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# Difficulties related to Probabilities

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*Dedicated to Marie-Louise Nykamp*

## Abstract

Probability theory is often used as it would have the same ontological status with, for instance, Euclidean Geometry or Peano Arithmetics. In this regard, several highly questionable aspects of probability theory are mentioned which have earlier been presented in two arxiv papers.

## Part I :

### Redundancies : An Omission in Probability Theory ?

## Abstract

It is shown that the standard Kolmogorov model for probability spaces cannot in general allow the elimination but of only a small amount of probabilistic redundancy. This issue, a purely theoretical weakness, not necessarily related to empirical reality, appears not to have received enough attention in foundational studies of Probability Theory.

## 1. The Need for a Match

Several recent works have brought to attention the events during the first decades of the 20th century which culminated in 1933 with Kolmogorov's foundation of modern Probability Theory in his *Grundbegriffe*, see Shafer & Vovk [1], Vovk & Shafer, von Plato, or Bilova et.al.

Ever since Probability Theory first started to emerge in the work of Jacob Bernoulli early in the 18th century, there has been an interest in the extent to which such a theory may in fact match with the empirical reality it is supposed to model.

The axiom regarding "total probability", namely

$$P(A \cup B) = P(A) + P(B)$$

for two events  $A$  and  $B$  which cannot happen simultaneously, or the axiom for "compound probability", namely

$$P(A \cap B) = P(A)P(B|A)$$

for two arbitrary events did not lead to significant problems when related to empirical reality.

What on the other hand led to such problems was the so called *Cournot Principle* which in one of its formulations, Cournot, stated that :

*The physically impossible event is therefore the one that has infinitely small probability, and only this remark gives substance - objective and phenomenal value - to the theory of mathematical probability.*

This idea, however, did not originate with Cournot. Jacob Bernoulli in his 1713 *Ars Conjectandi* proved what would later be called the Law of Large Numbers which in simple terms says that in a large enough sequence of independent trials of the same event there is a high probability that the *frequency* of the event will be close to its *probability*. And Bernoulli commented that we can consider that high probability as a certainty, and thus use frequency as an estimate of probability.

## 2. Falsifiability

Only recently in the history of science, and owing to Karl Popper, did the idea clearly arise and got expressed of a proper test of the possible match, or otherwise, between a given scientific theory and the empirical reality it is supposed to model. And the test of such a match which Popper suggested, and called "falsifiability", consists of a set of fundamental experiments clearly specified by the respective theory, experiments which can effectively be performed within the empirical reality. And in case any of them may happen to fail, then the theory as a whole is considered inadequate.

Obviously, two conditions should be met by any such set of fundamental experiments :

- the set of required experiments must be finite, and preferably small,

- each such experiment should only require a finite time to perform, and preferably, a short enough time at that.

When for instance, Einstein came up in 1915 with his theory of General Relativity, he himself suggest several such fundamental experiments. One of them was the celebrated bending of light rays near to a massive mass. And early in 1919, during a Sun eclipse, Arthur Eddington managed to show that General Relativity passed that test, since the amount of bending predicted by that theory was in good correspondence with the empirical bending measured.

Needless to say, the various theories of Physics, including General Relativity, have so far been required to satisfy experiments which happen to conform to the above two conditions.

In this regard, one of the major deficiencies of String Theory is precisely the fact that, so far, it cannot offer any experiment which could effectively be performed and respect the above two conditions.

### 3. The Special Situation of Probability Theory

As far as the mentioned Cournot Principle is concerned, it is quite obvious that any empirical experiments which would be able to test it in the sense of Popper's falsifiability are duty bound to fail the second above condition. Indeed, there is simply no a priori way of knowing how large a finite sequence of independent experiments of the same event should be, in order to give the possibility for obtaining a sufficiently reliable acceptance or rejection of that principle.

In this way, Probability Theory is in the special position to be unable to either confirm or falsify itself on effective empirical grounds. And this comes from the simple fact that aiming to establish a close enough connection between *frequency* and *probability* cannot lead to empirical tests which satisfy the above second condition, since any such close enough connection can only stand or fail at the *infinite limit*.

Needless to say, even if not clearly and explicitly enough, there has been an awareness of this problem, and consequently, a variety of solutions has been proposed, as seen for instance, in Shafer & Vovk [1].

Among the more recent major ideas in this regard, one can mention the following two :

- Relating probability to *complexity* in the sense of Kolmogorov.
- Relating probability to *games*, Shafer & Vovk [2].

The first idea, already about half a century old, has proved to be particularly seminal, and not only in Probability Theory, but also in several major branches

of Mathematics, as seen for instance in Chaitin.

The second idea, a more recent one, may at first appear as risking to lead well beyond what would conventionally be understood by randomness or probability. Indeed, Kolmogorov complexity is defined by the performance of the shortest possible program on a universal Turing machine. On the other hand, a game, as suggested for instance in Shafer & Vovk [2], Vovk & Shafer, comprises more than one *conscious* player. Thus one may become concerned that the processes which such a game can generate may in fact go beyond what one usually conceives of as being random or probabilistic.

#### 4. Redundancies : An Omission ?

It appears that, historically, the concerns related to Probability Theory have been focused upon its possible match with empirical reality, and dealt mainly, if not in fact, mostly with the issue of the relation between frequency and probability, a relation which as mentioned above, proves not to be within the realms of customary empirical tests.

And during this long ongoing process, it appears that an important *internal purely theoretical weakness* of Probability Theory - a weakness which is clearly there, even if no relation to empirical reality is involved - was not given attention. Namely, let us consider an arbitrary probability space

$$(1) \quad (\Omega, \mathcal{F}, P)$$

where  $\mathcal{F}$  is a  $\sigma$ -algebra of subsets of  $\Omega$ , while  $P : \mathcal{F} \longrightarrow [0, 1]$  is the respective probability. Further, let us assume, as customary, that

$$(1a) \quad \{ \omega \} \in \mathcal{F}, \quad \text{for } \omega \in \Omega$$

There are now two cases :

- the set  $\Omega$  is finite or countable,
- the set  $\Omega$  is uncountable.

In the first case, let us take

$$(2) \quad \Omega_1 = \{ \omega \in \Omega \mid P(\{ \omega \}) > 0 \}$$

in other words, we discard from  $\Omega$  all the points  $\omega$  which have probability zero, thus are *redundant probabilistically*. In this way  $\Omega_1$  represents the set of probabilistically non-redundant points. Then clearly

$$\Omega_1 \in \mathcal{F}$$

since  $\Omega_1 = \Omega \setminus \{ \omega \in \Omega \mid P(\{ \omega \}) = 0 \}$ , and the second set in the right hand term is at most countable, thus in view of (1a), it belongs to  $\mathcal{F}$ .

Further, corresponding to  $\Omega_1$ , let us take

$$(3) \quad \mathcal{F}_1 = \{ A \cap \Omega_1 \mid A \in \mathcal{F} \}$$

thus  $\mathcal{F}_1 \subseteq \mathcal{F}$ , and  $P(B) > 0$ , for  $B \in \mathcal{F}_1$ ,  $B \neq \phi$ .

Finally, let us take

$$(4) \quad P_1 : \mathcal{F}_1 \longrightarrow [0, 1] \quad \text{defined by} \quad P_1 = P|_{\mathcal{F}_1}$$

In this way we obtain the *probabilistic redundancy free* probability space

$$(5) \quad (\Omega_1, \mathcal{F}_1, P_1)$$

And now one can note that much of the probabilistic phenomena of interest are *the same* on the probability spaces  $(\Omega, \mathcal{F}, P)$  and  $(\Omega_1, \mathcal{F}_1, P_1)$ . And needless to say, it is more convenient to work on the latter, precisely owing to the lack of redundancies.

On the other hand, in the second case above, namely, when  $\Omega$  is uncountable, one *cannot* in general apply the above procedure in order to get rid of *probabilistic redundancies*. For instance, let us take the simplest usual case when

$$(6) \quad \Omega = [0, 1] \subset \mathbb{R}$$

while  $\mathcal{F}$  and  $P$  are, respectively, the set of Lebesgue measurable subsets of  $[0, 1]$  and the Lebesgue measure on  $[0, 1]$ . Indeed, in this case we obtain

$$(7) \quad \Omega_1 = \phi$$

since every point  $x \in [0, 1]$  has Lebesgue measure zero. Thus we simply cannot eliminate *all probabilistically redundant* points, since we do no longer remain with a nontrivial probability space.

This failure related to the elimination of probabilistic redundancy in the Kolmogorov model is one of the main reasons in the technical difficulties encountered in the study of stochastic processes with continuous time.

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## Part II :

### Quantum Foundations :

### Is Probability Ontological ?

#### Abstract

It is argued that the Copenhagen Interpretation of Quantum Mechanics, founded ontologically on the concept of probability, may be questionable in view of the fact that within Probability Theory itself the ontological status of the concept of probability has always been, and is still under discussion.

#### 1. Two Issues in Probability Theory

Two long unresolved - and in fact, seldom considered - issues have for more than two centuries by now accompanied modern Probability Theory.

The first one we shall only mention as an example of how fundamental aspects can be - and in fact, are - overlooked. The second one, which has not been completely overlooked, has nevertheless found itself side lined for longer, even if it has obvious major implications in Quantum Foundations.

As for the more general background on the various, and often severely conflicting views and interpretation of the basic concepts or methods of Probability Theory, a recent survey can be found in [1].

One of the two issues mentioned above, namely, the issue of *redundancies*, [2], seems not to have been the object of any wider awareness, although it has led to considerable technical difficulties in, among others, stochastic processes with continuous time. And as it happens, this issue has not been overcome even by the nonstandard approach to probabilities, specifically, by the introduction of Loeb measures and integration.

The second issue, which is known at least in principle, seems nevertheless similarly to be outside of the realms of general awareness, and certainly, of a more active pursuit in its possible implications in a large variety of applications of Probability



Theory. This issue concerns whether the axioms of Probability Theory, as given for instance in their formulation due to Kolmogorov, happen to have a deeper status than mere *epistemic convenience*.

## 2. Deterministic, versus Non-Deterministic Phenomena

Classical Physics, as well as Special and General Relativity, typically deal with *deterministic* phenomena, such as for instance those related to gravitation, electromagnetism, and so on.

What is important to note here is that, usually in such situations, the classification "deterministic" is not merely an epistemic choice of convenience, but it is rather seen as an essential ontological feature of the respective phenomena.

On the other hand, and not only in the realms of Physics, there are phenomena which obviously cannot be considered deterministic, be it from an ontological, or for that matter, epistemic point of view.

In this regard, until the 1960s, there has been one single subclass of non-deterministic phenomena specifically identified and studied as such, namely, the subclass composed of those phenomena which were considered to be *probabilistic*. And clearly, even if not always explicitly expressed, the subclass of probabilistic phenomena was not - and in fact, could not be - identified with the whole class of non-deterministic ones.

However, since the work of Zadeh in the 1960s, a second subclass of non-deterministic phenomena, namely, those called *fuzzy*, has been pointed out, even if not in as clear and rigorous mathematical formulation as that of the probabilistic ones. And needless to say, there is no claim that these two classes, namely, the probabilistic and the fuzzy one, may have a significant overlap. Similarly, there is no - and there could not possibly be - any claim that these two classes exhaust all the non-deterministic phenomena.

Not much later, in the 1970s, with the work of Feigenbaum, a further subclass of phenomena, namely, the *chaotic* ones got singled out. Here however, a somewhat surprising turn of events happened, as the chaotic phenomena studied ever since have typically been those that may be seen as "deterministic chaos", that is, chaotic phenomena produced - rather surprisingly - by certain deterministic nonlinear systems.

## 3. Ontological, or merely Epistemic ?

Being here mostly interested in the issue of the ontological, or on the contrary, merely epistemic status of modern Probability Theory, we shall only consider the following division in classes of phenomena :

- deterministic
- non-deterministic
  - probabilistic
  - fuzzy
  - the rest

Several remarkable facts, seldom, if at all of a wider concern, are worth mentioning about these classes.

First perhaps is the fact that the class of non-deterministic phenomena is simply defined by a mere *negation*. And as it is a well known elementary fact of Logic, such a definition by negation is hardly appropriate, since it opens up all the possibilities other than specified by the negated one, [4-6]. This is therefore a possible major source of the difficulties which may lurk at the bottom of all subsequent dealings with the resulting concept of non-deterministic phenomena. In particular, this definition by mere negation may negatively affect the attempts to set up definitions for the subclasses of probabilistic, fuzzy or chaotic phenomena, not to mention other possible non-deterministic subclasses.

And one of such important negative effects may be in the fact that, although we tend to see the concept of "deterministic" as having an essentially ontological status, the definition by a mere negation of the concept of "non-deterministic" may seriously weaken any hope for a similar status of that concept, and consequently, of its particular cases of concepts, such as "probabilistic", "fuzzy" "chaotic", and so on.

Focusing now on the concept of "probabilistic", it should be recalled that, back in the 1930s, soon after the setting up of the Kolmogorov model, De Finetti pointed out that, within such a model there are most serious issues as to the ontological status of the concept of probability. And in a somewhat shocking and provocative manner, De Finetti stated, [3], that "probability does not exist!" ...

As it happens, such a view is but a part of a larger trend or school of thought, called usually *Subjective Probability*, [1], a trend which incorporates the much older Bayesian subjectivity which is still in use in certain circles, as well as a number of other ones. And the main claim of that view is that the concept of probability is *lacking* any ontological status, being instead a mere *epistemic* choice of convenience, therefore subjective as such.

Needless to say, in the case of the fuzzy subclass of non-deterministic phenomena, the ontological status of that concept appears to have even a lesser likelihood than that of the concept of probability.

#### 4. Should Quanta Be Founded on Ontologically Uncertain Probabilities ?

Starting with the mid 1920s, when the modern version of Quantum Mechanics replaced the original one introduced in 1900 by Planck, and developed by Einstein, Bohr and a few others, it appeared to be a matter of a significant pride to Bohr, Heisenberg and Born to have originated such a totally unprecedented new theory of Physics, one which in its rather incredibly counterintuitive novelty went far beyond even of Einstein's General Relativity. And in the view of their originators, a view which was to become the so called Copenhagen Interpretation, a major, if not in fact, by far the most major novelty was what they considered to be the inevitable and irreducible involvement of randomness or probability in quantum phenomena.

This position of the founders of modern Quantum Mechanics was precisely that which created the extreme separation between them, and on the other hand, a few others, among them at the time Einstein, Schrödinger, De Broglie, and later, Bohm and Bell. In this regard, the often cited maxim of Einstein that "God does not play dice" expresses quite clearly the total opposition between the respective two views.

In other words, the founders of modern Quantum Mechanics postulated nothing short of an *ontological* status for probabilities in quantum phenomena. On the other hand, Einstein and others were only willing to accept for probabilities a mere *epistemic* status. In particular, in Einstein's view, the wave function, and the corresponding probability amplitude, were only describing an ensemble of quantum particles prepared in the same way, and by no means one single individual such particle. Thus Einstein's view that Quantum Mechanics, as advocated by the Copenhagen Interpretation, had to be incomplete, a view famously presented in the 1935 celebrated EPR paper.

But now, with hindsight, the following issue arises :

- Quantum Mechanics, in its Copenhagen Interpretation, is founded on the ontological status played in that theory by the concept of probability. On the other hand, that ontological status of the concept of probability has for long been questioned in Probability Theory. Therefore, should, or for that matter, can Quantum Mechanics have an ontology which insists on probabilities ?

The fact that the founders of modern Quantum Mechanics did not seem to consider the above issue is easy to explain. As most of those who apply Probability Theory, they were not much concerned with the inner affairs of that theory. Not to mention that the work of De Finetti only started to appear a decade later, in the 1930s.

What may, however, be less easy to explain is why even today no concern is shown related to the mentioned possibly ill-founded ontology of Quantum Mechanics in its Copenhagen Interpretation.

## 5. Infinity Again Causes Trouble ...

One of the basic problems - even if hardly at all known within wider circles of mathematicians or physicists - which keep troubling Probability Theory is the inevitable and essential involvement of *infinity*.

In this regard, it is worth recalling that Euclidean Geometry has for more than two millennia - that is, until Bolyai and Lobachevski introduced in the early 1800s non-Euclidean Geometry - been also troubled by infinity. Indeed, Euclid, with so many others after him, firmly believed that postulates, or what we call nowadays by the name of axiomxs, must be self-evident, this being the only basis for their acceptance. However, his Fifth Postulate on parallel lines was not only clearly more complex in its formulation than the other postulates, but on top of that, it was also the only one which involved infinity, and as such, it was not within the realms of direct empirical verification.

The effect was that for more than two millennia, it got singled out as being less self-evident than the other postulates. And as a result, the general attitude has been to question its status of being a postulate.

What happened, however, was that such a questioning could for long only make those involved aware of one single logical option. Namely, if the fifth postulate was not in fact a postulate, then it had to be a consequence of the other postulates, therefore, it could be proved based on the other postulates.

As we know, such an approach proved to be wrong, and a second logical possibility turned out to exist, namely that the fifth postulate was actually *independent* of the other ones. And then, one could build a Geometry in which the *negation* of the fifth postulate would hold, together with the other axioms of Euclidean Geometry. Bolyai and Lobachevski chose one of the two possible negations, namely that there are many different parallel lines to a given line which pass through a point outside of that line.

Now, in Probability Theory infinity appears also inevitably, and does so in several not unrelated ways. Here, for the sake of brevity, we mention two of them. Further details can be found in [1].

An important feature of both of these approaches is that they consider probability as being *ontological*, and not merely epistemic. Or in other words, they consider probability to exist as a reality in the world, and not only as a construct in our mind.

The *frequency* based view of probability is indeed most natural in case of a finite number of possible events. However, when attempted to extrapolate it to an *infinite* number of events, one faces difficulties which in their foundational aspects are not unlike those faced with the Fifth Postulate of Euclidean Geometry.

The *propensity* based view of probability seems also rather natural, even if somewhat more involved and subtle than the frequency based one. And similarly, it involves *infinity* inevitably and in an essential way.

In view of the above, what may indeed turn out - regardless of anything else - to be simply amusing about the Copenhagen Interpretation is the apparent total lack of awareness of its proponents and supporters of the fact that they are inevitably involving themselves in dealing with infinity, and doing so in ways Physics, or for that matter, physicists have never done it before.

Indeed, in all other branches of Physics, infinity only appears as a possible *quantity* of one or another of the physical entities. And the definition of such entities does not at all involve infinity. Also, empirically one can deal with such entities without having to get involved with infinity, except for special cases which are outside of by far most of the usual encounters with Physics, be they theoretical, experimental, or applicative.

On the other hand, according to the Copenhagen Interpretation, which sees probability as ontological in the realms of quanta, the inevitable and essential involvement of infinity is there, as mentioned above, and it is there in completely new - yet consciously hardly known - ways in Physics ...

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