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## Nonlinear Ultrasonic Spectroscopy Using ESAM and DORT Symbiosis

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Combination of DORT (Décomposition de l'Opérateur du Retournement Temporel) and ESAM (Excitation Symmetry Analysis Method) is presented as a powerful tool for detection and localization of defects by analysis of their nonlinear signature [1, 2]. The third order nonlinear responses are extracted from ESAM eigen-excitations and constitute the multistatic data matrix associated with the singular value decomposition. The symbiosis of ESAM-DORT provides a normalization process of singular values associated with scatterers, validated in simulations and experiments of acoustic propagation [3]. Piezoelectric transducers are used for both excitation and data acquisition. Transmitters consequently emit the excitation signals and corresponding responses are measured by the array of receivers. The amplitude and phase of excitation signals is variable as to separate the nonlinear parts of the measured signal. ESAM signal pre-processing is used for nonlinear parts extraction. Separated signal records form nonlinear multistatic data matrix. DORT method is applied on the data matrix to separate echoes of defects in the tested medium. Data obtained from DORT method are used for evaluation of nonlinear parameters corresponding to separated defects and also for their localization. The procedure is completed by visualization of nonlinear signatures of detected defects which is referred to as pseudotomographic imaging. This ESAM-DORT approach is compared to AE method and seems to improve the performance of the TR-NEWS methods, which can be applied to the tomography of structural defects.

### Introduction

Methods based on elastic wave propagation provide a very effective tool in non-destructive testing (NDT) for detection and quantification of damage in various materials. The most recent trend is to link time reversal (TR) and nonlinear (NEWS = Nonlinear Elastic Wave Spectroscopy) approaches of ultrasonic signal processing [4, 5]. On one hand, time reversal offers a bulk of advantages concerning signal-to-noise ratio enhancement, auto-focusing and heterogeneous medium compensation [6], on the other hand, nonlinear approach permits evaluation and quantification of nonlinear behavior typically generated by thin cracks and other defects like delamination or inhomogeneities [7], which are the major subject of interest of latest NDT, since they are hardly detectable by standard linear methods.

In this paper a new approach of TR-NEWS link will be presented. TR process is represented by DORT method (French acronym for *Décomposition de l'Opérateur de Retournement Temporel*, i.e. decomposition of time reversal operator), an approximative method which allows separation of invariants of time reversal process, i.e. the Green's functions (pure echoes) corresponding to well-resolved scatterers present in the tested sample [1]. The nonlinear signature of detected scatterers is evaluated by ESAM (*Excitation Symmetry Analysis Method*). ESAM is used to extract nonlinear terms from the acoustic response. In order to obtain these terms, different excitations are used and corresponding responses are treated according to the properties of point group  $C_3$  [2].

Combination of ESAM and DORT allows us formulate quantified nonlinear parameters associated to different scatterers or zones of the tested sample. With regard to the

sample geometry and positions of the transducers used for the data acquisition, the parameter values can be visualized to obtain an image of damaged zones.

## 1 ESAM-DORT Analysis

### 1.1 ESAM signal pre-processing

Medium with tiny cracks generally shows an enhanced 3<sup>rd</sup> order nonlinearity which could be extracted with the ESAM signal processing approach using optimized excitations [2]. ESAM uses nonlinear signature coming from the direct acoustic response as a pre-processing tool for the extraction of nonlinear signature.

If scatterers' responses are supposed to have cubic expansion in strain, the nonlinear response can be written as  $y(t) = N_1x(t) + N_2x^2(t) + N_3x^3(t)$ , where  $N_1$ ,  $N_2$  and  $N_3$  are nonlinear parameters. With use of the properties of point group  $C_3$  and its irreducible representation, ESAM permits extraction of these parameters. The responses  $y_E$ ,  $y_\epsilon$ ,  $y_{\epsilon^*}$  to the excitations  $x_E = x(t)$ ,  $x_\epsilon(t) = x(t)e^{\frac{2i\pi}{3}}$ ,  $x_{\epsilon^*}(t) = x(t)e^{-\frac{2i\pi}{3}}$  enable extraction of the cubic term  $s_3(t) = N_3x^3(t)$  by

$$s_3(t) = N_3x^3(t) = \frac{y_E(t) + y_\epsilon(t) + y_{\epsilon^*}(t)}{3}. \quad (1)$$

Term  $s_3(t)$  extracted by ESAM corresponds to the nonlinear behavior of the system, but also it still depends on the original excitation  $x(t)$  and its powers. Therefore, to obtain the unique coefficient  $N_3$ , the excitation dependent parts must be eliminated. In order to realize it, cubic energy related to the excitation, given by

$$E_{03} = \int_{-\infty}^{\infty} |x^3(t)|^2 dt, \quad (2)$$

is included in a calibration process. Energy corresponding to the nonlinear term extracted from the acoustic responses is then given by

$$E_3 = \int_{-\infty}^{\infty} |s_3(t)|^2 dt = N_3^2 E_{03}. \quad (3)$$

Hence the nonlinear coefficient can be evaluated by:

$$N_3 = \sqrt{\frac{E_3}{E_{03}}}. \quad (4)$$

## 1.2 DORT method

To distinguish the different scatterers in the medium, DORT method is added to the ESAM pre-processing. DORT is an approximative method which substitutes the iterative time-reversal process. With use of this method, it is possible to extract echoes corresponding to well-separated defects present in the media (their Green's functions), which can be used to focus on these defects [1]. Moreover, for each defect, information about its frequency dependant apparent reflectivity is obtained. This is associated to its nonlinear behavior extracted with ESAM.

DORT can be implemented by transmit-receive scheme in a single array or transducers of between two different transducer arrays [8], where the number of transmitters does not necessarily have to be equal to the number of receivers. Let us consider  $K$  transmitters  $E_k$ ,  $k \in \hat{K} = \{1, 2, \dots, K\}$ , and  $M$  receivers  $R_m$ ,  $m \in \hat{M} = \{1, 2, \dots, M\}$ . Usually in the TR-NEWS experimental configurations, the number of receivers is equal or less than the number of transmitters ( $M \leq K$ ). With use of these transducer arrays an inter-element  $K \times M$  matrix  $K(t)$  is constructed from the responses: matrix element  $K_{mk}(t)$  is the signal received on  $R_m$  after emission from  $E_k$ . After Fourier transform ( $\mathcal{F}$ ), the transfer matrix  $K(f) = \mathcal{F}[K(t)](f)$  is obtained, from which Time Reversal Operators  $T_{Tx}(f)$ ,  $T_{Rx}(f)$  are constructed according to the basis (transmit or receive) with

$$T_{Tx}(f) = K^*(f)K(f) \in \mathbb{C}^{K,K}, \quad (5)$$

$$T_{Rx}(f) = K(f)K^*(f) \in \mathbb{C}^{M,M}. \quad (6)$$

Generally,  $T_{Tx}(f)$  and  $T_{Rx}(f)$  are different, but both have the same rank, equal to the rank of  $K(f)$  ( $\text{rank}(K) = D$ ), with the same number of non-zero eigenvalues. Since they are both hermitian positives, they can be diagonalized. In practice, the diagonalization of  $T_{Tx}(f)$  and  $T_{Rx}(f)$  is not used, since it is mathematically equivalent to singular value decomposition

$$K(f) = U(f)\Sigma(f)V^*(f), \quad (7)$$

where  $U(f)$  is a  $M \times M$  unitary matrix ( $U^* = U^{-1}$ ),  $V(f)$  is a  $K \times K$  unitary matrix and  $\Sigma(f)$  is a  $M \times K$  diagonal positive semidefinite matrix. Columns of  $U(f)$  are the eigenvectors of  $T_{Rx}(f) = K(f)K^*(f)$  (the TRO in receive basis), columns of  $V(f)$  are the eigenvectors of  $T_{Tx}(f) = K^*(f)K(f)$  (the TRO in transmit basis). The singular values  $\sigma_d(f) = \Sigma_{dd}(f) \neq 0$ ,  $d \in \hat{D}$ , corresponding to the transfer matrix  $K(f)$  are the square roots of the TRO eigenvalues  $\lambda_d(f)$ , same in transmit and receive basis. Thus, the apparent reflectivity of the scatterers where their corresponding Green's functions  $\mathbf{g}(f)$  are given by the TRO eigenvectors[1]. For each frequency component  $f_0$  and scatterer  $S_d$ , a monochromatic Green's

function  $\mathbf{g}_d(f_0)$  is obtained in transmit  $\mathbf{g}_d^{(T)}(f_0)$  and receive  $\mathbf{g}_d^{(R)}(f_0)$  basis together with singular value  $\sigma_d(f_0)$ :

$$\mathbf{g}_d^{(R)}(f_0) = U_{\bullet d}(f_0) \in \mathbb{C}^M, \quad (8)$$

$$\mathbf{g}_d^{(T)}(f_0) = V_{\bullet d}(f_0) \in \mathbb{C}^K, \quad (9)$$

$$\sigma_d(f_0) = \Sigma_{dd}(f_0) \in \mathbb{R} - \{0\}, \quad (10)$$

where  $U_{\bullet d}$  denotes the  $d^{\text{th}}$  column of matrix  $U$  and  $V_{\bullet d}$  denotes the  $d^{\text{th}}$  column of matrix  $V$ . For nonlinear parameters evaluation it is necessary to reconstruct the spectrum of broadband Green's function  $\mathbf{g}(f)$  and link together singular values  $\sigma(f)$  corresponding to given scatterer [8, 9].

## 1.3 Nonlinear parameters of different scatterers

### 1.3.1 Construction of multistatic data matrix

For the extraction of nonlinear parameters corresponding to the scatterers, DORT is applied on the separated nonlinear terms of direct responses provided by ESAM. This procedure requires measurement of inter-element responses for each excitation necessary for ESAM application. From these responses the nonlinear term  $s_{3,mk}(t)$  corresponding to the trajectory  $|T_k R_m|$  is extracted and finally multistatic data matrix  $K^{(3)}(t)$  corresponding to the extracted nonlinear terms is constructed according to

$$K_{mk}^{(3)}(t) = s_{3,mk}(t). \quad (11)$$

### 1.3.2 Implementation of DORT method

DORT method is applied on multistatic data matrix  $K^{(3)}(t)$ , as described previously in subsection 1.2. Finally broadband Green's functions  $\mathbf{g}_d^{(3)}(f)$  and corresponding singular values  $\sigma_d^{(3)}(f)$  are obtained for each separated scatterer  $S_d$ .

### 1.3.3 Parameters evaluation

For nonlinear parameters evaluation it is necessary to reconstruct echo associated to every separated scatterer  $S_d$  and trajectory  $|T_k R_m|$ . First extracted singular values  $\sigma_d^{(3)}(f)$  undergo a normalization by corresponding Green's functions in transmit array ( $\mathbf{g}_d^{(T,3)}(f) \in \mathbb{C}^K$ ) and in receive array ( $\mathbf{g}_d^{(R,3)}(f) \in \mathbb{C}^M$ ):

$$\tilde{\sigma}_{d,km}^{(3)}(f) = \sigma_d^{(3)}(f) \cdot g_{d,k}^{(T,3)}(f) \cdot g_{d,m}^{(R,3)}(f), \quad (12)$$

where  $g_{d,k}^{(T,3)}(f)$  is the  $k^{\text{th}}$  element of  $\mathbf{g}_d^{(T,3)}(f)$ . This normalization introduces trajectory dependence (indices  $k$  and  $m$  correspond to trajectory  $|T_k R_m|$ ) into the nonlinear parameters. Fourier transform of normalized singular values  $\tilde{\sigma}_{d,km}^{(i)}(f)$  give the desired echo  $e_{d,km}^3(t)$ :

$$e_{d,km}^3(t) = \mathcal{F}^{-1}[\tilde{\sigma}_{d,km}^{(3)}(f)]. \quad (13)$$

The corresponding energy  $E_3^{(d,km)}$  is given by

$$E_3^{(d,km)} = \int_{-\infty}^{\infty} |e_{d,km}^3(t)|^2 dt, \quad (14)$$

and the nonlinear parameters  $N_3^{(d)}$  are finally evaluated for each trajectory  $|T_k R_m|$ :

$$N_3^{(d,km)} = \sqrt{\frac{E_3^{(d,km)}}{E_{03}}}, \quad (15)$$

where  $E_{03}$  is given by Eq (2). To enhance the parameter variation, the relevant parameters are brought to a square:  $(N_3^{(d,km)})^2$ .

## 2 Experimental tests

### 2.1 Testing sample

The previous theoretical principles were first tested by numerical simulations and simple experiment, which gave satisfactory results [10]. Therefore the method was used for evaluation of damage of a real part of plane chassis - Actuator Steering Bracket (ASB). This part underwent a series of loading cycles. During the loading acoustic emission (AE) was monitored. After localisation by neural networks [11] a cluster of AE sources was identified (Figure 1). This result shows a possibility of initialisation of a damage (crack) in the area of the cluster.

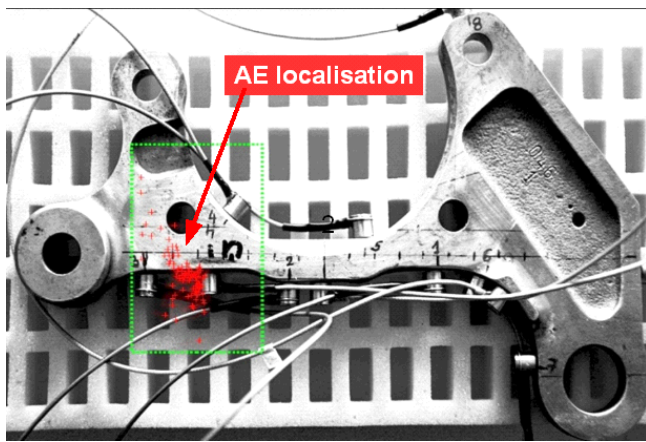


Figure 1: Localisation of a cluster of AE sources generated during loading cycles on ASB sample.

### 2.2 Experiment

The method presented in this paper was used to examine the nonlinear properties of damaged ASB. The hypothesis was that the enhanced nonlinear signature should be related to the spot, which was identified by AE as a possible origin of a crack.

Emitting array consisted of 6 transducers, used in transmit and receive mode. Basic excitation  $x(t)$  was 15 periods of sine wave of frequency  $f_0 = 500kHz$ , the phase shifts were:  $0$ ,  $\frac{2\pi}{3}$  and  $-\frac{2\pi}{3}$ . Sampling frequency was set to 10MHz. The ESAM-DORT procedure was carried out, resulting nonlinear parameters are summarized in Table 1. Better view of the nonlinear parameter distribution is given by visualisation (Figure 2).

The visualisation is coherent with the previous AE result - the nonlinearity is largely enhanced in the spot localised as a possible origin of a crack. This corresponds to the expectations and also shows a relevance of the results obtained by ESAM-DORT method.

## Conclusion

The link between nonlinear analysis and time-reversal represented by ESAM and DORT methods was established and

Table 1: Nonlinear parameters of the ASB sample

$(N_3)^2$	$T_2$	$T_3$	$T_4$	$T_5$	$T_6$
$T_1$	103	102	27	11	50
$T_2$	–	183	78	18	76
$T_3$	–	–	34	14	77
$T_4$	–	–	–	7	21
$T_5$	–	–	–	–	26

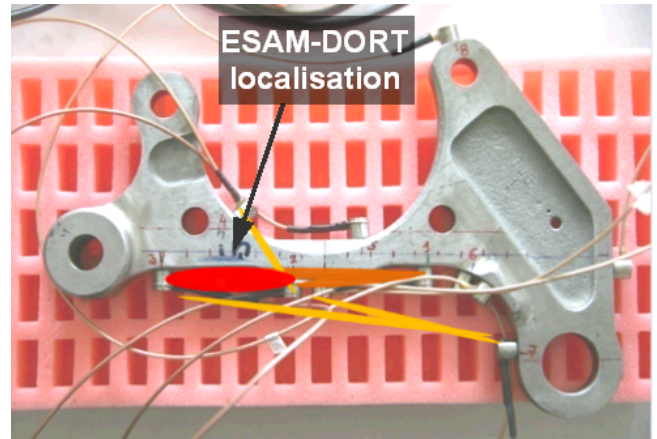


Figure 2: Visualisation of determined nonlinear parameters along the trajectories between the transducers. Red colour signifies high value, orange medium and yellow small value.

tested. Experiment on a damaged sample coming from aeronautic industry shows high potential of this method in detection and localisation of problematic zones, where cracks and other damage are likely to appear. The results of the method are coherent with another well known and widely used method, the acoustic emission (AE), which signifies high relevance of the obtained results. The implementation of the method is relatively easy, obtained parameters can be used for defective zones imaging. Improvements of the method mainly in the visualisation domain is a subject to further research. The ESAM-DORT processing is also promising for nonlinear imaging of complex structures. Medical application such as tooth, skin or brain could be proposed as an innovative perspective of ESAM-DORT pseudotomography.

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