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# A coupled Unscented Kalman Filter and modified Error in Constitutive Relation technique for structural dynamics identification

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**ABSTRACT:** This work arises from an industrial need for EDF (the French electric power company) to study the dynamics of complex structures, for which (noisy) measurements are used in an effort to alleviate the presence of Finite Element model uncertainties. Our work aims at formulating and implementing a strategy allowing optimal estimation of the structure dynamics and model parameters from available, imperfect, modelling and experimental information. One of the motivations of this work is to consider evolving model parameters. This introduces additional difficulties in the identification strategy, especially when evolution laws are *a priori* unknown. In the present work we propose a strategy to tackle with the above mentioned problem based in combining both the modified Error in Constitutive Relation (mECR) technique and the Unscented Kalman Filter (UKF). A numerical example of a reinforced concrete beam is presented to show how this strategy gives particularly good results to both locate and identify model errors such as structural damage and boundary conditions misspecification. Furthermore, we illustrate how this approach is adapted for the case of evolving damage with no prior knowledge of the evolution law.

**KEY WORDS:** Error in Constitutive Relation ; Unscented Kalman Filter ; Structural dynamics ; System identification

## 1 MOTIVATION AND GENERAL PURPOSE

This work arises from an industrial need for EDF (the French electric power company) to study the dynamics of complex structures, for which (noisy) measurements are used in an effort to alleviate the presence of Finite Element model uncertainties. Our work aims at formulating and implementing a strategy allowing optimal estimation of the structure dynamics and model parameters from available, imperfect, modelling and experimental information. One of the motivations of this work is to consider evolving model parameters (e.g. damage). This introduces additional difficulties in the identification strategy, especially when evolution laws are *a priori* unknown.

In the approach presented herein, we propose to take advantage of the interesting properties of both the modified Error in Constitutive Relation (mECR) and the Unscented Kalman filter (UKF) resulting into a coupled coupled mECR-UKF structural identification strategy. Thus, we propose to tackle the identification problem, for which *a priori* knowledge of modelling errors is not available, in a two-step strategy:

1. In the first step, the mECR method is used to spatially localize the regions of the structure where the assumed model is least consistent with available measurements.
2. The second step consists in introducing the mECR cost functions in a state-space formulation to be solved by means of the Unscented Kalman Filter.

Therefore, this paper first introduces both the mECR and the UKF methods to finally propose to solve the updating problem by coupling both techniques. A numerical example of a reinforced concrete beam is presented in which we seek to localize and identify simultaneously structural damage and imperfect boundary conditions modelling.

## 2 A MODIFIED ERROR IN CONSTITUTIVE RELATION OVERVIEW

The Error in Constitutive Relation (ECR) concept is at the root of energy-based cost functionals that allow to evaluate the discrepancy between a mathematical model describing the behavior of a given mechanical structure and a set of available measurements for this same structure. It was first introduced by Ladevèze in [1] as a means to *a posteriori* evaluate the accuracy of a finite element (FE) solution. Quickly thereafter, other developments of this approach were aimed at achieving model updating based on frequencies and eigenmodes in the free-vibration problem [2], and later extended to the forced-vibration case [3, 4]. More recent studies investigated the application of ECR in time-domain formulations [5–7].

Since then, the method has proved its ability to update the mass, damping and stiffness properties of mechanical static and dynamic models. In particular, works in this area have shown interesting properties of the ECR method which are particularly relevant in our approach:

- Excellent ability to locate structural parts where modelling is defective [8].
- Robustness of the method against high measurement noise [5].
- Good convexity properties of the cost functions [9].

### 2.1 Modified Error in Constitutive Relation formulation

Consider a structure within a domain  $\Omega$  during a time interval  $[0, T]$  and the following constitutive relations in the frequency domain:

$$\begin{cases} \sigma = K\varepsilon + i\omega C\varepsilon \\ \Gamma = -\rho\omega^2 u \end{cases} \quad (1)$$

where  $K$  and  $C$  are the elasticity and damping tensors,  $u, \varepsilon$  denote the displacement and linearized strain tensor, and  $\Gamma, \sigma$  are the inertial forces and stress.

The ECR formulation presented in this paper is based on the so-called Drucker error which can be written as:

$$\begin{aligned} \xi_{\Omega\omega}^2(u, v, w) &= \int_{\Omega} \frac{\gamma}{2} \text{tr}[(K + T\omega^2 C)(\varepsilon(v) - \varepsilon(u))^*(\varepsilon(v) - \varepsilon(u))] \\ &\quad + \frac{1-\gamma}{2} \rho \omega^2 (u-w)^*(u-w) d\Omega \end{aligned}$$

where  $*$  denotes the complex conjugate and the related kinematic and dynamic admissible fields are chosen as follows:

$$\begin{cases} u_c = u \\ \sigma_d = K\varepsilon(v) + j\omega C\varepsilon(v) \\ \Gamma_d = -\rho\omega^2 w \end{cases} \quad (2)$$

Thus, the discretization of the above-mentioned problem in a FE framework leads to the following modified ECR cost function [4]:

$$\begin{aligned} e_{\Omega\omega}^2(\{u\}, \{v\}, \{w\}) &= \frac{\gamma}{2} \{u-v\}^T [\mathbf{K} + T\omega^2 \mathbf{C}] \{u-v\} \\ &\quad + \frac{1-\gamma}{2} \{u-w\}^T \omega^2 [\mathbf{M}] \{u-w\} \\ &\quad + \frac{r}{1-r} \{\Pi u - \tilde{u}\}^T [\mathbf{G}_r] \{\Pi u - \tilde{u}\} \end{aligned} \quad (3)$$

where  $[\mathbf{M}], [\mathbf{C}]$  and  $[\mathbf{K}]$  are, respectively, the mass, damping and stiffness matrix of the FE model,  $\{\tilde{u}\}$  denotes a discrete set of measured displacements,  $\Pi$  the projection operator onto the observation space and  $[\mathbf{G}_r]$  a symmetric positive-definite weighting matrix. Since the choice of  $[\mathbf{G}_r]$  is not a critical issue, it is often chosen to be energy consistent. Here, we consider:

$$[\mathbf{G}_r] = \frac{\gamma}{2} ([\mathbf{K}_r] + T\omega^2 [\mathbf{C}_r]) + \frac{1-\gamma}{2} \omega^2 [\mathbf{M}_r] \quad (4)$$

where “ $r$ ” stands for the Guyan matrix reduction onto the set of observed DOFs.

When working in a frequency range  $[\omega_{min}, \omega_{max}]$  and assuming that the structure is divided into a set of substructures  $\Omega_s \subset \Omega$ , the normalized modified ECR cost function is finally given by

$$e_N^2 = \int_{\omega_{min}}^{\omega_{max}} \eta(\omega) \left( \sum_{\Omega_s \subset \Omega} \frac{e_{\Omega_s\omega}^2(\{u\}, \{v\}, \{w\})}{D_{\omega}^2(\{u\})} \right) d\omega \quad (5)$$

where  $\eta(\omega)$  is a weighting function such that

$$\int_{\omega_{min}}^{\omega_{max}} \eta(\omega) d\omega = 1 \quad \eta(\omega) \geq 0 \quad (6)$$

and  $D_{\omega}^2(\{u\})$  is a reference normalizing energy value defined by

$$\begin{aligned} D_{\omega}^2(\{u\}) &= \frac{\gamma}{2} \{u\}^T [\mathbf{K} + T\omega^2 \mathbf{C}] \{u\} + \frac{1-\gamma}{2} \{u\}^T \omega^2 [\mathbf{M}] \{u\} \end{aligned} \quad (7)$$

For a given measurement set  $\{\tilde{u}\}$  at a certain frequency, the admissible kinematic and dynamic admissible fields minimizing the functional (3) can be obtained by solving a system of linear equations [4].

## 2.2 Modified Error in Constitutive Relation updating problem

Based on the previously-mentioned properties of the ECR method, the updating problem arises into a two step algorithm. The first step consists in locating the regions of the structure where the value of  $e_N^2$  (representing the relative quality of the model with respect to the measurements in a certain frequency range) is above a user-defined threshold. This analysis allows us to choose a subset  $\theta$  of the set  $\Theta$  of model parameters that enter the definition of structural matrices  $[\mathbf{M}]$ ,  $[\mathbf{C}]$ ,  $[\mathbf{K}]$ , corresponding to those structural areas where  $e_N^2$  reveals inadequate modelling.

Thus, the updating problem can be formulated as

$$\text{Find } \hat{\theta} \in \Theta \text{ minimising } e_N^2(\theta) \quad (8)$$

In general, this leads to a nonlinear optimization problem and, in our approach, we propose to solve the above-mentioned updating problem (8) by means of the Unscented Kalman Filter.

## 3 THE UNSCENTED KALMAN FILTER FOR SOLVING THE MECR PROBLEM

The Kalman Filter (KF), introduced in the early 60's by Kalman [10], provides a recursive solution method for the linear optimal filtering problem in the presence of measurement and state uncertainties. It consists of a prediction-correction algorithm where the updated estimate is computed from the previous estimate (prediction) and it is corrected with the new input data (correction).

Since then, several different approaches have been proposed to address nonlinear filtering problems using variations on the KF formalism. The most popular approach to perform recursive nonlinear estimation is the Extended Kalman Filter (EKF). The derivation of the EKF exploits a linearization of the nonlinear state-space model on the assumption that deviations from linearity are of first order. Despite its easy implementation in a wide range of engineering problems, this assumption can lead in some cases to inaccuracy and divergence of the algorithm [11].

The Unscented Kalman Filter (UKF) was proposed in 1997 by Julier and Uhlmann [12, 13] as an alternative to the EKF allowing to take into account the effect of nonlinearity in the state-space model in a more accurate manner. The UKF proposes to sample the probability density function of the states by a minimal set of carefully chosen points, commonly known as sigma points. These sample points are propagated through the true nonlinear state-space model, and the state estimates and its error covariance matrix are computed using the corresponding sample statistics. The Unscented transformation thus gives a mathematical framework for achieving second order accuracy for any (sufficiently smooth) nonlinearity. A further advantage of the UKF is that it does not require the actual evaluation of derivatives of the nonlinear equations.

### 3.1 Unscented Kalman Filter formulation

Consider a random variable  $x \in R^n$  with mean  $\bar{x}$  and covariance  $P_x$ . Let  $S_x = \{X, W\}$  be the set of the so-called sigma points and corresponding weighting factors of  $x$ , defined by

$$\begin{aligned}
[\mathbf{X}]_0 &= \bar{x} \\
[\mathbf{X}]_i &= \bar{x} + (\sqrt{(n+\lambda)\mathbf{P}_x})_i \quad i = 1, \dots, n, \\
[\mathbf{X}]_i &= \bar{x} - (\sqrt{(n+\lambda)\mathbf{P}_x})_i \quad i = n+1, \dots, 2n
\end{aligned} \tag{9}$$

(where  $(\sqrt{\mathbf{P}_x})_i$  represents the  $i^{\text{th}}$  column of the Cholesky decomposition of matrix  $\mathbf{P}_x$ ) and

$$\begin{aligned}
\mathbf{W}_0^m &= \frac{\lambda}{n+\lambda} \\
\mathbf{W}_0^c &= \frac{\lambda}{n+\lambda} + 1 - \alpha^2 + \beta \\
\mathbf{W}_i^m &= \mathbf{W}_i^c = \frac{1}{2(n+\lambda)}, \quad i = 1, \dots, 2n.
\end{aligned} \tag{10}$$

where  $\lambda = \alpha^2(n+\kappa) - n$  is a scaling parameter. The constant  $\alpha$  determines the spread of the sigma points around  $\bar{x}$ , while the constant  $\beta$  is used to incorporate prior knowledge of the distribution of  $x$  and  $\kappa$  is a secondary scaling parameter.

Thus, the Unscented transformation approaches the posterior mean and covariance of the random variable  $z$  which undergoes the nonlinear transformation  $z = f(x)$  by the following posterior statistics

$$\begin{aligned}
\bar{z} &\approx \sum_{i=0}^{2n} \mathbf{W}_i^m [\mathbf{Z}]_i \\
\mathbf{P}_z &\approx \sum_{i=0}^{2n} \mathbf{W}_i^c ([\mathbf{Z}]_i - \bar{z})([\mathbf{Z}]_i - \bar{z})^T
\end{aligned} \tag{11}$$

where  $[\mathbf{Z}]_i = f([\mathbf{X}]_i)$  is the transformed  $i^{\text{th}}$  sigma point.

The UKF then consists of a straightforward application of the Unscented transformation to the KF framework, and is summarized now. Consider a dynamical discrete nonlinear state-space model given by

$$\begin{cases} x_{k+1} = F(x_k) + \mathbf{v}_k \\ y_k = h(x_k) + \eta_k \end{cases} \tag{12}$$

where  $x_k$  is the state variable vector at  $t = k\Delta t$ ,  $\mathbf{v}_k$  is the zero-mean process noise vector with covariance matrix  $\mathbf{Q}_k$ .  $y_k$  is the zero-mean observation vector at  $t = k\Delta t$  and  $\eta_k$  is the observation noise vector with corresponding covariance matrix  $\mathbf{R}_k$ .

Then, the UKF equations applied to (12) are the following. In order to initialize the algorithm, a prior estimation of the initial state and covariance are given

$$\begin{aligned}
\hat{x}_0 &= \mathbf{E}[\mathbf{x}_0] \\
\mathbf{P}_0 &= \mathbf{E}[(\mathbf{x}_0 - \hat{x}_0)(\mathbf{x}_0 - \hat{x}_0)^T]
\end{aligned} \tag{13}$$

The prediction step consists in propagating the sigma points through the nonlinear state-space model (12) and estimate the posterior mean and covariance with the statistics of the propagated points. Consequently, the prediction step for each time step  $k = 1, 2, \dots$  is as follows:

1. Build a set of sigma points around  $\hat{x}_{k-1}$

$$\mathcal{S}_x^{k-1} = \{\mathbf{X}_{k-1}, \mathbf{W}_{k-1}\} \tag{14}$$

2. Propagate the sigma points through the nonlinear dynamic equation

$$[\mathbf{X}_k]_i = F([\mathbf{X}_{k-1}]_i) \tag{15}$$

3. Calculate the posterior statistics

$$\hat{x}_k^- = \sum_{i=0}^{2n} \mathbf{W}_i^m [\mathbf{X}_k]_i \tag{16}$$

$$\mathbf{P}_k^- = \sum_{i=0}^{2n} \mathbf{W}_i^c (([\mathbf{X}_k]_i - \hat{x}_k^-)([\mathbf{X}_k]_i - \hat{x}_k^-)^T) + \mathbf{Q}_k \tag{17}$$

4. Build the predicted state in the observation space

$$\hat{y}_k = \sum_{i=0}^{2n} \mathbf{W}_i^m h([\mathbf{X}_k]_i) \tag{18}$$

In an analogous way to the KF, the UKF measurement-update equations are as follows (correction step)

5. Correction of the posterior mean

$$\hat{x}_k = \hat{x}_k^- + \mathbf{K}_k (y_k - \hat{y}_k) \tag{19}$$

6. Correction of the posterior covariance

$$\mathbf{P}_k = \mathbf{P}_k^- + \mathbf{K}_k \mathbf{P}_k^{yy} \mathbf{K}_k^T \tag{20}$$

where

$$\mathbf{K}_k = \mathbf{P}_k^{xy} (\mathbf{P}_k^{yy})^{-1}$$

$$\mathbf{P}_k^{xy} = \sum_{i=0}^{2n} \mathbf{W}_i^c (([\mathbf{X}_k]_i - \hat{x}_k^-)(h([\mathbf{X}_k]_i) - \hat{y}_k)^T)$$

$$\mathbf{P}_k^{yy} = \sum_{i=0}^{2n} \mathbf{W}_i^c (((h([\mathbf{X}_k]_i) - \hat{y}_k)(h([\mathbf{X}_k]_i) - \hat{y}_k))^T)$$

#### 4 SOLVING THE MEQR WITHIN THE UKF FRAMEWORK

In this section we discuss how the detailed equations of the UKF can be used to solve an optimization problem. In particular, the problem that we propose to solve is the updating problem defined in (8) in order to take advantage of the good properties of the mEQR mentioned in Sec. 2.

The key idea of this approach is to reconsider the state-space formulation proposed in (12) by changing the metrics of the observation space. For this, the discrepancy between the model solution and the available data is no longer measured by a least-square distance but by a more appropriate cost function, which in our case is chosen to be (5). In this way, we consider the state vector as  $\theta$  and the associated process equation becomes the identity. The updating problem (8) can then be written in a state-space formulation as

$$\begin{cases} \theta_{j+1} = \theta_j + w_j^\theta \\ \zeta_j = e_N^2(\theta_j) + \mathbf{v}_j \end{cases} \tag{21}$$

where  $w_j^\theta$  and  $\mathbf{v}_j$  represents zero-mean Gaussian processes playing an important role in this optimization scheme. Since the UKF will tend to minimize the distance  $(\zeta_j - e_N^2(\theta_j))^2$  we can interpret  $\zeta_j$  as a target value. Thus, for minimization purposes we will write

$$\zeta_j = 0 \quad \forall j \tag{22}$$

## 5 NUMERICAL RESULTS

The numerical experiment we present herein consists of locating and identifying a model misspecification by use of the coupled method UKF and modified ECR presented in Sec. 4. The reference FE model shown in figure 1 represents a reinforced concrete beam clamped at its end sections and excited by a white noise external force  $F$ . The structure has a length of 3,75m and a constant rectangular section (height 0,22m, width 0,15m), and the Young modulus, mass density and Poisson's ratio are respectively set to  $E = 20 \cdot 10^9 \text{N/m}^2$ ,  $\rho = 7800 \text{Kg/m}^3$  and  $\nu = 0,3$ .

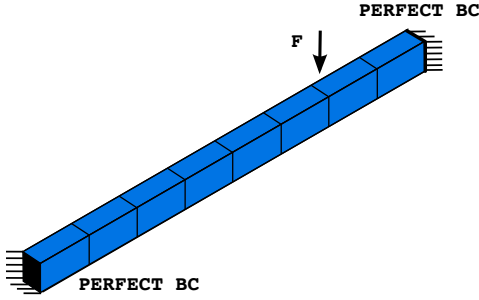


Figure 1. FE reference model with homogeneous material and perfectly clamped end sections.

Besides, the “true” beam deviates from these assumptions in that (a) one element is damaged, with its weakened Young modulus of the form

$$E = (1 - \delta)E_0,$$

(with  $\delta = 0.85$ ) and (b) one of its end sections is imperfectly clamped, its impedance  $H(\omega)$  being

$$H(\omega) = -\omega^2 m + k \quad (23)$$

(with  $k = 10^7 \text{N/m}$  and  $m = 33 \text{Kg}$ ).

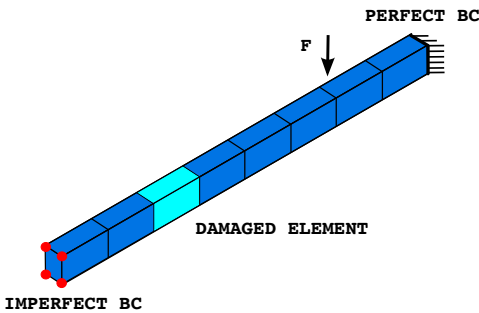


Figure 2. FE “true” model containing imperfect clamping and structural damage.

The available measurements are numerically simulated by computing the response of the damaged beam under the given load at a set of 7 points, playing the role of sensors, and adding a 5% Gaussian noise to these simulated measurements.

Thus, the numerical experience consists of locating and identifying the modelling discrepancies of the reference model by applying the proposed ERC-UKF approach to the set of simulated measurements.

### 5.1 Localizing imperfectly modelled regions

The first step of the proposed procedure consists of localizing the regions of the structure where the FE model deviates the most from the true structure. For this, we propose to compute the value of  $e_N^2$  proposed in (5) in the frequency range of interest  $\omega \in \mathcal{F} = [0 \text{ Hz}, 600 \text{ Hz}] \times 2\pi$ . In our case, any frequency range is considered to be equally relevant in  $\mathcal{F}$ , so the weighting function is simply taken as

$$\eta(\omega) = \frac{1}{600 \times 2\pi} \quad (24)$$

In Fig. 3, the solid line shows the spectrum of the  $e_N^2$  over  $\mathcal{F}$ . The highest values of the mECR appear around the frequency of 80Hz which is close to the eigenfrequency of the impedance (23), tuned to be around 87,5Hz. Drops in the mECR spectrum appear at frequencies where the dynamical response of the structure is orthogonal to the sensors direction, so no data is available to investigate about the quality of the model in this direction.

Besides, the dashed line in Fig. 3, illustrates the spectrum of the  $e_N^2$  with measurements obtained on a FE model without structural damage. It is interesting to see how the presence of the structural damage, present in one single element of the FE model, becomes relevant at a medium frequency range, while the boundary condition missmodelling remains the same in the low frequencies.

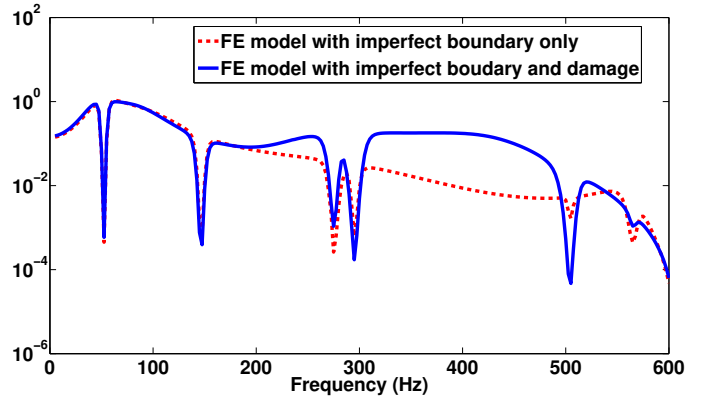


Figure 3. Initial modified ECR spectrum for a FE model with perfect clamping and lack of structural damage.

While this information improves our knowledge of the global error of the structure in the frequency domain, the figure 4 shows this distribution of the error over the structure for two significant frequencies. In other words, we calculate the value of

$$\frac{e_{K\omega}^2(\{u\}, \{v\}, \{w\})}{D_\omega^2(\{u\})} \quad (25)$$

where  $K$  is the region occupied by one finite element of the mesh. It clearly appears that the regions where the value of equation (25) is the highest correspond to those where the model differs the most to the reference structure. As a matter of fact, error at low frequencies appears to be concentrated in the element neighboring the imperfect clamping, while this error moves to the damaged element in the middle frequencies. The

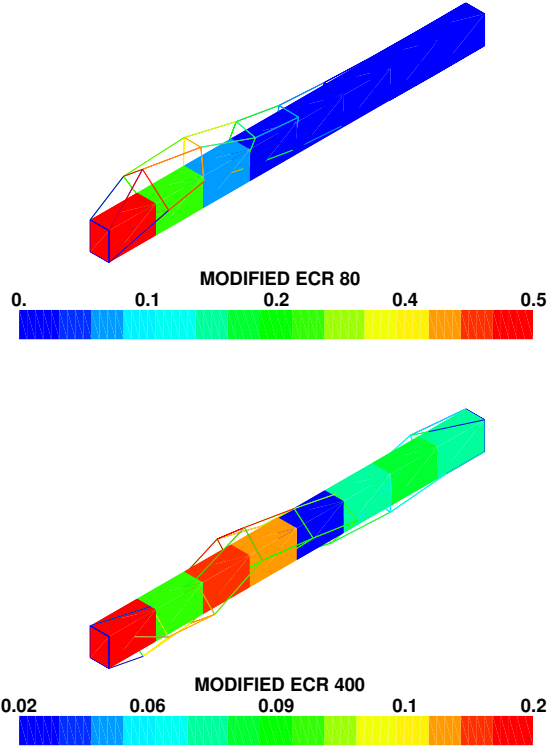


Figure 4. Comparison of the modified ECR distribution and the related  $\{u - v\}$  error field at 80Hz (top) and 400Hz (bottom).

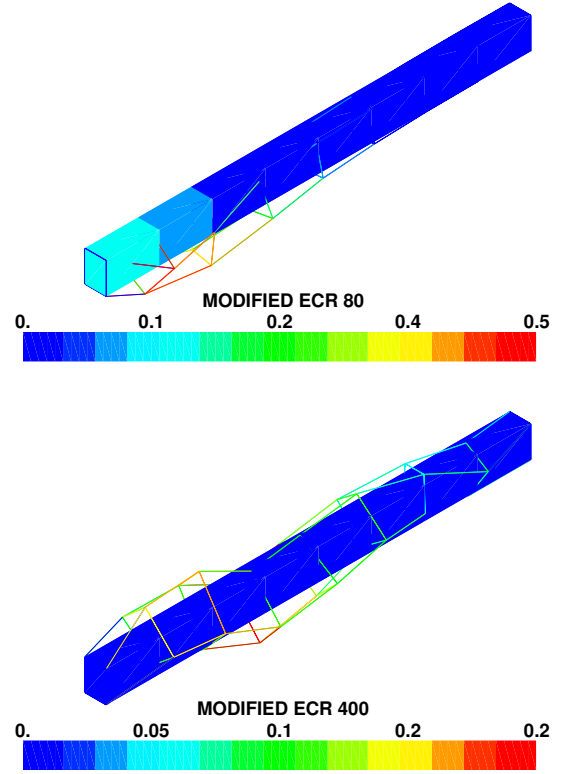


Figure 5. Comparison of the modified ECR distribution and the related  $\{u - w\}$  error field at 80Hz (top) and 400Hz (bottom).

modelling error indicator based on the modified ECR method seems to accurately locate the model errors over the structure, improving the knowledge of the model misspecification and leading to a reduced updating problem by chosen a proper subset  $\theta$  of structural parameters to be updated.

### 5.2 The UKF-mECR method to solve the updating problem

In the mECR analysis, the part of the  $e_N^2$  error introduced by  $\{u - v\}$  field is generally associated to a parameter discrepancy of the  $[\mathbf{K}]$  matrix, while the error introduced by the  $\{u - w\}$  field is often associated to a modelling error affecting the mass matrix  $[\mathbf{M}]$  (see [8] for a detailed study of such heuristic rules).

In our case, similarly to the error distributions shown figure 4, errors associated to the  $[\mathbf{K}]$  are mainly located at the element adjacent to the imperfect clamping and the element containing structural damage. In contrast, as shown in figure 5, errors associated to  $[\mathbf{M}]$  are not only lower in magnitude but also appear to be concentrated at the element adjacent to the imperfect clamping only. As a consequence, it seems to be natural to alleviate the erroneous boundary conditions by means of introducing an additional spring-mass element modelling the clamping conditions. Thus, the choice of the  $\theta$  parameter for the updating problem becomes in our case:

$$\theta = \{\delta_6 \quad k \quad m\}^T \quad (26)$$

This reduced parameter updating problem is now solved using the coupled UKF-mECR method proposed in Sec. 4.

Figure 6 shows how the UKF is capable of properly identifying the uncertain parameters of the initial model with satisfactory convergence.

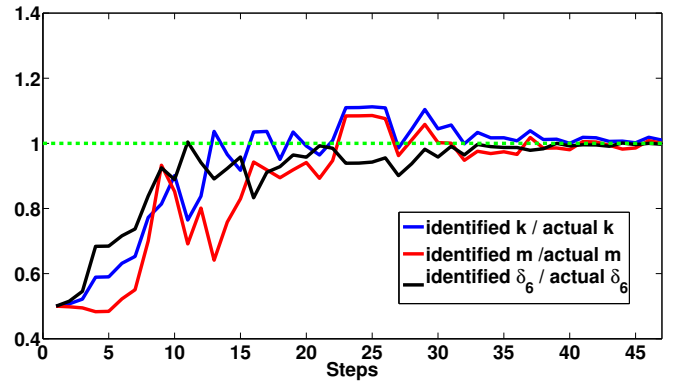


Figure 6. Damage and boundary impedance parameters identification using the mECR-UKF approach.

In a similar way, we have performed the study for the case where the structure suffers from evolutive structural damage. In that sense, we have considered an available sequence of measurements  $\{\tilde{u}\}_i$  at time  $t_i = i\Delta t$ , where the structural damage evolves over  $t$ . Thus, the iterations steps  $j$  from (21) are now considered as time steps and, as a consequence, measurements  $\{\tilde{u}\}_i$  are recursively introduced in  $e_N^2(\theta_i)$  as the identification process goes on. Hence, we can rewrite the the state-space

formulation in (21) for the evolution problem as

$$\begin{cases} \theta_{i+1} = \theta_i + w_i^\theta \\ \zeta_i = e_N^2(\theta_i, \{\tilde{u}\}_i) + v_i \end{cases} \quad (27)$$

In figure 7 we show the capability of the coupled mECR-UKF approach to track the damage evolution over time where the evolution law is *a priori* unknown.

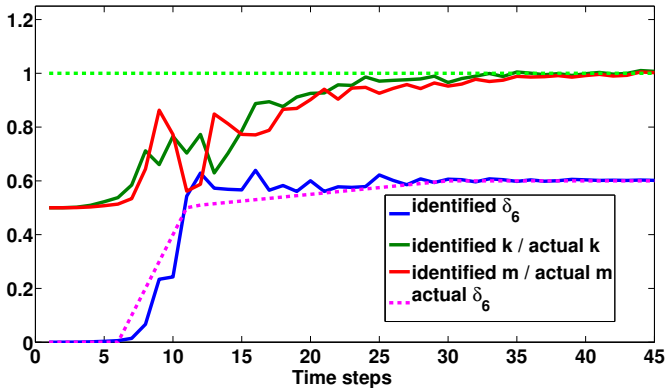


Figure 7. Evolutive damage and boundary impedance parameters identification using the mECR-UKF approach.

## 6 CONCLUSIONS

In this work he have proposed a solution strategy for identification problems associated with uncertain FE models in structural dynamics. The proposed method is based on combining the modified Error in Constitutive Relation (mECR) technique and the Unscented Kalman Filter (UKF). The mECR is first used to obtain prior information on the nature of the model errors and reduce the set of unknown structural parameters to be identified, taking advantage of its excellent properties to localize model errors. The UKF is then used to solve the updating problem, in a form where the observation equation is formulated again in terms of the mECR cost function.

A numerical example of a reinforced concrete beam has been used to show how this strategy gives particularly good results and accuracy to both locate and identify model errors such as structural damage and boundary conditions misspecification. The proposed formulation is particularly well suited to the usual FE framework of structural dynamics, where the use of the Unscented transformation avoids explicit evaluations of Jacobian and Hessian operators.

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