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## Negative refraction of zero order flexural Lamb waves through a two-dimensional phononic crystal

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We investigate experimentally the refraction of antisymmetric Lamb waves at frequencies in the second band of a two-dimensional phononic crystal. The heterostructure is made of a square lattice of air inclusions in a silicon plate. Our experimental scheme is based on the selective generation of narrow band elastic waves at a few megahertz. Both positive and negative refractions are observed. The comparison of our data with computed dispersion curves shows very good agreement. © 2010 American Institute of Physics. [doi:10.1063/1.3491290]

Whatever their physical nature, elastic or electromagnetic, waves are never negatively refracted when they go through the interface between two homogeneous media and the phenomenon of negative refraction (NR) had never been conceived before the pioneering Veselago's work<sup>1</sup> in the late 1960s. It has been established at that time, that optical metamaterials, built from the juxtaposition of two materials with, respectively, permittivity  $\varepsilon < 0$  and permeability  $\mu < 0$ , are likely to exhibit NR and that these "double-negative" optical materials are extremely interesting to design perfect optical lenses suitable to overcome the diffraction limit, and therefore, to focus into a perfect image all the Fourier components issuing from a two-dimensional (2D) object.

More recently, it has been recognized that the effective index can also be controlled in a periodic heterostructure through the band structure<sup>2</sup> and that NR of electromagnetic waves can be observed in photonic crystals as well. In that case NR is not related to the double negativity but rather to the bands folding resulting from the periodicity, and to the negative slope of some optical branches.

Similar approaches have been adopted with elastic waves but, in contrast to electromagnetic waves, NR was originally expected in periodic heterostructures. Indeed, although elastic metamaterials with mass density and compressibility simultaneously negative have now been realized,<sup>3</sup> the constraints on the physical properties of the constituents are so severe that this promising approach remains exploratory in a large extent. On the other hand, NR of elastic waves in phononic crystals (PC's) is now well established and even several experimental demonstrations of the phenomenon have already been given.<sup>4,5</sup> However, most of these studies deal with the refraction of bulk waves, performed in 2D PC's with a fluid matrix<sup>6,7</sup> or in holes structures drilled into a solid matrix.<sup>8</sup> We treat in the present work a different case since we address NR of Lamb waves propagating in a 2D phononic plate.

Studying slabs of PC's has become a topic of major interest since several groups have shown that these structures can exhibit absolute band gaps. This makes them suitable for

analogous applications as for 2D PC with the additional property of confinement in between the free surfaces. In this paper, we first recall the physical conditions under which NR is expected to occur. We then report on experimental data giving clear evidence that Lamb waves propagating in a phononic plate actually undergo NR.

The refractive properties can be predicted by considering both the band structures of the PC and of the adjacent medium. We have thus computed the dispersion of Lamb waves propagating in a 2D PC constituted by a square lattice of voids drilled throughout a silicon plate. The calculations were made using a super-cell (SC) plane waves expansion method described in Ref. 9, the physical and geometrical parameters being those of the sample described below. In short, the method consists in covering the phononic plate of thickness  $t$  with a slab of thickness  $d$  made of a fictitious low impedance material with very low density and very high speeds of sound modeling vacuum. This allows defining a three-dimensional SC with total thickness  $t+d$ . This SC is then periodically repeated along the three spatial directions and the equations of motion are Fourier transformed. Due to their very low impedance in comparison to that of the phononic slab, the fictitious material forbids the interaction between the vibrational modes of neighboring periodically repeated PC plates. The result for any  $\mathbf{k}$  vector along  $\Gamma X$  in the reduced Brillouin zone and for frequencies up to 11 MHz is shown in Fig. 1. The band structure of a phononic plate generally displays a great variety of modes and their multiple crossing makes it difficult exploring separately a one given mode. To minimize this unwanted effect, we have fixed the ratio  $t/a=0.6$ , where  $a$  is the lattice constant. For this ratio, there is no band overlapping in the frequency range of interest for the experiments described below. Still having several different modes, one had to analyze the surface displacements in order to identify the polarization of every mode (symmetric, antisymmetric or SH mode).<sup>10</sup> Moreover, being interested in modes having a significant out-of-plane component, which are the only experimentally detectable, we studied only the two branches corresponding to the zero order flexural Lamb wave (Fig. 1, first branch and folded branch). According to the calculated band structure displayed in Fig. 1, the crystal possess a first stop band along  $\Gamma X$  direction

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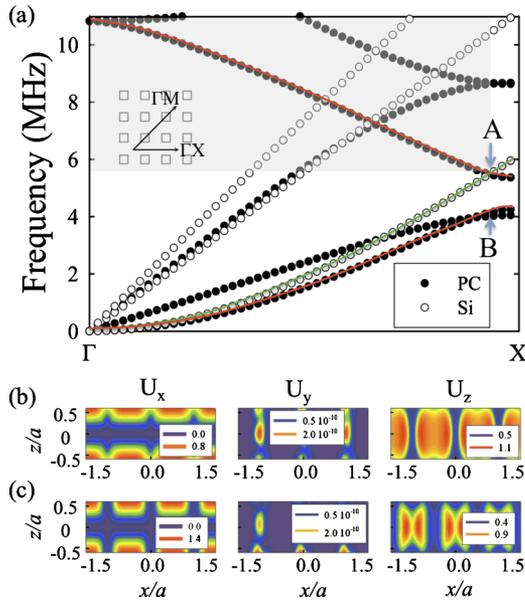


FIG. 1. (Color online) Filled circles: calculated band structure of PC (square array of holes in a silicon plate,  $t=160\ \mu\text{m}$ ,  $a=267\ \mu\text{m}$ ,  $f=0.4$ ). Both the first branch and the first folded branch are for the antisymmetric Lamb mode in the PC. Empty circles: band structure of a silicon plate  $160\ \mu\text{m}$  thick. The first branch is for the silicon line (a). Components of the displacement vector  $\mathbf{u}$  computed at point A:  $k=10\ \text{mm}^{-1}$ ,  $\nu=5.8\ \text{MHz}$  (b) and at point B:  $k=10\ \text{mm}^{-1}$ ,  $\nu=4\ \text{MHz}$  (c).

extending from 4.2 to 5.4 MHz and a first branch with a negative slope (wave vector is opposite to the group velocity) from 5.4 to 10.5 MHz. The Snell–Descartes law (conservation of the  $\mathbf{k}$  component along the surface of refraction) would result in the NR effect in the frequency region,<sup>6,7</sup> where the second branch is above the corresponding silicon line, marked by a gray rectangle in Fig. 1.

We show in Fig. 2 the experimental scheme. For this study, we have elaborated a 2D prism shaped PC in a silicon plate with thickness  $t=160\ \mu\text{m}$ . The plate was patterned with a square lattice of air holes drilled throughout, by means of photolithography processes and anisotropic chemical etching. The filling ratio and the lattice constant were respectively  $f=0.4$  and  $a=267\ \mu\text{m}$  ( $t/a=0.6$ ). The rows of voids were parallel to the crystallographic directions  $[110]$  and  $[\bar{1}10]$  of silicon and the overall shape of the array was an isosceles right triangle.

Our experimental technique is based on the laser generation and detection of acoustic pulses with a narrow spectrum. Short light pulses issued from a frequency-doubled Nd:YAG laser ( $\lambda=532\ \text{nm}$ , duration 30 ps, repetition rate 20 Hz) hit the sample after they have run across an amplitude mask and a spherical singlet ( $f:75\ \text{mm}$ ). The image of the mask on the sample results in a fringes lattice corresponding to a thermoelastic source with a spatially periodic profile and Gaussian time dependence. This thermoelastic source allows to selectively excite coherent acoustic waves propagating perpendicularly to the fringes, with a wave vector  $k=2\pi/\Lambda$ . The fringe spacing  $\Lambda$  being tunable from a few tens to a few hundreds of micrometer, elastic pulses with a carrier frequency in the megahertz range can be excited. In most of the experiments reported here, the central frequency was fixed to  $\nu=5.8\ \text{MHz}$ , with spectral width over  $\sim 4\ \text{MHz}$ . The thickness of the plate being of the same order of magnitude as  $\Lambda$ , Lamb waves are generated. Moreover, the thermoelastic

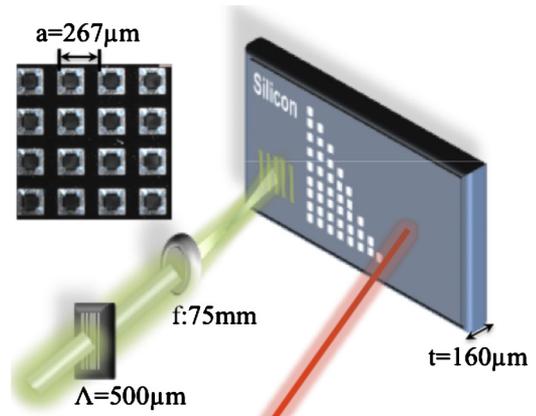


FIG. 2. (Color online) Experimental setup. Inset: image of the sample.

source being located on one face of the sample and not on the edge, zero order flexural Lamb waves were preferentially excited that way. These waves were generated beside the PC itself, in a uniform region of the sample free of air inclusions; they impinged the PC at normal incidence and propagated along  $\Gamma X$  without any change in the initial direction. The waves were then incident at  $\theta_i=45^\circ$ , on the interface between the PC and the uniform silicon where they were refracted.

We probed the refractive properties of the heterostructure, using an actively stabilized Michelson interferometer to monitor the time dependence of the surface displacements in the uniform medium behind the outgoing interface, on a large area lying on both sides of the normal to this interface (Fig. 2). It is to be noticed that this interferometric method is only sensitive to the normal component of the displacements but not to the in-plane components. The time data were then Fourier transformed, allowing for a frequency-position mapping of the out-of-plane displacement amplitude.

We show in Fig. 3 the refracted signal in the frequency domain, for frequencies in the range 0–10 MHz, recorded at regularly spaced positions, distant from each other by 0.75 mm, along three lines in parallel (see inset in Fig. 3—lines a, b, and c). The result clearly shows that the outgoing signal is refracted on both sides of the normal to the interface: in addition to the usual diffraction route (lower part of Fig. 3—positive  $y$ ), elastic waves are also measured in the NR zone (upper part of Fig. 3—negative  $y$ ). We compare in Fig. 4 the Fourier components of both refracted beams. Whereas, the wave packet in the positive side remains centered on  $\sim 5.8\ \text{MHz}$ , a small but still significant shift to higher frequencies is observed for the negatively refracted beam which is centered on  $\sim 6.4\ \text{MHz}$ . This is in accordance with the condition that NR occurs for frequencies above the silicon line that intersects the folded antisymmetric line at  $k_{\text{PC}}=11.2\ \text{mm}^{-1}$ ,  $\nu=5.5\ \text{MHz}$  (see Fig. 1). It is worth noting that the excitation processes involved in our experimental scheme basically consists in selecting one particular  $\mathbf{k}$  vector in the Brillouin zone, and therefore, disregarding the symmetry issues, all modes with this particular  $\mathbf{k}$  vector are able to be simultaneously excited into vibration. It is thus important to make sure that the flexural Lamb mode is actually the only one contributing to the observed signal. To this end, we have computed the components of the displacement vector  $\mathbf{u}$  for the negatively refracted mode at  $k=10\ \text{mm}^{-1}$

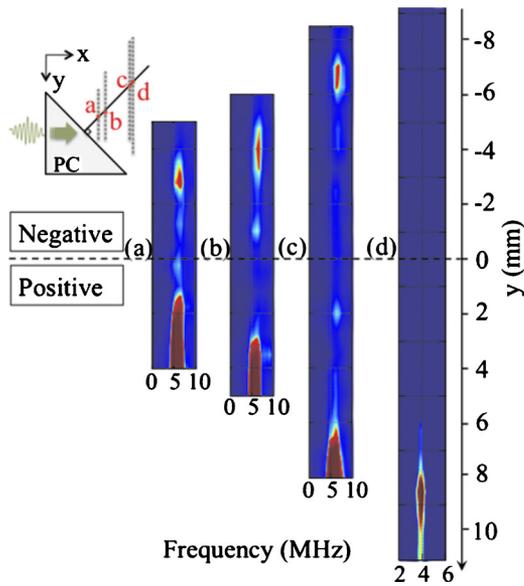


FIG. 3. (Color online) Refracted signal in the frequency-position domain, recorded along four lines in parallel. The distances along the normal to the interface are  $a \approx 2.8$ ,  $b \approx 4.2$ , and  $c \approx d \approx 8.5$  mm. The frequency of the acoustic pulse was centered either on 5.8 MHz [(a)–(c)] or on 4 MHz (d). Inset: scheme of the experiment. The arrow is for the direction of propagation of the Lamb waves in the PC. Two consecutive measurements along the vertical lines are distant by 0.75 mm.

and  $\nu = 5.8$  MHz [point A in Fig. 1(a)]. The comparison with the displacement vector computed at  $k = 10$   $\text{mm}^{-1}$  and  $\nu = 4$  MHz [point B in Fig. 1(a)] shows that both modes exhibit identical elliptical displacement fields in the sagittal plane, and thus the negatively refracted signal cannot be attributed to any modes coupling.

In another experiment, we have tuned the frequency to 4 MHz, i.e., in the first band where NR is not expected to occur, and we have measured the refracted signal on both

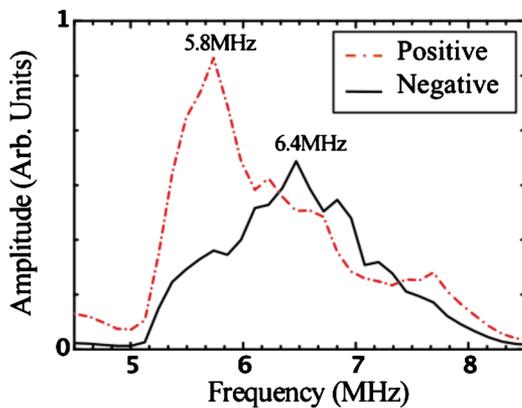


FIG. 4. (Color online) Fourier components of positive (dash-dot line) and negative (solid line) refracted beams.

sides of the normal to the interface (see inset in Fig. 3—line d). No signal was detected in the NR zone.

A comprehensive understanding of the experimental data can be obtained through a straightforward analysis based on Snell–Descartes law,  $k_{\text{PC}} \sin \theta_i = k_{\text{Si}} \sin \theta_r$ , where  $k_{\text{PC}}$  and  $k_{\text{Si}}$  are the magnitudes of the wave vectors in PC and in silicon, respectively;  $\theta_i$  is the angle of refraction and depends on the frequency. Owing to the periodicity of the PC, and for propagation along  $\Gamma X$ ,  $k_{\text{PC}}$  can assume any values differing from each other by an integer number of reciprocal vectors  $\mathbf{G}_{\Gamma X}$ , and reads  $\mathbf{k}_{\text{PC}} = \mathbf{k}_{\text{red}} + n\mathbf{G}_{\Gamma X}$ , where  $n$  is an integer and  $\mathbf{k}_{\text{red}}$  the wave vector in the reduced Brillouin zone. Therefore, the refractive angle can be deduced from:  $\sin \theta_r = (k_{\text{red}} + nG_{\Gamma X})k_{\text{Si}}^{-1} \sin \theta_i$ , where  $k_{\text{red}}$  can be calculated using a plane waves expansion method and takes negative values along folded branches. Whatever the frequency in the range 5.5–10 MHz, (i.e., above the silicon line), both positive and NRs are allowed since the value  $n=0$  (respectively  $n=1$ ) leads to a negative (respectively positive) value for  $\sin \theta_r$ . Moreover, good agreement is observed between measured and predicted angles:  $-25^\circ$  for the experimental value to be compared to  $-28^\circ$  for theoretically predicted angle for NR at 6.4 MHz and  $54^\circ$  to be compared to  $56^\circ$  for positive refraction at 5.8 MHz. The experimental values of the angles were measured at the maximum of the spectrum.

In summary, we have demonstrated the NR of antisymmetric Lamb waves in a 2D phononic crystal. Very directive beam is observed, experimental and theoretical angles of refraction are in a good agreement. The small discrepancy in values may result from the not sufficient spatial resolution of measurements and inevitable sample fabrication defects. Nevertheless, this first study of negative propagation of Lamb waves on solid-matrix PC's is one more step toward controlling of elastic phenomena at microscales and nanoscales.

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