



HAL
open science

A new Fermi liquid: the normal phase of a strongly interacting gas of cold atoms

Sylvain Nascimbène, Nir Navon, Sebastiano Pilati, Frédéric Chevy, Stefano Giorgini, Antoine Georges, Christophe Salomon

► **To cite this version:**

Sylvain Nascimbène, Nir Navon, Sebastiano Pilati, Frédéric Chevy, Stefano Giorgini, et al.. A new Fermi liquid: the normal phase of a strongly interacting gas of cold atoms. 2010. hal-00548927v1

HAL Id: hal-00548927

<https://hal.science/hal-00548927v1>

Preprint submitted on 20 Dec 2010 (v1), last revised 8 Jun 2011 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

A new Fermi liquid: the normal phase of a strongly interacting gas of cold atoms

S. Nascimbène^{1*}, N. Navon¹, S. Pilati², F. Chevy¹, S. Giorgini³, A. Georges^{4,5}, C. Salomon¹

¹ *Laboratoire Kastler Brossel, CNRS, UPMC, École Normale Supérieure, 24 rue Lhomond, 75231 Paris, France*

² *Theoretische Physik, ETH Zurich, CH-8093 Zurich, Switzerland*

³ *Dipartimento di Fisica, Università di Trento and INO-CNR BEC Center, I-38050 Povo, Trento, Italy*

⁴ *Centre de Physique Théorique, CNRS, Ecole Polytechnique, route de Saclay, 91128 Palaiseau Cedex, France*

⁵ *Collège de France, 11 place Marcellin Berthelot, 75005 Paris, France*

(Dated: December 20, 2010)

PACS numbers: 03.75.Ss; 05.30.Fk; 32.80.Pj; 34.50.-s

In 1956 Landau developed an elegant description of interacting Fermi systems at low temperature relying on the existence of long-lived quasiparticles. While this Fermi liquid theory (FLT) describes well Helium 3 and many solid-state materials above the superfluid temperature, there exist notable exceptions such as underdoped cuprates [1]. In dilute atomic fermionic gases, the Fermi liquid nature of the normal phase is currently under debate. On the one hand, recent photoemission spectroscopy experiments near the critical temperature were interpreted using a pseudogap model [2]. On the other hand, measurement of the temperature dependence of the specific heat displayed a linear behaviour compatible with Fermi liquid's prediction [3]. Here, we measure the magnetic susceptibility of a Fermi gas with tunable interaction in the low temperature limit and compare it to quantum Monte Carlo calculations. Experiment and theory are in excellent agreement and fully compatible with FLT. We also measure the condensation energy which is a key quantity measured in superconducting compounds. Temperature variation data and magnetic susceptibility are combined to deduce a full set of Fermi liquid parameters describing the microscopic quasiparticle properties of the normal phase. The photoemission spectrum calculated with these parameters is found in good agreement with the measurements reported in [2]. We conclude that all existing data to date on the normal phase of the unitary Fermi gas are well described by Landau's Fermi liquid theory.

Ultracold Fermi gases are a privileged tool to investigate the subtle interplay between the strength of interactions, temperature and external magnetic field on a strongly-interacting material. For a trapped spin-1/2 mixture with contact interactions described by an s -wave scattering length a , the strength of interactions can be

tuned using a Feshbach resonance, allowing to investigate the crossover between the weakly and strongly attractive regimes. While the superfluid state has been extensively studied [4], the normal state and its nature is still an open question. Several scenarios were proposed to describe the crossover from the Bose gas of pre-formed molecules (small and positive a) to the ideal Fermi gas (small and negative a) and were recently addressed for spin-balanced gases above the superfluid transition, through the measurement of equations of state [3, 6–8], the study of the single-particle excitation spectrum [2, 9] or of spin fluctuations [10]. All these experimental probes give access to the properties of the normal phase above the critical temperature. This limitation can be overcome by stabilizing the normal state at $T < T_c$ by imposing a spin population imbalance in the trapped gas [11–13]. Previous works focused on the highly-polarized limit where minority atoms behave as impurities: $n_2 \ll n_1$, where n_i is the density for species i [3, 5, 14–21]. In this Letter we interpret the spin imbalance as the application of an effective magnetic field to the unpolarized normal gas at very low temperature. Using a combination of Monte Carlo simulations and experimental results, we extract from the equation of state the magnetic spin response of the normal phase in the limit $T \ll T_c$.

The polarization dependence of the energy E of the system directly reflects the presence of spin-singlet dimers in the sample. Indeed, the presence of a gap in the spin excitation spectrum implies a linear dependence of the energy E with polarization p at low temperature, and hence a zero spin susceptibility (see supplementary discussions).

We have performed quantum Monte Carlo simulations of the partially polarized Fermi gas at $T = 0$ in the BEC-BCS crossover. We make use of the fixed-node diffusion Monte Carlo method that was employed in earlier studies of polarized Fermi gases [15, 19] (see Methods). The state of the system is forced to be in the normal phase by imposing the nodal surface of a many-body wave function incompatible with off-diagonal long-range order. We consider a system with fixed total number of particles N in a fixed volume V and we calculate its energy density E for different values of the interaction strength $1/k_F a$, where $k_F = (3\pi^2 n)^{2/3}$, $n = N/V$ and of the polarization

*present address: Ludwig-Maximilians Universität, Fakultät für Physik, Schellingstr. 4, 80799 München, Germany

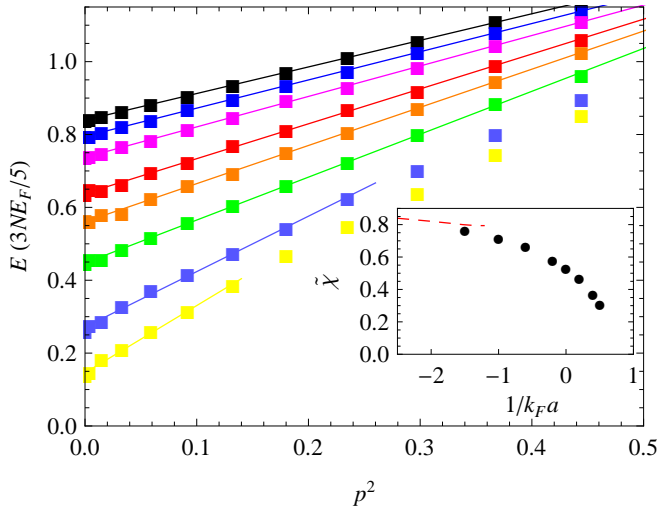


FIG. 1: Canonical equation of state of a two-component Fermi gas calculated using quantum Monte Carlo simulation, for $1/k_F a = -1.5, -1, -0.6, -0.2, 0, 0.2, 0.4, 0.5$ (from top to bottom). The solid lines are fits of the low-polarization data with equation (1). Inset: Extracted values of the susceptibility $\tilde{\chi}$ as a function of $1/k_F a$. The dashed red line is the result of a perturbation expansion valid up to order $(k_F a)^2$.

$p = (N_1 - N_2)/(N_1 + N_2)$. The results for the canonical equation of state are shown in Fig.1. They are well fitted by the energy functional

$$E(p) = \frac{3}{5} N E_F \left(\xi_N + \frac{5}{9} \tilde{\chi}^{-1} p^2 + \dots \right), \quad (1)$$

holding for a spin polarizable system at low temperature, where both ξ_N and the dimensionless spin susceptibility $\tilde{\chi}$ (in units of the susceptibility of an ideal Fermi gas $3n/2E_F$) depend on $1/k_F a$. The Monte Carlo method indicates the absence of spin gap, and thus of preformed molecules in the normal phase for $1/k_F a \lesssim 0.5$. Note that the extracted values of $\tilde{\chi}$ reported in the inset of Fig.1 show a rapid drop for positive values of a when entering the BEC side of the Feshbach resonance. A likely explanation is the binding of fermions into spin-singlet pairs for some positive value of the interaction strength $1/k_F a$. Monte Carlo calculations for values of $1/k_F a \geq 0.7$ show that $E(p)$ is indeed linear rather than quadratic in p , indicating the emergence of a gap. However, pairing fluctuations play a major role for such values of the coupling and the nodal surface of the Jastrow-Slater state (see Methods) is no longer sufficient to enforce the normal phase. This behavior is reminiscent of the pairing transition investigated in the framework of BCS theory [23], as well as in the normal phase of the attractive Hubbard model, extrapolated to a temperature range below the superfluid transition [24, 25], while in our work the extrapolation is made towards a small spin imbalance.

We now compare these simulations with the grand-canonical equation of state of a homogeneous system obtained experimentally in ref. [3, 5]. Using the local

density approximation, it is possible to relate simply the doubly integrated density profile of a cloud trapped inside a harmonic potential to the grand canonical equation of state $P(\mu_1, \mu_2)$ (see Methods). From dimensional analysis, the equation of state (EoS) of a spin-imbalanced Fermi gas can be written as

$$P(\mu_1, \mu_2, a) = P_0(\mu) h \left(\delta = \frac{\hbar}{\sqrt{2m\mu}a}, b = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right),$$

where $\mu = (\mu_1 + \mu_2)/2$ is the mean chemical potential and $P_0(\mu)$ is the pressure of a non-interacting unpolarized Fermi gas. δ is a grand-canonical analog of the interaction parameter $1/k_F a$, and b is a dimensionless number proportional to the effective ‘spin-polarizing field’ $\mu_1 - \mu_2$. Thanks to the variation of the chemical potentials with axial position, an *in situ* image provides a measurement of the function $h(\delta, b)$ along a line in the (δ, b) plane (see Fig.2). Except in the unitary limit, both δ and b vary along the cloud, according to $\delta \propto \sqrt{b}$.

At all values of the scattering length addressed in this work, the equation of state exhibits a clear discontinuity of its derivative at the critical field $b_c(\delta)$, indicating a first-order phase transition from a superfluid state for $b < b_c$ to a normal state for $b > b_c$ [5, 12]. Let us first analyze the equation of state in the superfluid phase. In the unitary limit, $h(\delta = 0, b)$ is constant in the superfluid phase $b < b_c = 0.80(4)$. Since the density difference $n_1 - n_2$ is proportional to $\partial h / \partial b|_\delta$, this directly shows that the superfluid phase is not polarized, whatever the chemical potential imbalance. Away from the unitary limit, since the interaction parameter varies with b , the derivative dh/db does not directly give access to the density difference. However, we observe on the absorption images of the two spin states that the doubly-integrated density difference is always constant in the region corresponding to the superfluid phase [5]. This is a direct signature of full pairing in the superfluid phase in the whole parameter space addressed in this work [22]. The superfluid equation of state is thus independent of b and can be written as $h(\delta, b) = h_S(\delta)$, a quantity that we measured in [5].

We now focus on the properties of the normal phase. From experimental data of Fig. 2, we observe that h is linear with b^2 , in agreement with the expansion

$$h(\delta, b) = h_N(\delta) \left(1 + \frac{15}{8} \tilde{\chi}^{\text{GC}}(\delta) b^2 + O(b^4) \right). \quad (2)$$

$h_N(\delta)$ is the grand-canonical equation of state in the normal state, extrapolated to a spin-symmetric configuration. $\tilde{\chi}^{\text{GC}}(\delta)$ is a grand-canonical analog of the magnetic susceptibility. For an ideal two-component Fermi gas, the functions h_N and $\tilde{\chi}^{\text{GC}}$ are equal to 1. Fitting our data in the normal phase with (2), we obtain the parameters $h_N(\delta)$ and $\tilde{\chi}^{\text{GC}}(\delta)$ in the BEC-BCS crossover shown in Fig.3 where we compare their values to the predictions of the Monte Carlo simulations. To this end, we

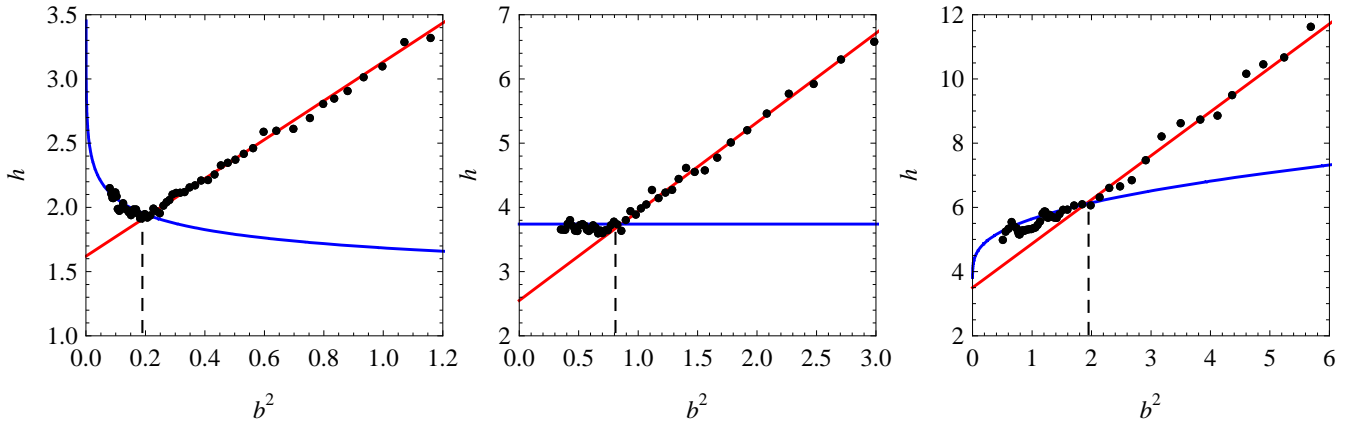


FIG. 2: Samples of thermodynamic function $h(\delta, b)$ measured at different magnetic fields $B_0 = 871, 834, 822$ G. For each magnetic field, the data from 10 to 20 images are averaged. The blue lines correspond to the superfluid equation of state $h_S(\delta)$ measured in [5]. The red lines are linear fits of the data in the normal phase, $b > b_c$. The dashed lines indicate the superfluid/normal phase transition ($b = b_c$).

fit the dependence with $1/k_F a$ of the parameters ξ_N and $\tilde{\chi}$ determined by Monte Carlo simulations, and perform a Legendre transform to obtain the grand-canonical EoS $h_N(\delta)$ of the normal phase and magnetic susceptibility $\tilde{\chi}^{\text{GC}}(\delta)$ measured experimentally. In the investigated parameter range, the agreement between theory and experiment is excellent. We also remark that our value for the susceptibility of the normal phase at unitarity is about twice larger than the MIT value, measured on a gas with a 35% condensate fraction, confirming a significant suppression of the spin susceptibility in the superfluid phase [10].

To make a closer connection with condensed matter systems, in the Supplementary Discussion, we convert our data at the unitary limit to usual variables of condensed matter systems, namely the spin polarization $p = (n_1 - n_2)/(n_1 + n_2)$ versus normalized chemical potential difference $(\mu_1 - \mu_2)/2E_F$.

We also show in Fig.3 the superfluid thermodynamic function $h_S(\delta)$ that we measured in [5]. Since for a spin-balanced gas the superfluid is the actual ground state, we necessarily have $h_S(\delta) > h_N(\delta)$. The difference $h_S(\delta) - h_N(\delta)$ quantifies the ‘condensation energy’ $E_c = P_0(\mu)V(h_S(\delta) - h_N(\delta))$, which is the value of the decrease in free energy associated with superfluidity. At the superfluid/normal transition $b = b_c(\delta)$, the superfluid and normal pressures are equal, leading to the relation

$$h_S(\delta) - h_N(\delta) = \frac{15}{8} h_N(\delta) \tilde{\chi}^{\text{GC}}(\delta) b_c^2(\delta),$$

which is analogous to the expression for the condensation energy of a superconductor as a function of the critical field H_c , $E_c = V \frac{1}{2} \mu_0 H_c^2$, where μ_0 is the permeability of free space. In the Supplementary Discussion, we show that, surprisingly, this condensation energy agrees in the strongly-interacting regime addressed in this work with

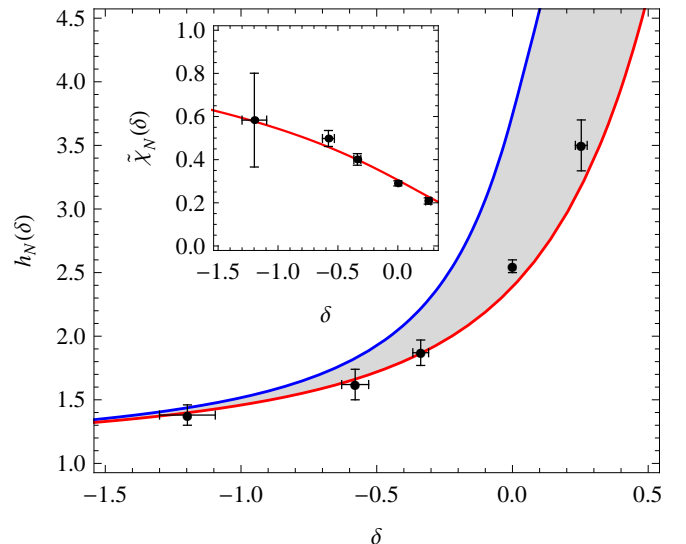


FIG. 3: Fermi liquid equation of state extrapolated to a spin-symmetric configuration $h_N(\delta)$. The black dots are the experimental data, and the red line is calculated from the Monte Carlo data. The blue line is the equation of state $h_S(\delta)$ of the actual (superfluid) ground state, fitted from the data in [5]. The difference $h_S(\delta) - h_N(\delta)$ (in gray) quantifies the condensation energy. Inset: Grand-canonical susceptibility $\tilde{\chi}^{\text{GC}}(\delta)$ of a Fermi gas in the BEC-BCS crossover. Experimental data (black dots), and Monte Carlo data (red line) are in excellent agreement.

the BCS prediction $E_c = \frac{3}{8} N \Delta^2 / E_F$, where Δ is the single-particle excitation gap measured in [26].

Our findings demonstrate that for $1/k_F a \lesssim 0.5$, the spin excitations of the system are not gapped in the normal phase which therefore does not support “true” molecules. However, a certain class of theories predicts a reminiscence of this gap in the form of a dip in the density of states over a range Δ^* around the Fermi level. Δ^* is of-

ten called the pseudogap, in relationship to some features of high-critical temperature superconductors. These theories predict a departure of $E(p)$ from its quadratic behavior when the Fermi levels of the two spin species reach the edges of the dip, $\mu_2 - \mu_1 \simeq \Delta^*$. (see supplementary materials). The absence of such an anomaly in Fig. 1 and 2 thus suggests that the dip is either extremely narrow or very broad: the density of state remains flat over the range of polarizations and interaction strength studied in our work. For instance, at unitarity this range covers $0 < b^2 < 3$. If a sizeable dip existed, then its width cannot be smaller than $\simeq (\mu_1 + \mu_2)\sqrt{3} \simeq 1.4E_F$ where we have used the unitary equation of state, $\mu = 0.41 E_F$ [5]. Such a large pseudogap is not compatible with the photoemission data of [2](See below). Furthermore, we would expect on physical grounds that Δ^* becomes smaller on the BCS side of the resonance. This is observed neither in the experimental data of Fig.2 nor in the Quantum Monte Carlo results of Fig.1.

On the contrary, Landau's theory of Fermi liquids is fully compatible with our observations. This theory assumes the existence of long-lived fermionic excitations above the Fermi surface. Combining the results of [3] with the magnetic susceptibility data presented here, we can fully characterize the parameters of the theory at the unitary limit. Indeed, in [3] the low-temperature compressibility κ and specific heat C_v were measured. Here we obtain from the magnetic response of the $T = 0$ unitary gas its magnetic susceptibility and another determination of κ . The two determinations of κ coincide within 5%, showing that the two approaches indeed probe the same Fermi liquid. From this set of thermodynamic quantities we derive, according to Landau's Fermi liquid theory, a complete characterization of the low-lying excitations of the unitary gas: besides their effective mass $m^* = 1.13 m$ and Landau parameters $F_0^s = -0.42$, $F_1^s = 0.39$ found in [3], we recover here $F_0^s = -0.40$ and obtain the new parameter $F_0^a = m^*/m\tilde{\chi}(0)^{-1} - 1 = 1.1(1)$. Note that $F_0^a > 0$ corresponds to magnetic correlations which do favor the singlet configuration. From these quantities we derive in the supplementary discussion a stability region of the normal phase in the spin-imbalance, temperature plane.

We can finally test FLT on the single-particle photoemission spectrum of a ^{40}K trapped gas prepared at the unitary limit and at the onset of superfluidity [2]. The experimental signal $\overline{A}(k, \omega)$ is directly proportional to the spectral function $A(k, \omega - \mu)$ averaged over the trap (see Methods).

In the vicinity of the Fermi surface, the dispersion relation of the Fermi liquid quasi-particles reads

$$\hbar\omega_k = \mu + \frac{\hbar^2 k^2 - \hbar^2 k_F^2}{2m^*}, \quad (3)$$

where $m^* = 1.13 m$. Assuming long-lived quasiparticles, we approximate $A(k, \omega)$ by $\delta(\omega - \omega_k)$ and perform the

integration over the trap to obtain $\overline{A}(k, \omega)$. In the harmonic trap, $A(k, \omega)$ becomes $\delta(\hbar\omega - \mu(r) - \frac{m}{m^*}(k_F(r)^2 - k^2))$. We calculate the density profile and hence the local Fermi momentum $k_F(r)$ of the trapped gas using the equation of state of the unitary gas at the onset of superfluidity measured in [3]. In order to make a direct comparison with the experimental data, we convolve our result with the experimental resolution in ω in [2], equal to $0.25E_F/\hbar$ and results for various values of k are shown in Fig. 4.

With no free parameter in the theory, FLT well reproduces the experimental spectra for $\overline{A}(k, \omega)$ in the region $k < k_F$, with an excellent agreement in the region $0.3k_F \leq k \leq k_F$ close to the most probable Fermi level in the trap ($\simeq 0.7k_F$) where FLT is expected to be more accurate. Interestingly, we observe that the width of the peak at $k/k_F = 0.6$ is well reproduced by our model meaning that the broadening of the line is not limited by the lifetime of the quasiparticles, but rather by trap inhomogeneity and measurement resolution. Significant deviations between experiment and FLT appear for $k > 1.1k_F$, far from the most probable Fermi wave-vector. However in this region the energy spectrum signal is very broad and weak, corresponding to an incoherent background in the spectral function. Our Fermi liquid description thus accounts for the coherent part of the excitation spectrum from [2].

In conclusion we have shown that the magnetic and thermal responses of the unitary Fermi gas support a description of the normal phase in terms of Fermi liquid theory despite the fact that this system exhibits a high critical temperature for superfluidity. This behaviour is in contrast with underdoped cuprate high T_c materials displaying anomalous magnetic susceptibility or pseudogap physics in the normal phase. Note however that recent quantum oscillation experiments on underdoped cuprates in high magnetic field, aiming at studying the incipient normal state (somewhat analogously to the present work) do suggest long-lived quasiparticles [28]. From thermodynamic quantities, we derived the Landau parameters of the low energy excitations and the FLT reproduces the excitation dynamics measured in [2]. The drop of the susceptibility on the BEC side of the resonance for $1/k_F a \gtrsim 0.5$ suggests the appearance of a spin gap in this regime that deserves further investigations. Finally, the magnetic susceptibility could be a key observable for characterizing the onset of itinerant ferromagnetism in a repulsive Fermi gas [27, 29].

-
- [1] P. Lee, N. Nagaosa, and X. Wen, Rev. Mod. Phys. **78**, 17 (2006).
 - [2] A. Perali, P. Pieri, F. Palestini, G. Strinati, J. Stewart, J. Gaebler, T. Drake, and D. Jin, arXiv:1006.3406 (2010).
 - [3] S. Nascimbène, N. Navon, K. Jiang, F. Chevy, and C. Sa-

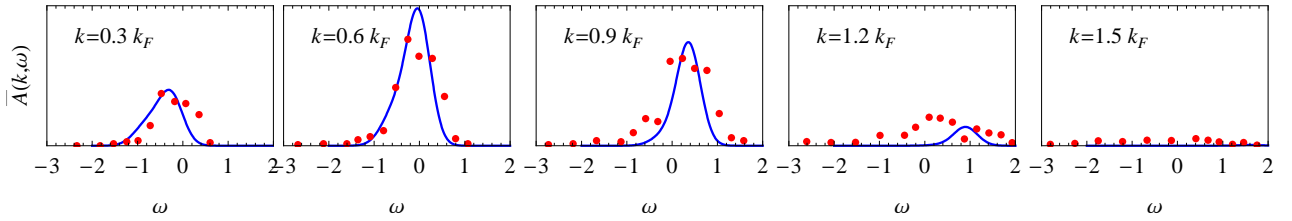


FIG. 4: Energy distribution data $\bar{A}(k, \omega)$ from [2] (red dots), compared with the prediction of Fermi liquid theory (blue lines), for $k/k_F = 0.3, 0.6, 0.9, 1.2, 1.5$.

- lomon, *Nature* **463**, 1057 (2010).
- [4] M. Inguscio, W. Ketterle and, C. Salomon, Ultra-cold Fermi gases, Proceedings of the International School of Physics Enrico Fermi, Course CLXIV, Varenna (2006).
- [5] N. Navon, S. Nascimbène, F. Chevy, and C. Salomon, *Science* **328**, 729 (2010).
- [6] J. Stewart, J. Gaebler, C. Regal, and D. Jin, *Phys. Rev. Lett.* **97**, 220406 (2006).
- [7] L. Luo, B. Clancy, J. Joseph, J. Kinast, and J. Thomas, *Phys. Rev. Lett.* **98**, 80402 (2007).
- [8] M. Horikoshi, S. Nakajima, M. Ueda, and T. Mukaiyama, *Science* **327**, 442 (2010).
- [9] J. Gaebler, J. Stewart, T. Drake, D. Jin, A. Perali, P. Pieri, and G. Strinati, *Nature Phys.* **6**, 569 (2010).
- [10] C. Sanner, E. Su, A. Keshet, W. Huang, and W. Ketterle, arXiv:1010.1874 (2010).
- [11] G. Partridge, W. Li, R. Kamar, Y. Liao, and R. Hulet, *Science* **311**, 503 (2006).
- [12] M. Zwierlein, A. Schirotzek, C. Schunck, and W. Ketterle, *Science* **311**, 492 (2006).
- [13] S. Nascimbène, N. Navon, K. Jiang, L. Tarruell, M. Teichmann, J. McKeever, F. Chevy, and C. Salomon, *Phys. Rev. Lett.* **103**, 170402 (2009).
- [14] F. Chevy, *Phys. Rev. A* **74**, 063628 (2006).
- [15] C. Lobo, A. Recati, S. Giorgini, and S. Stringari, *Phys. Rev. Lett.* **97**, 200403 (2006).
- [16] R. Combescot, A. Recati, C. Lobo, and F. Chevy, *Phys. Rev. Lett.* **98**, 180402 (2007).
- [17] Y. Shin, *Phys. Rev. A* **77**, 041603 (2008).
- [18] N. Prokof'ev and B. Svistunov, *Phys. Rev. B* **77**, 020408 (2008).
- [19] S. Pilati and S. Giorgini, *Phys. Rev. Lett.* **100**, 030401 (2008).
- [20] A. Schirotzek, C.-H. Wu, A. Sommer, and M. W. Zwierlein, *Phys. Rev. Lett.* **102**, 230402 (2009).
- [21] C. Mora and F. Chevy, *Phys. Rev. Lett.* **104**, 230402 (2010).
- [22] T. De Silva and E. Mueller, *Phys. Rev. Lett.* **97**, 70402 (2006).
- [23] R. Combescot, *Euro. Phys. J. D.* **32**, 69 (2005).
- [24] M. Capone, C. Castellani, and M. Grilli, *Phys. Rev. Lett.* **88**, 126403 (2002).
- [25] A. Toschi, P. Barone, M. Capone, and C. Castellani, *New Journal of Physics* **7**, 7 (2005).
- [26] A. Schirotzek, Y. Shin, C. Schunck, and W. Ketterle, *Phys. Rev. Lett.* **101**, 140403 (2008).
- [27] G. Jo, Y. Lee, J. Choi, C. Christensen, T. Kim, J. Thywissen, D. Pritchard, and W. Ketterle, *Science* **325**, 1521 (2009).
- [28] L. Taillefer, *J. Phys. Cond. Mat.* **21**, 164212 (2009).

[29] S. Pilati, G. Bertaina, S. Giorgini, and M. Troyer, *Phys. Rev. Lett.* **105**, 030405 (2010).

[30] Y. Shin, C. Schunck, A. Schirotzek, and W. Ketterle, *Nature* **451**, 689 (2008).

Acknowledgements We are grateful to G. Bruun, C. Lobo, P. Massignan, S. Stringari, T. Giamarchi and R. Combescot for insightful comments. We wish to thank D. Jin, T. Drake and J. Gaebler for providing us experimental data on the radio-frequency spectroscopy. We acknowledge support from ESF (FerMix), ANR FABIOLA, Région Ile de France (IFRAF), ERC Ferlodim, and Institut Universitaire de France. S.P. acknowledges support from the Swiss National Science Foundation, S.P. and A.G. from the Army Research Office with funding from the DARPA OLE program.

Author Information Correspondence and requests for materials should be addressed to S. N. (email: sylvain.nascimbene@physik.uni-muenchen.de).

METHODS SUMMARY

We recall here the procedure used to measure the equation of state $P(\mu_1, \mu_2, a)$, that was already employed in [5] after the method proposed by [9]. We prepare a deeply degenerate mixture of the two lowest internal states of ${}^6\text{Li}$, held in a cylindrically symmetric hybrid optical/magnetic trap, of radial (axial) frequency ω_r (ω_z , respectively). The bias magnetic field B_0 is chosen between 822 G and 981 G, allowing to tune the strength of interactions. The final atom number is 2 to 10×10^4 atoms per spin state, and the gas temperature is $0.03(3)T_F$, as measured from the fully-polarized wings of a trapped gas [30]. As shown in [3, 5, 9], the local gas pressure along the z axis can directly be obtained from its *in situ* image. In the framework of local density approximation, this provides the grand-canonical equation of state $P(\mu_1, \mu_2, a)$ at the local chemical potentials $\mu_{iz} = \mu_i^0 - \frac{1}{2}m\omega_z^2 z^2$, where μ_i^0 is the global chemical potential for species i . The global chemical potential μ_1^0 for the majority species is directly obtained from the Thomas-Fermi radius R_1 of the fully polarized phase, according to $\mu_1^0 = \frac{1}{2}m\omega_z^2 R_1^2$. Similarly to [5], we obtain the global chemical potential μ_2^0 by imposing that, at

the outer radius R_2 of the minority species, the chemical potential ratio μ_2/μ_1 is given by the resolution of the impurity problem [14–16, 18–20].

We make use of the fixed-node diffusion Monte Carlo method that was employed in earlier studies of polarized Fermi gases [15, 19]. The state of the system is forced to be in the normal phase by imposing the nodal surface of a many-body wave function incompatible with off-diagonal long-range order. A simple way to implement this requirement is by choosing the trial function of the Jastrow-Slater form

$$\psi_T(\mathbf{R}) = \prod_{i,i'} f(r_{ii'}) D(N_1) D(N_2), \quad (4)$$

where $\mathbf{R} = (\mathbf{r}_1, \dots, \mathbf{r}_N)$ is the spatial configuration vector of the N particles and D denotes the Slater determinant of plane waves in a cubic box of size L with periodic boundary conditions, accommodating the N_i particles of each species i . The positive Jastrow correlation term $f(r)$ is determined as described in Ref. [15]: at short distances it corresponds to the lowest-energy solution of the two-body problem, while it satisfies the boundary condition on its derivative $f'(r = L/2) = 0$. Finite-size effects have been reduced using the technique described in [29].

METHODS

Fixed-Node Monte Carlo simulation The Hamiltonian of the $N = N_1 + N_2$ atoms of the two species is given by

$$H = -\frac{\hbar^2}{2m} \left(\sum_{i=1}^{N_1} \nabla_i^2 + \sum_{i'=1}^{N_2} \nabla_{i'}^2 \right) + \sum_{i,i'} V(r_{ii'}), \quad (5)$$

where i, j, \dots and i', j', \dots label, respectively, majority and minority fermions. We model interspecies interatomic interactions using an attractive square well potential: $V(r) = -V_0$ if $r < R_0$ and zero otherwise ($V_0 > 0$). The short range R_0 is fixed by the condition $nR_0^3 = 10^{-6}$, where $n = n_1 + n_2$ is the total atom density. The depth V_0 is instead chosen as to give the proper value of the scattering length a along the BEC-BCS crossover. We consider a system with fixed total number of particles ($N = 66$) in a fixed volume $V = L^3$ and we calculate using fixed-node diffusion Monte Carlo method its energy for different values of the interaction strength $1/k_F a$, where $k_F = (3\pi^2 n)^{2/3}$, and of the polarization $p = (N_1 - N_2)/(N_1 + N_2)$, and obtain the data plotted in Fig.1.

Averaged spectral function of a Fermi liquid The trap-averaged spectral function $\bar{A}(k, \omega)$ is related to the spectral function $A(k, \omega)$ by [2]:

$$\bar{A}(k, \omega) = \frac{48k^2}{\pi^2} \int d^3r \frac{A(k, \omega - \mu(\mathbf{r})/\hbar)}{1 + \exp \frac{\hbar\omega - \mu(\mathbf{r})}{k_B T}}, \quad (6)$$

where $\mu(\mathbf{r})$ is the local chemical potential at position \mathbf{r} . In order to calculate the integrated spectral function $\bar{A}(k, \omega)$ of a Fermi liquid, we replace the spectral function by equation (3), where $k_F(r)$ is calculated from the equation of state of the unitary gas determined in [3], and perform the integral in (6). The temperature is chosen at the onset of superfluidity $k_B T/\mu^0 = 0.32$. In order to make a direct comparison with the experimental data, we finally convolve our result with the experimental resolution in ω , equal to $0.25E_F/\hbar$.

SUPPLEMENTARY DISCUSSION

Gap and spin susceptibility Let us consider a system containing N_\uparrow and N_\downarrow spin up and spin down particles. We define $M = N_\uparrow - N_\downarrow$ and $N = N_\uparrow + N_\downarrow$ the polarization and the total atom number and we note $E(N, M)$ the energy of the system. If one assumes that the energy can be expanded in M then by symmetry the linear term vanishes and one gets $E(N, M) = E(N, 0) + M^2/2\chi + \dots$. With this definition, χ is then the spin susceptibility of the system. Indeed, adding a magnetic field h contributes to a $-hM$ term to the energy and we immediately see that the energy minimum is shifted from $M = 0$ to $M = \chi h$.

This argument is no longer true in the case of a gapped system. Indeed, polarizing a spin balanced system costs the binding energy of the broken pairs. This definition applies to any system composed of spin-singlet dimers, from a fermionic superfluid composed of Cooper pairs, or a pure gas of uncondensed molecules, and leads to the following leading order expansion

$$E(N, M) = E(N, 0) + |M|\Delta + \dots$$

To evaluate the spin susceptibility, we add as above a magnetic field h changing the energy into $E - hM$. We see that for $h \neq 0$, the potential is tilted but the energy minimum stays located at $M = 0$ (as long as $|h| < \Delta$ corresponding to the Pauli limit pointed out by Clogston and Chandrasekhar in the case of superconductors [7, 8]).

Thermodynamic signature of the pseudogap. The pseudogap phenomenon can be defined as a dip in the density of state $\rho(\varepsilon)$ close to E_F reminiscent of the true cancellation of ρ inside the superfluid gap. We note Δ^* the width of the dip, and for the sake of simplicity we assume that $\Delta^* \ll k_B T_F$. In a simple model where one assumes that the excitations of the system are described by the Fermi-Dirac distribution, one can show that the spin susceptibility of the system is the inverse of the density of state at the Fermi level. For small imbalances, the Fermi levels of the two spin states lie within the dip. The spin susceptibility is thus $\chi^{-1} = \rho_0^*$. When the imbalance is larger, the two Fermi surfaces are outside the dip (when $E_{F1} - E_{F2} \gg \Delta^*$), and the pseudogap excitations do not contribute anymore to the thermodynamics of the system. In this case, the spin susceptibility is given by $\chi^{-1} = \rho_0 > \rho_0^*$ (see Fig. 6).

Polarizability of the unitary Fermi gas It is interesting to express our data for the unitary Fermi gas in the usual variables of condensed matter physics, namely the polarization $p = (n_1 - n_2)/(n_1 + n_2)$ as a function of $(\mu_1 - \mu_2)/2E_F$, where E_F is the Fermi energy. These quantities are calculated from the thermodynamic function at unitarity $h_0(b) = h(\delta = 0, b)$ according to

$$p = \frac{h'_0(b)}{\frac{5}{2}h_0(b) - b h'_0(b)}, \quad \frac{\mu_1 - \mu_2}{2E_F} = \frac{b}{(h_0(b) - \frac{2}{5}b h'_0(b))^{2/3}}.$$

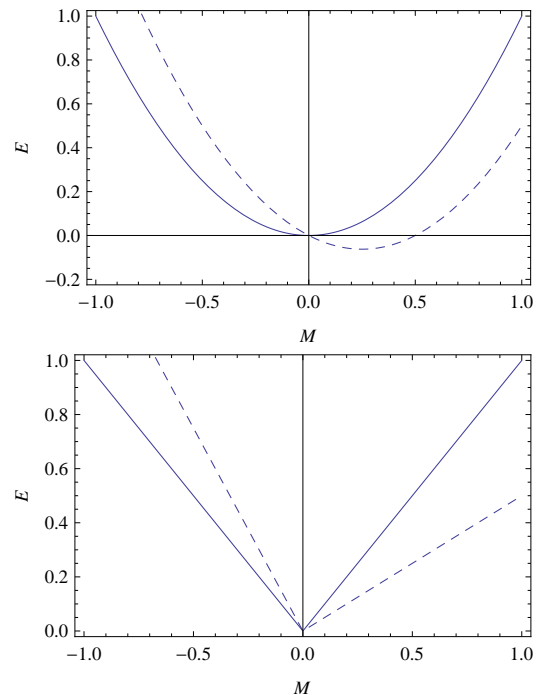


FIG. 5: Dependence of energy E with spin population imbalance M for gapless (top) and gapped (bottom) systems. Top. Full line: The dependence with spin imbalance is quadratic and the curvature is equal to the inverse of the spin susceptibility $\tilde{\chi}$. Dashed line: In the presence of a spin polarizing field h , the energy minimum is shifted to $M = \tilde{\chi}h$. Bottom: gapped system. Full line: Energy in the absence of external spin polarizing field. The slope is equal to the gap Δ . Dashed line: in the presence of a spin polarizing field, the energy profile is tilted but the minimum remains located at $M = 0$.

This requires to take the derivative of our experimental data, which decreases the signal-to-noise ratio. We obtain the data plotted in Fig.7. In the superfluid phase the polarization remains equal to 0 and jump to $p \simeq 0.4$ at the superfluid/normal transition (for $\mu_1 - \mu_2 \simeq 0.4 \cdot 2E_F$). The polarization then increases linearly with the magnetic field over a large polarization range, according to $p = \frac{3}{2}\chi_N(\mu_1 - \mu_2)/2E_F$, with $\tilde{\chi} = 0.54$.

Condensation energy and excitation gap BCS theory predicts that the condensation energy associated with superconductivity is related to the single-particle excitation gap through

$$E_c = \frac{3}{8}N \frac{\Delta^2}{E_F}.$$

The transposition to the canonical equations of state $E = \frac{3}{5}NE_F\xi_\alpha(1/k_F a)$ in the superfluid ($\alpha = S$) and normal ($\alpha = N$) states reads

$$\xi_N - \xi_S = \frac{5}{8} \frac{\Delta^2}{E_F}. \quad (7)$$

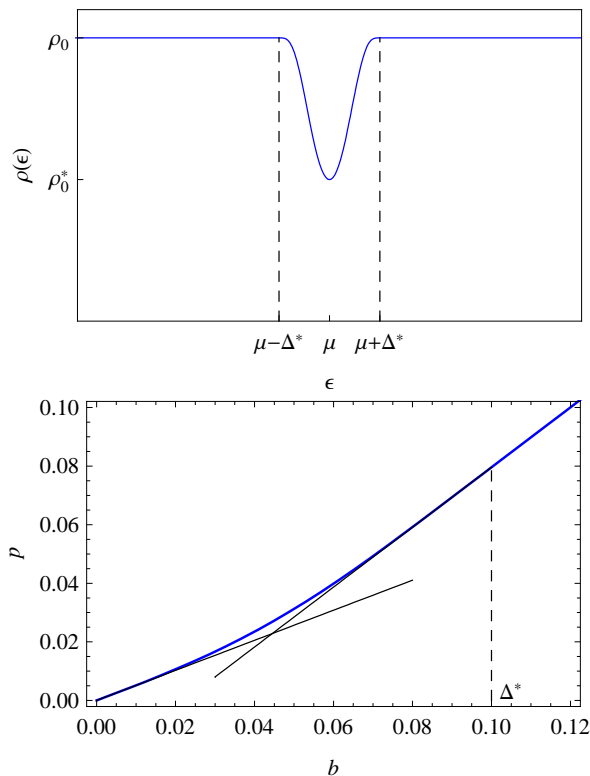


FIG. 6: Top: sketch of the density of state in a pseudogap model. A footprint of the molecular state appears as a dip of width Δ^* and depth $\rho_0 - \rho_0^*$ in the density of state. Bottom: Polarization p as a function of the magnetic field b , with $\rho_0^* = 0.5\rho_0$ and $\Delta^* = 0.1\mu$. At low imbalance, the Fermi levels of the two spin species lie inside the dip. This results in a depletion of spin excitations and a reduction of the spin susceptibility.

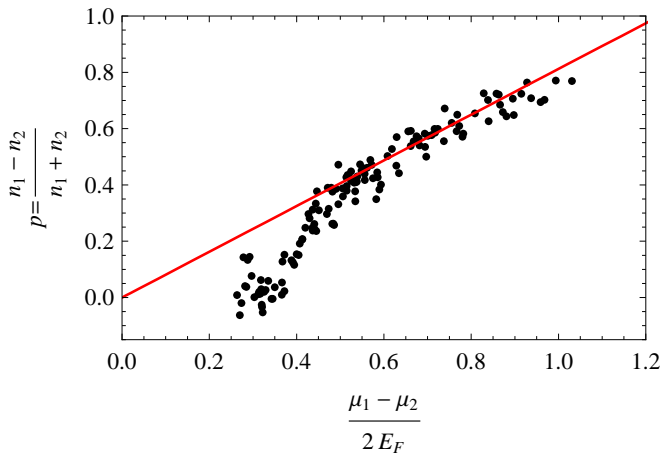


FIG. 7: Polarization p of the unitary Fermi gas as a function of the magnetic field $\mu_1 - \mu_2$, normalized to the Fermi energy $E_F = \hbar^2/2m(3\pi^2n)^{2/3}$. The red line corresponds to the relation $p = \frac{3}{2}\tilde{\chi}(\mu_1 - \mu_2)/2E_F$, with $\chi = 0.54$.

In Fig.8 we compare $\xi_N - \xi_S$ determined from Monte

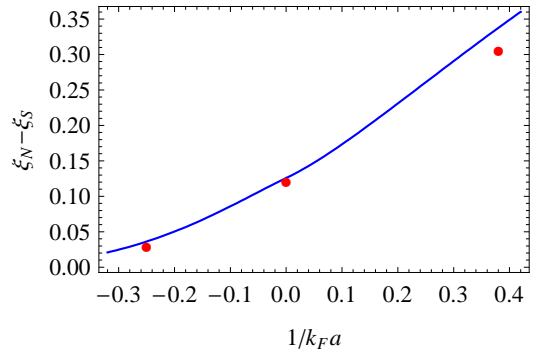


FIG. 8: Condensation energy expressed as $\xi_N - \xi_S$ as a function of $1/k_F a$, determined from a fit of Monte Carlo calculations (blue line). The red dots are equal to $\frac{3}{8}\Delta^2/E_F$, with the experimental gap values from [26].

Carlo simulations [1–3] to the experimental values of the gap from [26]. Surprisingly, we observe that equation (7) seems valid in the strongly-interacting regime addressed in this work.

Lower bound on the superfluid/normal transition

From the low-temperature and low-magnetic field behav-

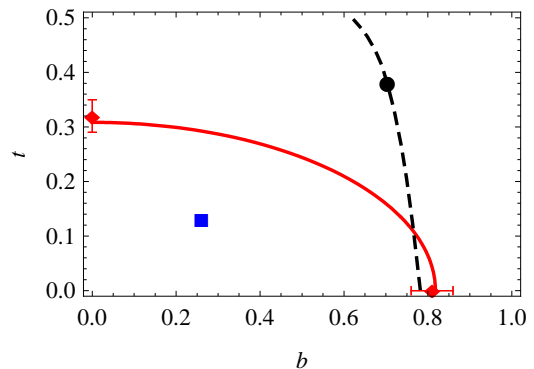


FIG. 9: Lower bound on the superfluid/normal transition in the (b, t) magnetization, temperature plane deduced from a Fermi liquid description of the normal phase (red line). The red diamonds correspond to the $t = 0$ and $b = 0$ superfluid/normal transitions [3]. The dashed black line is a Nozières-Schmitt prediction from [4], the black dot is the corresponding tri-critical point. The blue square is the tri-critical point predicted from a renormalization group theory [5].

iors measured respectively in [3] and in this work, we can write the equation of state of the normal phase of the unitary gas at finite temperature and magnetic field:

$$\frac{P_N(\mu_1, \mu_2, T)}{P_0(\mu)} = \xi_N^{-3/2} + \frac{5\pi^2}{8}\xi_N^{-1/2}\frac{m^*}{m}t^2 + \frac{15}{8}\xi_N^{-1/2}\tilde{\chi}^{GC}b^2,$$

where $t = k_B T/\mu$. In the superfluid phase, since $\partial P_S/\partial T|_{\mu_1, \mu_2}$ is the entropy and is positive, one has $P_S(\mu_1, \mu_2, T) \geq P_S(\mu_1, \mu_2, T = 0) = 2\xi_S^{-3/2}P_0(\mu)$. At the superfluid to normal transition, $P_S = P_N$, which

leads to the following inequality:

$$\xi_N^{-3/2} + \frac{5\pi^2}{8}\xi_N^{-1/2}\frac{m^*}{m}t^2 + \frac{15}{8}\xi_N^{-1/2}\tilde{\chi}^{\text{GC}}b^2 \geq \xi_S^{-3/2},$$

which provides a lower bound on the stability region of the Fermi liquid state (see Fig.9). At low temperature the bound coincides with the actual superfluid/normal transition ($b \simeq 0.8, t = 0$) by construction. Interestingly, it is also very close to the transition point of a spin-unpolarized gas ($b = 0, t = 0.32$) determined in [3]. This shows that temperature effects are rather small in the superfluid state. While the prediction of Nozières-Schmitt-Rink theory from [4] for the transition line is correctly above our lower bound, it is surprising to observe that the prediction of renormalization group theory for the tricritical point largely violates it [5]. Finally we can give a lower bound on the curvature of T_c at low magnetic field: $d^2t_c/db^2 > -0.46$. The Monte Carlo calculation

$d^2t_c/db^2 = 0.25(40)$ from [6] satisfies this condition.

-
- [1] S. Chang, V. Pandharipande, J. Carlson, and K. Schmidt, Phys. Rev. A **70**, 43602 (2004).
 - [2] G. Astrakharchik, J. Boronat, J. Casulleras, and S. Giorgini, Phys. Rev. Lett. **93**, 200404 (2004).
 - [3] S. Pilati and S. Giorgini, Phys. Rev. Lett. **100**, 30401 (2008).
 - [4] M. Parish, F. Marchetti, A. Lamacraft, and B. Simons, Nature Phys. **3**, 124 (2007).
 - [5] K. Gubbels and H. Stoof, Phys. Rev. Lett. **100**, 140407 (2008).
 - [6] O. Goulko and M. Wingate, Phys. Rev. A **82**, 053621 (2010).
 - [7] A. M. Clogston, Phys. Rev. Lett. **9**, 266 (1962).
 - [8] B. S. Chandrasekhar, App. Phys. Lett. **1**, 7 (1962).
 - [9] T. Ho and Q. Zhou, Nature Phys. **6**, 131 (2009).