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# Induction Motor Control through AC/DC/AC Converters

## Formal analysis of speed regulation performances and power factor correction

A. Elfadili, F. Giri, H. Ouadi, A. Magri, L. Dugard, A. Abouloifa

**Abstract**— We are considering the problem of controlling inductions motors driven through AC/DC rectifiers and DC/AC inverters. The control objectives are threefold: (i) forcing the motor speed to track a reference signal, (ii) regulating the DC Link voltage, (iii) assuring a satisfactory power factor correction (PFC) with respect to the power supply net. First, a nonlinear model of the whole controlled system is developed in the Park-coordinates. Then, a nonlinear multi-loop controller is synthesized using the backstepping design technique. A formal analysis based on Lyapunov stability and average theory is developed to describe the control system performances. In addition to closed-loop global asymptotic stability, it is proved that all control objectives (motor speed tracking, DC link voltage regulation and unitary power factor) are asymptotically achieved up to unavoidable but small harmonic errors (ripples).

### I. INTRODUCTION

It is widely recognized that the induction motor is going to become the main actuator for industrial purposes. Indeed, as compared to the DC machine, it provides a better power/mass ratio, simpler maintenance (as it includes no mechanical commutators) and a relatively lower cost. However, the problem of controlling the induction motor is more complex, as its model is multivariable and highly nonlinear. A considerable progress has been made in power electronics technology leading to reliable power electronic converters and making possible varying speed drive of inductions machines. Indeed, speed variation can be achieved for these machines by acting on the supply net frequency. Until the development of modern power electronics, there was no effective and simple way to vary the frequency of a supply net. On the other hand, in the electric traction domain, the used power nets are either DC or AC but mono-phase. Therefore, three-phase DC/AC inverters turn out to be the only possible interface (between railway nets and 3-phase AC motors) due to their high capability to ensure flexible voltage and frequency variation.

The above considerations illustrate the major role of modern power electronics in the recent development of electrical traction applications (locomotives, vehicles ...). As mentioned above, a 3 phase DC/AC inverter used in traction is supplied by a power net that can be either DC or mono-phase AC. In the case of AC supply, the (mono-phase) net is connected to the three-phase DC/AC inverter through a transformer and an AC/DC rectifier. The connection line between the rectifier and the inverter is called DC link.

The control problem at hand is to design a controller ensuring a wide speed range regulation for the system including the AC/DC converter, the DC/AC inverter and the induction motor. The point is that such system behaves as a nonlinear load seen by the AC supply line. Then, undesirable current harmonics are likely to be generated in the AC line. These harmonics reduce the rectifier efficiency, induce voltage distortion in the AC supply line and cause electromagnetic compatibility problems. The pollution caused by the converter may be reduced resorting to additional protection equipments (transformers, condensers...) and/or over-dimensioning the converter and net elements. However, this solution is costly and may not be sufficient. To overcome this drawback, the control objective must not only be motor speed regulation but also current harmonics rejection. The last objective is referred to power factor correction (PFC) [1].

Previous works on induction machine speed control simplified the control problem neglecting the dynamics of the AC/DC rectifier i.e. focusing only on the set 'DC/AC inverter - Motor'. This simplified system has been dealt with using several control strategies ranging from simple techniques, e.g. field-oriented control [5], to more sophisticated nonlinear approaches, e.g. direct torque control [6]. A control strategy that ignores the presence of the AC/DC rectifier suffers at least from two drawbacks. First, the controller design relies on the assumption that the DC voltage (provided by the AC/DC rectifier) is perfectly regulated; the point is that perfect regulation of the rectifier output voltage can not be ensured ignoring the rectifier load which is nothing other than the set 'DC/AC inverter - Motor'. The second drawback lies in the entire negligence of the PFC requirement. That is, from a control viewpoint, it is not judicious to consider separately the inverter-motor association, on one hand, and the power rectifier, on the

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other hand.

In the present work, we will develop a new control strategy dealing simultaneously with both involved subsystems i.e. the AC/DC inverter and the combination ‘DC/AC inverter-motor’. Our control strategy is featured by its multi-loops nature. First, a current loop is designed so that the coupling between the power supply net and the AC/DC rectifier operates with a unitary power factor. Then, a second loop is designed to regulate the output voltage of the AC/DC rectifier so that the DC-link between the rectifier and the inverter operates with a constant voltage despite changes of the motor operation conditions. Finally, a bi-variable loop is designed to make the motor velocity track its varying reference value and to regulate the rotor flux norm to its nominal value. All control loops are designed using the Lyapunov and backstepping techniques [3]. It will be formally proved that the proposed multi-loop controller actually stabilizes (globally and asymptotically) the controlled system and does meet its tracking objectives with a good accuracy.

The paper is organized as follows: the system under study (i.e. the AC/DC/AC converter and induction motor association) is modeled and given a state space representation in Section 2; the controller design and the closed-loop system analysis are presented in Section 3; the controller performances are illustrated through numerical simulations in Section 4.

## II. MODELING THE ‘AC/DC/AC CONVERTER-INDUCTION MOTOR’ ASSOCIATION

The controlled system is illustrated by Fig 1. It includes an AC/DC boost rectifier, on one hand, and a combination ‘inverter-induction motor’, on the other hand. The inverter is a DC/AC converter operating, like the AC/DC rectifier, according to the known Pulse Wide Modulation (PWM) principle.

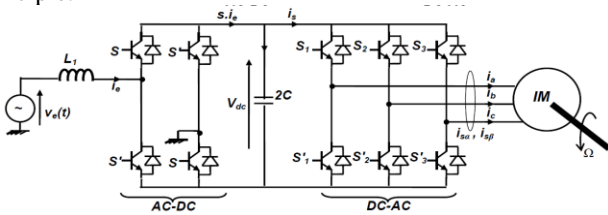


Fig.1. AC/DC/AC drive circuit with three-level inverter

### A. AC/DC rectifier modeling

The power supply net is connected to a H-bridge converter which consists of four IGBT’s with anti-parallel diodes for bidirectional power flow mode. This is expected to accomplish two main tasks: (i) providing a constant DC link voltage; (ii) providing an almost unitary power factor. Applying Kirchoff’s laws, this subsystem is described by the following set of differential equations:

$$\dot{i}_e = (v_e - s v_{dc}) / L_1 \quad (1a)$$

$$\dot{v}_{dc} = (s i_e - i_s) / 2C \quad (1b)$$

where  $i_e$  is the current in inductor  $L_1$ ,  $v_{dc}$  denotes the

voltage in capacitor  $2C$ ,  $i_s$  designates the input current inverter,  $v_e = \sqrt{2}.E.\cos(\omega_e t)$  is the sinusoidal net voltage (with known constants  $E, \omega_e$ ) and  $s$  is the switch position function taking values in the discrete set  $\{-1, 1\}$ . Specifically:

$$s = \begin{cases} 1 & \text{if } S \text{ is ON and } S' \text{ is OFF} \\ -1 & \text{if } S \text{ is OFF and } S' \text{ is ON} \end{cases} \quad (1c)$$

The above (instantaneous) model describes accurately the physical inverter. Then, it is based upon when constructing converter simulators. However, it is not suitable for control design due to the switched nature of the control input  $s$ . As a matter of fact, most existing nonlinear control approaches apply to systems with continuous control inputs. Therefore, control design for the above inverter will be performed using the following average version of (1a-b) [4]:

$$\dot{x}_1 = (v_e - u_1 x_2) / L_1 \quad (2a)$$

$$\dot{x}_2 = (u_1 x_1 - \bar{i}_s) / 2C \quad (2b)$$

$$\text{where: } x_1 = \bar{i}_e, x_2 = \bar{v}_{dc}, u_1 = \bar{s} \quad (2c)$$

are the average values over cutting periods of  $i_e$ ,  $v_{dc}$  and  $s$ , respectively

### B. Inverter-Motor modeling

The induction model is based on the motor equations in rotating  $\alpha$ -and- $\beta$  axes and reads as

$$\dot{\Omega} = -c\Omega + m(i_{s\beta}\phi_{r\alpha} - i_{s\alpha}\phi_{r\beta}) - T_L / J \quad (3a)$$

$$\dot{i}_{s\alpha} = ba\phi_{r\alpha} + bp\Omega\phi_{r\beta} - \gamma i_{s\alpha} + m_1 v_{s\alpha} \quad (3b)$$

$$\dot{i}_{s\beta} = ba\phi_{r\beta} - bp\Omega\phi_{r\alpha} - \gamma i_{s\beta} + m_1 v_{s\beta} \quad (3c)$$

$$\dot{\phi}_{r\alpha} = -a\phi_{r\alpha} + aM_{sr}i_{s\alpha} - p\Omega\phi_{r\beta} \quad (3d)$$

$$\dot{\phi}_{r\beta} = -a\phi_{r\beta} + aM_{sr}i_{s\beta} + p\Omega\phi_{r\alpha} \quad (3e)$$

with

$i_{s\alpha}, i_{s\beta}, \phi_{r\alpha}, \phi_{r\beta}, \Omega$ , and  $T_L$ , are the stator currents, rotor fluxes, angular speed, and load torque, respectively. The subscripts  $s$  and  $r$  refer to the stator and rotor, respectively. The parameters  $a, b, c, \gamma, \sigma, m$ , and  $m_1$  are defined as:  $a = R_r / L_r$ ,  $c = f / J$ ,  $b = M_{sr} / \sigma L_s L_r$ ,  $m = pM_{sr} / (JL_r)$ ,  $\gamma = (L_r^2 R_s + M_{sr}^2 R_r) / \sigma L_s L_r^2$ ,  $\sigma = 1 - (M_{sr}^2 / L_s L_r)$ ,  $m_1 = 1 / \sigma L_s$ , respectively.  $R_s$  and  $R_r$  are the resistances.  $L_s$  and  $L_r$  are the self-inductances, and  $M_{sr}$  is the mutual inductance between the stator and rotor windings.  $p$  is the number of pole-pair.  $J$  is the inertia of the system (motor and load), and  $f$  is the viscous damping coefficients.

The  $v_{s\alpha}, v_{s\beta}$  denote the stator voltage in  $\alpha\beta$ -coordinate (Park’s transformation of the 3 phase stator voltages). The inverter is featured by the fact that the stator  $\alpha$ - and  $\beta$ -voltage can be controlled independently. To this end, [7]:

$$v_{s\beta} = v_{dc} u_3 \quad (4a)$$

$$v_{s\alpha} = v_{dc} u_2 \quad (4b)$$

$$\bar{i}_s = (u_2 x_4 + u_3 x_5) \quad (4c)$$

where:  $u_2, u_3$  represent the average  $\alpha$ - and  $\beta$ -axis (Park's transformation) of the 3 phase duty ratio system  $(s_1, s_2, s_3)$ .

The latter are defined (1c) replacing there  $s$  and  $s'$  by  $s_i$  and  $s'_i$  ( $i = 1, 2, 3$ ). Now, let us introduce the state variables:

$$x_3 = \bar{\Omega}, \quad x_4 = \bar{i}_{s\alpha}, \quad x_5 = \bar{i}_{s\beta}, \quad x_6 = \bar{\phi}_{r\alpha}, \quad x_7 = \bar{\phi}_{r\beta}, \quad (5)$$

Substituting (4a-e) in (3a-e), the state space equations obtained up to now are put together to get a state-space model of the whole system including the AC/DC/AC converters combined with the induction motor. For convenience, the whole model is rewritten here for future reference:

$$\dot{x}_1 = (v_e - u_1 x_2) / L_1 \quad (6a)$$

$$\dot{x}_2 = (u_1 x_1 - (u_2 x_4 + u_3 x_5)) / 2C \quad (6b)$$

$$\dot{x}_3 = -cx_3 + m(x_5 x_6 - x_7 x_4) - T_L / J \quad (6c)$$

$$\dot{x}_4 = bax_6 + bpx_3 x_7 - \gamma x_4 + m_1 u_2 x_2 \quad (6d)$$

$$\dot{x}_5 = bax_7 - bpx_3 x_6 - \gamma x_5 + m_1 u_3 x_2 \quad (6e)$$

$$\dot{x}_6 = -ax_6 + aM_{sr} x_4 - px_3 x_7 \quad (6f)$$

$$\dot{x}_7 = -ax_7 + aM_{sr} x_5 + px_3 x_6 \quad (6g)$$

### III. CONTROLLER DESIGN

#### A. Control objectives

There are two operational control objectives:

- (i) Speed regulation: the machine speed  $\Omega$  must track, as closely as possible, a given reference signal  $\Omega_{ref}$ .
- (ii) PFC requirement: the rectifier input current  $i_e$  must be sinusoidal and in phase with the AC supply voltage  $v_e$ .

As there are three control inputs at hand, namely  $u_1, u_2$  and  $u_3$ , we will further seek two additional control objectives:

- (iii) Controlling the continuous voltage  $v_{dc}$  making it track a given reference signal  $v_{dcref}$ . This generally is set to a constant value equal to the nominal voltage entering the inverter.

- (iv) Regulating the rotor flux norm  $\Phi_r = \sqrt{(x_6^2 + x_7^2)}$  to a reference value  $\Phi_{ref}$ , equal to its nominal value.

#### B. AC/DC rectifier control design

**1) Controlling rectifier input current to meet PFC:** The PFC objective means that the input current rectifier should be sinusoidal and in phase with the AC supply voltage. We therefore seek a regulator that enforces the current  $x_1$  to track a reference signal  $x_1^*$  of the form:

$$x_1^* = k v_e \quad (7)$$

At this point  $k$  is any real parameter that is allowed to be time-varying. Introduce the current tracking error:

$$z_1 = x_1 - x_1^* \quad (8)$$

In view of (6a), the above error undergoes the following equation:

$$\dot{z}_1 = (v_e - u_1 x_2) / L_1 - \dot{x}_1^* \quad (9)$$

To get a stabilizing control law for this first-order system, consider the quadratic Lyapunov function  $V_1 = 0.5z_1^2$ . It can be easily checked that the time-derivative  $\dot{V}_1$  is a negative definite function of  $z_1$  if the control input is chosen to be:

$$u_1 = L_1 (c_1 z_1 + (v_e / L_1) - \dot{x}_1^*) / x_2 \quad (10)$$

with  $c_1 > 0$  a design parameter.

The properties of such control law are summarized in the following proposition.

**Proposition 1:** Consider the system, next called current (or inner) loop, composed of the current equation (6a) and the control law (10) where  $c_1 > 0$  is arbitrarily chosen by the user. If the reference  $x_1^* = k v_e$  and its first time derivative are available then one has the following properties:

The current loop undergoes the equation  $\dot{z}_1 = -c_1 z_1$  where  $z_1 = x_1 - x_1^*$ . As  $c_1$  is positive this equation is globally exponentially stable i.e.  $z_1$  vanishes exponentially, whatever the initial conditions.

If in addition  $k$  converges (to a finite value), then the PFC requirement is asymptotically fulfilled in average i.e. the (average) input current  $x_1$  tends (exponentially fast) to its reference  $k v_e$  as  $t \rightarrow \infty$   $\square$

**2) DC link voltage regulation:** The aim is now to design a tuning law for the ratio  $k$  in (7) so that the rectifier output voltage  $x_2 = \bar{v}_{dc}$  is steered to a given reference value  $v_{dcref}$ . As mentioned above,  $v_{dcref}$  is generally (not mandatory) chosen to be the constant nominal inverter input voltage amplitude (i.e the nominal stator voltage).

**a) Relationship between  $k$  and  $x_2$ :** The first step in designing such a loop is to establish the relation between the ratio  $k$  (control input) and the output voltage  $x_2$ . This is the subject of the following proposition.

**Proposition 2.** Consider the power rectifier described by (6a-b) together with the control law (10). Under the same assumptions as in Proposition 1, one has the following properties:

- 1) The output voltage  $x_2$  varies, in response to the tuning ratio  $k$ , according to the equation:

$$\dot{x}_2 = (k v_e^2 + z_1 v_e) / 2C x_2 - (u_2 x_4 + u_3 x_5) / 2C \quad (11)$$

- 2) The squared voltage ( $y = x_2^2$ ) varies, in response to the tuning ratio  $k$ , according to the equation:

$$\dot{y} = k v_e^2 / C + z_1 v_e / C + \chi(x, t) \quad (12)$$

$$\text{with } \chi(x, t) = -x_2 (u_2 x_4 + u_3 x_5) / C \quad (13)$$

$\square$

**b) Squared DC-link voltage regulation:** The ratio  $k$  stands up as a control signal in the first-order system defined

by (12). As we said before, the reference signal  $y_{ref} \stackrel{def}{=} v_{dcref}^2$  (of the squared DC-link voltage  $x_2 = \bar{v}_{dc}$ ) is chosen to be constant (i.e.  $\dot{y}_{ref} = 0$ ), it is given the nominal inverter input voltage value. Then, it follows from (12) that the tracking error  $z_2 = y - y_{ref}$  undergoes the following equation:

$$\begin{aligned} \dot{z}_2 &= E^2 k / C + E^2 k \cos(2\omega_e t) / C \\ &+ \sqrt{2} E z_1 \cos(\omega_e t) / C + \chi(x, t) - \dot{y}_{ref} \end{aligned} \quad (14)$$

where we have used the fact that  $v_e = \sqrt{2} E \cdot \cos(\omega_e t)$  and  $v_e^2 = E^2 (1 + \cos(2\omega_e t))$ . To get a stabilizing control law for the system (15), consider the following quadratic Lyapunov function:  $V_2 = 0.5 z_2^2$  (15)

It is easily checked that the time-derivative  $\dot{V}_2$  can be made negative definite in the state  $z_2$  by letting:

$$\begin{aligned} k E^2 + k E^2 \cos(2\omega_e t) + \sqrt{2} E z_1 \cos(\omega_e t) \\ = C (-c_2 z_2 - \chi(x, t)) + C \dot{y}_{ref} \end{aligned} \quad (16)$$

Bearing in mind the fact that the first derivative of the control ratio  $k$  must be available (Proposition 1), we suggest  $\dot{k} + d k = d C (-c_2 z_2 - \chi(x, t)) / E^2 + d C \dot{y}_{ref} / E^2$  (17)

The  $(d, c_2)$  are any positive real constants.

### C. Motor speed and rotor flux norm regulation

We are interested in the problem of controlling the rotor speed and flux norm for the induction machine described by (6c-g). The speed reference  $\Omega_{ref}$  is any bounded and derivable function of time and its two first derivatives are available and bounded. These properties can always be achieved filtering the reference through second-order linear filters. The flux reference  $\Phi_{ref}$  is fixed to its nominal value. The controller design will now be performed in two steps using the backstepping technique [3]. First, introduce the tracking errors:

$$z_3 = \Omega_{ref} - x_3 \quad (18)$$

$$z_4 = \Phi_{ref}^2 - (x_6^2 + x_7^2) \quad (19)$$

**Step 1.** It follows from (6c) and (6f-g) that the errors  $z_3$  and  $z_4$  undergo the differential equations:

$$\dot{z}_3 = \dot{\Omega}_{ref} - m(x_6 x_5 - x_7 x_4) + T_L / J + c x_3 \quad (20)$$

$$\dot{z}_4 = 2\Phi_{ref} \dot{\Phi}_{ref} - 2aM_{sr}(x_6 x_4 + x_7 x_5) + 2a(\Phi_{ref}^2 - z_4) \quad (21)$$

In (20) and (21), the quantities  $m(x_6 x_5 - x_7 x_4)$  and  $2aM_{sr}(x_6 x_4 + x_7 x_5)$  stand up as virtual control signals. If these were the actual control signals, the error system (20)-(21) could be globally asymptotically stabilized letting  $m(x_6 x_5 - x_7 x_4) = \mu_1$  and  $2aM_{sr}(x_6 x_4 + x_7 x_5) = \nu_1$  with:

$$\mu_1 \stackrel{def}{=} c_3 z_3 + \dot{\Omega}_{ref} + T_L / J + c(\Omega_{ref} - z_3) \quad (22)$$

$$\nu_1 \stackrel{def}{=} c_4 z_4 + 2\Phi_{ref} \dot{\Phi}_{ref} + 2a(\Phi_{ref}^2 - z_4) \quad (23)$$

where  $c_3$  and  $c_4$  are any positive design parameters. Indeed, considering the Lyapunov function:

$$V_3 = 0.5(z_3^2 + z_4^2) \quad (24)$$

One would get from (20)-(21), letting  $m(x_6 x_5 - x_7 x_4) = \mu_1$  and  $2aM_{sr}(x_6 x_4 + x_7 x_5) = \nu_1$ :

$$\dot{V}_3 = -c_3 z_3^2 - c_4 z_4^2 \quad (25)$$

This would prove the global asymptotic stability of the system (20)-(23). As the quantities  $m(x_6 x_5 - x_7 x_4)$  and  $2aM_{sr}(x_6 x_4 + x_7 x_5)$  are not the actual control signals, they cannot be let equal to  $\mu_1$  and  $\nu_1$ , respectively. Nevertheless, we retain the expressions of  $\mu_1$  and  $\nu_1$  as first stabilizing functions and introduce the new errors:

$$z_5 = \mu_1 - m(x_6 x_5 - x_7 x_4) \quad (26)$$

$$z_6 = \nu_1 - 2aM_{sr}(x_6 x_4 + x_7 x_5) \quad (27)$$

Then, using the notations (22) to (27), the dynamics of the errors  $z_3$  and  $z_4$ , initially described by (20)-(23), can be rewritten as follows:

$$\dot{z}_3 = -c_3 z_3 + z_5, \dot{z}_4 = -c_4 z_4 + z_6 \quad (28)$$

Similarly, the time-derivative of  $V_3$  can be expressed in function of the new errors as follows:

$$\dot{V}_3 = -c_3 z_3^2 - c_4 z_4^2 + z_3 z_5 + z_4 z_6 \quad (29)$$

**Step 2:** The second design step consists in choosing the actual control signals,  $u_2$  and  $u_3$ , so that all errors  $(z_3, z_4, z_5, z_6)$  converge to zero. To this end, we should make how these errors depend on the actual control signals  $(u_2, u_3)$ . We start focusing on  $z_5$ ; it follows from (26) that:

$$\dot{z}_5 = \dot{\mu}_1 - m(\dot{x}_6 x_5 + x_6 \dot{x}_5 - \dot{x}_7 x_4 - x_7 \dot{x}_4) \quad (30)$$

For convenience, the above equation is given the following compact form:

$$\dot{z}_5 = \mu_2 + m m_1 x_2 (x_7 u_2 - x_6 u_3) \quad (32)$$

with  $\mu_2 = c_3 (-c_3 z_3 + z_5) + \ddot{\Omega}_{ref} + \dot{T}_L / J$

$$\begin{aligned} &+ m(c + a + \gamma)(x_6 x_5 - x_7 x_4) \\ &- c T_L / J - c^2 x_3 + m p x_3 (x_7 x_5 + x_6 x_4) + m p x_3 (x_6^2 + x_7^2) \end{aligned} \quad (33)$$

Similarly, it follows from (27) that,  $z_6$  undergoes the following differential equation:

$$\dot{z}_6 = \dot{\nu}_1 - 2aM_{sr}(\dot{x}_6 x_4 + x_6 \dot{x}_4 + \dot{x}_7 x_5 + x_7 \dot{x}_5) \quad (34)$$

Equation (34) is in turn given the following compact form:

$$\dot{z}_6 = \nu_2 - 2aM_{sr} x_2 m_1 (x_6 u_2 + x_7 u_3) \quad (36a)$$

with

$$\begin{aligned} v_2 = & (c_4 - 2a)(-c_4 z_4 + z_6) + 2\Phi_{ref} \ddot{\Phi}_{ref} + 2\dot{\Phi}_{ref}^2 + 4a\Phi_{ref} \dot{\Phi}_{ref} \\ & + 2aM_{sr} p(x_7 x_4 - x_5 x_6) + 2aM_{sr} (a + \gamma)(x_4 x_6 + x_5 x_7) \\ & - 2(aM_{sr})^2 (x_4^2 + x_5^2) - 2a^2 bM_{sr} (\Phi_{ref}^2 - z_4) \end{aligned} \quad (36b)$$

To analyze the error system, composed of equations (28), (32) and (36a), let us consider the following augmented Lyapunov function candidate:

$$V_4 = V_3 + 0.5(z_5^2 + z_6^2) \quad (37)$$

Its time-derivative along the trajectory of the state vector  $(z_3, z_4, z_5, z_6)$  is:

$$\dot{V}_4 = z_3 \dot{z}_3 + z_4 \dot{z}_4 + z_5 \dot{z}_5 + z_6 \dot{z}_6 \quad (38)$$

Using (29), (32) and (36a), equation (28) implies:

$$\begin{aligned} \dot{V}_4 = & -c_3 z_3^2 - c_4 z_4^2 + z_5 (z_3 + \mu_2 + mm_1 x_2 (x_7 u_2 - x_6 u_3)) \\ & + z_6 (z_4 + v_2 - 2aM_{sr} x_2 m_1 (x_7 u_3 + x_6 u_2)) \end{aligned} \quad (39a)$$

Adding  $c_5 z_5^2 - c_5 z_5^2 + c_6 z_6^2 - c_6 z_6^2$  to the right side of (39a) and rearranging terms, yields:

$$\begin{aligned} \dot{V}_4 = & -c_3 z_3^2 - c_4 z_4^2 - c_5 z_5^2 - c_6 z_6^2 \\ & + z_5 (z_3 + \mu_2 + c_5 z_5 + mm_1 x_2 (x_7 u_2 - x_6 u_3)) \\ & + z_6 (z_4 + v_2 + c_6 z_6 - 2aM_{sr} x_2 m_1 (x_7 u_3 + x_6 u_2)) \end{aligned} \quad (39b)$$

where  $c_5$  and  $c_6$  are new arbitrary positive real design parameters. Equation (39b) suggests that the control signals  $u_2, u_3$  must set to zero the two quantities between curly brackets (on the right side of (41)). Letting these quantities equal to zero and solving the resulting second-order equation system with respect to  $(u_2, u_3)$ , gives the control law:

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \lambda_0 & \lambda_1 \\ \lambda_2 & \lambda_3 \end{bmatrix}^{-1} \begin{bmatrix} -z_3 - c_5 z_5 - \mu_2 \\ -z_4 - c_6 z_6 - v_2 \end{bmatrix} \quad (40)$$

with:  $\lambda_0 = mm_1 x_7 x_2, \lambda_1 = -mm_1 x_6 x_2, \lambda_2 = -2aM_{sr} x_6 x_2,$   
 $\lambda_3 = -2aM_{sr} x_7 x_2$  (41)

It worths to note that the matrix  $\begin{bmatrix} \lambda_0 & \lambda_1 \\ \lambda_2 & \lambda_3 \end{bmatrix}$  is nonsingular.

Indeed, it is easily checked that its determinant is  $D = \lambda_0 \lambda_3 - \lambda_1 \lambda_2 = -2mm_1 aM_{sr} x_2^2 (x_6^2 + x_7^2)$  which never vanish in practice because of the machine remnent flux. Substituting the control law (42) to  $(u_2, u_3)$  on the right side of (41) yields:

$$\dot{V}_4 = -c_3 z_3^2 - c_4 z_4^2 - c_5 z_5^2 - c_6 z_6^2 \quad (42)$$

As this is a negative definite function of the state vector  $(z_3, z_4, z_5, z_6)$ , it follows that the speed and flux tracking errors (i.e.  $z_3 = \Omega_{ref} - x_3$  and  $z_4 = \Phi_{ref}^2 - (x_6^2 + x_7^2)$ ) converge globally asymptotically to zero. This result will now be used to establish the performances of the whole

control system. To this end, let us first introduce the following notations:

$$Z_1 = [z_1 \quad z_2 \quad k]^T; Z_2 = [z_3 \quad z_4 \quad z_5 \quad z_6]^T$$

$$Z = [Z_1 \quad Z_2]^T \quad (43a)$$

$$a_2 = \sqrt{2}EC^{-1}; a_3 = dE^{-2}; \varepsilon = \omega_e^{-1} \quad (43b)$$

$$A = \begin{pmatrix} A_1 & \mathbf{O}_{(3,4)} \\ \mathbf{O}_{(4,3)} & A_2 \end{pmatrix} \in \mathbb{R}^{7 \times 7} \quad (43c)$$

with

$$A_1 = \begin{pmatrix} -c_1 & 0 & 0 \\ 0 & 0 & a_0 \\ 0 & -a_1 & -d \end{pmatrix} A_2 = \begin{pmatrix} -c_3 & 0 & 1 & 0 \\ 0 & -c_4 & 0 & 1 \\ -1 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & -c_6 \end{pmatrix} \quad (43d)$$

$$f(Z, t) = \begin{bmatrix} 0 & (a_0 k \cos(2\omega_e t) + a_2 z_1 \cos(\omega_e t)) & 0 & 0_4^T \end{bmatrix} \in \mathbb{R}^7 \quad (43e)$$

$$g = \begin{bmatrix} 0 & -C^{-1} & a_3 & 0_4^T \end{bmatrix} \in \mathbb{R}^7 \quad (43f)$$

$$h = \begin{bmatrix} 0 & 1 & -d & CE^{-2} & 0_4^T \end{bmatrix} \in \mathbb{R}^7 \quad (43g)$$

where  $0_4$  denotes the null vector of  $\mathbb{R}^4$

$$\sigma(Z_2, t) = \frac{j}{m^2 m_1 \Phi_r^2} (z_5 - \mu_1) - \frac{l}{(2aM_{sr})^2 \Phi_r^2} (v_1 - z_6) \quad (43h)$$

$$\begin{aligned} j = & -(z_3 + c_5 z_5 + c_3(-c_3 z_3 + z_5)) + \ddot{\Omega}_{ref} + \dot{T}_L / J \\ & + (c + a + \gamma)(\mu_1 - z_5) - cT_L / J - c^2(\Omega_{ref} - z_3) \\ & + mp(\Omega_{ref} - z_3)(v_1 - z_6) / 2aM_{sr} \\ & + mp(\Omega_{ref} - z_3)(\Phi_{ref}^2 - z_4) \end{aligned} \quad (43i)$$

$$\begin{aligned} l = & -(z_4 + c_6 z_6 + (c_4 - 2a)(-c_4 z_4 + z_6)) \\ & + 2\Phi_{ref} \ddot{\Phi}_{ref} + 2\dot{\Phi}_{ref}^2 + 4a\Phi_{ref} \dot{\Phi}_{ref} + 2aM_{sr} p(z_5 - \mu_1) \\ & + (a + \gamma)(v_1 - z_6) / m - (2(aM_{sr})^2 (z_5 - \mu_1)^2 / m^2 \\ & + (v_1 - z_6)^2 / 2) / \Phi_r^2 - 2a^2 bM_{sr} (\Phi_{ref}^2 - z_4) \end{aligned} \quad (43j)$$

**Theorem 1 (main result).** Consider the system including the AC/DC/AC power converters and the induction motor connected in tandem, as shown in Fig.1. For control design purpose, the system has been represented by its average model (6a-g). Consider the controller defined by the control laws (10), (17) and (40) where all design parameters, namely  $c_1, c_2, c_3, c_4, c_5, c_6$  and  $d$  are positive. Then, one has the following results:

1) The resulting closed-loop system undergoes the state-space equation:

$$\dot{Z} = AZ + f(Z, t) + g\sigma(Z_2, t) + h\dot{y}_{ref} \quad (44)$$

2) Let the reference signals be chosen such that  $v_{dcref} > 0, \Omega_{ref} \geq 0$  and  $\Phi_{ref}$  equal to its nominal value.

Then, there exists a  $\varepsilon^*$  so that, if  $0 < \varepsilon < \varepsilon^*$  then:

a) The tracking error  $z_2 = y - y_{ref}$  and the tuning parameter  $k$  are harmonic signals continuously.

b) Furthermore, one has:

$$(i) \lim_{\varepsilon \rightarrow 0} z_2(t, \varepsilon) = 0 ; \quad (ii) \lim_{\varepsilon \rightarrow 0} k(t, \varepsilon) = \bar{\sigma}(0_4) / a_0 C \quad (45)$$

where  $\bar{\sigma}(0_4)$  denotes the mean value of the periodic time function  $\sigma(0_4, t)$   $\square$

#### IV. SIMULATION

The experimental setup has been simulated, with the following characteristics:

..AC/DC/AC converters:  $L_1 = 15\text{mH}$  ;  $C = 1.5\text{mF}$  ;

. numerical values of considered motor characteristics are chosen as nominal power,  $P_n = 7.5\text{KW}$  , The following values of the controller design parameters proved to be suitable:  $c_1 = 1000$  ,  $c_2 = 30$  ,  $c_3 = 100$  ,  $c_4 = 400$  ,  $c_5 = 500$  ,  $c_6 = 1000$  ,  $d = 100$  .

The controller performances are illustrated by Figs 2 to 5. Fig. 2 shows that the DC-link voltage  $x_2 = v_{dc}$  is well regulated and quickly settles down after each change in the speed reference or the load torque. Figs 2 and 4 show that the motor speed and the rotor flux norm converge to their respective references perfectly. The tracking quality is quite satisfactory for both controlled variables ( $\Omega$ ,  $\Phi_r$ ). The resulting input current  $i_e$  is all time in phase with the supply net voltage complying with the PFC requirement. Fig 5 shows that the ratio  $k$  takes a constant value after transient periods following the changes in references signals and load torque.

#### V. CONCLUSION

We have addressed the problem of controlling associations including an AC/DC rectifier, a DC/AC inverter and induction motor. The system dynamics have been described by the averaged seventh order nonlinear state-space model (6a-h). We have formally established that the proposed controller achieves the objectives it has been designed to: (i) almost unitary power factor; (ii) well regulated DC-link voltage ( $v_{dc}$ ); (iii) satisfactory rotor speed reference tracking over a wide range of load torque variation; (iv) tight regulation of the rotor flux norm. These results have been confirmed by a simulation study.

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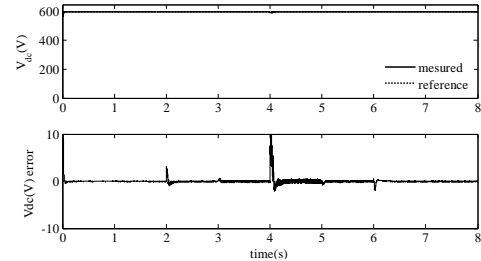


Fig. 2. DC-link voltage  $v_{dc}$  Upper: reference and mesured DC-link voltage; lower: error control

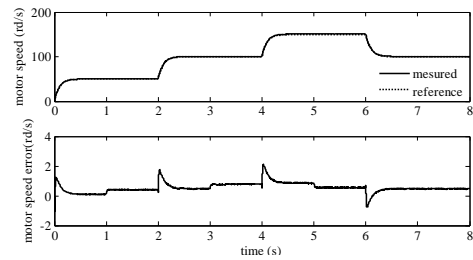


Fig. 3. Rotor speed  $\Omega$  (rd/s) Upper: reference and mesured speed; lower: speed error control

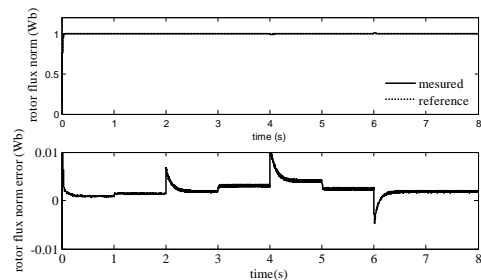


Fig. 4. Rotor flux norm (Wb). Upper: reference and mesured flux norm; lower: error control

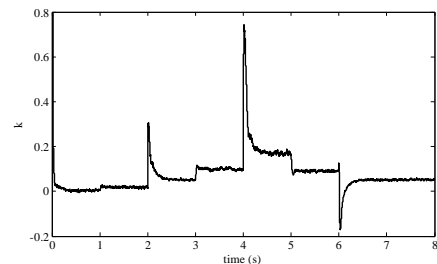


Fig. 5. Tuning parameter  $k$  response after each change in the speed reference or the load torque.