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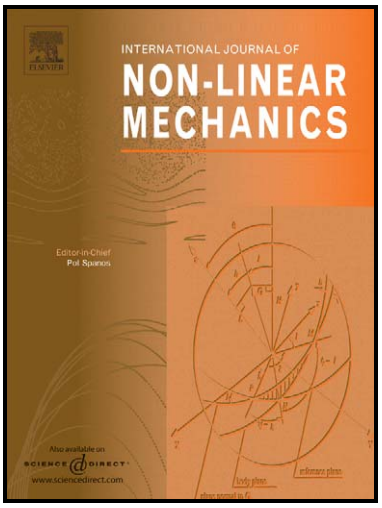

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Non-linear closed-form computational model of cable trusses

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Abstract

In this paper the non-linear closed-form static computational model of the pre-stressed suspended biconvex and biconcave cable trusses with unmovable, movable, or elastic yielding supports subjected to vertical distributed load applied over the entire span and over a part (over the half) of the span is presented. The paper is an extension of the previously published work of authors [1]. Irvine's linearized forms of the deflection and the cable equations are modified because the effects of the non-linear truss behaviour needed to be incorporated in them. The concrete forms of the system of two non-linear cubic cable equations due to the load type are derived and presented. From a solution of a non-linear vertical equilibrium equation for a loaded cable truss, the additional vertical deflection is determined. The computational analytical model serves to determine the response, *i.e.* horizontal components of cable forces and deflection of the geometrically non-linear biconvex or biconcave cable truss to the applied loading, considering effects of elastic deformations, temperature changes and elastic supports. The application of the derived nonlinear analytical model is illustrated by numerical examples. Resulting responses of the symmetric and asymmetric cable trusses with various geometries (shallow and deep profiles) obtained by the present non-linear closed-form solution are compared with those obtained by Irvine's linear solution and those by the non-linear finite element method. The conditions for the use of the linear and non-linear approach are briefly specified.

Keywords: Cable truss; Non-linear closed-form solution; System of cubic cable equations; Linear and non-linear behavior

1. Introduction

Light weight pre-stressed cable trusses offer an economical and efficient alternative for many structural problems. Their use is widely accepted in projects that require coverage of large areas. A review of computational methods for tension structures can be found in [2].

Most of the recent methods of non-linear analysis of cable trusses are based on the discretization of the equilibrium equations using the finite element method (FEM) and solving the resulting non-linear algebraic equations by numerical methods [3-8]. Kassimali and Parsi-Feraidoonian [4] investigated the non-linear behaviour and the ultimate strength of pre-stressed cable trusses including consideration of the effects of large displacements, slackening of members and inelastic material properties. Kanno *et al.* [9] derived a special method for friction and friction-less analysis of non-linear elastic cable structures based on second-order cone programming. Brew and Lewis [10] proposed an efficient numerical tool for a better integration of the design and analysis with the manufacture of tension membrane structures. The common approach to these investigations is to study the cable structure as a geometrically non-linear system. There have been only a few published analytical studies on non-linear solutions. Due to the mathematical derivation difficulties that can arise in the non-linear closed-form solution, numerical methods are by far the most popular. Nevertheless, some linear analytical work has been attempted.

Comprehensive analytical treatments on the cable trusses have been given by Schleyer [11], Møllmann [12,13] and by Irvine [14]. Their linearized approximate theories provide consistent methods for finding the static response of a perfectly flexible and elastic, hinge-suspended cable truss to applied loads. However, little attention is paid to the closed-form non-linear analysis of a cable truss suspended on elastic supports. Rakowski [15] proposed a special non-linear closed-form solution for cable trusses: a non-linear task replaced by the linear one. For this purpose the equivalent loading parameters were derived and used. Sultan *et al.* [16] presented the general pre-stressability conditions for tensegrity structures, which can be analytically solved.

The approximate analytical methods on a linear static analysis of the cable trusses are presented in further works, *e.g.*, Baron and Venkatesan [17], Urelius and Fowler [18], Moskalev [19], Kadlcak [20] and Buchholdt [21]. Their results lead to simple methods for preliminary dimensioning of various types of cable structures. An interesting discussion of the relative merits of the truss configurations used to support large span roofs can be found in Krishna *et al.* [22]. Kmet [23] proposed a linearized solution of pre-stressed cable trusses with rheological properties. Raoof and Davies [24] theoretically demonstrated that, in view of the rather small axial load perturbations under serviceability state conditions, use of the more appropriate no-slip stiffness (as opposed to the traditionally used full-slip value) leads to practically significant reductions in the estimated values of the vertical deflections of the cable truss.

Irvine [14] investigates a static response of the cable trusses using the linearized engineering analytical theory of the suspended cable. He neglected all second-order terms that appear in the differential equations of equilibrium and in the cable equations. However, significant nonlinearities can occur in a response of the truss with different initial geometries and material properties of the carrying and stabilizing cables. That is why the authors focus on these problems, and elaborating on them they start with the work of Irvine [14], which has been further complemented. The present paper is an extension of the previously published work of authors [1]. Compared to a previous work [1], new load types such as a load applied over a part and over the half of the truss span have been added.

In this paper a non-linear closed-form static computational model of a biconvex and biconcave suspended cable truss subjected to various types of static load is presented. Concrete forms of the system of two non-linear cubic cable equations and the deflection functions are derived. The application of the derived non-linear analytical model is illustrated by numerical examples. Resulting responses of the symmetric and asymmetric cable trusses with various geometries (shallow and deep profiles) obtained by the present non-linear closed-form solution are compared with those obtained by Irvine's linear solution and those by the non-linear finite element method, when COSMOS/M software [25] is used. The conditions for use the linear and non-linear approach are briefly specified.

2. Initial assumptions

The profile geometries of the biconcave and biconvex cable trusses are shown in Fig. 1. The profiles of the bottom and top chords, respectively, are assumed to be parabolic and given by

$$z_b = 4(d_b - b_b)\frac{x}{l}\left(1 - \frac{x}{l}\right) + b_b$$

$$z_t = 4(d_t - b_t)\frac{x}{l}\left(1 - \frac{x}{l}\right) + b_t \tag{1}$$

where l is the span of the cable truss. In the case of biconcave truss, sag of the carrying cable is given as $s = b_t - d_t$ and camber of the stabilizing cable is $c = b_b - d_b$. For the biconvex

truss, sag of the carrying cable is given as $s = d_b - b_b$ and camber of the stabilizing cable is $c = d_t - b_t$.

The non-linear closed-form static analysis of the biconvex or biconcave cable trusses, initially symmetric about a vertical axis at mid-span, will proceed on the following assumptions: perfectly flexible cables, working only in tension and having zero stiffness in compression and bending. The relatively small weight of the cables and the spacers will be ignored, so that the initial free-hanging geometry will be specified by the cable pretensions, the lengths of the spacers, and the span. It will be assumed that the slopes of the chords are, and remain small, so that the maximum difference between spacer length and distance of the chords at the supports should be less than one-quarter of the span. In practical applications this requirement will always be met. Only trusses with vertical spacers or ties will be considered. In the analysis the spacers and the ties will be replaced by a continuous diaphragm whose adjacent vertical elements may slide freely with respect to each other. Each vertical element of the diaphragm is considered inextensible. The small longitudinal movements of the chords associated with the vertical movements of the truss under load must be allowed to occur freely. The analysis will be hold for biconcave systems under uniformly distributed load symmetrical about mid-span if the chords are clamped together at mid-span. The derivations will be performed for the biconvex cable truss, all results obtained can be equally applied to the biconcave case.

3. Vertical deflections for some characteristic loading types

Suppose that under applied vertical loading q, the shear force at some cross section x along the span is Q. Following Irvine [14], vertical equilibrium at a cross section of the biconvex cable truss further requires that (see Fig. 2)

$$(H_{0b} + \Delta H_b)\frac{d(z_b + w)}{dx} - (H_{0t} - \Delta H_t)\frac{d(z_t - w)}{dx} = Q \tag{2}$$

where H_{0b} and H_{0t} are the horizontal components of the pretensions in the bottom and top chords, respectively, ΔH_b and ΔH_t are the additional horizontal components of cable tension owing to the applied load, z_b and z_t are the initial profiles of the chords given by Eqs. (1), and w is the additional vertical deflection. Consideration of the internal equilibrium of the

unloaded truss, given by the expression $H_{0b}\,dz_b/dx = H_{0t}\,dz_t/dx$, allows Eq. (2) to be reduced to

$$(H_{0b}+H_{0t})\frac{dw}{dx}+(\Delta H_b-\Delta H_t)\frac{dw}{dx}+\Delta H_b\frac{dz_b}{dx}+\Delta H_t\frac{dz_t}{dx}=Q \tag{3}$$

The non-linear differential equation (3) may be integrated directly, and after the boundary conditions have been applied, the equation for the vertical deflection of a biconvex cable truss is obtained in the form

$$w=\frac{1}{H_{0b}+\Delta H_b+H_{0t}-\Delta H_t}\left\{M-4\Delta H_b(d_b-b_b)\left(\frac{x}{l}-\frac{x^2}{l^2}\right)-4\Delta H_t(d_t-b_t)\left(\frac{x}{l}-\frac{x^2}{l^2}\right)\right\} \tag{4}$$

Under a vertical uniformly distributed load q applied over the entire span of the truss (Fig. 3a), the bending moment M in Eq. (4) is

$$M=\frac{ql}{2}x\left(1-\frac{x}{l}\right) \tag{5}$$

Under a uniformly distributed load q extending from $x=a$ to $x=b$ along the span of the truss (Fig. 3b), the bending moments are

$$M=\frac{qx}{2l}(b-a)^2+\frac{qx}{l}(b-a)(l-b) \tag{6}$$

for $x\in\langle 0,a\rangle$,

$$M=\frac{qx}{2l}(b-a)^2+\frac{qx}{l}(b-a)(l-b)-\frac{qx^2}{2}+qax-\frac{qa^2}{2} \tag{7}$$

for $x\in\langle a,b\rangle$, and

$$M=\frac{qx}{2l}(b-a)^2+\frac{qx}{l}(b-a)(l-b)-qx(b-a)+\frac{qb^2}{2}-\frac{qa^2}{2} \tag{8}$$

for $x\in\langle b,l\rangle$.

Under a uniformly distributed load q applied over the left half of the span from $x=0$ to $x=l/2$ (Fig. 3c), the bending moments are

$$M=ql\left(\frac{3x}{8}-\frac{x^2}{2l}\right) \tag{9}$$

for $x\in\langle 0,l/2\rangle$, and

$$M = \frac{ql}{8}(l - x) \tag{10}$$

for $x \in \langle l/2, l \rangle$.

4. System of cubic cable equations for some characteristic loading types

To complete the solution, ΔH_b and ΔH_t must be evaluated. Use is made of the cable equations that incorporate Hooke's law and temperature effects to provide closure conditions relating the changes in cable tensions to the changes in cable geometries when the bottom and top cables are displaced (in plane) from their original initial equilibrium profiles. The geometry of these displacements for a biconvex truss is shown in Fig. 4. If ds_b and ds_t are the original lengths of the bottom and top element, respectively, and $d\bar{s}_b$ and $d\bar{s}_t$ are their new lengths, then $ds_b^2 = dx^2 + dz_b^2$, $d\bar{s}_b^2 = (dx + du_b)^2 + (dz_b + dw)^2$, $ds_t^2 = dx^2 + dz_t^2$ and $d\bar{s}_t^2 = (dx - du_t)^2 + (dz_t - dw)^2$ where u_b, u_t and w are the longitudinal and vertical components of the displacements for the corresponding cables, respectively. If the profiles of the bottom and top cables are flat so that the ratio of their sag to span (and/or camber to span) is *1:8* or less (due to the application of the engineering theory of suspended cables with the parabolic profiles), fractional changes in their lengths, corrected to the second order, are

$$\frac{d\bar{s}_b - ds_b}{ds_b} = \frac{du_b}{ds_b}\frac{dx}{ds_b} + \frac{dw}{ds_b}\frac{dz_b}{ds_b} + \frac{1}{2}\left(\frac{dw}{ds_b}\right)^2$$

$$\frac{d\bar{s}_t - ds_t}{ds_t} = \frac{du_t}{ds_t}\frac{dx}{ds_t} + \frac{dw}{ds_t}\frac{dz_t}{ds_t} - \frac{1}{2}\left(\frac{dw}{ds_t}\right)^2 \tag{11}$$

Here the Hooke's law requires that

$$\frac{d\bar{s}_b - ds_b}{ds_b} = \frac{\Delta H_b}{E_b A_b}\frac{ds_b}{dx}$$

$$\frac{d\bar{s}_t - ds_t}{ds_t} = \frac{\Delta H_t}{E_t A_t}\frac{ds_t}{dx} \tag{12}$$

where E_b and E_t are the moduli of elasticity of the bottom and top chords, respectively, and A_b and A_t are the cross-sectional areas of the cable chords.

If the effects of a uniform temperature difference of $\Delta T_b = T_b - T_{0b}$ and/or $\Delta T_t = T_t - T_{0t}$ (where T_0 and T are the initial and design temperatures, respectively) are considered, terms $\varepsilon_{Tb} = \alpha \Delta T_b$ and $\varepsilon_{Tt} = \alpha \Delta T_t$ need to be added to the elemental equations, where α is the coefficient of expansion. On the basis of Eqs. (11) and (12), cable equations for the bottom and top elements can be written as

$$\frac{\Delta H_b}{E_b A_b}\frac{ds_b}{dx} + \alpha \Delta T_b = \frac{du_b}{ds_b}\frac{dx}{ds_b} + \frac{dw}{ds_b}\frac{dz_b}{ds_b} + \frac{1}{2}\left(\frac{dw}{ds_b}\right)^2$$

$$\frac{\Delta H_t}{E_t A_t}\frac{ds_t}{dx} + \alpha \Delta T_t = \frac{du_t}{ds_t}\frac{dx}{ds_t} + \frac{dw}{ds_t}\frac{dz_t}{ds_t} - \frac{1}{2}\left(\frac{dw}{ds_t}\right)^2 \tag{13}$$

After respective multiplication of equations (13) by $(ds_b/dx)^2$ and $(ds_t/dx)^2$, one obtains

$$\frac{\Delta H_b}{E_b A_b}\left(\frac{ds_b}{dx}\right)^3 + \alpha \Delta T_b \left(\frac{ds_b}{dx}\right)^2 = \frac{du_b}{dx} + \frac{dw}{dx}\frac{dz_b}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

$$\frac{\Delta H_t}{E_t A_t}\left(\frac{ds_t}{dx}\right)^3 + \alpha \Delta T_t \left(\frac{ds_t}{dx}\right)^2 = \frac{du_t}{dx} + \frac{dw}{dx}\frac{dz_t}{dx} - \frac{1}{2}\left(\frac{dw}{dx}\right)^2 \tag{14}$$

If the effects of elastic cable deformations (assuming Hooke's law, temperature change and the fractional change in length of the bottom and top cables) are corrected to the second order, cable equations for a biconvex cable truss follow in the integrated form as

$$\frac{\Delta H_b L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{Tb} = u_b(l) - u_b(0) + \int_0^l \left(\frac{dw}{dx}\frac{dz_b}{dx}\right)dx + \frac{1}{2}\int_0^l \left(\frac{dw}{dx}\right)^2 dx$$

$$\frac{\Delta H_t L_{et}}{E_t A_t} + \alpha \Delta T_t L_{Tt} = u_t(l) - u_t(0) + \int_0^l \left(\frac{dw}{dx}\frac{dz_t}{dx}\right)dx - \frac{1}{2}\int_0^l \left(\frac{dw}{dx}\right)^2 dx \tag{15}$$

where $u_b(l)$ and $u_b(0)$ and/or $u_t(l)$ and $u_t(l)$ are the longitudinal movements of the bottom and top supports, respectively. The left side members L_{eb}, L_{et}, L_{Tb} and L_{Tt} of Eqs. (15) characterizing the lengths of the unloaded cables are given by

$$L_{eb,t} = \int_0^l \left(\frac{ds_{b,t}}{dx}\right)^3 dx \cong l\left[1 + 8\left(\frac{d_{b,t} - b_{b,t}}{l}\right)^2\right]$$

$$L_{Tb,t} = \int_0^l \left(\frac{ds_{b,t}}{dx} \right)^2 dx = l \left[1 + \frac{16}{3} \left(\frac{d_{b,t} - b_{b,t}}{l} \right)^2 \right] \tag{16}$$

where $ds_{b,t}^2 = dx^2 + dz_{b,t}^2$, $ds_{b,t}/dx = \sqrt{1 + \left(dz_{b,t}/dx\right)^2}$, and the first two terms of the binomial series are considered.

The additional horizontal components of cable tension ΔH_b and ΔH_t, caused by the applied distributed load q, are found from the system of two non-linear coupled cable equations given by Eqs. (15). Because dw/dz is continuous along the span, the selected entries on the right side of Eqs. (15) can be evaluated by integration by parts and Eqs. (15) can be rewritten as

$$\begin{aligned} \frac{\Delta H_b L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{Tb} &= -\int_0^l \frac{d^2 z_b}{dx^2} w dx - \frac{1}{2} \int_0^l \frac{d^2 w}{dx^2} w dx + B_b \\ \frac{\Delta H_t L_{et}}{E_t A_t} + \alpha \Delta T_t L_{Tt} &= -\int_0^l \frac{d^2 z_t}{dx^2} w dx + \frac{1}{2} \int_0^l \frac{d^2 w}{dx^2} w dx + B_t \end{aligned} \tag{17}$$

where parameters B_b and B_t characterize the boundary conditions at the supports of bottom and top cables, respectively, in the longitudinal direction. For unmovable supports $B_b = 0$ and $B_t = 0$, for movable supports in horizontal direction $B_b = u_b(l) - u_b(0)$ and $B_t = u_t(l) - u_t(0)$ if the horizontal support movements of $u_b(0)$ and $u_b(l)$ and/or $u_t(0)$ and $u_t(l)$ occur at each bottom and top end of the truss. For elastic supports in the horizontal direction $B_b = f_{xb}(0) + f_{xb}(l)$ and $B_t = f_{xt}(0) + f_{xt}(l)$, where the horizontal support flexibilities of $f_{xb}(0)$ and $f_{xb}(l)$ and/or $f_{xt}(0)$ and $f_{xt}(l)$ respectively occur at each bottom and top end of the truss.

Substituting the deflection equation (4) into Eqs. (17) and performing the necessary integration, the following coupled system of cubic cable equations for ΔH_b and ΔH_t is found as

$$\begin{aligned} &\Delta H_b^3 + c_{b1} \Delta H_b^2 + c_{b2} \Delta H_t^2 + c_{b3} \Delta H_b^2 \Delta H_t + c_{b4} \Delta H_b \Delta H_t^2 + \\ &+ c_{b5} \Delta H_b \Delta H_t + c_{b6} \Delta H_b + c_{b7} \Delta H_t + c_{b8} = 0 \end{aligned} \tag{18}$$

$$\begin{aligned} &\Delta H_t^3 + c_{t1} \Delta H_b^2 + c_{t2} \Delta H_t^2 + c_{t3} \Delta H_b^2 \Delta H_t + c_{t4} \Delta H_b \Delta H_t^2 + \\ &+ c_{t5} \Delta H_b \Delta H_t + c_{t6} \Delta H_b + c_{t7} \Delta H_t + c_{t8} = 0 \end{aligned}$$

The coefficients c_{bi} and c_{ti} for $i = 1,2...8$ in Eqs. (18) depend on the type of the applied load and are defined in the Appendix (Eqs. (A1) and (A2) for a vertical uniformly distributed load q applied over the entire span of the truss). The deflection equation (4) and the cubic cable equations (18) are sufficient to obtain a non-linear closed-form solution for the dependent variables w, ΔH_b and ΔH_t.

In the case of a vertical uniformly distributed load q applied from $x = a$ to $x = b$ along the span of the truss cable equations (15) can be written as

$$
\begin{aligned}
&\frac{\Delta H_b L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{Tb} = \\
&= -\int_0^a \left(\frac{d^2 z_b}{dx^2} \right) w_1 dx - \int_a^b \left(\frac{d^2 z_b}{dx^2} \right) w_2 dx - \int_b^l \left(\frac{d^2 z_b}{dx^2} \right) w_3 dx - \\
&- \frac{1}{2} \int_0^a \left(\frac{d^2 w_1}{dx^2} \right) w_1 dx - \frac{1}{2} \int_a^b \left(\frac{d^2 w_2}{dx^2} \right) w_2 dx - \frac{1}{2} \int_b^l \left(\frac{d^2 w_3}{dx^2} \right) w_3 dx + \\
&+ \frac{1}{2} \left[\frac{dw_1}{dx} w_1 \right]_0^a + \frac{1}{2} \left[\frac{dw_2}{dx} w_2 \right]_a^b + \frac{1}{2} \left[\frac{dw_3}{dx} w_3 \right]_b^l + \\
&+ \left[\frac{dz_b}{dx} w_1 \right]_0^a + \left[\frac{dz_b}{dx} w_2 \right]_a^b + \left[\frac{dz_b}{dx} w_3 \right]_b^l + B_b
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
&\frac{\Delta H_t L_{et}}{E_t A_t} + \alpha \Delta T_t L_{Tt} = \\
&= -\int_0^a \left(\frac{d^2 z_t}{dx^2} \right) w_1 dx - \int_a^b \left(\frac{d^2 z_t}{dx^2} \right) w_2 dx - \int_b^l \left(\frac{d^2 z_t}{dx^2} \right) w_3 dx + \\
&+ \frac{1}{2} \int_0^a \left(\frac{d^2 w_1}{dx^2} \right) w_1 dx + \frac{1}{2} \int_a^b \left(\frac{d^2 w_2}{dx^2} \right) w_2 dx + \frac{1}{2} \int_b^l \left(\frac{d^2 w_3}{dx^2} \right) w_3 dx - \\
&- \frac{1}{2} \left[\frac{dw_1}{dx} w_1 \right]_0^a - \frac{1}{2} \left[\frac{dw_2}{dx} w_2 \right]_a^b - \frac{1}{2} \left[\frac{dw_3}{dx} w_3 \right]_b^l + \\
&+ \left[\frac{dz_t}{dx} w_1 \right]_0^a + \left[\frac{dz_t}{dx} w_2 \right]_a^b + \left[\frac{dz_t}{dx} w_3 \right]_b^l + B_t
\end{aligned}
$$

where the deflections w_1, w_2 and w_3, obtained from Eq. (4), correspond to the individual characteristic parts of the truss span: $w_1 \in \langle 0, a \rangle$, $w_2 \in \langle a, b \rangle$ and $w_3 \in \langle b, l \rangle$. Substituting the individual deflection Eq. (4) into Eqs. (19) and performing the necessary integration the identical coupled system of cubic cable equations (18) for ΔH_b and ΔH_t can be found. The coefficients of Eqs. (18) are defined by Eqs. (A1) and (A2) in the Appendix, except c_{b7}, c_{b8}, c_{t6} and c_{t8} which are defined by Eqs. (A3) and (A4).

In the case of a vertical uniformly distributed load q applied over the left half of the span from $x = 0$ to $x = l/2$ cable equations (15) are written as

$$\begin{aligned}
&\frac{\Delta H_b L_{eb}}{E_b A_b} + \alpha \Delta T_b L_{Tb} = \\
&= -\int_0^{l/2} \left(\frac{d^2 z_b}{dx^2} \right) w_1 dx - \int_{l/2}^{l} \left(\frac{d^2 z_b}{dx^2} \right) w_2 dx - \\
&-\frac{1}{2} \int_0^{l/2} \left(\frac{d^2 w_1}{dx^2} \right) w_1 dx - \frac{1}{2} \int_{l/2}^{l} \left(\frac{d^2 w_2}{dx^2} \right) w_2 dx + B_b \\
&\frac{\Delta H_t L_{et}}{E_t A_t} + \alpha \Delta T_t L_{Tt} = \\
&= -\int_0^{l/2} \left(\frac{d^2 z_t}{dx^2} \right) w_1 dx - \int_{l/2}^{l} \left(\frac{d^2 z_t}{dx^2} \right) w_2 dx + \\
&+\frac{1}{2} \int_0^{l/2} \left(\frac{d^2 w_1}{dx^2} \right) w_1 dx + \frac{1}{2} \int_{l/2}^{l} \left(\frac{d^2 w_2}{dx^2} \right) w_2 dx + B_t
\end{aligned} \tag{20}$$

where the deflections w_1 and w_2, obtained from Eq. (4), correspond to the individual characteristic parts of the truss span: $w_1 \in \langle 0, l/2 \rangle$ and $w_2 \in \langle l/2, l \rangle$. Substituting the individual deflection Eq. (4) into Eqs. (20) and performing the necessary integration the identical coupled system of cubic cable equations (18) for ΔH_b and ΔH_t can be found. The coefficients of Eqs. (18) are defined by Eqs. (A1) and (A2) in the Appendix, except c_{b7}, c_{b8}, c_{t6} and c_{t8} which are defined by Eqs. (A5) and (A6).

To solve the nonlinear cable equations (18) Newton-Raphson iterations were applied.

All the derived equations and results are equally applicable to biconvex and biconcave cable trusses. In the case of a biconvex truss $(d_b - b_b)$ and/or $(d_t - b_t)$ are positive and ΔH_b and ΔH_t calculated from the Eqs. (18) are also positive. For the resulting values of the bottom and top horizontal components of cable forces holds true

$$H_b = H_{0b} + \Delta H_b$$
$$H_t = H_{0t} - \Delta H_t \tag{21}$$

In the case of a biconcave truss $(d_b - b_b)$ and/or $(d_t - b_t)$ are negative and ΔH_b and ΔH_t calculated from Eqs. (18) are also negative. Substituting $-\Delta H_b$ and $-\Delta H_t$ into Eqs. (21), the resulting values of horizontal components of cable forces are in the form

$$H_b = H_{0b} - \Delta H_b$$
$$H_t = H_{0t} + \Delta H_t \tag{22}$$

In the deflection Eq. (4) it is necessary to substitute all terms with the corresponding signs. The additional deflection is of course still positive.

5. Comparison with finite element results

In order to verify the accuracy of the nonlinear analytical model developed in this study, solutions have been generated for the cable truss with immovable supports previously examined by Kassimali and Parsi-Feraidoonian [4]. The non-linear finite element method was used in [4]. The structure analyzed was a symmetric biconcave cable truss with a span $l = 60\,\mathrm{m}$ and the cross-sectional areas of the bottom and top cables $A_b = 1.3.10^{-3}\,\mathrm{m}^2$ and $A_t = 2.0.10^{-3}\,\mathrm{m}^2$, shown in Fig. 5. The moduli of elasticity of both cables are $E_b = E_t = 1.48135.10^{8}\,\mathrm{kNm}^{-2}$. The initial horizontal components of pretension in the

bottom and top chord are $H_{0b} = H_{0t} = 588.603\,\text{kN}$. There is a vertical uniformly distributed load $q = 8.91\text{kNm}^{-1}$ applied over the left half of the span. Seven different loading levels q, $2q$, $3q$, $4q$, $5q$, $6q$ and $7q$ were considered. The following data for the geometrical quantities are specified: $d_b = d_t = 0.5\,\text{m}$ and $b_b = b_t = 4.52\,\text{m}$ (see Fig. 5).

In the case of the closed-form analysis, five ties symmetrically situated along the truss span are replaced by a continuous diaphragm.

Vertical deflections w in the third of the truss span versus applied loads obtained by the present non-linear closed-form solution (when Eqs. (18) with the coefficients from Eqs. (A1), (A2), (A5) and (A6) and Eq. (4) and Eq. (9) are used) and by non-linear FEM [4] are shown in Fig. 6. Results confirmed a good agreement.

The deflection course of the cable truss under a load $q = 8.91\text{kNm}^{-1}$ obtained by the present solution (when Eq. (4) and Eqs. (9) and (10) are used) is shown in Fig. 7.

6. Linear and non-linear behaviour of cable trusses

Consider a biconcave initially symmetric and asymmetric cable truss about the longitudinal axis (see Fig.1) with a span $l = 60\text{m}$ and cross-sectional areas of the bottom and top cables $A_b = 1.3.10^{-3}\,\text{m}^2$ and $A_t = 2.0.10^{-3}\,\text{m}^2$. The moduli of elasticity of both cables are $E_b = E_t = 1.5.10^8\,\text{kNm}^{-2}$. There is a vertical uniformly distributed load $q = 10.0\text{kNm}^{-1}$ applied over the entire span. In the case of the closed-form analysis, the ties with a large axial stiffness are replaced by a continuous diaphragm.

Resulting responses of the symmetric and asymmetric cable trusses with various geometries (shallow and deep profiles) obtained by the present non-linear closed-form solution are compared with those obtained by Irvine's linear solution and those by the non-linear FEM, when COSMOS/M software [25] is used.

In the case of a *symmetric truss* the initial horizontal components of pretensions in the bottom and top chord are $H_{0b} = H_{0t} = 600.0\,\text{kN}$. Eight different span-to-sag ratios of the top carrying cable that are equal to eight span-to-camber ratios of the bottom stabilizing cable, $l/s = l/c = 7.5$, 10, 12.5, 15, 17.5, 20, 22.5 and 25 are considered. The following data for the geometrical quantities are specified: $b_b = b_t = 8.5$, 6.5, 5.3, 4.5, 3.93, 3.5, 3.17 and 2.9 m and $d_b = d_t = 0.5\,\text{m}$.

Results, *i.e.* horizontal components of cable forces in the bottom $H_b = H_{0b} - \Delta H_b$ and top chord $H_t = H_{0t} + \Delta H_t$ (see Fig. 8 and Fig. 9) and vertical deflections w in the mid-span of the truss under applied load versus the span-to-sag ratio l/s of the carrying cables (see Fig. 10) obtained by the present non-linear closed-form solution are compared with those obtained by the linear solution (when Irvine's linear analytical model is applied) and those by non-linear FEM (when software COSMOS/M is used).

In the case of an *asymmetric truss* eight different span-to-sag ratios of the top carrying cable, $l/s = 7.5$, 10, 12.5, 15, 17.5, 20, 22.5 and 25 were considered. The span-to-camber ratio of the bottom stabilizing cable was kept constant at $l/c = 25$ for all mentioned cases. The following data for the geometrical quantities are specified: $b_b = 2.9$ m, $b_t = 8.5$, 6.5, 5.3, 4.5, 3.93, 3.5, 3.17 and 2.9 m, and $d_b = d_t = 0.5$ m. The initial horizontal component of pretension in the bottom chord is $H_{0b} = 600.0$ kN. The initial horizontal component of pretension in the top chord H_{0t} is calculated according to the changing value of sag (at the middle of span) of the carrying cable from the equation of internal equilibrium for the initial pre-stressed unloaded asymmetric truss, which can be written as $H_{0t} = H_{0b}\, c/s$.

Results, *i.e.* horizontal components of cable forces in the bottom $H_b = H_{0b} - \Delta H_b$ and top chord $H_t = H_{0t} + \Delta H_t$ (see Fig. 11 and Fig. 12) and vertical deflections w in the mid-span of the truss under applied load versus the span-to-sag ratio l/s of the carrying cables (see Fig. 13) obtained by the three mentioned approaches are compared. This demonstrates that there are differences between the responses of the investigated cable trusses obtained by the linear and non-linear analyses.

As shown in Fig. 8 and Fig. 11 the differences between the resulting horizontal components of cable forces at the bottom chords of trusses and between the deflections (Fig. 10 and Fig. 13) obtained by the linear and non-linear solution are more significant as the span-to-sag ratio l/s of the carrying cables increases. Deflection response is overestimated, which is conservative, while additional tension in the chords is underestimated when the linear theory is assumed.

In the case of a symmetric truss the differences in the resulting deflections begin at the span-to-sag ratio $l/s = 17.5$ as shown in Fig. 10. In the case of an asymmetric truss are these differences more significant and they already occur at the span-to-sag ratio $l/s = 12.5$ as shown in Fig.13.

When a symmetric or asymmetric truss is initially flat (shallow truss with the small sag and camber), *i.e.* when the span-to-sag ratio of carrying cables increases the differences between the results obtained by the linear and non-linear theory can not be ignored as they are significant and the non-linear solution need to be performed.

Results show that the responses obtained by the present non-linear closed-form computational model are of a very good agreement with those obtained by the nonlinear FEM when COSMOS/M software is used.

7. Conclusions

In this paper the non-linear closed-form static solution of the pre-stressed suspended biconvex and biconcave cable trusses with unmovable, movable, or elastic yielding supports subjected to vertical distributed load applied over the entire span and over a part (over the half) of the span has been presented. Irvine's linearized forms of the deflection and the cable equations were modified because of the effects of the non-linear truss behaviour. The concrete forms of the system of two non-linear cubic cable equations due to the load type were derived and presented. From a solution of a non-linear vertical equilibrium equation for a loaded cable truss, the additional vertical deflection was determined. The computational analytical model serves to determine the response, *i.e.* horizontal components of cable forces and deflection of the geometrically non-linear biconvex or biconcave cable truss to the applied loading, considering effects of elastic deformations, temperature changes and elastic supports.

The application of the derived non-linear analytical model was illustrated by numerical examples. Verification of the results was performed and the behaviour of the symmetric and asymmetric pre-stressed cable trusses with various initial geometries was investigated. Results, *i.e.* horizontal components of cable forces in the bottom and top chord and vertical deflections in the mid-span of the truss under applied load versus the span-to-sag ratio of the carrying cables obtained by the present non-linear closed-form solution were compared with those obtained by the linear solution when Irvine's linear analytical model was applied and those by non-linear FEM when software COSMOS/M was used.

The obtained results confirm the correctness of the derived equations and their mathematical and physical importance.

To compare the non-linear structural analysis results obtained by the other analytical or numerical methods and to perform a quick reliability assessment of cable trusses: that is why this new closed-form model is useful.

It is believed that the non-linear solution presented will lead to an improved closed-form analysis of cable trusses in the non-linear range.

Acknowledgements

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Appendix

In the case of a vertical uniformly distributed load q applied over the entire span of the truss, the individual coefficients of Eqs. (18) are

- for the bottom cable

$$c_{b1} = 2\left(H_{0b} + H_{0t}\right) + \frac{E_b A_b}{L_{eb}}\left[\frac{16}{6l}\left(d_b - b_b\right)^2 + \alpha\Delta T_b L_{Tb} + B_b\right]$$

$$c_{b2} = \frac{E_b A_b}{L_{eb}}\left[-\frac{16}{3l}\left(d_b - b_b\right)\left(d_t - b_t\right) - \frac{16}{6l}\left(d_t - b_t\right)^2 + \alpha\Delta T_b L_{Tb} + B_b\right]$$

$$c_{b3} = -2$$

$$c_{b4} = 1$$

$$c_{b5} = -\frac{E_b A_b}{L_{eb}}\left[\frac{16}{3l}\left(d_b - b_b\right)^2 + 2\alpha\Delta T_b L_{Tb} + 2B_b\right] - 2\left(H_{0b} + H_{0t}\right) \qquad \text{(A1)}$$

$$c_{b6} = \frac{E_b A_b}{L_{eb}}\left[\frac{16}{3l}\left(d_b - b_b\right)^2\left(H_{0b} + H_{0t}\right) + 2\alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right) + \right.$$
$$\left. + 2B_b\left(H_{0b} + H_{0t}\right)\right] + \left(H_{0b} + H_{0t}\right)^2$$

$$c_{b7} = \frac{E_b A_b}{L_{eb}}\left[\frac{16}{3l}\left(d_b - b_b\right)\left(d_t - b_t\right)\left(H_{0b} + H_{0t}\right) + K_1\left(d_b - b_b\right) + K_1\left(d_t - b_t\right) - \right.$$
$$\left. - 2\alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right) - 2B_b\left(H_{0b} + H_{0t}\right)\right]$$

$$c_{b8} = \frac{E_b A_b}{L_{eb}}\left[-K_1\left(d_b - b_b\right)\left(H_{0b} + H_{0t}\right) - K_2 + \alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right)^2 + B_b\left(H_{0b} + H_{0t}\right)^2\right]$$

- for the top cable

$$c_{t1} = \frac{E_t A_t}{L_{et}}\left[\frac{16}{3l}(d_b - b_b)(d_t - b_t) + \frac{16}{6l}(d_b - b_b)^2 + \alpha\Delta T_t L_{Tt} + B_t\right]$$

$$c_{t2} = -2(H_{0b} + H_{0t}) + \frac{E_t A_t}{L_{et}}\left[-\frac{16}{6l}(d_t - b_t)^2 + \alpha\Delta T_t L_{Tt} + B_t\right]$$

$$c_{t3} = 1$$

$$c_{t4} = -2$$

$$c_{t5} = \frac{E_t A_t}{L_{et}}\left[\frac{16}{3l}(d_t - b_t)^2 - 2\alpha\Delta T_t L_{Tt} - 2B_t\right] + 2(H_{0b} + H_{0t}) \qquad \text{(A2)}$$

$$c_{t6} = \frac{E_t A_t}{L_{et}}\left[\frac{16}{3l}(d_b - b_b)(d_t - b_t)(H_{0b} + H_{0t}) - K_1(d_t - b_t) - K_1(d_b - b_b) + \right.$$
$$\left. + 2\alpha\Delta T_t L_{Tt}(H_{0b} + H_{0t}) + 2B_t(H_{0b} + H_{0t})\right]$$

$$c_{t7} = \frac{E_t A_t}{L_{et}}\left[\frac{16}{3l}(d_t - b_t)^2(H_{0b} + H_{0t}) - 2\alpha\Delta T_t L_{Tt}(H_{0b} + H_{0t}) - \right.$$
$$\left. - 2B_t(H_{0b} + H_{0t})\right] + (H_{0b} + H_{0t})^2$$

$$c_{t8} = \frac{E_t A_t}{L_{et}}\left[K_2 - K_1(d_t - b_t)(H_{0b} + H_{0t}) + \alpha\Delta T_t L_{Tt}(H_{0b} + H_{0t})^2 + B_b(H_{0b} + H_{0t})^2\right]$$

where $K_1 = \frac{2}{3}ql$ and $K_2 = \frac{q^2 l^3}{24}$.

In the case of a vertical uniformly distributed load q applied from $x = a$ to $x = b$ along the span of the truss, are the coefficients of Eqs. (18) defined by Eqs. (A1) and (A2) except c_{b7}, c_{b8}, c_{t6} and c_{t8} which are

- for the bottom cable

$$c_{b7} = \frac{E_b A_b}{L_{eb}}\left[- K_3\left(\left(d_b - b_b\right) + \left(d_t - b_t\right)\right) - K_5\left(\left(d_b - b_b\right) + \left(d_t - b_t\right)\right) + \frac{16}{3l}\left(d_b - b_b\right)\left(d_t - b_t\right).\right.$$
$$\left.\left(H_{0b} + H_{0t}\right) - 2\alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right) - 2B_b\left(H_{0b} + H_{0t}\right)\right] \tag{A3}$$

$$c_{b8} = \frac{E_b A_b}{L_{eb}}\left[K_3\left(d_b - b_b\right)\left(H_{0b} + H_{0t}\right) + K_4 + K_5\left(d_b - b_b\right)\left(H_{0b} + H_{0t}\right) + \right.$$
$$\left. + K_6 + K_7 + B_b\left(H_{0b} + H_{0t}\right)^2\right]$$

- for the top cable

$$c_{t6} = \frac{E_t A_t}{L_{et}}\left[K_3\left(\left(d_b - b_b\right) + \left(d_t - b_t\right)\right) + K_5\left(\left(d_b - b_b\right) + \left(d_t - b_t\right)\right) + \frac{16}{3l}\left(d_b - b_b\right)\left(d_t - b_t\right).\right.$$
$$\left.\left(H_{0b} + H_{0t}\right) + 2\alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right) + 2B_b\left(H_{0b} + H_{0t}\right)\right] \tag{A4}$$

$$c_{t8} = \frac{E_t A_t}{L_{et}}\left[K_3\left(d_t - b_t\right)\left(H_{0b} + H_{0t}\right) - K_4 + K_5\left(d_t - b_t\right)\left(H_{0b} + H_{0t}\right) - \right.$$
$$\left. - K_6 - K_7 + B_t\left(H_{0b} + H_{0t}\right)^2\right]$$

where $K_3 = \frac{qa^2}{l}\left[-\frac{4a}{3l} + 2\right]$, $K_4 = \frac{qa^2}{l}\left[-\frac{qal}{3} + \frac{qa^2}{8}\right]$, $K_5 = \frac{qb^2}{l}\left[\frac{4b}{3l} - 2\right]$,

$K_6 = \frac{qb^2}{l}\left[-\frac{qbl}{6} + \frac{qb^2}{8}\right]$ and $K_7 = q^2 ab\left[-\frac{ab}{4l} + \frac{a}{2}\right]$.

In the case of a vertical uniformly distributed load q applied over the left half of the span from $x = 0$ to $x = l/2$, are the coefficients of Eqs. (18) defined by Eqs. (A1) and (A2) except c_{b7}, c_{b8}, c_{t6} and c_{t8} which are

- for the bottom cable

$$c_{b7} = \frac{E_b A_b}{L_{eb}}\left[K_9\left(\left(d_b - b_b\right) + \left(d_t - b_t\right)\right) + \frac{16}{3l}\left(d_b - b_b\right)\left(d_t - b_t\right)\left(H_{0b} + H_{0t}\right) - \right.$$
$$\left. - 2\alpha\Delta T_b L_{Tb}\left(H_{0b} + H_{0t}\right) - 2B_b\left(H_{0b} + H_{0t}\right)\right] \tag{A5}$$

$$c_{b8} = \frac{E_b A_b}{L_{eb}}\left[- K_8 - K_9\left(d_b - b_b\right)\left(H_{0b} + H_{0t}\right) + B_b\left(H_{0b} + H_{0t}\right)^2\right]$$

- for the top cable

$$c_{t6} = \frac{E_t A_t}{L_{et}}\left[- K_9\left((d_b - b_b) + (d_t - b_t)\right) + \frac{16}{3l}(d_b - b_b)(d_t - b_t)(H_{0b} + H_{0t}) + \right.$$
$$\left. + 2\alpha \Delta T_t L_{Tt}(H_{0b} + H_{0t}) + 2B_t(H_{0b} + H_{0t})\right]$$

(A6)

$$c_{t8} = \frac{E_t A_t}{L_{et}}\left[K_8 - K_9(d_t - b_t)(H_{0b} + H_{0t}) + B_t(H_{0b} + H_{0t})^2\right]$$

where $K_8 = \frac{5}{384} q^2 l^3$ and $K_9 = \frac{ql}{3}$.

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Figure Captions

Fig.1. Profile geometry for biconcave and biconvex cable truss.

Fig. 2. Vertical equilibrium of a loaded biconvex cable truss.

Fig. 3. Characteristic loading types.

Fig. 4. Displacements of elements of bottom and top cables in a loaded biconvex cable truss.

Fig. 5. Geometry and loading of a biconcave cable truss [4].

Fig. 6. Vertical deflections of a cable truss at various loads obtained using the present non-linear closed-form model and discrete finite element method proposed by Kassimali and Parsi-Feraidoonian [4].

Fig.7. Vertical deflection course of a cable truss obtained using the present non-linear closed-form computational model.

Fig. 8. Horizontal component of cable force in bottom chord of symmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

Fig. 9. Horizontal component of cable force in top chord of symmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

Fig. 10. Mid-span vertical deflection of symmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

Fig. 11. Horizontal component of cable force in bottom chord of asymmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

Fig. 12. Horizontal component of cable force in top chord of asymmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

Fig. 13. Mid-span vertical deflection of asymmetric cable truss at various span-to-sag ratios obtained using linear and non-linear approaches.

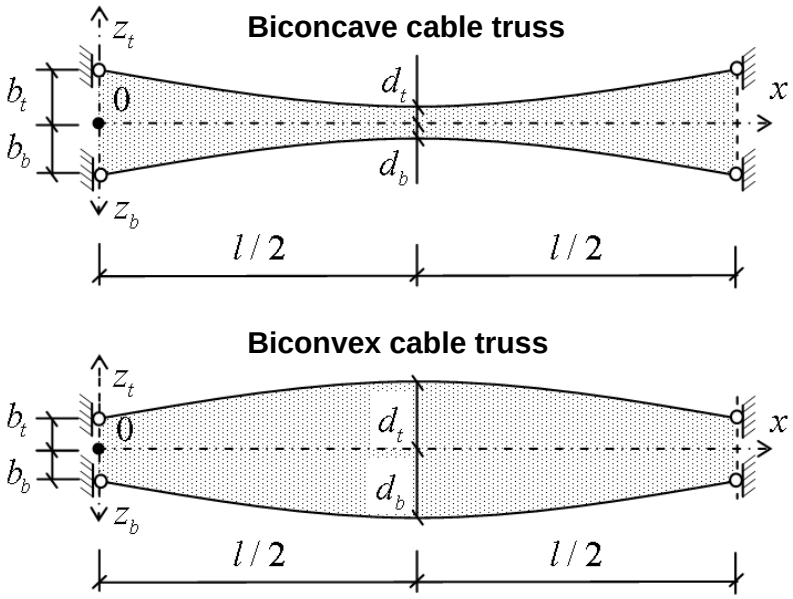

Fig. 1

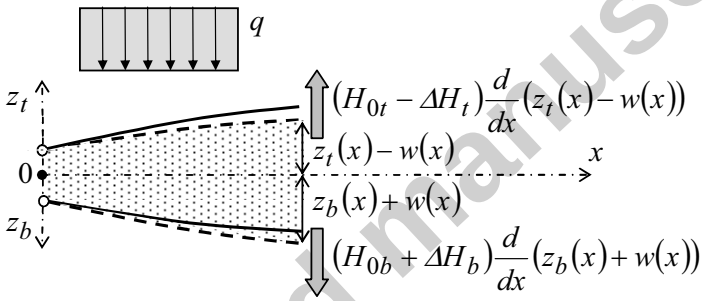

Fig. 2

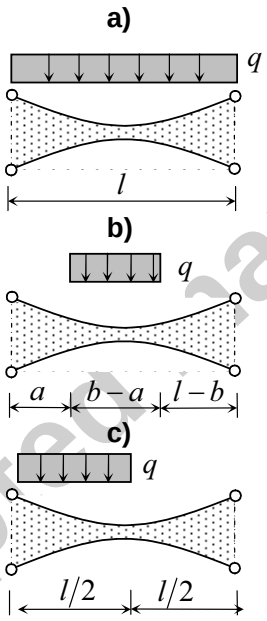

Fig. 3

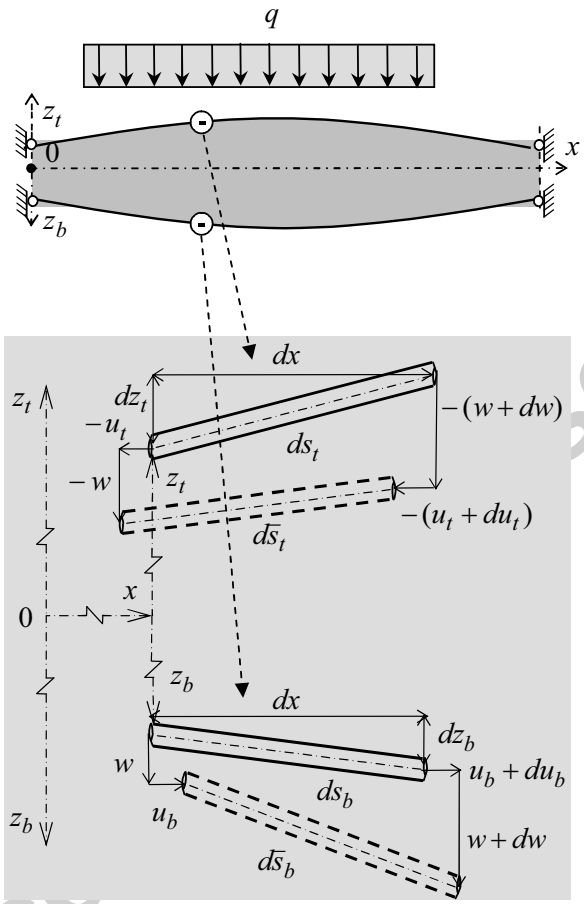

Fig. 4

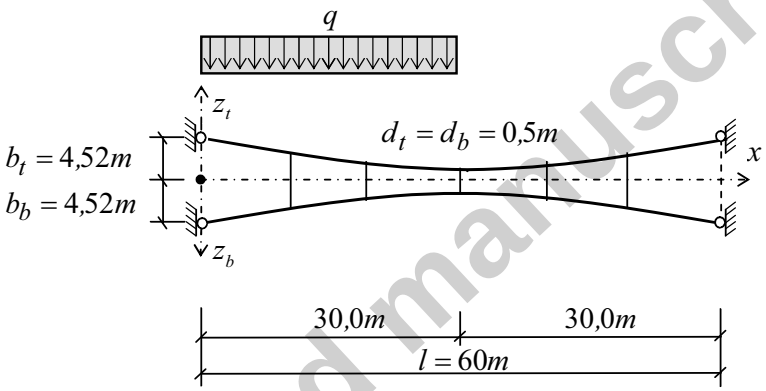

Fig. 5

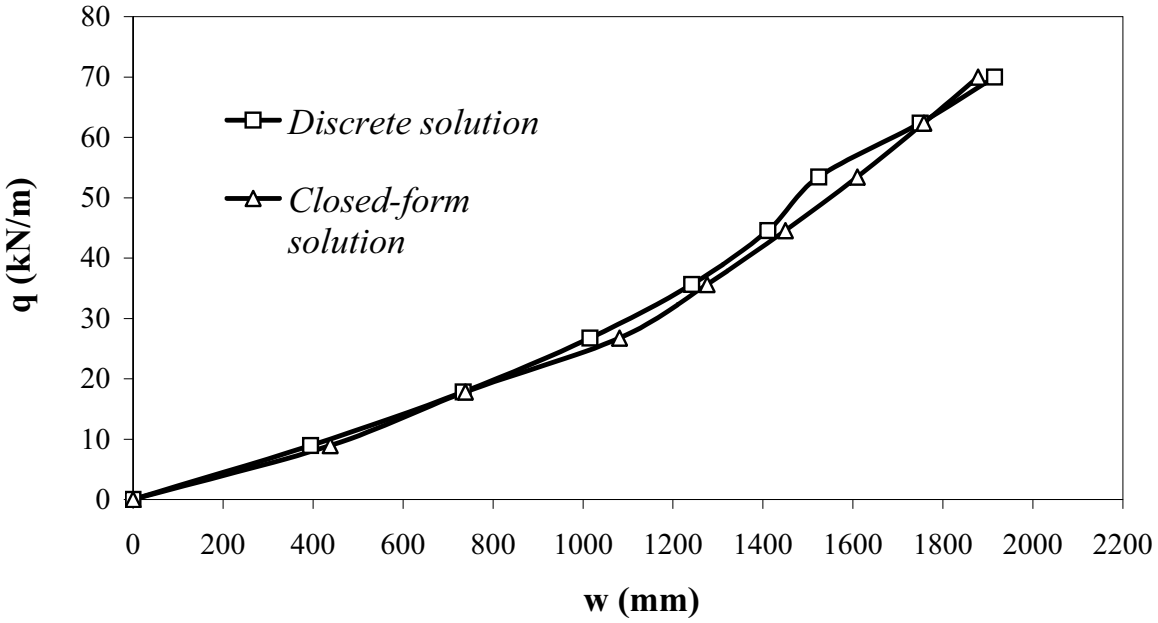

Fig. 6

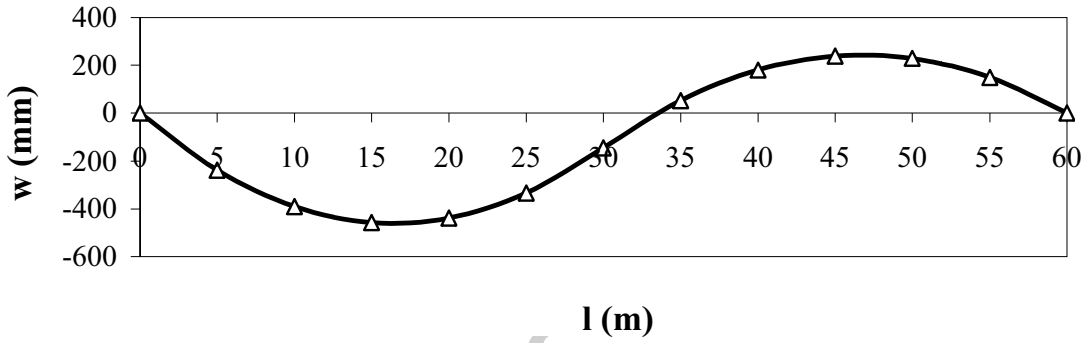

Fig. 7

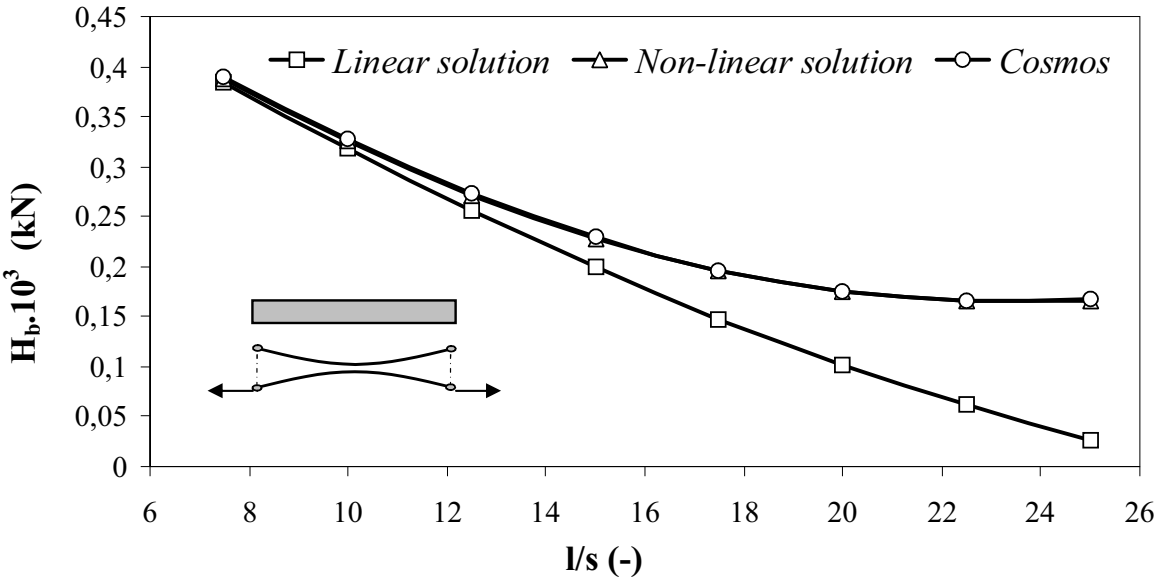

Fig.8

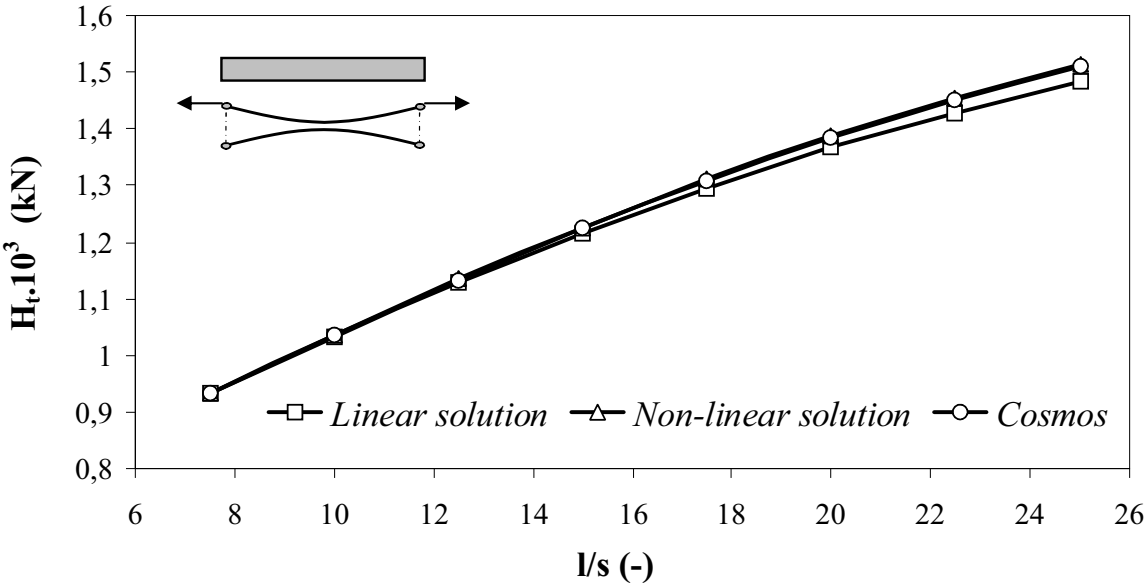

Fig.9

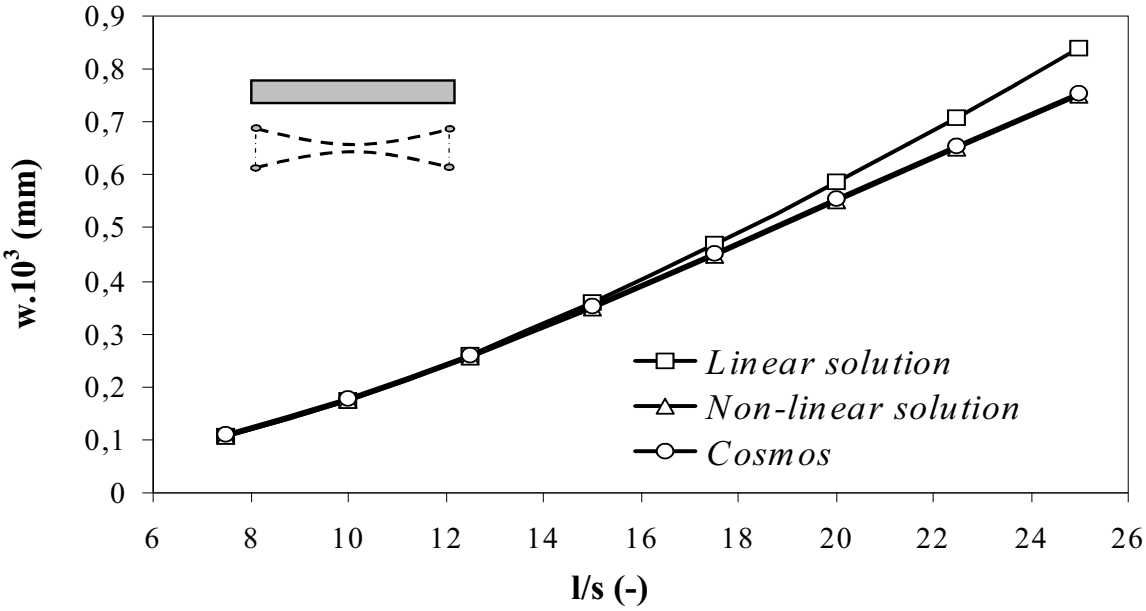

Fig.10

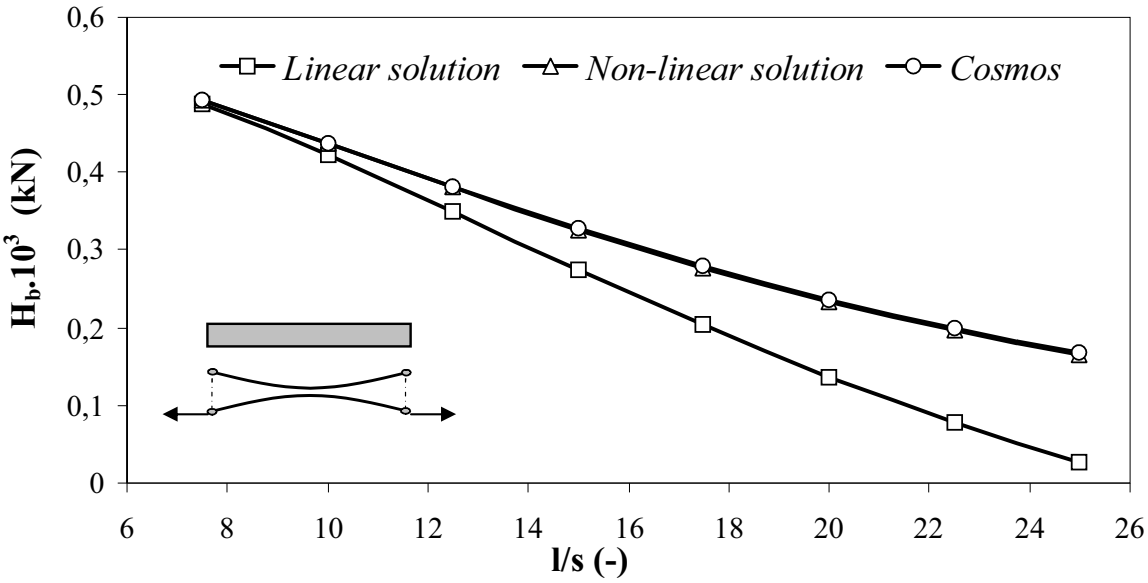

Fig. 11

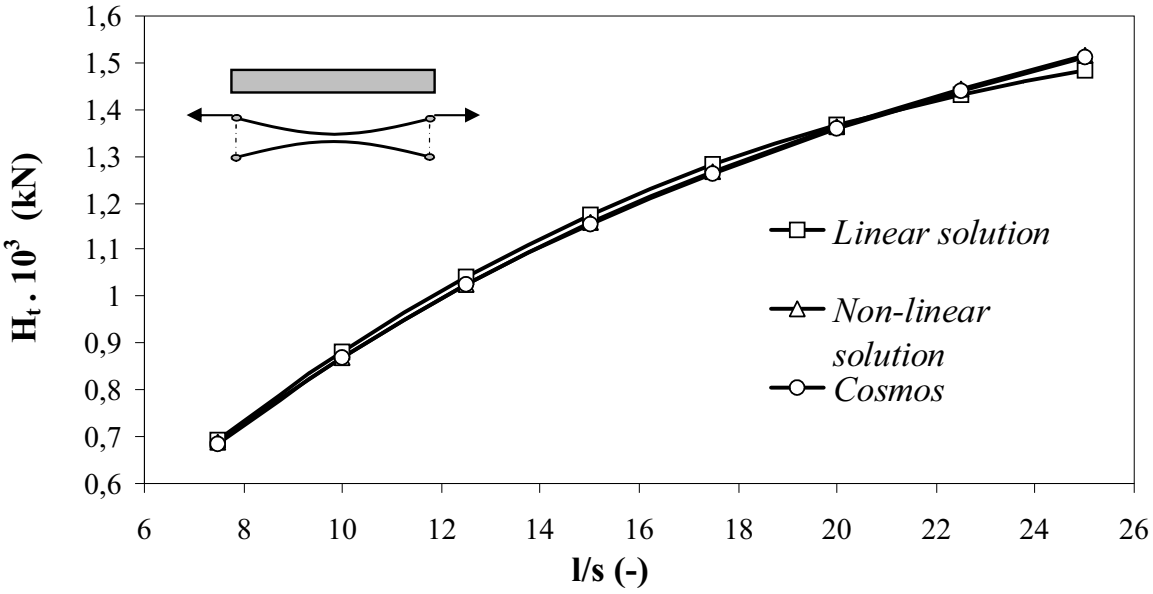

Fig.12

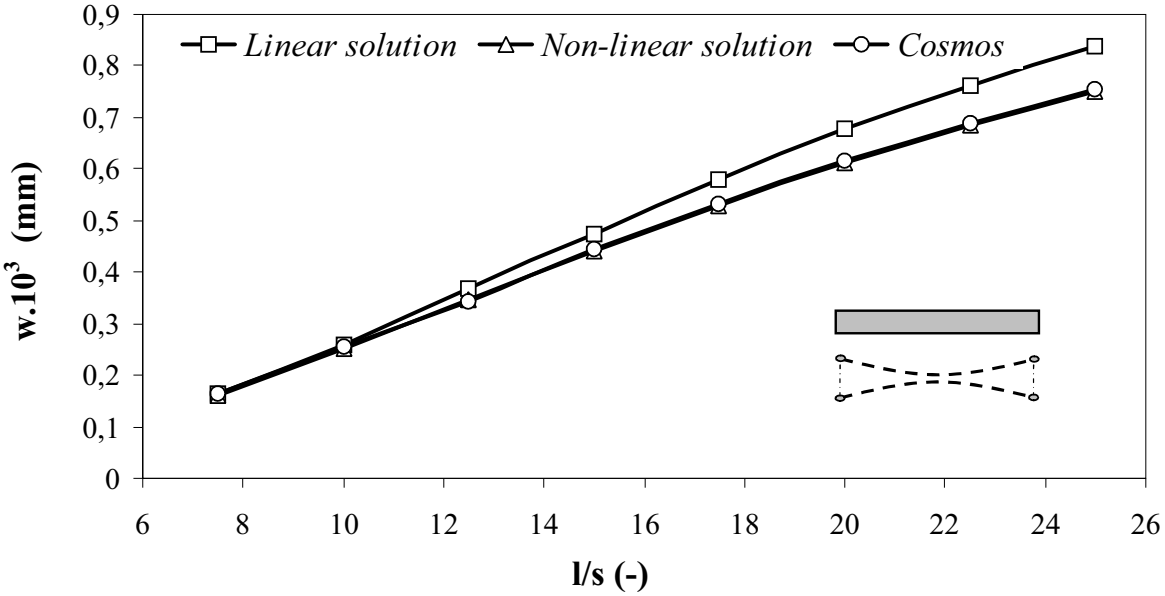

Fig.13