2D fuzzy spatial relations: New way of computing and representation

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Abstract In existing methods, fuzzy topological relations are based on computing topological relations between fuzzy objects. These types of fuzzy topological relations are due to the imprecision in observed phenomenon and objects have the weak contour. This imprecision can be found in semantics of relationships or it represents the fuzzy semantics. In such a situation, fuzzy topological relations are needed between crisp objects. These relations are much less developed.

we propose a method for computing fuzzy topological and directional relations which is called combined topological and directional relations (CTD) method. More over a single method is used to derive the fuzzy topological and fuzzy directional relations and this method deals with the second order fuzziness. A matrix method to represent the topological relations along with directional contents is proposed, it is a quantitative method for finding the topological and directional relations while its evaluation approach is fuzzy and at the end an algorithm is proposed to know the final topological and directional relation between a 2D object pair. For method validation, a number of experiments are performed on artificial data. This method can be used to answer the complex queries in spatial databases design and managements.

Keywords Fuzzy science · Fuzzy semantics · second order fuzziness · Fuzzy matrix of spatial relations.

Introduction

The space can be studied through the objects which are contained into it and spatial relationship between them. Spatial relations between objects provide information

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regarding the image contents. These spatial relations provide information about topological structure, orientation and distance between them or more generally they provide information about their relative location.

Topological relations are derived from geometric description. An uncertain relation is a relation which exists with a certain probability. There are two well-known models for finding the topological relations between spatial regions. 9-intersections and Region Connected Calculus (RCC) model[10,21,22]. 9-intersections (3×3) model depends upon the point set topology where topological parts (interior, boundary and exterior) of an object participate and topological structure is studied by the empty ($\emptyset$) and non-empty ($\neg\emptyset$) intersection of topological parts. Eight basic topological configurations between $2D$ objects in $\mathbb{R}^2$ are observed out of $2^9 = 512$ possibilities. Many extensions have been proposed in the 9-intersection model[5,34,7,6,12]. This method is extended to deal with fuzzy objects and 44 useful topological relations were developed using 9-intersections between objects with extended boundaries[4]. This method is also extended to 16 (4×4) intersections and a set of 152 useful configurations between simple fuzzy regions in $\mathbb{R}^2$ is realized[31]. In this theory no algebraic function is involved so this theory cannot deal fuzziness at relation's level.

On the other hand, RCC calculus provides us information about the topological structure of an image which are corresponding to the eight basic topological relations. RCC models are applied to regular topological spaces. This calculus is based upon the well established axiomatic theory and regions are primitives. This theory also supports a set of functions that define boolean composition of regions and all relations are based on a single relation $C(x, y)$, called connection[3]. This calculus is also extended to fuzzy theory[14,19,26,27,29]. In this method a fuzzy membership value is assigned to each point (pixel) then fuzzy relations are realized depending on these membership values and a set of 46 useful topological configurations are considered. These fuzzy topological relations represent the imprecision or fuzziness induced in image regions or objects where the contour is not strong or fuzziness due to the rough segmentation. This theory is also extended to deal fuzziness at relation's semantics and fuzzy connection relation is based on nearness (distance function) between two objects[26].

Topological relations ignore the directional contents between objects. 9-intersections model for directional relations was developed where objects are considered by their minimum bounding rectangle (MBR) and then 2D projections are used[9]. To know the position of an object inside the other object, a method of internal cardinal directional relations (ICD) was introduced[15,30]. This model divides the central tile of 9-intersections model for directional relations between extended objects into four, nine or 13 sub-tiles and internal directional relation between object pair is calculated by the intersection of sub-tile and argument object. 9-intersection model for directional relations and internal cardinal directional relations models describe the extended objects regarding their relative position and they don't answer the question that where in the space a topological relation holds. Another type of spatial relation are distance relations and these relations provide the distance between closet pixels of two objects in an image. Distance relation holds only when the topological relation between two objects is disjoint and for all the other topological relations this relation doesn't hold. Once a topological relation is established then the next question arises that where a topological relation in the space holds? As an example, it is described that object A overlaps object$B$, then next question will be, either object A overlaps object B from north direction, east or west direction. One single model cannot be used to perfectly describe object in the embedding space. 9-intersections model for topological and directional relations are based on two different approaches as a result some information are lost due to change in object approximation. Fuzzy reasoning about spatial relationship can be viewed fuzzy description of object's relative position. In qualitative domain mostly topological relations are studied between vague or fuzzy objects such as in RCC and 9-intersection models. In these methods fuzzy topology is used to develop topological relations between fuzzy objects. In such topological relations, category of a topological relation depends upon the existence of an object, but in languages people use fuzziness in relation's semantics for example *"Both objects are approximately equal"* or *"Both objects are almost equal"*, in this case both objects are crisp, but the fuzziness is involved in relation. A method which can describe fuzzy spatial relations between crisp objects is required. Fuzzy objects can be treated as crisp objects by considering the *egg's* boundary of *egg-yolk* model for representing fuzzy objects. This method represents the fuzziness at relation's level.

Idea of combined directional and topological relations is not new, fuzzy methods[16, 20, 25] can be used to model the fuzzy directional and topological relations at the same time. Allen relations create 13 partitions around the reference interval or segment corresponding to each relation and these partitions represent eight topological relations. To represent fuzziness at relation's level along with the direction information, fuzziness is introduced at Allen's relations. This method deals with the positional fuzziness in topological relations. These fuzzy topological relations are used to model the positional uncertainty present according to the orientation viewpoint of relative position of object pair in a spatial domain, in this paper the idea is to specify fuzzy topological relationship between 2D objects along with directional information, for this purpose each relation is split into several components and 1D Allen relations are used in spatial domain due to their direct isomorphism between time structure and 1D spatial structure. We hope our work will be helpful to answer questions that where in space a topological relation holds.

This paper is structured as follows, section 2 discuss in detail the different terms and necessary computations for 1D Allen relations. In section 3, our method for computing the topological relations along with directional aspects and their interpretation is given, results for different situation is given in section 4 and section 5 concludes the paper.

Related work

One of the development trends in Geographic information science is a move from determinate geographic science to the fuzzy geographic science and spatial relations is a major part of the geographic information science. Information on the spatial organization in an image are useful. Methods of extracting and representing these information greatly effect the obtained results. In early years fuzzy directional relations are studied separately and different approaches were adopted like mathematical morphology[2, 11] and quantitative approaches [18] where evaluation approach for directional relations was fuzzy and these relations represents uncertainty at relation's level. It is the first numerical description of object relative position, called angle histogram and force histograms[17] were the extension of angle histogram which deals only *disjoint* objects. In all these approaches, quantitative or fuzzy directional relations are studied and less attention has been paid to fuzzy topological relations while the topological, directional and distance relations are considered essential to understand image configuration, modeling common sense knowledge and spatial reasoning. In most of the existing topological relations methods uncertainty is represented at object level but uncertainty can also exist at relation's level. This type of uncertainty can be handled by assigning fuzzy membership value to a relation. According to the best of our knowledge, S. Schockaert et al.[26] are the premier in this field who throw the first stone into the theory of RCC and defined the fuzzy "Connection" relation based on nearness. In spite of all this the basic question, that where in the space a topological relation holds remain unanswered. For example, we say that Belgium touches the France from north direction rather than this that Belgium touches France then next question arises that France has common boundary with Spain, Monaco, Italy, Switzerland, Germany, Luxembourg also, then in which direction Belgium touches France? To answer this question that where a topological relation exist the idea of combined topological and directional relations information was initiated. The idea of combined topological and directional relations was first initiated by J. Malki et al.[16] by using the 1D Allen[13] relations of temporal domain in spatial domain. This work was revisited by Matsakis and Nikitenko[20] and fuzziness in the 1D Allen relations was introduced and algorithm for fuzzification of segments of a longitudinal section is replaced by *t-conorms* along with polygonal approximation of objects. The use of fuzzy connectors along with polygonal approximation decreases its computational complexity to $O(nN)$ where n number of directions and N represents the number of vertexes of polygonal objects[25].This work is related to fuzzy spatial aspects where the topological and directional relations are evaluated according to fuzzy set theoretical viewpoint.

The method addressed fuzziness at two levels, in case of topological relations and fuzziness according to the directional viewpoint. Method of combined fuzzy topological and directional relations (CTD) has two fold impacts, Allen relations are combined in such a way that whole space can be analyzed by using the directions $[0, \pi]$ and this model answer well the question that where a topological relation exists in space. This model will be helpful to answer the a query completely in managing database, and detecting the small changes in scene descriptions.

1 Preliminary definitions

1.1 Fuzzy set Theory

1. A fuzzy set A in a set X is a set of pairs $(X, \mu_A(x))$ such that $A = \{(x, \mu_A(x)|x \in X)\}$
2. A membership function μ in a set X is a function .i.e., $\mu : X \to [0, 1]$. Different fuzzy membership functions are developed according to the requirement of an application. Trapezoidal membership function is defined as

$$\mu(x; a, b, c, d) = \begin{cases} 0 & \text{if } x \leq \text{ a} \\ \frac{x-a}{b-a} & \text{if } a < x \leq b \\ 1 & \text{if } b < x \leq c \\ \frac{d-x}{d-c} & \text{if } c < x \leq d \\ 0 & \text{if} d < x \end{cases}$$

or using the max. and min. operators it could be defined as

$$\mu(x; a, b, c, d) = max(min(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c}), 0)$$

and for simplicity it is written as $\mu_{(a,b,c,d)}(x)$where $\vee x, a, b, c, d \in \mathbb{R}$ and $a < b \leq c < d$

3. T-norms: t-norm is an operator defined as:

$$\top : [0,1] \times [0,1] \to [0,1]$$

which is commutative, associative, increasing in both variables and admits 1 as identity. It is a conjunction operator and generalizes the *intersection* or logical *AND* operator. Examples are min (a,b), ab, max(a+b-1,0).

4. S-norms: s-norm (t-conorm) is an operator defined as:

$$\perp : [0,1] \times [0,1] \to [0,1]$$

which is commutative, associative, increasing in both variables and its identity is 0. It is a disjunction operator. It generalizes the *Union*, or logical *OR* operator. Typical examples are max(a, b), a+b-ab, min(a+b,1).

5. Uncertain Relation: An uncertain relation is a relation which exists with a certain probability[33].
6. Object: Objects are considered as regular closed sets. A set A is regular closed if $A = \overline{\mathring{A}}$ (Å= interior of A and $\overline{A}$ represents closure of A).
7. Fuzzy topological relations: Fuzzy topological relations can be divided into two classes.
   (a) Fuzzy topological relations between fuzzy regions: This type of topological relations are developed between fuzzy image regions and largely they depend upon the fuzzy topological spaces. Where any point x (pixel) of a fuzzy image region, $\mu(x)$ is a membership value to which x belongs to fuzzy image region, then the fuzzy topological relations are computed using the fuzzy topology. Different approaches are considered to develop these type of spatial relations such as (3×3) intersections method with intermediate boundaries, (4×4) intersections method in point set topological approach or fuzzy *RCC* calculus. These types of topological relations deals with image regions where contour is not strong or rough segmentation is used.
   (b) Fuzzy topological relations between crisp objects: This type of fuzzy topological relations between crisp objects are much less developed. These type of relations represent the fuzziness at relations semantics. Fuzzy objects can be treated by considering the egg's boundary as the boundary of a fuzzy object and thus fuzzy objects could be treated as crisp objects.
      – Fuzzy neighborhood relation:(Fuzzy connected) Fuzzy connected relation between two regions is assumed the extension of crisp connected relation. Crisp connected relation between two regions exist if at least they share one boundary point. This relation can be defined as two points x and y are neighbors, such that a decreasing function of distance between two points can be defined [1].
      $n_{xy} = \frac{1}{1+d_\delta(x,y)}$ or $n_{xy} = \frac{1}{1+exp^{b(\frac{d_\delta(xy)-1}{\delta}-1)}}$ Where d_δ is euclidian distance and δ, b are positive parameters. A fuzzy relation α is said to be fuzzy adjacency if it is reflexive and symmetric, s.t. $U_\alpha(c,d)$ is non increasing function of distance between pints c, d [32].

$$U_\alpha(c,d) = \begin{cases} \frac{1}{1+k_1\sqrt{\sum_{i=1}^{n}(c_i-d_i)^2}} & \text{if } \sum_{i=1}^{n}(c_i-d_i)^2 \leq n \\ 0 & \text{otherwise} \end{cases}$$

where k_1 is a non negative constant and for $2D$ regions $n = 2$ and this relation can be extended for objects.

- A connection relation in RCC theory based on nearness of two objects by using the resemblance relation

$$R_{(\alpha,\beta)}(x,y) = \begin{cases} 1 & \text{if } d(x,y) \leq \alpha \\ 0 & \text{if } d(x,y) > \alpha\beta \\ \frac{\alpha+\beta-d(x,y)}{\beta} & \text{otherwise} \end{cases}$$

is called fuzzy connection relation where X, Y are two objects and α is distance that how long for two objects we consider that object are near to each other and β is a smooth transition from nearness to for-off.

8. Conceptual Neighbor: Two relations between pairs of events are *(conceptual)* neighbors, if they can be directly transformed into one another by continuously deforming (i.e. by shortening, lengthening or moving) events (in topological sense) and a set of relations between pair of events forms a *conceptual* neighborhood if its elements are path connected through *conceptual* neighbor relations[8].
9. Fuzzy directional relations: In numerical methods, the angle between vector $\overrightarrow{ba}$ and x-axis of the coordinate frame is computed, i.e., $\angle(\overrightarrow{ba}, \mathbf{i})$. This angle constitute the domain on which primitive spatial relations are defined, then this value of θ is multiplied by a fuzzy membership value of a specified direction to get the degree of a directional relation.

1.2 Force histograms

In this subsection some basic definitions are given. These definitions are frequently used in this paper. More details can be found in[17].

- ϕ- histogram: ϕ_r is defined as a real valued function

$$\phi : \mathbb{R} \rightarrow \mathbb{R}_+$$

such that

$$\phi_r(y) = \begin{cases} \frac{1}{y^r} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

where r and y are real numbers. This function is used for the treatment of points.

- f- histogram: The function f deals with the segments. It is a real valued function.

$$f : \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$$

This function is defined as

$$f(x_I, y^\theta_{IJ}, z_J) = \int_{x_I+y^\theta_{IJ}}^{x_I+y^\theta_{IJ}+z_J} \int_0^{z_J} \phi(u-w)dw)du$$

x_I represents the length of segment (I) of argument object A, z_J denotes the length of the reference segment (J) of reference object y^θ_{IJ} represents the difference between the minimum value of argument segment and maximum value of reference segment for a fixed θ and variables (x, y, z) are explained in figure 1.

– F- histogram: F function is used to treat the longitudinal section. It is a real valued function.

$$F : \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$$

If there exist a longitudinal section for object A or B for a given line $\Delta_\theta(v)$ (for example in figure1 object A has two segments for a line $\Delta_\theta(v)$), and this histogram is directly associated to f-histograms.

$$F(\theta, A_\theta(v), B_\theta(v)) = \sum_{i=1..n, j=1..m} f(x_{Ii}, y^{\theta}_{IiJj}, z_{Jj})$$

Where n, m represents the number of segments of object A and object B respectively. then the histogram of forces attaches a weight to the argument that object A lies *after* B in direction θ. It is defined as

$$F^{AB}(\theta) = \int_{-\infty}^{+\infty} F(\theta, A_\theta(v), B_\theta(v))dv$$

These are the definitions of Force histograms, directly depending upon the definition of the function ϕ. $F^{AB}(\theta)$ is actually a real valued function.

2 Terminology used for computation of fuzzy Allen relations

This section describes the different terminology used to decompose the space and computation of different terms used in this paper.

2.1 Oriented lines, segments and longitudinal sections

A and B be two spatial objects and $(v, \theta) \in \mathbb{R}$, where v is any real number and $\theta \in [0, 2\pi]$. $\Delta_\theta(v)$ is an oriented line at orientation angle θ. $A \cap \Delta_\theta(v)$ is the intersection of object A and oriented line $\Delta_\theta(v)$. It is denoted by $A_\theta(v)$, called segment of object A and length of its projection interval on x-axis is x. Similarly for object B where $B \cap \Delta_\theta(v) = B_\theta(v)$ is segment and length of its projection interval on x-axis is z. y is the difference between the minimum of projection points of $A \cap \Delta_\theta(v)$ and maximum value of projection points of $B \cap \Delta_\theta(v)$(for details[20]). In case of polygonal object approximation (x, y, z) can be calculated from intersecting points of line and object boundary, oriented lines are considered which passes through at least one vertex of two polygons. If there exist more than one segment, then it is called longitudinal section as in case of $A_\theta(v)$ in figure 1.

In this paper all the 180 directions are considered with an angle increment of one degree and lines are drawn by 2d Bresenham digital line algorithm. For drawing the line simple mathematical formula (slop intercept formula)

$$y = mx + C$$

is used where $m = tan^{-1}(\theta)$ and $C = v$ is the intercept on y-axis. A polygonal object approximation is taken and lines passing through polygon vertices are taken into account. Segments are computed and all pairs of segments can be treated simultaneously. Fuzzy Allen relations are computed for each segment.

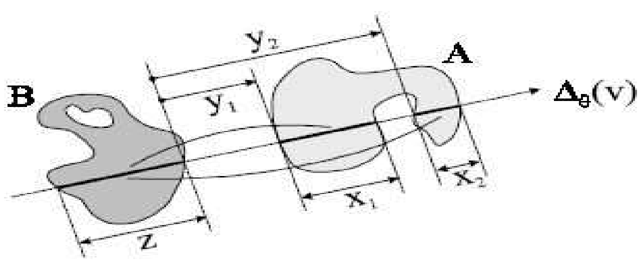


Fig. 1: Oriented line $\Delta_\theta(v)$, segment as in case of object B, longitudinal section as in case of object A[20].

2.2 1D Allen relations in space

Allen[13] introduced the well-known 13 jointly exhaustive and pairwise disjoint (JEPD) interval relations based on temporal interval algebra. These relations are arranged as $\mathcal{A} = \{<, m, o, s, f, d, eq, d_i, f_i, s_i, o_i, m_i, >\}$ with meanings *before, meet, overlap, start, finish, during, equal, during_by, finish_by, start_by, overlap_by, meet_by, and after*. All the Allen relations in space are conceptually illustrated in figure (2). These relations have a rich support for the topological and directional relations. In the neighborhood

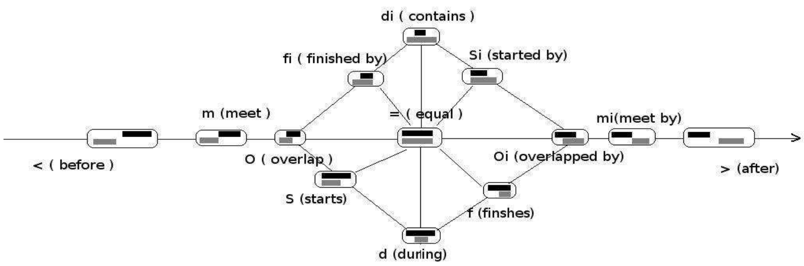


Fig. 2: Black segment represents the reference object and gray segment represents argument object

graph of Allen relations, three paths can be found following the distance transformation (because we consider solid objects conserving size and shape during movement, due to this reason other possible paths are ignored). Depending upon the neighborhood graph of Allen relations, inverse of an Allen relation can be divided into two categories.

1. Inverse when objects interchange: There are three continuous paths in this graph between object pair, e.g.
   - $<\rightarrow m \rightarrow o \rightarrow f_i \rightarrow d_i, \rightarrow s_i \rightarrow o_i \rightarrow m_i \rightarrow>$
   - $<\rightarrow m \rightarrow o \rightarrow = \rightarrow o_i \rightarrow m_i \rightarrow>$
   - $<\rightarrow m \rightarrow o \rightarrow s \rightarrow d, \rightarrow f \rightarrow o_i \rightarrow m_i \rightarrow>$

   When objects are interchanged, A becomes reference object and B becomes argument object, then order of relation is also changed, i.e., when in one direction they move from one edge to the other edge through relations
   $<\rightarrow m \rightarrow o \rightarrow f_i \rightarrow d_i, \rightarrow s_i \rightarrow o_i \rightarrow m_i \rightarrow>$

after change the object role, they follow the other path, i.e. $>\rightarrow m_i \rightarrow o_i \rightarrow f \rightarrow d, \rightarrow S \rightarrow o \rightarrow m \rightarrow <$. This change in path shows that Allen relations $\{<, m, o, s, f, d, =\}$ and their inverses according to the object commutativity are $\{>, m_i, o_i, s_i, f_i, d_i =\}$, in this case there are seven atomic Allen relations.
2. Inverse about directions: When the direction is reversed Allen relations and their inverses also change, in this case there are eight Allen relations and their inverse. These inverses are also called reorientation of a relation.we can write $\mathcal{A}_1 = \{<, m, o, s, d, f_i, d_i, =\}$ and their inverses as $\mathcal{A}_2 = \{>, m_i, o_i, f, d, s_i, d_i, =\}$. This shows that the relations $d, =, d_i$ have their own reorientation and s relation is reorientation of f relation similarly s_i relation reorientation of f_i relation.

Eight relations are possible combination of 8 independent Allen relations in 1D. These relations and their reorientation show that the whole 2D space can be explored with the help of 1D Allen relations using the oriented lines varying from $(0, \pi)$.

2.3 Fuzzification of Allen relations

Fuzzy Allen relations are used to represent the fuzzy topological relations where vagueness or fuzziness is represented at the relations level. The different approaches are used to fuzzify the Allen temporal relations such as in [28]. Human defined variables are used in fuzzification for temporal domain, for the qualitative aspects of temporal knowledge and qualitative temporal reasoning processes. There is a homeomorphism between the Allen's temporal relations and $1D$ spatial relations, due to this homeomorphism, Allen relations are also used for extracting the combined directional and topological relations information. Fuzzification process of Allen relations do not depend upon particular choice of fuzzy membership function. Trapezoidal membership function is used due to flexibility in shape change. Let $r(I, J)$ be an Allen relation between segments I and J where $I \in A$(argument object) and $J \in B$ (reference object), r' is the distance between $r(I, J)$ and it's conceptional neighborhood. We consider a fuzzy membership function $\mu : r' \longrightarrow [0, 1]$. The fuzzy Allen relations defined by Matsakis and Nikitenko [20] are:

- $f_<(I, J) = \mu_{(-\infty,-\infty,-b-3a/2,-b-a)}(y)$,
- $f_>(I, J) = \mu_{(0,a/2,\infty,\infty)}(y)$
- $f_m(I, J) = \mu_{(-b-3a/2,-b-a,-b-a,-b-a/2)}(y)$,
- $f_{mi}(I, J) = \mu_{(-a/2,0,0,a/2)}(y)$
- $f_o(I, J) = \mu_{(-b-a,-b-a/2,-b-a/2,b)}(y)$,
- $f_{oi}(I, J) = \mu_{(-a,-a/2,-a/2,0)}(y)$
- $f_f(I, J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x))$
- $f_{fi}(I, J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$
- $f_s(I, J) = min(\mu_{-b-a/2,-b,-b,-b+a/2}(y), \mu_{(-\infty,-\infty,-b,-(b+a)/2)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)$
- $f_{si}(I, J) = min(\mu_{(-(b+a)/2,-a,-a,+\infty)}(y), \mu_{(-3a/2,-a,-a,-a/2)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$
- $f_d(I, J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(-\infty,-\infty,z/2,z)}(x)$
- $f_{di}(I, J) = min(\mu_{(-b,-b+a/2,-3a/2,-a)}(y), \mu_{(z,2z,+\infty,+\infty)}(x)$

where $a = min(x, z), b = max(x, z)$ and x is the length of segment (I) of argument object A and z is the length of segment (J) of reference object B and y is the difference between the minimum value of projection points of $A_\theta(v)$ and maximum value of projection points of $B_\theta(v)$. Most of relations are defined by one membership like $d(during)$, $d_i(during\_by)$, f $(finish)$, f_i $(finished\_by)$. In fuzzy set theory, sum of all the relations is one, this gives the definition for fuzzy relation *equal*. Fuzzy Allen relations are not Jointly Exhaustive and Pairwise Disjoint (JEPD) because there exist at least two relations between two spatial objects. These relations are represented as $f(x, y, z) = (> m_i\ o_i\ f\ s_i\ d\ eq\ d_i\ s\ f_i\ o\ m <)^t$. All these equations assign a numeric value to a spatial relation.

2.4 Treatment of longitudinal section

During the decomposition process of an object into segments, there can be multiple segments for a line depending on object shape and boundary that is called longitudinal section. Different segments of a longitudinal section are at a certain distance and these distances might affect end results. In polygonal object approximation, for each segment fuzzy Allen relation is a member of fuzzy set, fuzzy *T-norms*, *T-conorms* and fuzzy weighted operators are used for fuzzy integration of available information, here for simplicity only *T-conorm* is used.

$$\mu_{(OR)}(u) = max(\mu_{(A)}(u), \mu_{(B)}(u))$$

When fuzzy operator *OR* is used, only one fuzzy value contributes for the resultant value that is *maximum*, in this case each Allen relation has a fuzzy grade and objective is to accumulate the best available information. The choice for this operator is discussed in [23]. Suppose that longitudinal section of object A has two segments such that $x = x_1 + x_2$ where x_1 is the length of first segment and x_2 is the length of second segment and z is length of longitudinal section. Let $f(x_1, y_1, z)$ defines value of fuzzy Allen relations with the first segment and $f(x_2, y_2, z)$ represents value of fuzzy Allen relations with the second segment where y_1 and y_2 represent distances between object B and two segments of A. Now fuzzy *OR* operator is used to get consequent information obtained from two sets of fuzzy Allen relations, here we consider the different cases which can arise, obviously the relations between two segments depend upon the value of y.

- **C-1** Let us consider that $y_1 > \frac{a}{2}$ and $y_2 \leq \frac{a}{2}$. For $y > \frac{a}{2}$, fuzzy Allen relation behaves like a crisp relation, there exist only $>$ relation and for $y \leq \frac{a}{2}$ the relations are divided between *meet_by* (mi) and *after* ($>$) (definition of $mi, >$ in section 2.3). Let us consider $f_{mi} = 0.7, f_{>} = 0.3$, then the relation vectors will be

$$f(x_1, y_1, z) = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

  and

$$f(x_2, y_2, z) = (.3, .7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

  (Because a relations vector is $f(x, y, z) = (>, m_i, o_i, f, s_i, d, eq, d_i, s, f_i, o, m, <)^t$). Then fuzzy OR (fuzzy Sum operator) will produce the following results

$$F_{OR}(x, y, z) = (1, .7, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

- **C-2** Let us consider that $y_1 > \frac{a}{2}$ and $-\frac{a}{2} \leq y_2 \leq 0$. When $y > \frac{a}{2}$ then fuzzy Allen relation behaves like a crisp relation, there exist only $>$ relation and for $y \leq \frac{a}{2}$ the relations are divided between the oi, mi (definition of $oiandmi$ in section 2.3). In such a case, let us consider $f_{mi} = 0.5, f_{oi} = 0.5$, then the relation vectors will be

$$f(x_1, y_1, z) = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

and

$$f(x_2, y_2, z) = (0, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

will produce the following results

$$F_{OR}(x, y, z) = (1, 0.5, 0.5, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

– **C-3** Let us consider that $y_1 > \frac{a}{2}$ and $-b - \frac{3a}{2} \leq y_2 \leq -b - a$. When $y > \frac{a}{2}$ then fuzzy Allen relation behaves like a crisp relation, there exist only $>$ relation and for $-b - \frac{3a}{2} \leq y_2 \leq -b - a$ the relations are divided between the $< andm$ (definition of $<, m$ in section 2.3). In such a case, let us consider $f_m = 0.2, f_< = 0.8$, then the relation vectors will be

$$f(x_1, y_1, z) = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)^t$$

and

$$f(x_2, y_2, z) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.2, 0.8)^t$$

will produce the following results

$$F_{OR}(x, y, z) = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.2, 0.8)^t$$

2.5 Normalized fuzzy histogram of Allen relations

Histogram of fuzzy Allen relations represents the total area of subregions of A and B that are facing each other in given direction θ. Mathematically it can be written as [24]

$$F_r^{AB}(\theta) = \int_{-\infty}^{+\infty} F_r(\theta, A_\theta(v), B_\theta(v)dv$$

and

$$F_r(\theta, A_\theta(v), B_\theta(v))dv = r(I_k, J_k)$$

In discrete space this integral can be written as sum of the surface areas.

$$F_r^{AB}(\theta) = (X + Z) \sum_{k=1}^{n} r(I_k, J_k)$$

where Z is the area of reference object and X is area of augmented object in direction θ, n is total number of segments to be treated and $r(I_k, J_k)$ is an Allen relation for segments (I_k, J_k) and $k = 1, 2, .., n$. These histograms can easily be normalized by dividing all Allen relations by sum of all the Allen relations for every θ. It is represented by $\lceil F_r^{AB}(\theta) \rfloor$ where $r \in \mathcal{A}$. $\lceil F_r^{AB}(\theta) \rfloor = \frac{F_r^{AB}(\theta)}{\sum_{\rho \in A} F_\rho^{AB}(\theta)}$. Each fuzzy Allen relation has its own weight in a specified direction θ. These normalized weights can be used to define the quantitative fuzzy directions.

3 Topological and directional relations

This section consists of five subsections. In first subsection fuzzy membership functions for directional relations are depicted, second subsection describes the possible combination of 1D Allen relations, third subsection discusses the relationship between 1D Allen and topological relations. Their representation is given in fourth subsection and in final subsection proposed algorithm for finding 2D topological relation is given.

3.1 Fuzzy membership function for directional relations

The system of equations defined in section (2.3) assign a numeric value to a spatial relation in a direction θ. To asses the qualitative directions, directional fuzzy sets are used. A number of fuzzy membership functions has been proposed for assessing the directional relations, these functions include the trigonometric functions, triangular function, trapezium membership function and by means of favorable and unfavorable forces, In this paper we prefer to use trigonometric functions which are easy to implement. The function $f(\theta)$ to model the direction *Right_of* for four directional system, should have conditions, $f(+\frac{\pi}{2}) = 0 = f(-\frac{\pi}{2})$ and $f(0) = 1$ and this function should be increasing in the interval $(-\frac{\pi}{2}, 0)$ and decreasing in the interval $(0, \frac{\pi}{2})$. At $\frac{\pi}{4}$ both relations *Above* or *North* and *Right_ of* have equal values and similarly at $\frac{-\pi}{4}$ both relations *Below* or *South* and *Right_ of* have equal values. $Cos^2(\theta) = Sin^2(\theta) = \frac{1}{2}$ at $\pm\frac{\pi}{4}$ are the only choices, here, the third condition is more stronger and limits the choice of functions, this shows that every even power could not be used to assess the directional relations. To formulate the eight directions, the straightway process is too narrow the interval, i.e., $f(+\frac{\pi}{4}) = 0 = f(-\frac{\pi}{4})$ and $f(0) = 1$ for relation *Right_of.* For this purpose double angle trigonometric functions are used which obeys all the above cited conditions. Directions are represented as $\{E, NE, N, NW, W, SW, S, SE\}$ with meanings *East, North_East, North, North_West, West, South_West, South* and *South_East.* To assess these fuzzy directional relations, two trigonometric functions $cos^2 2\theta$ and $Sin^2 2\theta$ are used for even and odd directions ($\{E, N, W, S\}$ and $\{NE, NW, SW, SE\}$). The angle distribution is taken to the half plane so opposite Allen relations are used to define the opposite directions except the direction East and West where union of both relations are used. This exception is used due to the reorientation of relations and domain of *East* and *West* relations lie in different Allen relations (domain of *East* directional relation is $[\frac{-\pi}{4}, \frac{\pi}{4}]$ and $f_<(\theta) = f_>(\theta+\pi)$, using this combination of relations results in the decrease of time complexity due to angle distribution from $[0, \pi]$). Mathematically these relations can be written as

- $f_E = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_{r_2} \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_{r_1} \times cos^2(2\theta)$
- $f_W = \sum_{\theta=0}^{\frac{\pi}{4}} \mathcal{A}_{r_1} \times cos^2(2\theta) + \sum_{\theta=\frac{3\pi}{4}}^{\pi} \mathcal{A}_{r_2} \times cos^2(2\theta)$
- $f_N = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \mathcal{A}_{r_2} \times cos^2(2\theta)$
- $f_S = \sum_{\theta=\frac{\pi}{4}}^{\frac{3\pi}{4}} \mathcal{A}_{r_1} \times cos^2(2\theta)$
- $f_{NE} = \sum_{\theta=0}^{\frac{\pi}{2}} \mathcal{A}_{r_2} \times sin^2(2\theta)$
- $f_{NW} = \sum_{\theta=\frac{\pi}{2}}^{\pi} \mathcal{A}_{r_2} \times sin^2(2\theta)$
- $f_{SW} = \sum_{\theta=0}^{\frac{\pi}{2}} \mathcal{A}_{r_1} \times sin^2(2\theta)$
- $f_{SE} = \sum_{\theta=\frac{\pi}{2}}^{\pi} \mathcal{A}_{r_1} \times sin^2(2\theta)$

Where $\mathcal{A}_{r_i} \in \mathcal{A}_i, i = 1, 2$ given in section (2.2) are the fuzzy Allen relations with $\mathcal{A}_{r_2}$ is the reorientation of $\mathcal{A}_{r_1}$ and f is a topological relation and these topological relations in *RCC* theory are written as $\{DC, EC, PO, TPP, NTPP, TPPI, NTPPI, EQ\}$ with meaning *Disconnected, Externally connected, Partially overlap, Tangent proper part, Non Tangent proper part, Tangent proper part Inverse, Non Tangent proper part Inverse* and *Equal.* In this way the relations are manipulated as a (8×8) matrix where row hold the topological relations and columns have the qualitative directional aspects of the 2D scene information. Relations are expressed in numeric values where each value represents the specific topological relation in that direction.

3.2 Relationship between topological and Allen relations

Simple $1D$ definitions cannot be extended and applied directly to a $2D$ space, some assumptions have to adopt. The RCC theory does not impose conditions on regions, each time an interpretation is given that how the regions are assumed and relations are explained based on this interpretation. In the RCC regions are primitives, but the definition of C"connection" itself depends upon the points. i. e., if d is defined as pseudo metric space in the regular closed topological space, then

$$C(A,B) = \text{A is connected to B} \Leftrightarrow \{d(a,b) = 0, a \in A, b \in B\}$$

S. Schockaert et al. [26] define the fuzzy connection relation based on closeness.[1] In such case, number of topological relations don't change between objects only degree is associated to a topological relation.

In point set topological approach the topological primitives of an object (interior, boundary and exterior) participate and relations are developed based on the empty ($\emptyset$) or non empty ($\neg\emptyset$) intersection of topological parts of objects. These relations don't depend upon any algebraic function as a result the distance based approaches cannot be introduced in this theory. In case of merging the topological and directional relations some of the topological relations depend upon a finite direction and limited set of points such as *EC, PO, TPP, TPPI* (equivalently called *Meet, Covers* and *Covered_by* in point set topology). Relations like *NTPP, NTPPI, EQ* hold if the relation holds in all directions. We have to justify that a particular topological relation exists in the 2D space or not. The topological relations which exist in a finite directions, they share with another topological relation existing in another direction. The relations which exist in all directions, they share directional relations. Temporal Allen relations represent the eight topological relations in spatial domain $\mathbb{R}$, these relations can be extended to spatial domain $\mathbb{R}^2$ through the logical implication. These relations are defined as:

1. Disjoint $D(A,B)$: In point set topology, disjoint topological relation is defined as: *two objects don't share the boundary and interior and they have non empty intersection of exteriors.*
   Fuzzy topological relations based on distance are not defined in this theory. In RCC theory two objects are disjoint when there doesn't exist a connection relation (e.g., $Disjoint(A,B) \Rightarrow \neg C(A,B)$). A fuzzy connection relation based on nearness is defined by using the *resemblance relation*, while defining the fuzzy semantics, it is stated that the objects are at a certain distance and resemblance relation is zero degree then objects will be disjoint. In this system, functions f_m and f_{mi} plays the same role as the resemblance relation $R_{\alpha,\beta}(A,B)$ with variable $\alpha = 0$ for the $1D$ interval. In such a case both functions, $f_>$ and $f_<$, capture the semantics of $\neg R_{\alpha,\beta}(A,B)$, representing disjoint topological relation between two intervals on a

[1] S. Schockaert et al. [26] considered all of three,i. e., relation R, and both objects (A, B) are fuzzy, but the connection will remain fuzzy if we consider the resemblance relation R as fuzzy relation and A, B as crisp regions.

real line, along with additional information that argument interval either *before* or *after* the reference interval. In implication logic

$$\text{(A,B) are disjoint} \Leftrightarrow \{\vee\theta \in [0,\pi] \text{ (A,B) are disjoint}\} \tag{1}$$

where A, B are $2D$ objects or equivalently this can be stated as

$$\text{(A,B) are not disjoint} \Leftrightarrow \{\exists \text{at least one } \theta \in [0,\pi] \text{ s.t. (A, B) are not disjoint}\} \tag{2}$$

Now follow the rule 1, we claim that objects have fuzzy disjoint topological relation in a direction θ if all parallel segments are topologically disjoint in direction θ. A fuzzy disjoint topological relation exists in 2D domain if it exists in all directions, i. e., all the $f_<$ or $f_>$ behave like crisp functions and have identity value along all the oriented lines in $[0,\pi]$ and all the other functions have zero values. These relations are explained in table1.

2. $Meet(A,B)$:According to the topological view point, two objects have a $Meet$ (equivalently called EC in RCC theory and *adjacency relation* in literature) topological relation when they share at least one boundary point and they don't share the interior regions of two objects, i.e., regions are not internally connected. In our system of defining the topological relations, two functions are introduced f_m and f_{mi}. Both functions play the similar role like the resemblance relation defined by S. Schockaert et al. in [26], where the degree of closeness is one, when the intervals share a common point and smooth transition from *closeness to apart from* depends upon the size of the smaller interval. Both functions capture some additional semantics regarding position of the interval, either the interval is after or before the reference interval. To make the sense in the $2D$ scene, overall EC relation holds if at least one f_m or f_{mi} has some non zero value for any $\theta \in [0,\pi]$ and all the other directions have the *Disjoint* topological relation. These relations are explained in table2.

3. $PO(A,B)$: Partially overlap relation in topology (some time called simply overlap) exists when two objects share their interior region, in such a case their boundaries intersect at least from two points. In $\mathbb{R}$ the functions f_o and f_{oi} capture the overlap semantics on the interval along with the directional information. When $2D$ objects are decomposed into $1D$ segments, each pair of segments may have the different topological relations in different directions, e.g., the object's segments in direction θ_i may have overlap relation while in direction θ_j may have *meet* topological relation and in direction θ_k both the segments may be disjoint, where $i \neq j \neq k \wedge \theta_i, \theta_j, \theta_k \in [0,\pi]$. These relations are explained in table3.

4. $TPP(A,B)$ and $TPPI(A,B)$: $TPP(A,B)$ topological relation holds in $2D$ space when $A \subset B$ and they share a common point on the boundary. In $1D$ spaces the relation f_s or f_f shares the same semantics, if f is a crisp relation. In case of f is a fuzzy relation they represent the fuzzy semantics. When a $2D$ object is decomposed into $1D$ segments, in a limited number of directions they have the relation f_s or f_f while in other directions object is contained in the container, d (*during*) Allen relation exists. Similarly in case of $TPPI$ topological relation, in

some directions the $1D$ segments share the common boundary point and f_{si} and f_{fi} fuzzy Allen relations exist while in other directions the d_i Allen relation exists. In our system of defining the topological and directional relations information, TPP ($TPPI$) relation exist in some direction while in other directions $NTPP$ (respectively $NTPPI$) relation exists. These relations are explained in tables5,6.

5. $NTPP(A, B)$: This is also called *INSIDE* in point set topological methods. This topological relation holds when argument object is contained in reference object ($A \subset B$) and both objects don't share the boundary. It means that object is contained in the container object in all directions. When $1D$ Allen relations are applied to the spatial domain, the relation d captures the same semantics. If each segment of an argument object is contained in the segment of a reference object in all directions, then argument object is topologically inside the reference object and they don't share boundary, while representing the fuzzy semantics, it is observed that this relation holds when there is a certain distance between the boundaries of both objects. Situations having such relations are explained in first row in table 4 of section 5.4.

6. $NTPPI(A, B)$: This topological relation holds when the container is an argument object and reference object is contained in the argument object, e. g., object B is a reference and A is an argument object and object B is contained in object A ($B \subset A$), both objects don't share boundary points. It means that the object is contained in the container object in all directions. When the $1D$ relations are applied to the spatial domain, the relation d_i captures the same semantics. If each segment of an argument object is contained in the segment of a reference object and this relation holds in all the directions, then there exist topological relation $NTPPI$ for a pair of $2D$ objects. As explained in second row in table 4 of section 5.4.

7. Equal (A,B): Two objects A and B are equal if they share the interior, exterior and boundary, but interior of the one object doesn't intersect with the exterior and boundary of the other object. Semantically both objects have the same interior, boundary and exterior. In *RCC* system equal topological relations are defined as $EQ(A, B) \equiv_{def} P(A, B) \wedge P(B, A)$. Geometrically two regions are called equal when both objects seem equal in all direction. Function $f_{=}$ in our system captures the semantics equal if two intervals are equal. When this relation is applied to $1D$ segments of $2D$ objects, segments of both objects must be equal in all directions. These results that degree of EQ topological relation distributed equally in all directions. As explained in third
row in table 4 of section 5.4.

3.3 Algorithm

Numerical values for a relation are stored in a matrix called fuzzy matrix of relations. The relations are manipulated in (8×8) matrix where rows show topological relations and columns show the directional distribution of each topological relation.

Each entity of this matrix represents the percentage surface area of two objects having a topological relation in a specific direction. C(i,j) represents the i^{th} topological relation in j^{th}. Rows and columns of the representation matrix are explained below[2].

Explanation of rows and columns in representation matrix								
i	1	2	3	4	5	6	7	8
Topological relation	*DC*	*EC*	PO	*TPP*	*NTPP*	*TPPI*	*NTPPI*	*EQ*
j	1	2	3	4	5	6	7	8
Directional relation	E	NE	N	NW	W	SW	S	SE

These relations are not jointly exhaustive and pairwise disjoint (JEPD), i.e., there exist multiple relations between $1D$ segments of $2D$ objects. To approximate 2D topological and directional relation, first extract topological relation then proceed for directional relation. Since the fuzzy directional relations are distributed over multiple directions so we choose the maximum numerical value of directional relations. *Overlap* topological relation has the preference over other topological relations, due to this reason extraction method must be started from overlap relation. Proposed algorithm for 2D topological relation from 1D is given below.

Algorithm 1 Algorithm for approximating 2D fuzzy topological relations with directional contents

```
Require: Fuzzy matrix of topological and directional relations
  for i, j = 1 to 8 do
    if C(3, j) ≠ 0 then
      topological relation is overlap and directional relation is max_j(C(3, j), C(4, j), C(6, j))
    else if C(4, j) ≠ 0 then
      topological relation is TPP and directional relation is max_j(C(4, j))
    else if C(6, j) ≠ 0 then
      topological relation is TPPI and directional relation is max_j(C(6, j))
    else if all C(5, :) are approximately equal for all j then
      topological relation will be NTPP
    else if all C(7, :) are approximately equal for all j then
      topological relation is NTPPI
    else if all C(8, :) are approximately equal for all j then
      topological relation will be EQ
    else if C(i, j) = 0 ∨ i ≥ 3 then
      topological relation is meet EC and directional relation is max_j(C(2, j))
    else
      topological relation is Disjoint and directional relation is max_j(C(1, j))
    end if
  end for
Ensure: Fuzzy topological and directional relation
```

4 Neighborhood Graph

In figure 3, it is shown that every point of a neighborhood graph has eight possible direction edges, here relations are represented by a pair (α, β) where α represents the topological relation and β represents the orientation relation. In this neighborhood

[2] It is only representation and rows and columns explain how the relations are labeled

graph each node has eight edges, circular edges represent the directional neighbors and radial edges represents the topological neighbors and diagonal edges represent both topological and directional neighbors.

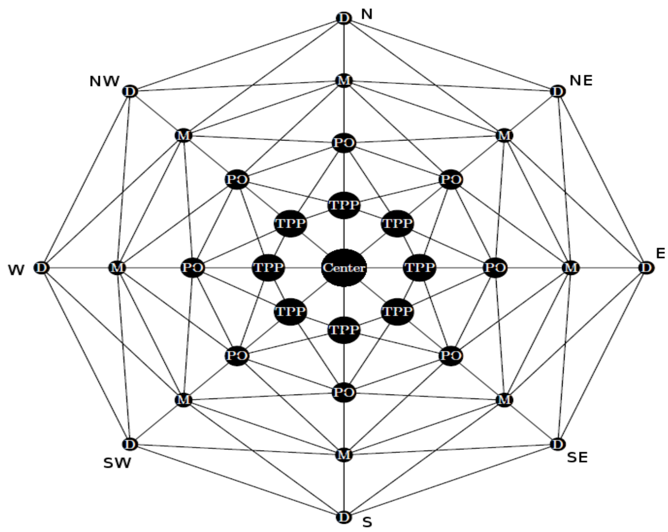


Fig. 3: Neighborhood graph in the system of combined topological and directional relations (CTD method) where center represents the NTPP, NTPPI, and EQ. For relation NTPPI, inner shell represnts the relation NTPP despite of TPP, in case of EQ relation inner most shell disappears. Rigid objects are considered, only distance transformation are considered and size transformation are not considered.

5 Experiments

At first step fuzzy Allen relations are computed for each segment then directional distribution is evaluated by a fuzzy membership function. When the directional relations are evaluated by fuzzy techniques, they can share more than one direction and their directional relations are represented by a degree. This degree of truth value represents the percentage of object which lies in that direction. The relations are manipulated in 8×8 matrix where rows show the topological relations and columns show the directional distribution of each topological relation. Each entity of this matrix represents the percentage surface area of two objects having a topological relation in a specific direction. C (i,j) represents the i^{th} topological relation in j^{th} direction. Values in each cell represents the strength of the relation between each pair of the objects. Throughout this paper reference object B is represented by dark grey color and light grey object represents the argument object A.

5.1 Fuzzy disjoint (DC) topological relation

In this section the fuzzy disjoint topological relations in different directions are considered. As in table 1, First column represents the object spatial position and second

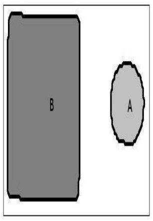
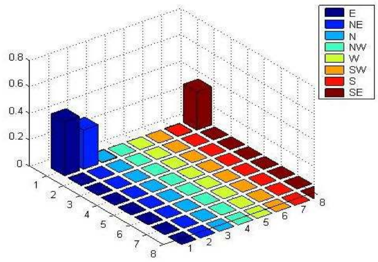
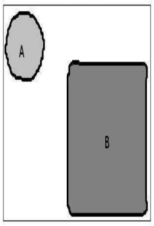
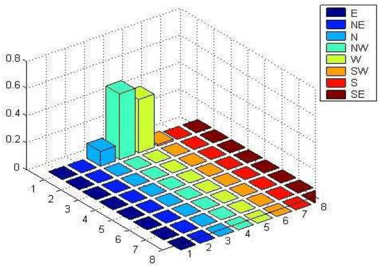
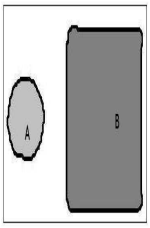
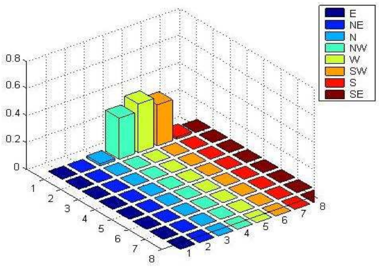
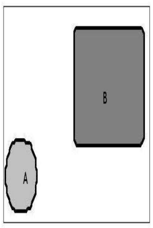
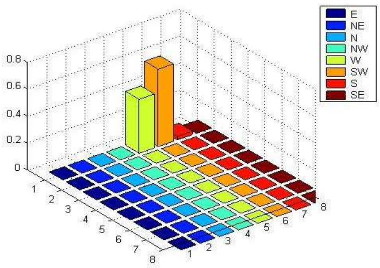
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation= Disjoint Direction= East
		Topological relation= Disjoint Direction= North_West
		Topological relation= Disjoint Direction= West
		Topological relation= Disjoint Direction= South_West

Table 1: Topological relation D

column represents the overall 2D topological and directional relations and in third column the generated topological and directional relation by algorithm(described in section 1) are depicted. As object A changes its position, its directional relation also changes, in first row of table 1, argument object lies in E to the reference object B. The Algorithm generates the topological relation *Disjoint* and directional relation *East*. In second row, argument object lies in direction $North\_West$, it histogram representation shows that in the $North\_West$ direction, it has the maximum membership value, as a result, algorithm allocates it a directional relation *North_west*.

5.2 Fuzzy meet (EC) topological relation

Fuzzy meet EC relations in topology exist when the exactly meeting or very close to each other and it seems that they are sharing the boundary. The table 2 represents the fuzzy *meet* topological relation (EC). First column shows the object locations at different orientations of argument object A with respect to the reference object B. First rows shows that the argument object A touches the object B from the *East* direction (first column). Second column shows its histogram representation of its relations where the relations are shared between the *Disjoint* and *Externally_Connected* (second column) and their 2D topological relation generated by the algorithm. Similarly second row represents the argument object touches the reference object from *North* direction, for this object pair algorithm produces the result that EC topological relation with *North* directional relation. In the third row argument object seems touching from the north direction, hence the output of algorithm shows that EC topological relation holds with directional relation *North_West*.

5.3 Fuzzy overlap (PO) topological relation

In this example we consider the overlapping objects in different directions. The object relative position, topological and directional relations and the topological and directional relations generated by the algorithm are described in table 3.

In the first column, object pairs are represented having the topological relation *Partially_Overlap* in different directions, second column represents topological and directional relations in a histogram representation. Third column represents the results generated by algorithm. As soon as object changes their position, topological and directional relations matrix also changes.

5.4 Fuzzy *NTPP, NTPPI* and *EQ* topological relations

In these examples we consider the all those topological relations which must exist in all the directions.

Here it is explained that a single topological relation must be held in all the directions for reference the object pairs represented in first column of the table 4. Argument object A lies inside the reference object B as a result the $NTPP$ relations holds equally in all directions, these results are represented in first row of the table, here the algorithm generates the directional relation *All* which means that this relation holds in all directions. In second row reference object B lies inside the argument object, as a result inverse topological relations hold in all directions. Third rows shows the situation, when both objects are equal in size, thus fuzzy equal relation holds if both objects have equal size in all directions.

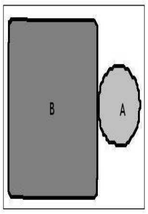
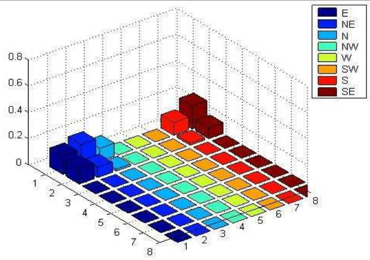
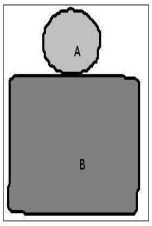
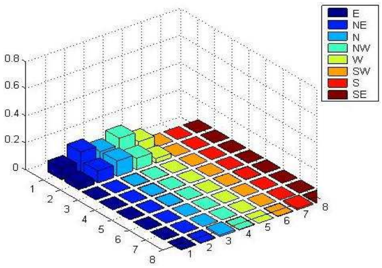
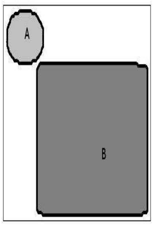
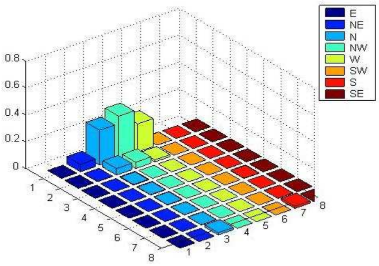
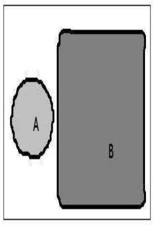
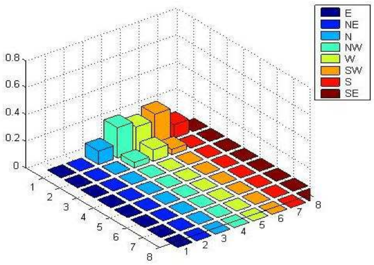
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation= EC Direction= East
		Topological relation= EC Direction= North
		Topological relation= EC Direction= North_West
		Topological relation= EC Direction= West

Table 2: Topological relation EC

5.5 Fuzzy *TPP* topological relation

In crisp topological relations, this relation holds when the argument object lies inside the reference object and share the boundary with the reference object. In fuzzy relations, this relation (*TPP*) holds when the argument object lies inside the reference object near the edge. In this method an entity in the matrix represents the degree of topological relation in a particular direction, as a result this relation holds in a particu-

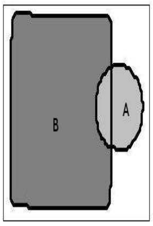
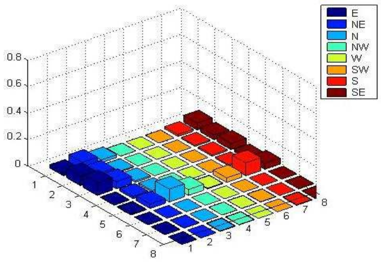
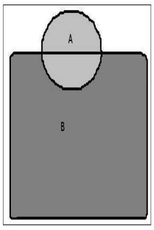
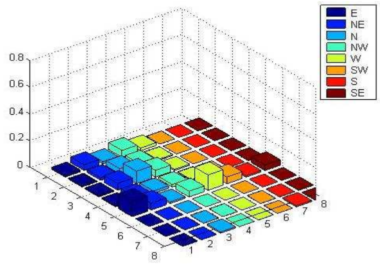
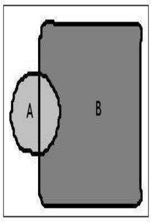
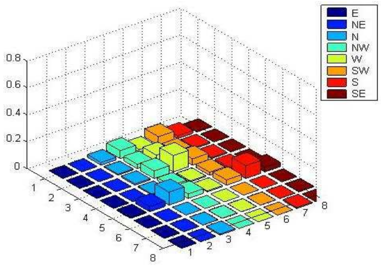
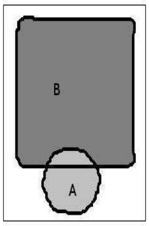
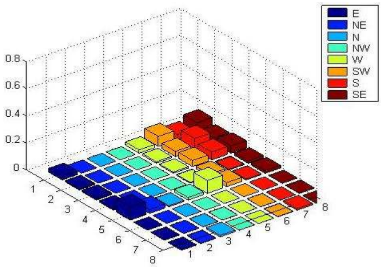
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation= PO Direction= East
		Topological relation= PO Direction= North
		Topological relation= PO Direction= West
		Topological relation= PO Direction= South

Table 3: Object pairs with *PO* topological relation

lar direction along with the other fuzzy topological *NTPP* relation in other directions, for example in first row of the table 5 where object lies near the eastern edge of reference object, there relations show that highest value of topological relation *TPP* exists in direction East while in all other directions fuzzy topological relation *NTPP* holds.

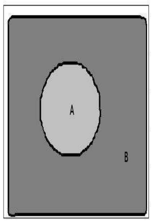
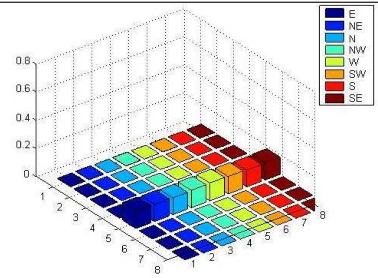
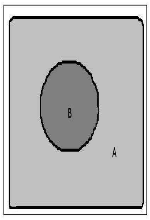
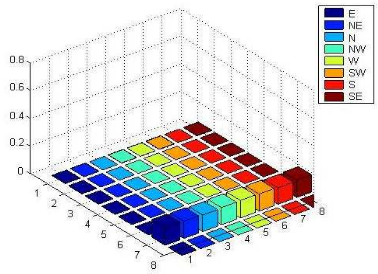
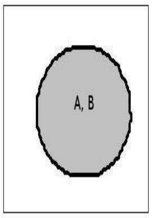
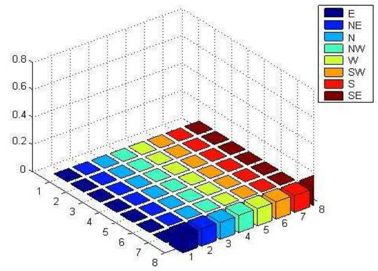
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation= NTPP Direction= All
		Topological relation= NTPPI Direction= All
		Topological relation= EQ Direction= All

Table 4: Topological relations NTPP, NTPPI and EQ

5.6 Fuzzy *TPPI* topological relation

In this example object B reference object is considered inside the argument object A. The object pairs are shown in first column of the table 6. Second column shows the histogram representation of relations and third column shows the results produced by the algorithm. Obviously it is an inverse relation as a result visually reference object seems to be in opposite direction of the directional relation. In first row of the table, visually reference object lies near the West edge of the argument object, but its relation is East, this is due to the inverse topological relation. When the objects commute, the topological and directional relations become inverse, for example consider the object pairs in third row of table 5 and first row of table 6, both represents the same object pair, when the objects commute, both the topological and directional relations become inverse to each other. Similarly for the other object pairs in table 5 and table 6, same object is used to represent the object pair when objects commute the topological and directional relations become inverse.

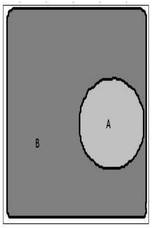
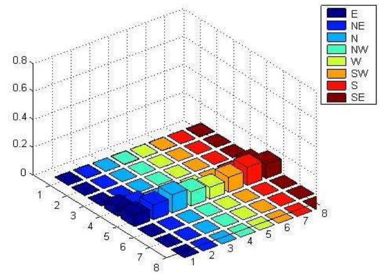
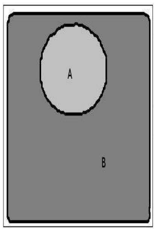
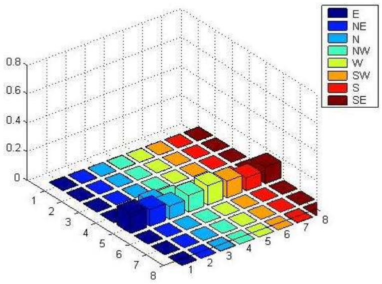
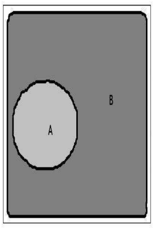
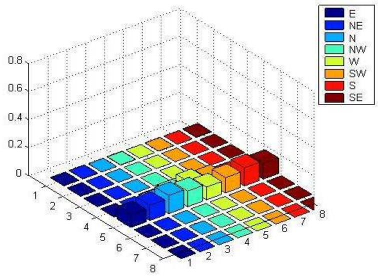
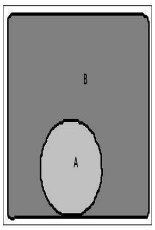
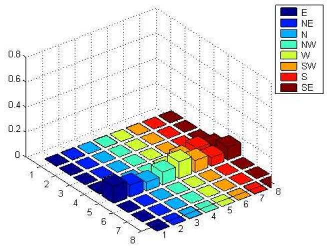
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation = TPP Direction= East
		Topological relation= TPP Direction= North
		Topological relation= TPP Direction= West
		Topological relation= TPP Direction= South

Table 5: Object pairs with *TPP* topological and their directional relations

6 Conclusion and future work

In this paper a new method for finding fuzzy topological and directional relations was proposed and all the topological relations are generated by using fuzzy Allen relations and directions are evaluated with the help of trigonometric functions. This method

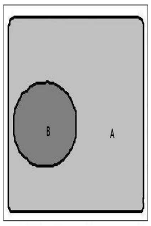
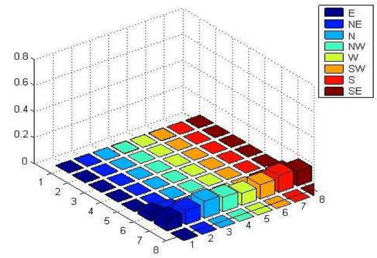
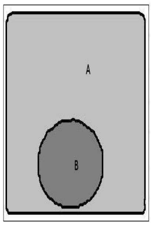
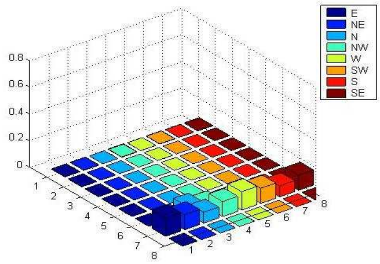
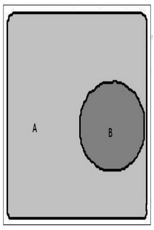
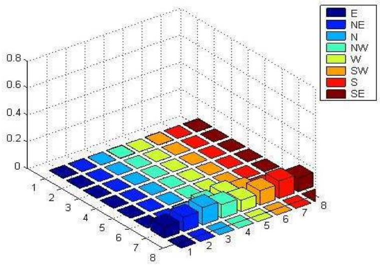
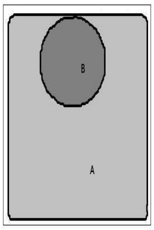
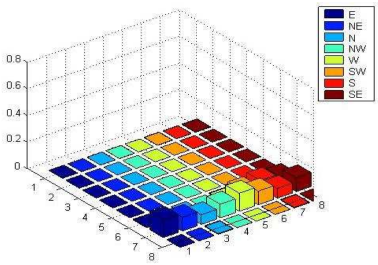
Object pairs	Matrix rep. of relations	Algo. Output
		Topological relation= TPPI Direction= East
		Topological relation= TPPI Direction= North
		Topological relation= TPPI Direction= West
		Topological relation= TPPI Direction= South

Table 6: Object pairs with *TPPI* topological and their directional relations

deals with fuzziness at two levels, fuzziness in the topological relations due to their geometrical description and fuzziness in directional relations. This method also deals the objects with holes or convex objects when such objects are decomposed into 1D segments, there exist the longitudinal section and a method to deal with such a situation is given in section 2.4. It is a numerical description of relative position of objects and value in each cell represents the strength of the relation between the object pair. It deals with the second order fuzziness in semantics of spatial relations. This method can be used in designing and managing spatial database, where one single model can answer perfectly a query. This method can detect small changes in a spatial scene when implemented to the same pair of objects at two different time instant, in this way this method can replace the implementation of four methods (topological, directional, distance and internal cardinal directional (ICD) relations) of spatial relations which are used to compare a scene. An algorithm is also given such that we can estimate the 2D fuzzy topological relation along with the directional components. Spatio-temporal relations are the emerging issue in GIS and other sciences and hopefully these results will be helpful in extending this work to a spatio-temporal aspects and fuzzy spatio-temporal reasoning and natural language processus. These results will be used in future to develop the motion verbs.

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