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Channel Shortening for Bit Rate Maximization in DMT Communication Systems

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***Abstract*—This paper considers the problem of impulse response shortening for discrete multitone (DMT) transceivers. The proposed channel shortening filter shortens the effective channel impulse response (CIR) while concentrating energy in a predetermined window. This approach is based on convolution operation and filter coefficients decomposition which over constraint the effective CIR and allows achieving better performance. Simulation results show that the proposed algorithm outperforms the classical channel shortening approachs in terms of achievable bit rate and computational complexity.**

I. Introduction

Discrete multitone is one of the most popular multicarrier modulation used for high speed data transmission over twisted pair lines. Many standards of communication use this modulation including assymetric digital subscriber line (ADSL) and very high data rate digital subscriber line (VDSL) as described in [1]. In DMT, the channel is divided into orthogonal subchannels. However, channel dispertion can destroy the subchannel orthogonality which causes inter-symbol interferences (ISI). To mitigate ISI, a guard interval, known as a cyclic prefix (CP), is inserted between successive symbols. In general, the CP should be larger than the delay spread of the channel, resulting in a loss of bandwidth and power efficiency. To avoid using large CP, a channel shortening filter (CSF) can be used to shorten the effective CIR to a desired length.

In DMT systems, researchers have developed different approachs to determine the CSF coefficients. The most popular design is the maximum shortening signal to noise ratio (MSSNR) [2]. In this approach the ratio of energy inside the cyclic prefix to the energy outside this window is maximized. The MSSNR technique reduces the complexity of the minimum mean squared error (MMSE) proposed in [3] but still requires expensive computations. In [4], an efficient design for minimizing the delay spread (MDS) of the channel was developed. As well as the Min-IBI approach proposed in [5], this approach uses polynomial weighting functions which provides higher complexity.

The technique proposed in this paper attempts to divide the CSF coefficients into two parts. The first part is dedicated to concentrate energy in the desired length. Simultanously, the coeffcients of the second part cancels the effective channel coefficients outside the allowed delay spread of the channel. We show that our method improves the achievable bit rate for the DMT systems at a low-complexity cost.

Section II gives a brief overview of the system model used in this paper. Section III proposes a new method for channel shortening to optimize bit rate performance. Section IV provides simulation results and section V concludes.

II. System Model

The structure of a DMT system, including a CSF at the receiver side, is shown in Fig. 1. The DMT transmitter divides the frequency spectrum into independent subchannels by using the inverse fast Fourier transform (IFFT). The cyclic prefix, consisting of the last ν samples of each data block, is added to the beginning of the block. At the receiver, after removing the CP, the data stream is recovered by using the fast Fourier transform (FFT). The output of the effective channel is given by,

$$y(k) = w(k) * (x(k) * h(k) + n(k)) \tag{1}$$

where $\boldsymbol{w} = [w(0), w(1), ..., w(L_w - 1)]$ represents the CSF of length L_w, $x(k)$ is the transmitted symbol, $\boldsymbol{h} = [h(0), h(1), ..., h(L_h - 1)]$ is the channel impulse response of length L_h and $n(k)$ is the zero-mean additive white gaussian noise added to channel.

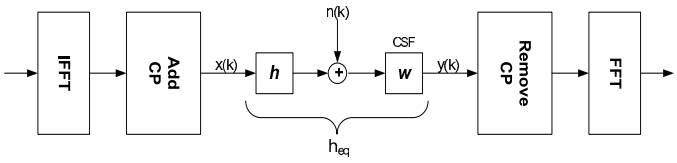

Fig. 1. Discrete Multitone transceiver structure.

III. A Reduced Complexity CSF Algorithm

The proposed CSF is designed to shorten the delay spread of the DMT channels and to simultanously concentrate energy in a predetermined temporal window. To attempt these purposes, we divide the CSF into two parts and decompose intermediate convolution matrix into four parts.

A. The CSF Decomposition

Classically, studies on CSF aim at maximizing a performance criterion (SNR, SINR...) that leads to shorten the channel. The objective of our approach is twofold: energy maximization and channel shortening using a single CSF.

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For the sake of the proposed algorithm, the CSF vector of length L_w is divided into two parts as follows,

$$\boldsymbol{w} = [\boldsymbol{w}_{max}\ ,\ \boldsymbol{w}_{min}] \tag{2}$$

where

$$\boldsymbol{w}_{max} = [w_{max}(0), \cdots, w_{max}(L-1)] \tag{3}$$

$$\boldsymbol{w}_{min} = [w_{min}(0), \cdots, w_{min}(L_w - L - 1)] \tag{4}$$

and L is the length of $\boldsymbol{w}_{max}$.

Similary, we divide the original channel $\boldsymbol{h}$ into two parts,

$$\boldsymbol{h}_{max} = [h(0), \cdots, h(L-1)] \tag{5}$$

$$\boldsymbol{h}_{min} = [h(L), \cdots, h(L_h - 1)] \tag{6}$$

Defining appropriate notations, the convolution of the CSF $\boldsymbol{w}$ and the original channel $\boldsymbol{h}$ gives the effective channel $\boldsymbol{h}_{eq}$ of length $L_{eq} = L_h + L_w - 1$ desribed with the following,

$$\boldsymbol{h}_{eq} = [\boldsymbol{h}_{eq_max}\ ,\ \boldsymbol{h}_{eq_min}] \tag{7}$$

It is desired that the major energy of $\boldsymbol{h}_{eq}$ is concentrated in the $\boldsymbol{h}_{eq_max}$ window. The length of $\boldsymbol{h}_{eq_max}$ is equal to the target delay spread length L that can be adjusted to optimize performance criterion related to the system requirements. Ideally, the energy of $\boldsymbol{h}_{eq_max}$ should be greater than the $\boldsymbol{h}_{max}$ one. From (2) and (7) we can write $\boldsymbol{h}_{eq_max}$ as follows,

$$h_{eq_max}(k) = \sum_{j=0}^{k} w_{max}(j) h(k-j) \tag{8}$$

where $0 \leq k \leq L-1$, which shows that $\boldsymbol{h}_{eq_max}$ only depends on $\boldsymbol{w}_{max}$ coefficients of the CSF.

On the other hand, to restrain $\boldsymbol{h}_{eq}$ to be confined within L coefficients, the energy of $\boldsymbol{h}_{eq_min}$ should be less than the $\boldsymbol{h}_{min}$ one.

$\boldsymbol{h}_{eq_min}$ can then be expressed as,

$$h_{eq_min}(k) = \sum_{j=0}^{L-1} w_{max}(j) h(k-j+L) + \sum_{j=0}^{k} w_{min}(j) h(k-j) \tag{9}$$

where $0 \leq k \leq L_{eq} - L - 1$

To determine the $\boldsymbol{w}_{max}$ and $\boldsymbol{w}_{min}$ coefficients while meeting these two criteria, we introduce the $\boldsymbol{M}_{int}$ matrix given by Fig. 2. This matrix, used as an intermediate matrix convolution is obtained by decomposing the convolution operation between $\boldsymbol{w}$ and $\boldsymbol{h}$ into four parts,

- The upper triangular matrix $\boldsymbol{A}$ of size $L \times L$ which elements only depend on $\boldsymbol{h}_{max}$ and $\boldsymbol{w}_{max}$.

$$\boldsymbol{A} = \begin{bmatrix} w_{max}(0)h(0) & w_{max}(0)h(1) & \cdots & w_{max}(0)h(L-1) \\ 0 & w_{max}(1)h(0) & \cdots & \cdots \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & w_{max}(L-1)h(0) \end{bmatrix}$$

- The matrix $\boldsymbol{B}$ of size $L \times L_{eq} - L$ which elements only depend on $\boldsymbol{w}_{max}$ and $\boldsymbol{h}$.

$$\boldsymbol{B} = \begin{bmatrix} w_{max}(0)h(L) & \cdots & 0 & 0 \\ w_{max}(1)h(L-1) & \cdots & w_{max}(1)h(L_h-1) & \vdots \\ \vdots & \vdots & \ddots & 0 \\ w_{max}(L-1)h(1) & \cdots & w_{max}(L-1)h(L_h-2) & w_{max}(L-1)h(L_h-1) \end{bmatrix}$$

- The matrix $\boldsymbol{S}$ of size $L_w - L \times L_{eq} - L$ which elements only depend on $\boldsymbol{w}_{min}$ and $\boldsymbol{h}$.

$$\boldsymbol{S} = \begin{bmatrix} w_{min}(0)h(0) & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & w_{min}(1)h(L_h-1) & \cdots & \vdots \\ \vdots & \ddots & \vdots & \vdots & 0 \\ 0 & \cdots & w_{min}(L_w-L-1)h(0) & \cdots & w_{min}(L_w-L-1)h(L_h-1) \end{bmatrix}$$

- A null matrix of size $L_w - L \times L$.

The position of the threshold represented by the dashed line separating matrix $\boldsymbol{A}$ and matrix $\boldsymbol{B}$ in Fig. 2 is defined by L and can be adjusted.

For a given L, each $\boldsymbol{h}_{eq}$ coefficient is obtained by summing the coefficients of each $\boldsymbol{M}_{int}$ column. So, meet the criteria outlined above, means that,

- Maximizing the energy in $\boldsymbol{h}_{eq_max}$ only depends on the choice of $\boldsymbol{w}_{max}$,
- Minimizing the energy in $\boldsymbol{h}_{eq_min}$ depends on the combination of the $\boldsymbol{B}$ and $\boldsymbol{S}$ matrix, and therefore the choice of $\boldsymbol{w}_{min}$ assuming that $\boldsymbol{w}_{max}$ is defined following the previous point.

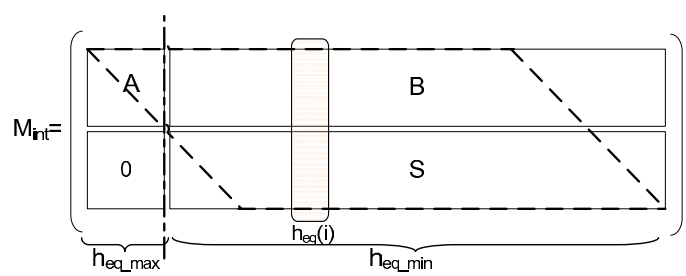

Fig. 2. Intermediate convolution matrix decomposed in three matrix A, B and S.

B. Energy Concentration

Signal decomposition is widely used to concentrate energy in image/video compression standard. The basic idea consists in concentrating the most of energy in the few first coefficients. In this paper, to concentrate the effective channel energy within the allowed delay spread window, we use the discrete cosine transform (DCT) [6] described as follows,

$$DCT(i) = \begin{cases} \sqrt{\frac{1}{L}} \cos\left[\frac{\Pi(2i+1)}{2L}\right], & 0 \leq i < L-1 \\ \sqrt{\frac{2}{L}}, & i = 0 \end{cases} \quad (10)$$

Since $\boldsymbol{h}_{eq_max}$ only depends on the coefficients of $\boldsymbol{w}_{max}$, to concentrate energy within the L coefficients, we initialize $\boldsymbol{w}_{max}$ with the DCT coefficients,

$$w_{max}(i) = DCT(i), \qquad 0 \leq i \leq L-1 \quad (11)$$

Note that other initialization of $\boldsymbol{w}_{max}$ coefficients could be used, according to constraints defined by the system.

C. Impulse Response Tail Suppression

Given the length of $\boldsymbol{w}_{max}$ and the original impulse response $\boldsymbol{h}$ we want to determine $\boldsymbol{w}_{min}$ coefficients to cancel energy outside the range of cyclic prefix. Thus, the matrix S is calculated to minimize $\boldsymbol{h}_{eq_min}$.

We call $\boldsymbol{s}_{vect}$ the resulting vector from the convolution of $\boldsymbol{h}$ and $\boldsymbol{w}_{min}$ nearby the vector calculated by the column-wise sum of $\boldsymbol{S}$.

$$s_{vect}(k) = \sum_{j=0}^{k} w_{min}(j)h(k-j) \quad (12)$$

where $0 \leq k < L_{eq} - L$

Similarly, we call $\boldsymbol{b}_{vect}$ the vector computed from the sums of each column of the matrix $\boldsymbol{B}$.

$$b_{vect}(k) = \sum_{j=0}^{L-1} w_{max}(j)h(k-j+L) \quad (13)$$

where $0 \leq k < L_{eq} - L$

We present $\boldsymbol{h}_{eq_min}$ as follows,

$$\boldsymbol{h}_{eq_min} = \boldsymbol{b}_{vect} + \boldsymbol{s}_{vect} \quad (14)$$

For the effective impulse response to have zero taps outside the window of size L, it is necessary for the $\boldsymbol{b}_{vect}$ rows to be the opposite of the $\boldsymbol{s}_{vect}$ ones,

$$\boldsymbol{h}_{eq_min} = \boldsymbol{0} \Leftrightarrow \boldsymbol{w}_{min} * \boldsymbol{h} = \boldsymbol{s}_{vect} = -\boldsymbol{b}_{vect} \quad (15)$$

where $\boldsymbol{0}$ is a null vector of size $1 \times L_{eq} - L$.

We can solve this equation in the frequency domain. Therefore, we convert each vector to the frequency domain using the FFT,

$$W_{min} \odot H = S_{vect} = -B_{vect} \quad (16)$$

where W_{min}, H, S_{vect} and B_{vect} are, respectively, the fast Fourier transforms of $\boldsymbol{w}_{min}$, $\boldsymbol{h}$, $\boldsymbol{s}_{vect}$ and $\boldsymbol{b}_{vect}$. The $\odot$ denotes the element by element multiplication.

Finally, we compute the coefficients of $\boldsymbol{w}_{min}$ by applying the IFFT as follows,

$$w_{min}(k) = \frac{1}{N} \sum_{n=0}^{N-1} \frac{B_{vect}(n)}{H(n)} \exp(\frac{2i\pi}{N} kn) \quad (17)$$

where N is the B_{vect} and H length and $0 \leq k \leq L_w - L - 1$.

IV. Performance Analysis

A. Complexity

The complexity of the proposed CSF algorithm can be estimated as follows,

1) The calculation of DCT coefficients is in the order of $O(Llog(L))$,
2) The calculation of $\boldsymbol{b}_{vect}$ requires
   - $(L_h - L)L + L_w + \sum_{i=1}^{L-1}(i) - 1$ additions.
   - $(L_h - L)L + \sum_{i=1}^{L-1}(i)$ multiplications.
3) The calculation of $\boldsymbol{w}_{min}$ requires 2 FFT with $L_h log(L_h)$ computations and 1 IFFT with $L_w log(L_w)$ computations.

Table I shows the computational complexity for conventional channel shortening approachs [7] in terms of number of multiply and accumulate (MAC) operations required to generate the CSF coefficients. N_Δ is the optimum delay used by the shortening filter.

Algorithm	Complexity
MMSE [3]	$(\frac{2}{3}\nu^3 + L_w^2 + L_w\nu)N_\Delta + 2\nu L_w^2$
MSSNR [2]	$\frac{8}{3}L_w^3 N_\Delta + L_h L_w + 2L_w^2$
MinIBI [5]	$\frac{8}{3}L_w^3 N_\Delta + L_{eq} L_w^2$
MDS [4]	$\frac{8}{3}L_w^3 N_\Delta + 2L_{eq} L_w^2$
Proposed	$2(L_h - L)L + L_w(1 + logL_w) + LlogL + L_h logL_h$ $+2\sum_{i=1}^{L-1} i$

TABLE I
Computational Complexity for Different CSF Designs.

As we can see, the computational complexity of the conventional channel shortening techniques is in the order of L_w^3. Thus, for large values of L_w, these techniques require expensive computations. On the other hand, the computational complexity of the proposed technique is in the order of $2L_h L$, namely $O(2L_h L)$, which is independent of the CSF order. Thus, unlike other approachs, the computational complexity of our design will remain the same even if we change the CSF order. Hence, conventional channel shortening designs, for which the computational complexity increase with the CSF order, require computationally intensive calculations.

B. Achievable Bit Rate

In this section, we apply the proposed method to the eight Carrier Serving Area (CSA) loops [8] in an ADSL

environment with perfect channel state information at the receiver. The system parameters are decribed as follows,

- Channel length of L_h = 512 samples with white additive gaussian noise,
- CP length of ν = 32,
- FFT size of N = 512,
- We assume that the transmitter and receiver are perfectly synchronized.

In Fig. 3, we compare the impulse response of the carrier-serving-area loop number 6 with the shortened channel by the proposed CSF. It can be seen that the proposed method shortened the channel quite well. The energy of the effective impulse response is concentrated into just few taps.

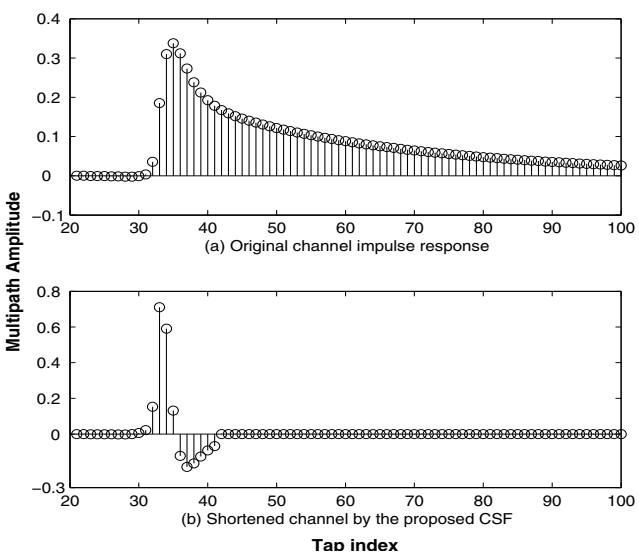

Fig. 3. Impulse response of (a) the original channel, (b) the shortened channel using the proposed filter

In ADSL systems, the achievable bit rate b_{DMT} for a fixed probability of error [9] is used as performance metric,

$$b_{DMT} = \sum_i log_2(1 + \frac{SNR_i}{\Gamma}) \tag{18}$$

where SNR_i is the signal to noise ratio of the ith subchannel and Γ is the SNR gap for achieving channel capacity with Γ = 9.8 dB.

To analyse the performance of the proposed method, we apply the MSSNR, MMSE, MDS, Min-IBI and the proposed CSF to the CSA loop number 6 when varying the CSF length at 60 dB SNR. We compare the obtained bit rates with the matched filter bound (MFB) which is the maximum achievable bit rate [10].

For bit rate calculation, we assume that a coding gain of 4.2 dB and a margin of 6 dB are used. As shown in Fig. 4, all methods need a CSF length greater than 40 to obtain the highest performance. However, as discussed previously, the computational complexity of conventional channel shortening approachs increases with the CSF length. Therefore, for these methods, it is preferable to use a short CSF. Unlike the proposed method, the bit rates of all other methods, especially the Min-IBI and the MMSE methods, oscillates as L_w increases. We should note that, for a CSF length higher than 50, our proposed method remains constant and comes very close to the MFB bit rate. The MSSNR design performs close to the proposed method but it requires intensive computationally complexity.

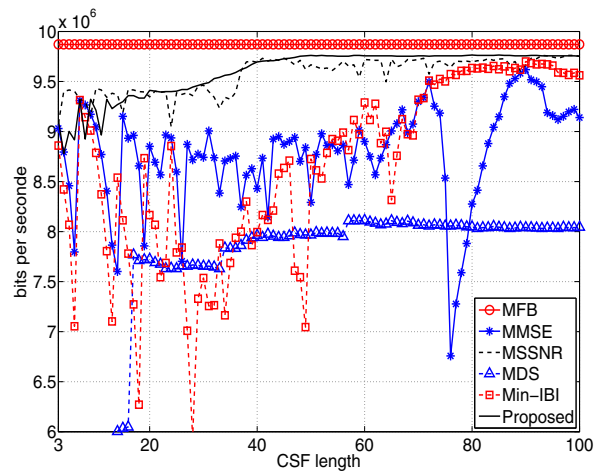

Fig. 4. Achievable bit rates in Mbps of MMSE, MSSNR, MDS, Min-IBI and Proposed designs vs. CSF length for the CSA loop number 6 at 60 dB SNR.

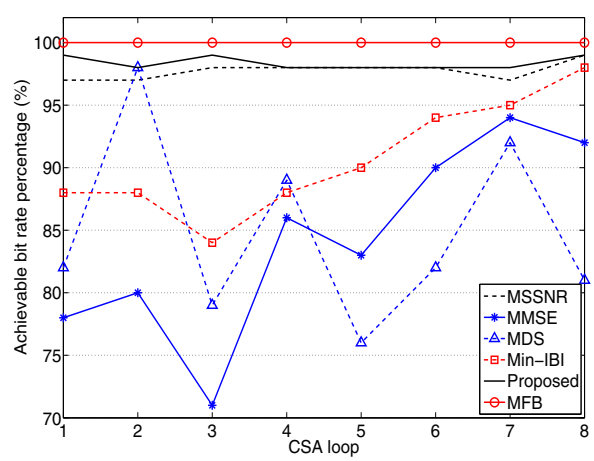

Fig. 5. Achievable bit rates for the eight CSA loops at 60 dB SNR using a 50-tap CSF.

Fig. 5 shows the bit rate, computed as a percentage of the MFB maximum bit rate, achieved by each method on the 8 CSA loops for L_w=50 taps. The MDS, MMSE, Min-IBI designs achieve respectively, 70-94%, 79-97% and 84-98% of the MFB maximum bit rate. On the other hand, the MSSNR design achieves better bit rate with 97-98% but does not perform as well as the proposed method which achieves

99% for the csa loop #1, #3 and #8 and does not fall below 98%.

From Fig. 6, it might be concluded that, for any CSF order, the proposed method outperforms the classical channel shortening techniques for all SNR value. Observe that for low SNR, the MSSNR, MMSE, Min-IBI and the proposed method perform very close. However, for SNR value less than 40, the performance of the MDS design degrades. On the other hand, for high SNR value, only the proposed and the MSSNR methods perform well. Nevertheless, the proposed method provides the closet Bit Rate to the MFB achievable Bit Rate.

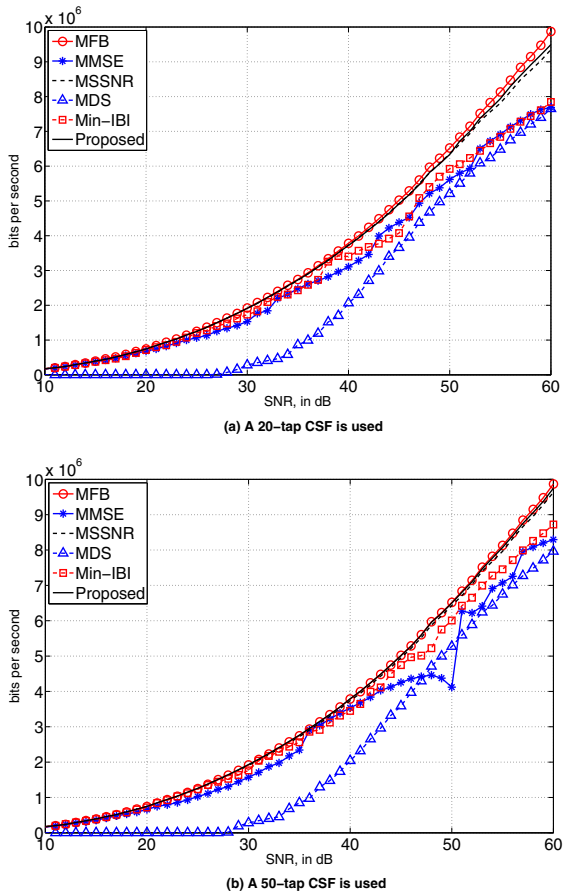

Fig. 6. Achievable bit rates of MMSE, MSSNR, MDS, Min-IBI and Proposed designs vs. SNR for the CSA loop number 6.

V. Conclusion

In this paper, a novel channel shortening method is proposed and applied for DMT communication systems. The main interest of this new approach is to efficiently compute the channel shortening filter coefficients while improving the bit rate performance. The experimental results show that the proposed method outperforms the conventional channel shortening approchs in terms of bit rate with a comparatively reduced computational complexity.

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