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A survey of near-mean-motion resonances between Venus and Earth

Á. Bazsó · R. Dvorak · E. Pilat-Lohinger · V. Eybl ·
Ch. Lhotka

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Abstract It is known since the seminal study of Laskar (1989) that the inner planetary system is chaotic with respect to its orbits and even escapes are not impossible, although in time scales of billions of years. The aim of this investigation is to locate the orbits of Venus and Earth in phase space, respectively, to see how close their orbits are to chaotic motion which would lead to unstable orbits for the inner planets on much shorter time scales. Therefore, we did numerical experiments in different dynamical models with different initial conditions—on one hand the couple Venus–Earth was set close to different mean motion resonances (MMR), and on the other hand Venus’ orbital eccentricity (or inclination) was set to values as large as $e = 0.36$ ($i = 40^\circ$). The couple Venus–Earth is almost exactly in the 13:8 mean motion resonance. The stronger acting 8:5 MMR inside, and the 5:3 MMR outside the 13:8 resonance are within a small shift in the Earth’s semimajor axis (only 1.5 percent). Especially Mercury is strongly affected by relatively small changes in initial eccentricity and/or inclination of Venus, and even escapes for the innermost planet are possible which may happen quite rapidly.

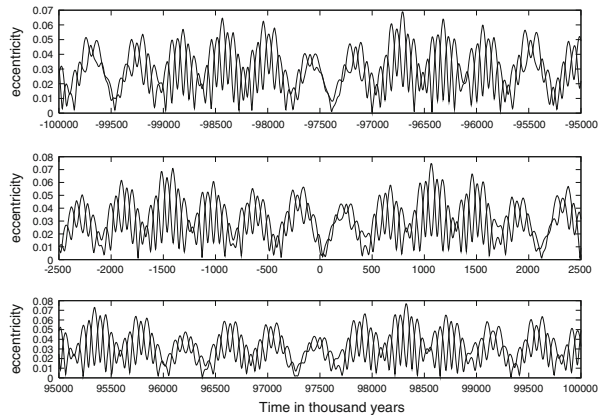
Keywords Mean motion resonances · Venus · Earth couple · Mercury’s escape · Inner Solar System

1 Introduction

Mean Motion Resonances are essential in Solar System Dynamics not only for the planets but also for the motion of the asteroids. The appearance of chaotic motion in the asteroid belt detected by Wisdom (1981)—the 3:1 MMR of an asteroid with Jupiter—was the first one discovered after the seminal discovery by Henon and Heiles (1964) for galactic dynamics. But the structure of the asteroid belt is also created by the Secular Resonances (SR), where the

Á. Bazsó (✉) · R. Dvorak · E. Pilat-Lohinger · V. Eybl · Ch. Lhotka
Institute of Astronomy, University of Vienna, Türkenschanzstr. 17, Vienna 1180, Austria
e-mail: bazso@astro.univie.ac.at

Fig. 1 The coupling in the eccentricities of Venus and Earth; from -100 to 95 Myrs (*upper graph*), from -2.5 to 2.5 Myrs (*middle graph*) and from 95 to 100 Myrs (*lower graph*), after Dvorak and Gamsjäger (2003)



motion of the perihelia and the nodes can be in resonance for different planets in combination with the asteroid's perihelion, respectively, node.

For the planets it was found by Laskar (1989, 1990, 1994), Dvorak and Süli (2002) that especially the Inner Solar System (ISS) is in a chaotic state. The appearance of SR together with diffusion in the chaotic zone may even shift the orbit of Mercury into a near-unstable one with probability of $\leq 2\%$ (Laskar 2008), if one does not take into account general relativity. Batygin and Laughlin (2008) performed a long-term numerical integration for the full solar system and they, too, found possible solutions for the escape or collision of Mercury with other planets. Following these studies Laskar and Gastineau (2009) then included general relativity and concluded that with a probability of 1% among the studied trajectories escapes and collisions for Mercury are still possible.

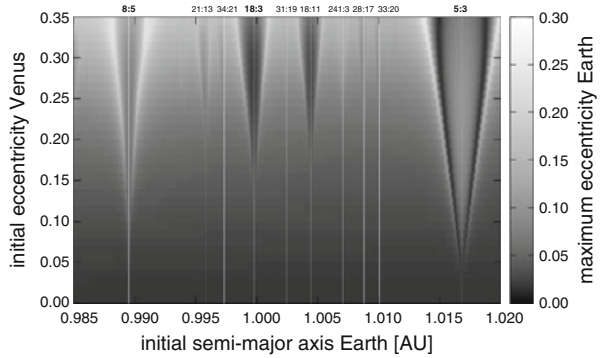
For the Outer Solar System it has been found by numerical integrations that chaos is present (Applegate et al. 1986; Sussman and Wisdom 1992), and many MMR are acting, e.g. the near $5:2$ MMR between Jupiter and Saturn. Investigations by Michtchenko and Ferraz-Mello (2001) and Varadi et al. (1999) show how close their motion is to a chaotic state with severe implications for the other planets. In the study of Murray and Holman (1999) it was shown that the chaos mainly results from the overlapping of mean motion resonances between Jupiter, Saturn and Uranus, but despite that fact the dynamical lifetime of Uranus exceeds 10^{18} years.

One can notice a coupling with respect to the inclinations and eccentricities of Venus and Earth due to mutual secular interactions (Fig. 1). As the actual orbits of Venus and Earth are quite close to the $13:8$ MMR¹ we investigated the phase space close to this high order resonance to determine the effect of MMR on that coupling.

The order of a resonance is defined as the difference q when we characterize a MMR by $p : (p + q)$. The value q is connected to the exponent in front of the Fourier term in the expansion of the disturbing function; the higher the exponent in a small quantity in front of the Fourier term is, the weaker is the influence on the dynamics (e.g. Murray and Dermott). Thus for the $13:8$ MMR we have to deal with the order 5. It is interesting to note that a relatively small shift of the Earth to a smaller semimajor axis would bring the couple Venus–Earth into the $8:5$ MMR (order 3); a shift to a larger semimajor axis would bring both planets into the $5:3$ MMR (order 2).

¹ The periods of Venus and Earth are almost in the ratio $13:8$.

Fig. 2 The MMRs between Venus and Earth; along the x-axis the initial semi-major axis of the Earth is changed, along the y-axis the initial values for the eccentricity of Venus are varied. The greytone indicates the maximum eccentricity, the vertical white lines show the position of the MMR up to high orders



In Fig. 2 the results of the computations in the model of the spatial full three body problem Sun-Venus-Earth are presented to show the location of the resonances and also their shape. We integrated both Venus and Earth with their actual masses and orbital elements, except for the changes in Earth's semi-major axis and Venus' eccentricity. The integration time spans 10^4 orbital periods for the Earth, thus is short enough to show the influence of the MMRs and shorter than the timescale based on the secular eigenfrequency g_5 of Earth (with a period of approx. 70000 years).

Besides extensive numerical integration of the motions of the planets in different dynamical models, we also used chaos indicators and analysed the frequencies involved, to understand the structure of phase space where Venus and Earth are embedded.

The content of the paper is as follows: after a careful test of the dynamical models which we used (Sect. 2) we roughly describe the three MMR mentioned above and show the structure determined with the aid of a chaos indicator (Sect. 3). We then report on the results of different numerical experiments where we show the dependency of the stability of the inner planetary system on the eccentricities and the inclinations of Venus (Sect. 4). In the conclusions we discuss the possible consequences for the stability of the inner planetary system.

2 The dynamical models and the numerical setup

To choose quite a realistic model we first did integrations with the actual initial conditions of Venus and Earth in different dynamical models,

- M5** the Inner Solar System (ISS, 5-bodies)
- M6** the ISS + Jupiter (6-bodies)
- M7** the ISS + Jupiter + Saturn (7-bodies)
- M8** the ISS + Jupiter + Saturn + Uranus (8-bodies)
- M9** the ISS + outer Solar system (9-bodies).

We applied a numerical integration scheme based on Lie-series (Hansmeier and Dvorak 1984; Delva 1984; Lichtenegger 1984). This numerical method was recently compared to other common methods (both symplectic and non-symplectic ones) with respect to some important properties—as for example the handling of conserved quantities—by Egl and Dvorak (2010). The results for the conservation of angular momentum are shown in Fig. 3, where an advantage in favour of the two symplectic integrators (the lower 'jumping' lines

Fig. 3 Comparison of the conservation of angular momentum in the 2-body problem for different types of numerical integration schemes, after [Eggl and Dvorak \(2010\)](#). In the online color version of this figure the coding is: *Green*: Radau; *Red*: Bulirsch-Stoer; *Cyan*: Runge-Kutta; *Magenta*: Lie Series; *Blue*: hybrid; *Gray*: Candy

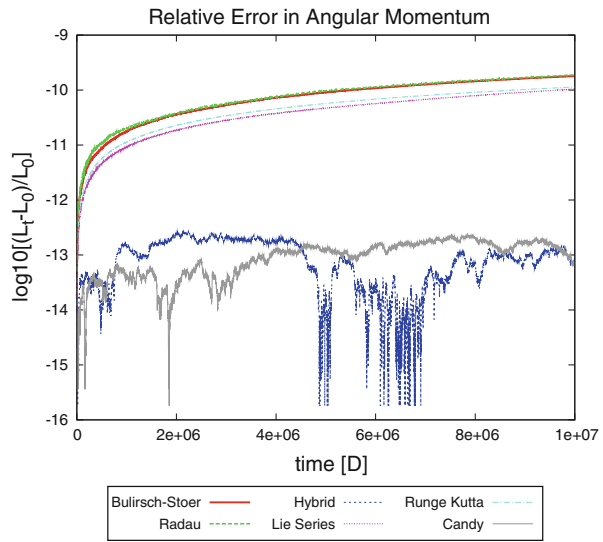
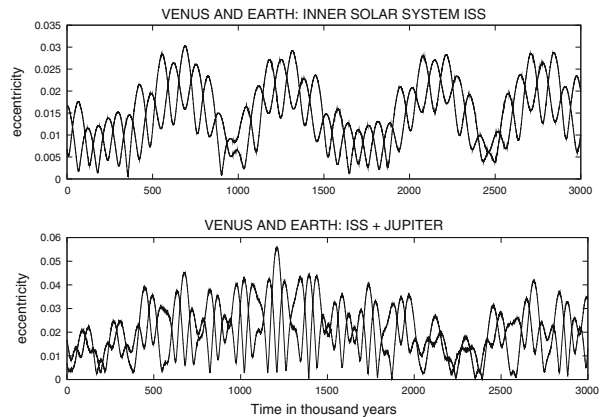


Fig. 4 Coupling in the eccentricities of Venus and Earth for the model **M5** (*upper graph*) and in the model **M6** for 3 million years

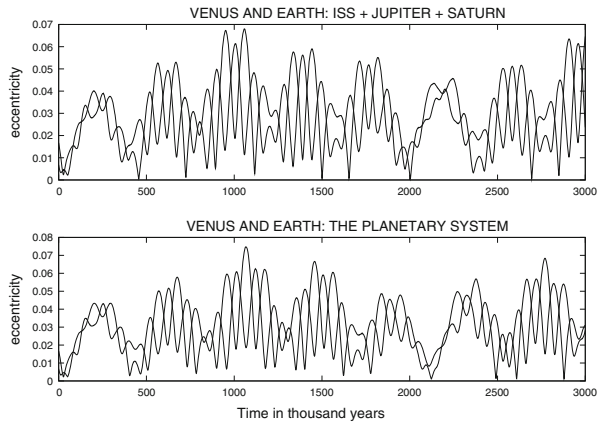


in Fig. 3)—the hybrid symplectic integrator ([Chambers 1999](#)) and the 4th order symplectic integrator ([Candy and Rozmus 1991](#))—is evident. Nevertheless the Lie-series method is very competitive among the non-symplectic methods, which can be seen from Fig. 3 (the lowest smooth line). For details about the integration schemes see [Eggl and Dvorak \(2010\)](#) and the references therein.

We restricted ourselves to point masses (i.e. no gravitational harmonics neither for the Sun nor for the planets) and neglected the effects of general relativity on the planets. The “Earth” we will refer to throughout this survey is actually the Earth–Moon barycentre with the combined masses.

The different behaviour between the models consisting only of the inner planets (**M5**) and the one including also Jupiter (**M6**) is visible in Fig. 4: the very regular variation limited by a maximum value of $e = 0.03$ (*upper graph*)—is completely destroyed by the perturbation of Jupiter (Fig. 4: *lower graph*). One can see three distinct sections: (i) sometimes the eccentricities almost do not change for up to 10^5 years (e.g. between 2.25 and 2.35×10^6 years

Fig. 5 Coupling in the eccentricities of Venus and Earth for the model **M7** (upper graph) and in the model **M9** for 3 million years



for Venus and 2.55 and 2.65×10^6 years for the Earth); (ii) sometimes the changes are in antiphase (like in the upper graph of Fig. 4); and (iii) sometimes they are in phase.

Including now Saturn as perturber in model **M7** changes the picture a lot: the development of the eccentricities is always in antiphase, reaches even larger maximum values, but looks ‘smoother’ for both planets compared to model **M6** (Fig. 5 upper graph). Now by comparing the dynamical evolution of Venus and Earth including all planets of the Solar System (model **M9**, Fig. 5 bottom) with **M7** it is interesting to note that for the first 10^6 years even quantitatively the behaviour is very similar. Later (up to 3×10^6 years) at least qualitatively both models show an analogous behaviour.²

This behaviour can be understood in the frame of the classical Laplace-Lagrange secular theory. The secular coupling in the eccentricities and inclinations of Venus and Earth in model **M5** is modified by introducing the secular eigenfrequency g_5 of Jupiter in model **M6**. This adds a considerable perturbation due to the dominating mass of Jupiter among the planets. By adding Saturn in model **M7** its eigenfrequency g_6 again changes the dynamical behaviour, so that one must conclude that both secular eigenfrequencies g_5 and g_6 are important for the dynamics of the ISS.

In addition to the former computations we compare for the five different models the dominating frequencies, which were derived by the program SIGSPEC³ (Reegen 2007). SIGSPEC incorporates the frequency and phase of the signal appropriately and takes into account the properties of the time-domain sampling which provides an unprecedentedly accurate peak detection (Kallinger et al. 2008). Although the present investigations deal with equidistantly sampled time series, the benefit of frequency- and phase-resolved statistics and the capability of full automation are indispensable also in this application.

In Fig. 6 we compare the most prominent frequencies arising in the element $k = e \cos \varpi$ in the time series of Venus and Earth for the five different models as described before. On the left panel the spectrum of Venus is shown, on the right panel the one of the Earth (from top to bottom for the models **M5**–**M9**). Comparing the different graphs it is clear that ignoring

² Model **M8**, not shown here, is not significantly better than **M7**.

³ This code is a new method in time series analysis and it is the first technique to rely on an analytic solution for the probability distribution produced by white noise in an amplitude spectrum (Discrete Fourier Transform). Returning the probability that an amplitude level is due to a random process, it provides an objective and unbiased estimator for the significance of a detected signal.

Fig. 6 Comparison of the 5 dominating frequencies in the dynamics of Venus (*left panel*) and Earth (*right panel*) for the five different models **M5–M9** (from *top to bottom*). The periods (x-axes) are plotted versus the amplitudes (y-axes)

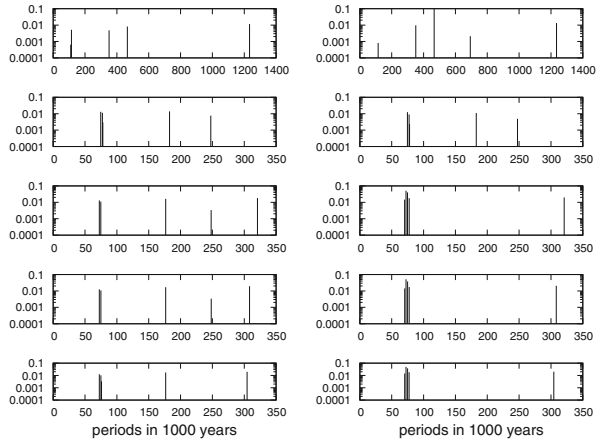
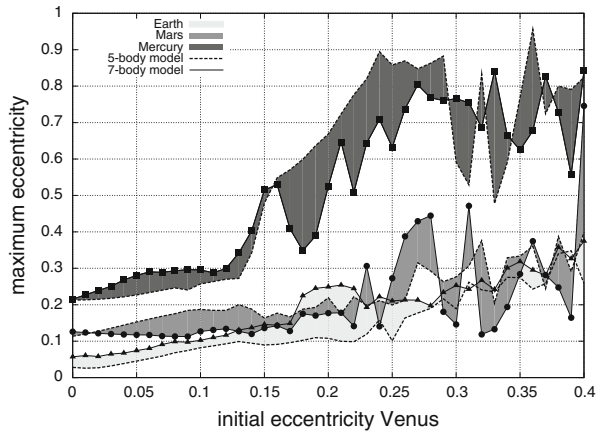


Fig. 7 Results of integrations for 10^6 years for initially different eccentricities of Venus (x-axis) starting in the 13:8 MMR with Earth: the grey-filled areas correspond to differences in maximum eccentricity for each individual planet in 2 models (**M5**: dashed lines, **M7**: full lines), the maximum eccentricities are plotted on the y-axis. Light grey: Earth, grey: Mars, dark grey: Mercury



Jupiter in the model **M5** (top row) shows a completely different dynamical behaviour than the one including the perturbations of Jupiter (model **M6**, 2nd row). Then, comparing the more realistic models (model **M7**, 3rd row; **M8**, 4th row; and the full Solar System model **M9**, 5th row) to each other it is visible that, in spite of some minor differences, **M7** seems to be a good approximation. We decided to use this model for our numerical experiments, because the inclusion of any additional massive planet prolongates the necessary CPU time significantly.

One additional test was made by checking the maximum eccentricities of the orbits (e_{max}) of the inner planets when changing the initial eccentricity of Venus. The respective integrations for 10^6 years shown in Fig. 7 point out the differences in e_{max} of each individual planet in the **M5** and **M7** models for the 13:8 MMR. In contrast to the other planets one can see that the Earth (light grey) has almost always a higher maximum eccentricity due to the influence of Jupiter and Saturn than without these two planets. For Mercury and Mars no clear dependence is observable, the maximum values of their eccentricities change with the initial eccentricity of Venus. It is remarkable that Mercury achieves very high eccentricities, and eventually escapes for longer integrations (see next chapter).

3 Description and analysis of the 8:5, 13:8 and 5:3 MMR

In Fig. 2 we showed a detailed graph of the neighbourhood of the 13:8 MMR in the spatial full 3-body problem Sun-Venus-Earth: in addition to the 13:8, the 8:5 and the 5:3 MMR many high order MMRs are visible. To derive a simple picture of the phase space structure close to the 13:8 MMR only two initial parameters were changed: on the x-axis the semimajor axis of the Earth was varied and on the y-axis the initial eccentricity of the perturbing planet Venus is plotted. In this graph, where the maximum eccentricity of an Earth-orbit during the integration time is shown on a grey scale, one already can see the inequality of the perturbations acting in the resonances.

- The 8:5 MMR (order 3, $a = 0.989501$ AU)⁴ has a relatively narrow triangular structure with highly perturbed wings down to $e_{Venus} = 0.12$ on both sides, whereas in the middle of the resonance the orbits are very regular with small eccentricities.
- The 13:8 MMR (order 5, at $a = 0.999782$ AU) is the weakest of the three MMR we are studying. The wings on both sides of the resonance go down to $e_{Venus} = 0.15$. Again in the middle of the resonance—here broader than in the 8:5 resonance—the orbits are only marginally perturbed; the eccentricities stay well below 0.075.⁵
- The 5:3 MMR (order 2, $a = 1.016799$ AU) is by far the strongest one: it is quite broad from large eccentricities of Venus on down to small eccentricities like $e = 0.05$. It shows a big ‘quiet’ central region and in the wings the Earth’s orbit suffers from relatively large eccentricities $e_{Earth} \sim 0.3$

Another high order MMR, the 18:11 resonance, is visible close to the 13:8 resonance, which we will not discuss here because its action is evidently weaker (order 7).

To use the results of direct numerical integrations one has to carry out orbital computations over a very long time. In order to save computation time it is advisable to use a so-called chaos indicator, that shows the state of motion quite fast and allows us to reduce the integration time significantly. Therefore, we used the Fast Lyapunov Indicator (FLI, see Froeschlé et al. 1997; Froeschlé and Lega 2000), which is quite a fast tool to distinguish between regular and chaotic motion. Given a set of differential equations for $\mathbf{x} = (x_1, x_2, \dots, x_n)$, the evolution of a set of deviation vectors $w_i(t) \in R^n$ with $\|w_i(0)\| = 1 (i = 1 \dots n, n$ denotes the dimension of the phase space, the $w_i(0)$ form the n -dimensional orthonormal basis of R^n) is followed by means of the (linear) variational equations, so we have the system of differential equations

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &= F(t, \mathbf{x}) \\ \frac{d\mathbf{w}(t)}{dt} &= \left(\frac{\partial F}{\partial \mathbf{x}} \right) \mathbf{w} . \end{aligned}$$

According to the definition

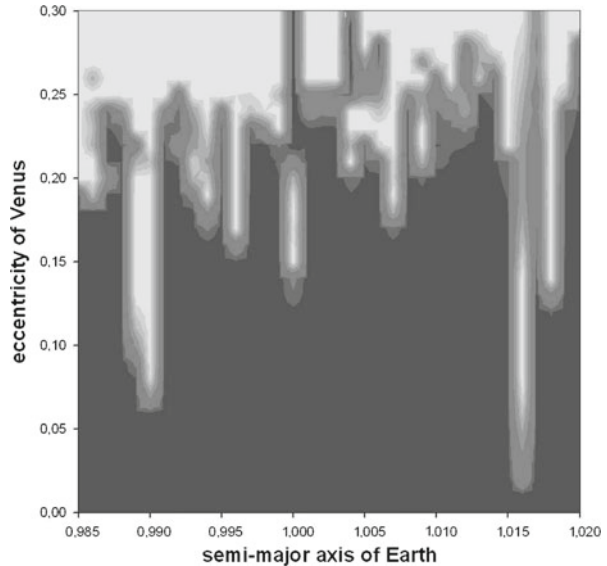
$$\psi(T) = FLI(\mathbf{x}(0), \mathbf{w}(0), T) = \sup_{0 < t \leq T} \log \|w_i(t)\| \quad (i = 1 \dots n)$$

ψ is the length of the largest tangent vector. It is obvious that chaotic orbits can be found very quickly because of the exponential growth of this vector in the chaotic region. This method has often been applied to studies of Extra-solar Planetary Systems (Pilat-Lohinger et al. 2008; Schwarz et al. 2009). In this study the FLIs were computed for about 5×10^4 years. The

⁴ with $a_{Venus} = 0.723330$ AU

⁵ The Earth with a semimajor axis $a = 1$ is just a little outside of this MMR.

Fig. 8 FLI map: semi-major axes of the Earth versus different eccentricities of Venus. Visible from left to right the 8:5, the 13:8 and the 5:3 MMR. *Black* regions indicate quasi-periodic orbits separated by a transition layer from the *light-grey* chaotic domains



resulting maps are shown in Figs. 8 and 9, where the black region marks the quasi-periodic motion whereas grey and bright areas show chaotic regions; the different colours depend on the values of the FLI, which range from 10^4 in the quasi-periodic area to 10^{30} in the chaotic region. Fig. 8 displays the state of motion from 0.98 to 1.02 AU for different eccentricities of Venus. One can clearly see perturbations in the stable region even for low eccentric motion of Venus due to MMR with Earth. The three MMRs—8:5, 13:8 and 5:3—that are analyzed in this study are clearly visible in Fig. 8. A magnification of the 13:8 MMR is shown in Fig. 9, which indicates chaotic behaviour for the position of the Earth if $e_{Venus} > 0.1$. The interesting fine structures inside the MMR (symmetric with respect to the central line of the resonance) unveil a central region for $e = 0.2$ which still shows regular motions of the Earth, while close to it chaotic motion is present (white in Fig. 8).

4 Dependence on the initial conditions of Venus

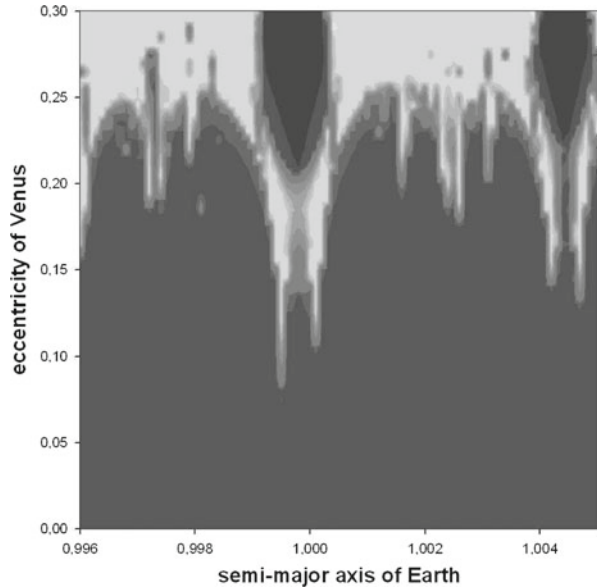
To check the sensitivity with respect to different initial conditions of the ‘planet twins’ Venus–Earth concerning the stability of the inner Solar System we performed a series of numerical integrations in the model M7 for 10^7 years where we checked the three MMR described before. Two cuts through the exact location of the MMR have been undertaken, one where we changed the eccentricity of Venus and another one where we changed its inclination. Both results were analysed through the simple test of the largest eccentricity of the 4 inner planets during the integration.

4.1 Dependence of the Earth’s orbit on the eccentricity of Venus

– The 8:5 MMR (Fig. 10, upper graph)

The orbits of Venus and the Earth develop qualitatively in the same way, and no dramatic changes are visible in their orbits. Mercury is the planet which suffers—even for

Fig. 9 Zoom of the 13:8 MMR, captions like in Fig. 8



relatively small initial eccentricities of Venus ($e_{Venus} \leq 0.12$)—from large eccentricities. For all larger values of e_{Venus} Mercury is in quite a chaotic orbit. Mars remains longer with orbital elements which do not allow close encounters to the Earth, but from $e_{Venus} \geq 0.2$ on its orbit is also destabilized.

– **13:8 MMR** (Fig. 10, middle graph)

Up to $e_{Venus} \leq 0.15$ the inner planetary system seems to be in a relatively ‘quiet’ region of phase space; from that on, Mercury and Mars getting larger and larger eccentricities, the system is destabilized. Venus and Earth are still developing in quite a similar way without dramatic changes of their orbits.

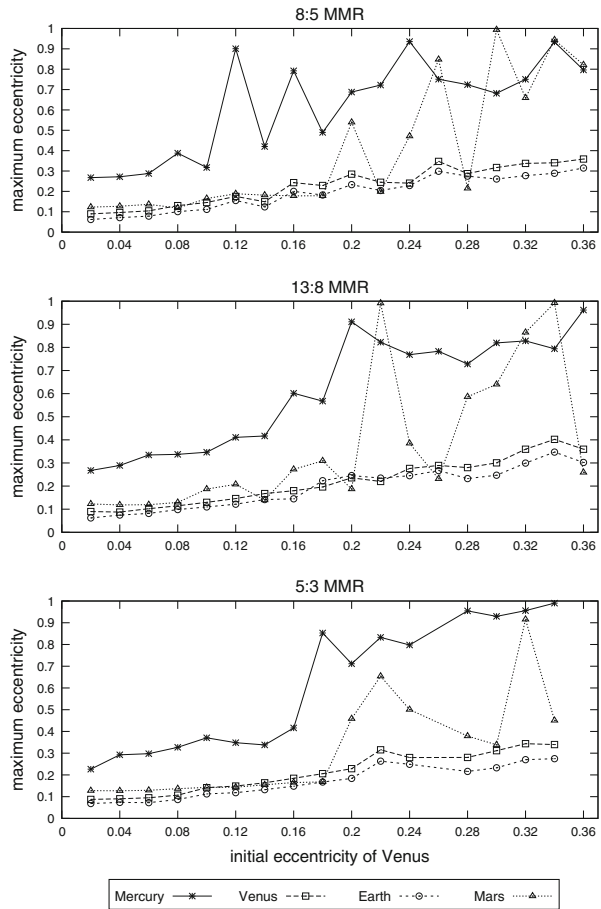
– **The 5:3 MMR** (Fig. 10, lower graph)

The orbit of the Earth suffers more and more from larger and larger eccentricities depending on the one of Venus very similar to the other two resonances. The two planets Mercury and Mars achieve such large eccentricities that encounters with Venus, respectively, Earth cause them to leave their orbits. The chaotic behaviour of the orbits of the inner planetary system is already visible in the FLI diagram for $e_{Venus} \geq 0.17$ on (compare Fig. 8).

The dynamical behaviour of orbits inside these three resonances is quite similar. Though the 5:3 and 8:3 MMR are of a lower order compared to the 13:8 MMR and one expects stronger perturbations on the orbits of the planets, in the respective plots this is not visible.

Since Mercury is always achieving large eccentricities above an initial eccentricity of Venus of $e = 0.16$ (cf. Figs. 10, 11) we investigated the reason for this. As stated in [Batygin and Laughlin \(2008\)](#), [Laskar and Gastineau \(2009\)](#), Mercury is close to the ν_5 linear secular resonance to Jupiter, and there are some influences by other planets (mainly Venus, Earth and Saturn) on the longitude of perihelion ([Dufey et al. 2008](#)). We checked for the 13:8 MMR if the increase in Mercury’s eccentricity is due to that secular resonance and found this to be

Fig. 10 e_{max} cuts through the centres of the 8:5, 13:8 and 5:3 MMR (from *top to bottom*) depending on the eccentricity of the perturbing planet Venus (x-axis)



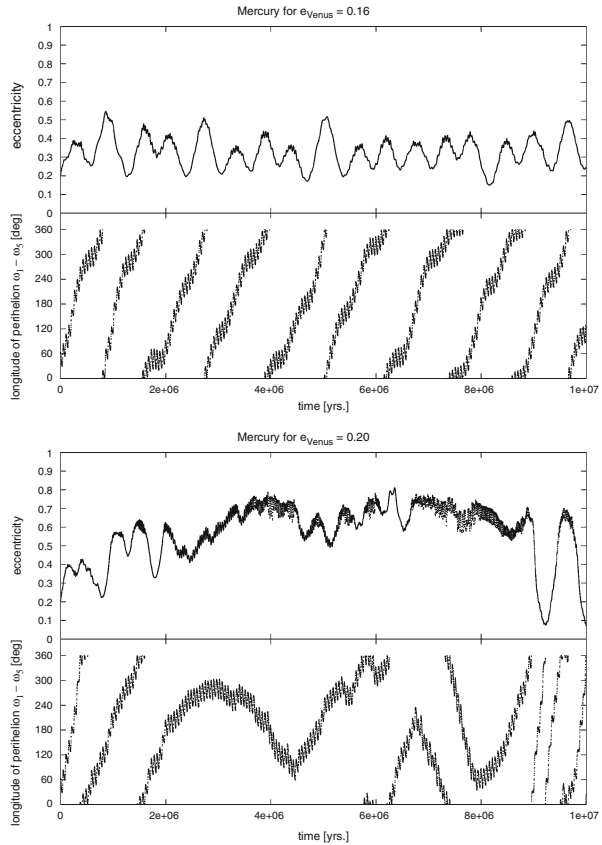
the case for initial Venus eccentricities higher than $e = 0.16$. The upper part of Fig. 11 shows the evolution of Mercury's eccentricity together with the difference between the longitudes of perihelion of Mercury and Jupiter over a period of 10^7 years. The eccentricity stays mostly below the critical value of $e = 0.5$, above which close encounters with Venus could occur, and only circulation of the critical angle is observed. At higher values of Venus' initial eccentricity Mercury enters into the secular resonance with Jupiter quite fast (after only 2 Myrs.). This is visible in the lower part of Fig. 11 when the transition from circulation to libration around 180° occurs, and Mercury's eccentricity is pumped up to values high enough to allow for close encounters with Venus and even Earth, and subsequently its evolution is of purely chaotic nature.

4.2 Dependence of the Earth's orbit on the inclination of Venus

– The 8:5 MMR (Fig. 12, upper graph)

In this resonance it is evident that Mercury is thrown out from its orbit already for moderate initial inclinations of Venus, although a small window of more stable orbits appears for $11^\circ \leq i \leq 12^\circ$. Earth and Venus are strongly perturbed: from $i = 29^\circ$ on eccentricities up to $e = 0.35$ for Venus and $e = 0.3$ for the Earth are possible. Mars obviously is not

Fig. 11 Evolution of Mercury’s eccentricity and longitude of perihelion with respect to Jupiter during 10^7 years for different initial eccentricities of Venus located in the 13:8 MMR with Earth. In the upper graph—just below the critical value of $e_{Venus} = 0.16$ —Mercury’s eccentricity stays lower than 0.5 for most of the time, preventing it from close encounters with Venus. In contrast for higher e_{Venus} the circulation in the longitude of perihelion changes to libration (*lower part*) and Mercury is in linear secular resonance with Jupiter, which drives its eccentricity to values high enough for close encounters with Venus and even Earth



so much affected by the Earth, only for $i = 37^\circ$ a maximum value of $e = 0.4$ is reached, which brings that planet into an Earth-crossing orbit. For longer integration time again an escape seems possible. Then, surprisingly, Mars for $i = 38^\circ$ and $i = 39^\circ$ (within the integration time of 10^7 years!) still stays in a moderately elliptic orbit.

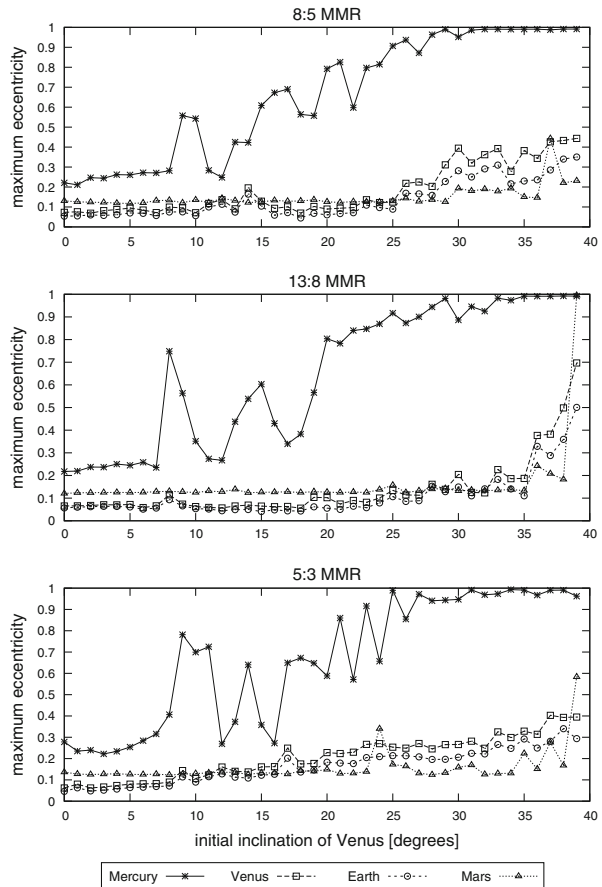
– **The 13:8 MMR** (Fig. 12, middle graph)

It is quite remarkable that even a small shift of the initial inclination to $i \sim 8^\circ$ of Venus may lead to an escape of Mercury (after $e = 0.75$ a subsequent escape from its orbit because of encounters with Venus is expected). A second ‘dangerous’ inclination is visible around $i = 15^\circ$; then, from $i \geq 20^\circ$ on, Mercury would suffer from close approaches to Venus. There seems to be little influence on the orbits of Venus, Earth and Mars up to $i_{Venus} = 35^\circ$.

– **The 5:3 MMR** (Fig. 12, lower graph)

The strongest of the three MMR shows in fact a dynamical evolution which is dominated by the very strong perturbations on Mercury for inclinations of Venus around 10° and then, from 15° on the large eccentricities of Mercury ($e \geq 0.6$) will always lead to escapes of this planet. Mars is not affected dramatically in this resonance, Venus and Earth stay within moderate eccentricities for all initial inclinations of Venus.

Fig. 12 e_{max} cuts through the centres of the 8:5, 13:8 and 5:3 MMR (from top to bottom) depending on the inclination of the perturbing planet Venus (x-axis)



5 Conclusions

In this investigation we explored the environment of the 13:8 MMR, which is very close to the actual position of Venus and Earth. After a careful test of a simplified dynamical model of our planetary system appropriate for our task we discussed the main structures of the three MMR in the vicinity of the two planets using the results of a study of the FLI.

In another step, using extensive numerical integrations we changed the ratio of the semi-major axes Venus–Earth;⁶ the orbit of the Earth was given different values of a to cover the proximate neighbourhood of the MMR where the twin-planets are in. Then, in two different runs the eccentricity of Venus was set to eccentricities up to $e = 0.36$ and in the dynamical model, consisting of the inner Solar System with Jupiter and Saturn, the equations of motion were integrated for 10^7 years. The same was done in a second run where we changed the inclination of Venus up to $i = 40^\circ$. These two experiments were undertaken for each of the three MMR 8:5, 13:8 and 5:3 and the results analysed with respect to the maximum eccentricities of the inner planets.

⁶ Venus' semi-major axis was fixed.

It turned out that only Mercury is the planet which would suffer from such perturbations, that it would achieve very large values of its eccentricity and thus its orbit would be subject to dramatic changes. As Mercury is close to the ν_5 secular resonance a slight change in Venus' eccentricity or inclination already can put it into resonance. According to our results Mars is not as strongly affected by a larger inclination or eccentricity of Venus as Mercury. Venus and also the Earth are only affected for larger initial inclinations of Venus, but the planets stay more or less in their orbits although sometimes achieving eccentricities as large as $e = 0.3$.

As a final statement we may say that according to our results although the couple of Venus and Earth is close to a MMR we do not expect big changes on the orbits of the inner planets unless Venus will come into an orbit of an inclination of about 7 degrees or will achieve an eccentricity of 0.2. Then, in fact, Mercury would be on an unstable orbit which could finally throw it either into the Sun or far out into the main belt of asteroids!

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