide range of precipitate sizes. It is interesting to note that the microstructure is periodically aligned along $\langle 201\rangle_{\mathrm{DO} 22} /\langle 110\rangle_{\mathrm{A} 1}$. Moreover, if only one superlattice reflection is selected to image only one variant, laths of precipitates are seen (figure 2 b ), indicating that $\mathrm{DO}_{22}$ variants are not randomly distributed. Precipitates of one of the variants are clearly aligned along $<110\rangle$ directions and depletion bands aligned along the same direction are observed.

After 10 h of aging at $800^{\circ} \mathrm{C}$ (Figure 2c), the "chessboard" microstructure has vanished. Only one variant of the $\mathrm{DO}_{22}$ phase remains, and precipitates are quadrangular-prism shaped. Moreover, at this step of the coarsening process, the orientation of precipitates is slightly tilted so that the angle between the invariant plane and $\langle 110\rangle_{\mathrm{A} 1}$ is $8^{\circ}\left( \pm 2^{\circ}\right)$.

## 4. Simulations.

### 4.1 Selective growth of $\mathbf{D} 0_{22}$ variants in $\mathrm{Ni}-\mathrm{V}$ alloys.

The kinetic equation (1) was numerically solved in reciprocal space using the explicit forward Euler technique. Two-dimensional simulation was performed with a 768x768 sites box, corresponding to $135 \mathrm{~nm} \times 135 \mathrm{~nm}$. Periodic boundary conditions were applied. Nominal Vanadium composition was fixed at $20 \%$ at. The system was started from a homogeneous disordered state with 1500 randomly distributed small $\mathrm{DO}_{22}$ precipitates ( x and y-variants each with 4 translation variants). According to the experimental results presented in [17], we have used the following set of elastic parameters: $\mathrm{T}=1000^{\circ} \mathrm{C}, \mu=367 \mathrm{meV} / \AA^{3}, v=1 / 3, \delta_{11}=-$ 0.0053 and $\delta_{33}=0.0106$. The exchange energies were fitted to the experimental phase diagram [18]. The free energies of the $\mathrm{DO}_{22}$ ordered phase as a function of composition at a given temperature are obtained by minimisation of chemical free energy (5) with respect to the order parameters. Finally, the equilibrium compositions of ordered and disordered phases were determined numerically by the common tangent construction. A reasonably good fit was obtained using the following chemical interaction parameters with a third-neighbour interaction model: $\omega_{1}=93.33 \mathrm{meV} ; \omega_{2}=-27.92 \mathrm{meV} ; \omega_{3}=-25.10 \mathrm{meV}$. The free energy curves for the ordered and disordered phases as a function of composition at $T=800^{\circ} \mathrm{C}$ are shown in Fig. 3. The simulated $\mathrm{Ni}-20 \% \mathrm{~V}$ alloy are situated in the region of phase diagram where the free energy of the tetragonal phase is lower than that of the cubic disordered phase at the same composition. As was noted in [6], in this region the transformation can start as a congruent (diffusionless) crystal lattice rearrangement. However, from our previous 3D atom probe analyses [19] the composition heterogeneities were observed at the very early stages of precipitation. Therefore, we will consider only the diffusion path for the cubic to tetragonal transformation in the Ni-V systems.

Figure 3

We assumed that diffusion jumps take place only between nearest-neighbour lattice sites, so that $\mathrm{L}\left(\mathbf{r}-\mathbf{r}^{\prime}\right)$ is equal to $\mathrm{L}_{1}$ if sites $\mathbf{r}$ and $\mathbf{r}^{\prime}$ are first-neighbour and 0 for further neighbour order. To initiate the simulation, precipitates were artificially introduced in the simulated box. This implies that information on the early stages of nucleation, like the incubation time, is not available from such simulations. The simulation time is measured in reduced time $\left(t^{*}=\mathrm{L}_{1} \mathrm{t}\right)$ where $\mathrm{L}_{1}$ can be related to the diffusion coefficient. Our emphasis is on the influence of elastic interactions on the coarsening kinetics, and in particular on the relative stability of orientation variants of $\mathrm{Ni}_{3} \mathrm{~V}$ precipitates.

The microstructural evolution in a $\mathrm{Ni}-20 \%_{\text {at }} V$ alloy is presented in Fig. 4. The x - and y variants are represented in white and grey respectively. Disordered fcc matrix is in black. The initial stage microstructure ( $4 \mathrm{a}: \mathrm{t}^{*}=4$ ) consists of rectangular ordered domains with characteristic alignment along the $\langle 110\rangle$ directions. Both orientational variants are present but bands of monovariants aligned along the $<110>$ direction start to be formed. At $\mathrm{t}^{*}=30$ (Fig. 4b), the microstructure is composed of alternating monovariant bands aligned along the $<110>$ direction. At this stage we also observe the independent coarsening of ordered domains in each monovariant bands. We would like to remark that some of these domains are separated by antiphase boundaries. At $t^{*}=120$ (Fig. 4c), local coarsening is finished, and $\mathrm{DO}_{22}$ precipitates form chessboard-like structure. The next stage of kinetics involves a nonlocal coarsening phenomenon. At this stage, the entire rows of $y$-variant precipitates disappear, and the structure becomes mono-variant (Fig. 4d).

Figure 4
The evolution of volume fraction of each variant is represented in fig.5. This figure demonstrates that $y$-variant starts to disappear from $t^{*}>200$, when the local coarsening in each variant road is finished.

Figure 5

These simulation results reproduce quite well the experimentally observed microstructure evolution in Ni-V alloy aging from 1 h to 10 h at $800^{\circ} \mathrm{C}$ (see Figure 2).

### 4.2 Influence of misfit ratio on chessboard-like microstructure.

Figure 6 gives a comparison of simulated and experimental chessboard-like structures. Microstructures including the size of precipitates ( $\sim 50 \mathrm{~nm}$ ) from the experiments and simulations are similar. However, there is a disagreement in elastically soft directions. In contrast to simulation (fig. 6a), TEM image (fig. 6b) reveals a slight misorientation of facets of precipitates. The angle of facets with respect to $\langle 110\rangle_{\mathrm{A} 1}$ is $8^{\circ}\left( \pm 2^{\circ}\right)$. As has been shown by Le Bouar et al. in [8], there is a relation between this angle and the misfit ration d:

Figure 6

$$
\begin{equation*}
\sin ^{2} \alpha_{x}=(1+v) \frac{-d}{1-d} \tag{11}
\end{equation*}
$$

where $\alpha_{x}$ is the soft direction angles of $x$-variant, with respect to the [100] axis and $v$ the Poisson coefficient. The Poisson coefficient was taken equal to $1 / 3$.

Figure 7
Figure 7 shows the dependence of x -variant soft direction with respect to the misfit ratio d (equation (11)). On this figure we indicated (grey square) the misfit measured by A. Suzuki et al [13]. This misfit corresponds to $\alpha_{x} \approx 42^{\circ}$. In our TEM images we find $\alpha_{x} \approx 37^{\circ}$ (black square on Figure 7). The observed discrepancy between two measurements can be explained by two reasons. Firstly, there is normal experimental incertitude in measurements of misfit and secondly, as was observed in [13] the angles $\alpha_{x}$ can vary during coarsening.

In order to better reproduce elastically soft direction in our simulations, a small correction of misfit was applied: we chose $\mathrm{d}=\delta_{11} / \delta_{33}=-0.35$ and $\delta_{11}-\delta_{33}$ was arbitrarily set to a constant. New misfits used in simulation were $\delta_{11}=-0.0041$ and $\delta_{33}=0.0118$ (instead of $\delta_{11}=-$ 0.0053 and $\delta_{33}=0.0106$ previously). Figure 8 shows the new microstructure simulated with $\mathrm{d}=-0.35$. Orientations of facets of precipitates are now quite well reproduced and correspond to the chessboard-like microstructure observed experimentally (see Figure 6b).

Figure 8
Microstructure evolution at early stages was found to be quite similar for both values of d. After forming monovariant bands, the precipitates coarsen independently in each band and create the chessboard-like structure. Once formed, the two-variant chessboard-like structure again is unstable. However, the disappearance of the last variant is much slower than the previous simulation. Our simulation has never reached the entire disappearance because of the longer time needed. By comparing the disappearance rates of the two simulations at the same variant volume fraction $\Phi_{\mathrm{x}}=0.39$ (see figure 4 d ), we observed that chessboard-like structure in the second simulation evolves slower than first simulation.

By comparing both simulations ( $\mathrm{d}=-0.5$ with $\delta_{11}=-0.0053$ and $\delta_{33}=0.0106$ and $\mathrm{d}=-$ 0.35 with $\delta_{11}=-0.0041$ and $\delta_{33}=0.0118$ ), we conclude that the final microstructure strongly depends on the misfit ration d . The decrease in misfit ratio significantly reduces the rate of disappearance of one of the $\mathrm{DO}_{22}$ variants. This strong dependence of the stability of the twovariant chessboard-like structure on misfit has been showed experimentally and by phase field modelling [20]. Suzuki et al. [13,14] have studied the alloying effect on the stability of multivariant structures in $\mathrm{Ni}_{3} \mathrm{~V}$ at elevated temperatures. They examined microstructures in several alloys (Ni-V or Ni-V-M where M is an alloying element such as Iron, Niobium and Cobalt). It was shown that the stability of the chessboard-like structure increases with the decrease in the magnitude of misfit $\delta_{11}$. The same tendency was found in our simulations. The chessboardlike structure is more stable for $\delta_{11}$ equal to 0.0041 than 0.0053 .

To understand better the influence of the misfit on final microstructure we simulated the kinetic evolution in $\mathrm{Ni}-\mathrm{V}$ system with the value of misfit obtained for $\mathrm{Ni}-\mathrm{V}-\mathrm{Nb}$ and $\mathrm{Ni}-\mathrm{V}-\mathrm{Co}$ systems [13]. In these ternary alloys the Nb and Co atoms occupy the same site in the crystal lattice as V and participate in the formation of $\mathrm{DO}_{22}$ structure. It was shown in [13] that Nb atoms drastically decrease the misfit between $\mathrm{DO}_{22}$ phase and the f.c.c. matrix. The misfit ratio d in this case is approximately 0 .

Figure 9

Figure 9 shows the microstructure at $\mathrm{t}^{*}=150$. This simulation reveals that for $\mathrm{d}=0$ the volume fraction of each orientation variant remains constant and $x$ - and $y$-variant coexist during all simulation time. The topology of microstructure is chessboard-like, but the ordered precipitates aligned along the $<100>$ and $<010>$ directions. This microstructure reproduces quite well the dark field images for $\mathrm{Ni}-15 \% \mathrm{~V}-5 \% \mathrm{Nb}$ obtained in [13].

### 4.3 Elastic terms

The elastic energy of the two-variant system can be written as a sum of three terms:

$$
\begin{equation*}
E_{\text {elas }}=E_{X X}+E_{Y Y}+E_{X Y} \tag{12}
\end{equation*}
$$

where

$$
\left\{\begin{array}{l}
\mathrm{E}_{\mathrm{XX}}=\frac{1}{2} \int^{\prime} \frac{\mathrm{d}^{3} \overrightarrow{\mathrm{k}}}{(2 \pi)^{3}} \mathrm{~B}_{\mathrm{xx}}(\overrightarrow{\mathrm{n}})\left|\eta_{\mathrm{x}}(\overrightarrow{\mathrm{k}})\right|^{2} \\
\mathrm{E}_{\mathrm{YY}}=\frac{1}{2} \int^{\prime} \frac{\mathrm{d}^{3} \overrightarrow{\mathrm{k}}}{(2 \pi)^{3}} \mathrm{~B}_{\mathrm{yy}}(\overrightarrow{\mathrm{n}})\left|\eta_{\mathrm{y}}(\overrightarrow{\mathrm{k}})\right|^{2} \\
\mathrm{E}_{\mathrm{XY}}=\frac{1}{2} \int^{\prime} \frac{\mathrm{d}^{3} \overrightarrow{\mathrm{k}}}{(2 \pi)^{3}} \mathrm{~B}_{\mathrm{xy}}(\overrightarrow{\mathrm{n}})\left[\eta_{\mathrm{x}}(\overrightarrow{\mathrm{k}}) \eta_{\mathrm{y}}^{*}(\overrightarrow{\mathrm{k}})+\eta_{\mathrm{y}}(\overrightarrow{\mathrm{k}}) \eta_{\mathrm{x}}^{*}(\overrightarrow{\mathrm{k}})\right]
\end{array}\right.
$$

The first and second terms $E_{X X}\left(E_{Y Y}\right)$ in elastic energy take into account the elastic interactions between the same variant domains. The third term $E_{X Y}$ reproduces elastic interaction energy between the two different variants. The misfits used in simulation are $\delta_{11}=-$ 0.0041 and $\delta_{33}=0.0118$.

To understand the influence of each elastic term on the microstructural evolution and on the shape of precipitates, two simulations have been realised. The first simulation takes into account the $\mathrm{E}_{\mathrm{XX}}+\mathrm{E}_{\mathrm{YY}}$ terms only, $\mathrm{E}_{\mathrm{XY}}$ being put zero.

Figure 10
Figure 11
As shown in Figure 10, if only elastic interactions between the same variants are taken into account, the chessboard-like structure is not observed, and one of the variants is observed vanish rapidly(less than 10 unities of reduced time). This mono-variant structure is similar to the maze structure observed in TEM images (Figure 2c). Figure 11 shows the
evolution of $E_{X X}, E_{Y Y}$ and $E_{X X}+E_{Y Y}$ during the simulation. The minimization of elastic energy due to the variant disappearance is clearly visible. The total elastic energy ( $\mathrm{E}_{\mathrm{XX}}+\mathrm{E}_{\mathrm{YY}}$ ) decreases by half after one of the variant disappeared. It clearly shows that $E_{X X}$ and $E_{Y Y}$ elastic terms are responsible for the formation of mono-variant microstructure.

In the second simulation, only $E_{X Y}$ terms were taken into account ( $\mathrm{E}_{\mathrm{XX}}=\mathrm{E}_{\mathrm{YY}}=0$ ). Figure 12 shows the microstructural evolution of two-variant structures at different reduced times. Figure $13(\mathrm{a}-\mathrm{b})$ represents the evolution of the volume fractions of each variant and variation with reduced time of $E_{X Y}$, respectively. In this case, the chessboard-like structure is clearly generated. Each precipitate of one of variants tends to be surrounded by 4 precipitates of the other variant along $<100>$ directions. The shape of precipitates is nearly spherical. This indicates that $E_{X Y}$ elastic term is responsible only for alignments of precipitates. As we can see in Figure 13(a), the volume fractions of each variant are equal and stay constant and there is no competitive growth between two variants. As we can see on Figure 13(b), the $\mathrm{E}_{\mathrm{XY}}$ elastic contribution is negative due to the opposite sign between $\delta_{11}$ and $\delta_{33}$ values. So if one variant will disappear then the $E_{X Y}$ term tends to zero. We can conclude that the $E_{X Y}$ term makes the two-variant structures stable.

## Figure 12

Figure 13
These two simulations in this section demonstrate that the $\mathrm{E}_{\mathrm{XY}}$ term is mainly responsible for the creation and the stability of the two variant chessboard-like structure while the $E_{X X}+E_{Y Y}$ elastic terms induce the instability of the two variant structure and push the system to form mono-variant microstructure. The real microstructure is formed by minimisation of total elastic energy which is a result of a competition between $\mathrm{E}_{\mathrm{xx}}\left(\mathrm{E}_{\mathrm{yy}}\right)$ and $\mathrm{E}_{\mathrm{xy}}$ terms as well as chemical free energy.

In order to better understand the influence of $E_{X X}$ and $E_{Y Y}$ terms on the shape of precipitates we calculated the value of $\mathrm{B}_{\mathrm{XX}}$ as function of angle $\theta$ which defines the angle between the unit vector $\vec{n}$ normal to matrix/precipitate interface and $<100\rangle$ direction of cubic lattice. Figure 14a represents the typical shape of $x$-variant precipitate of the chessboard-like structure. On this figure we indicated the vectors normal to the habit planes. Figure 14b shows that the function $\mathrm{B}_{\mathrm{XX}}$ reaches minima at $\theta=35^{\circ}$ and in the perpendicular direction. This is a reason why a precipitate will try to have a maximum interface in this direction. The directions $<100\rangle$ and $<010\rangle$ correspond to the maximum value of $\mathrm{B}_{\mathrm{xx}}$. Therefore the interfaces along these directions are energetically unfavourable. In this case the minimisation of elastic energy leads to decreasing of the surface of these interfaces and forms the corners. This configuration will increase the total interface of ordered particle and consequently the chemical interface energy. The minimisation of chemical interface energy will try to smooth the corners of precipitate. The competition between these two phenomena will give the equilibrium shape of precipitate.

## Figure 14

The function $\mathrm{B}_{\mathrm{Xx}}$ in fig. 14 b was evaluated with input parameters of $\mathrm{Ni}-\mathrm{V}$ system and with the misfit ration $\mathrm{d}=-0.35$. It should be noted that the height of two maxima is different for two perpendicular directions. To check the influence of the values of these maxima on the shape of corners of precipitates we performed the simulation where from $t^{*}=400$ the shape of $\mathrm{B}_{\mathrm{XX}}$ curve was artificially modified as shown in fig. 15.

Figure 15

Fig.16a presents the microstructure at $\mathrm{t}^{*}=400$. We can remark that two adjacent corners of ordered particles have different curvature. Figure 16 b shows the microstructure at $\mathrm{t}^{*}=500$, after 100 step of simulation with modified function $\mathrm{B}_{\mathrm{xx}}$. We can note that the curvature of all fourth corners of precipitates is the same.

Figure 16

These results show that we can predict the behaviour of the function $\mathrm{B}(\vec{n})$ from TEM observations. TEM images can give not only the information about elastically soft directions but also the values of maxima of the $\mathrm{B}(\vec{n})$.

## 5. Summary and conclusions

TEM investigations of the early stages of aging indicate that the spacing between precipitates is relatively large so that the multivariant structure is observed. Our simulations reveal that at this stage a two variant structure is formed and these two $\mathrm{DO}_{22}$ variants align along the $<110\rangle$ directions. During the later stages of coarsening, experimental data and simulations show that major variant grows at the expense of the minor variant and a monovariant structure is eventually observed. The habit plane makes an angle of $37^{\circ}$ with respect to the $[100]_{\text {DO22 }}$ (a-axis). It has been shown that the misfit ratio $d=\delta_{11} / \delta_{33}$ has a significant effect on the stability of two-variant chessboard-like microstructures. Simulations results reproduce quite well TEM observed microstructures.

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Figure captions

Figure 1 : SAED patterns performed along [001] f.c.c. axis in the $\mathrm{Ni}-19.5 \mathrm{at} . \% \mathrm{~V}$ alloy aged 15 $\min (\mathrm{a}), 1 \mathrm{~h}(\mathrm{~b}), 3 \mathrm{~h}(\mathrm{c})$ and $10 \mathrm{~h}(\mathrm{~d})$ at $800^{\circ} \mathrm{C}$. Superlattice reflections corresponding to the 3 variants x , y and z of the $\mathrm{DO}_{22}$ phase are labelled I, II and III respectively.

Figure 2 : (a) Dark field image showing the two $\mathrm{DO}_{22}$ variants in the alloy aged 3 h at $800^{\circ} \mathrm{C}$.
(b) dark field image showing only one $\mathrm{DO}_{22}$ variant ( x ) in the same alloy, (c) Dark field image showing the only variant remaining after 10 h aging at $800^{\circ} \mathrm{C}$.

Figure 3 : Free energy of disordered f.c.c. phase (in grey) and ordered $\mathrm{DO}_{22}$ phase (in black) versus composition for Ni-V system for $\mathrm{T}=800^{\circ} \mathrm{C}$.

Figure 4 : Morphological evolution of $\mathrm{DO}_{22}$ precipitates from 2 D simulations in a $\mathrm{Ni}-20 \%{ }_{\mathrm{at}} \mathrm{V}$ at different reduced time : (a) $t^{*}=4$; (b) $t^{*}=30$; (c) $t^{*}=120$ and (d) $t^{*}=750$. $x$ and y-variant are represented in white and grey, respectively. Disordered fcc matrix is in black. Computational domain size is 135 nmx 135 nm . Periodic boundary conditions have been applied to construct these images.
Figure 5: Evolution of volume fraction of $x$-(in black) and $y$-(in grey) orientation variants of $\mathrm{DO}_{22}$ structure.

Figure 6: Comparison of precipitate morphologies obtained by simulations (a) and experiments (b): a)simulation at $t^{*}=150$ (b) dark field image showing the two $\mathrm{DO}_{22}$ variants in $\mathrm{Ni}-19.5 \%_{\mathrm{at}} \mathrm{V}$ aged 3 h at $800^{\circ} \mathrm{C}$. x - and y -variant are represented in white and grey respectively. Disordered fcc matrix is in black.

Figure 7 : Evolution of elastically soft direction $\alpha_{x}$ for $x$-variant with respect to the misfit ratio $d$ given by equation (11). Poisson coefficient is fixed at $1 / 3$. The grey square corresponds to the measured misfits by [13]. The black square corresponds to the measured soft direction $\left(\alpha_{x}=37^{\circ}\right)$ from our TEM images.

Figure 8: Simulated microstructure at $t^{*}=750$. In this simulation the misfit parameters are $\delta_{11}=-0.0041$ and $\delta_{33}=0.0118$ and $\mathrm{d}=\delta_{11} / \delta_{33}=-0.35$. x - and y -variant are represented in white and grey respectively. Disordered fcc matrix is in black. The image represents 1.5 the length of simulation box. To obtain this image the periodic boundary conditions have been applied.

Figure 9: Simulated microstructure at $t^{*}=150$. In this simulation the misfit ratio $d=0 . x-$ and $y-$ variant are represented in white and grey respectively. Disordered fcc matrix is in black. The image represents 1.5 the length of simulation box. To obtain this image the periodic boundary conditions have been applied.

Figure 10: Microstructure evolution with $\mathrm{E}_{\mathrm{XY}}=0 .\left(\delta_{11}=-0.0041\right.$ and $\left.\delta_{33}=0.0118\right)$ at different reduced time $(a) t^{*}=2.5(b) t^{*}=5$, (c) $t^{*}=10$ and (d) $t^{*}=20$.
Figure 11: Evolution of the elastic energy $E_{X X}, E_{Y Y}$ and $E_{X Y}$ as function of reduced time in $\mathrm{Ni}-20 \% \mathrm{~V}$ aged at $800^{\circ} \mathrm{C}\left(\delta_{11}=-0.0041\right.$ and $\left.\delta_{33}=0.0118\right)$.

Figure 12: Microstructure with $\mathrm{E}_{\mathrm{XX}}=\mathrm{E}_{\mathrm{YY}}=0 .\left(\delta_{11}=-0.0041\right.$ and $\left.\delta_{33}=0.0118\right)(\mathrm{a})$, (b) and (c): correspond to $t^{*}=2.5,10$ and 20 respectively.

Figure 13: Simulation with $\mathrm{E}_{\mathrm{XX}}=\mathrm{E}_{\mathrm{YY}}=0$. ( $\delta_{11}=-0.0041$ and $\delta_{33}=0.0118$ ) (a): variation of volume fractions with time (b): time evolution of the $E_{X Y}$ elastic term.

Figure 14: a) Shape of $x$-variant precipitate. The arrows 1,2 and 3,4 indicate the directions where $B_{x x}$ has minima and maxima, respectively, b) function $B_{x x}$ versus angle $\theta$.

Figure 15: Function $\mathrm{B}_{\mathrm{xx}}$ versus angle $\theta$. Initial (in black) and modified (in grey).
Figure 16. : Microstructure with (a) $\mathrm{B}_{\mathrm{Xx}}$ of Ni-V system, (b) $\mathrm{B}_{\mathrm{xx}}$ modified. White lines indicate the orientations of the facets of precipitates.

$313 \times 105 \mathrm{~mm}(96 \times 96$ DPI)

(a) variants $x$ and $y$

(b) variant $x$

(c) variant x

$215 \times 279 \mathrm{~mm}(600 \times 600 \mathrm{DPI})$

$237 \times 220 \mathrm{~mm}(96 \times 96$ DPI)

$215 \times 279 \mathrm{~mm}(600 \times 600 \mathrm{DPI})$
(a)

(b)

$231 \times 108 \mathrm{~mm}(96 \times 96$ DPI)


$118 \times 118 \mathrm{~mm}(96 \times 96$ DPI)

$58 \times 58 \mathrm{~mm}(150 \times 150 \mathrm{DPI})$

$253 \times 75 \mathrm{~mm}(96 \times 96 \mathrm{DPI})$

$215 \times 279 \mathrm{~mm}(600 \times 600$ DPI)

(a)

(b)

(c)
$245 \times 87 \mathrm{~mm}(96 \times 96$ DPI)


$$
108 \times 45 \mathrm{~mm}(600 \times 600 \mathrm{DPI})
$$


(a)

(b)


$$
197 \times 139 \mathrm{~mm}(600 \times 600 \text { DPI })
$$


$100 \times 54 \mathrm{~mm}(150 \times 150 \mathrm{DPI})$

