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# Experimental demonstration of the negative refraction of a transverse elastic wave in a two-dimensional solid phononic crystal

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The negative refraction of transverse elastic waves is demonstrated experimentally in a two-dimensional phononic crystal (PC) made of a square lattice of cylindrical air cavities in an aluminum matrix. Dispersion curves of elastic waves in this PC exhibit a unique branch with phase and group velocities of opposite signs in a broad frequency range. Measurement of refraction angles through prismatic PC included in an aluminum block demonstrates negative refraction of elastic transverse wave. © 2010 American Institute of Physics. [doi:10.1063/1.3302456]

Left-Handed (LH) materials were first theoretically studied by Veselago<sup>1</sup> by considering media with both negative dielectric permittivity and magnetic permeability. For electromagnetic plane wave propagating in LH material, Poynting vector and wave vector, which are, respectively, associated to energy flux and phase velocity, are in opposite directions. Negative refraction, which occurs at the interface between media of different handedness, is one of the unusual properties associated to LH materials. It was experimentally demonstrated that artificial periodic materials including local resonators may display appropriate dielectric and magnetic effective properties in a limited frequency range.<sup>2</sup> The availability of negative refraction material has important consequences for imaging as a flat lens with relative refraction index equal to  $-1$  would have two important properties: (1) rays issued from a point source converge to a point image with exact phase compensation (2) evanescent waves can contribute to the formation of the image in the near field and thus lead to subwavelength focusing.<sup>3–6</sup> Negative refraction of acoustic waves also been studied recently and has been experimentally demonstrated in three-dimensional (3D) (Ref. 7) and two-dimensional (2D) (Refs. 8–12) phononic crystals (PCs) with a fluid (air or liquid) matrix. Based on recently published theoretical results,<sup>13</sup> the present letter demonstrates experimentally the negative refraction of transverse elastic waves in a PC made of air-filled cylindrical cavities in an aluminum matrix. This first experimental observation of negative refraction of the elastic waves in a 2D elastic PC is a first step toward the construction of a flat lens which will employ negative refraction to focus elastic waves emitted by a point source.

The 2D PC considered in this study is made of a square lattice of air-filled cylindrical cavities in aluminum matrix. The density of aluminum is  $\rho=2808 \text{ kg m}^{-3}$  and the longitudinal and shear velocities are, respectively,  $V_{L-\text{al}}=6337 \text{ m s}^{-1}$  and  $V_{T-\text{al}}=3130 \text{ m s}^{-1}$ . Distance between axes of two adjacent holes is  $a=3.9 \text{ mm}$  and hole diameter is  $d=3.2 \text{ mm}$ . This leads to a filling factor  $f$  of each hole de-

fined as the ratio between the cross-sectional area of a hole and the surface of the primitive unit cell equals to 53%. The well known plane wave expansion (PWE) method is used for the calculation of the dispersion curves (Fig. 1), presented in the first Brillouin zone, on the  $\Gamma\text{XM}$  path. The results are presented in terms of frequency in kilohertz versus reduced wave vector  $\vec{K}=\vec{k}a/2\pi$  (where  $\vec{k}$  is the wave vector). The band structure exhibits several branches with a negative slope, i.e., with phase velocity opposite to the group velocity. Thus, negative refraction is expected to occur in the region [470 kHz to 530 kHz], marked by the gray rectangle on Fig. 1 where the unique branch of negative slope corresponds to a mixed mode with predominantly transverse behavior.<sup>13</sup> Calculations have shown that the largest frequency range in which negative refraction occurs is around  $f=50\%$ .<sup>13</sup> We can also point out that equi frequency surfaces (EFS) exhibit a quasircular shape in the frequency domain of interest. These EFS are obtained by the intersection of the 3D dispersion curves with a horizontal plane, i.e., at fixed frequency. This condition is necessary to obtain a focusing effect of an ultrasonic source through a PC slab.<sup>12</sup> The group velocity  $\partial\omega/\partial k$  (where  $\omega$  is the angular frequency and  $k$  the wave-number) remains approximately constant in the frequency range of interest (2100 m/s), whereas the phase velocity  $V_{\text{PC}}=\omega/k$  varies from 6000 m/s in the lowest part of the

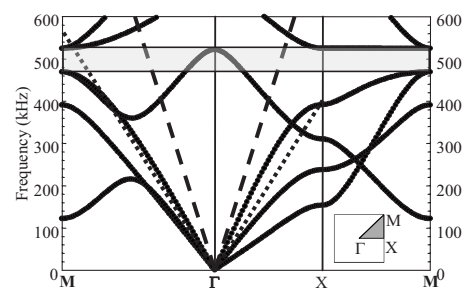


FIG. 1. PWE elastic band structures for the 2D PC made of a square array of holes in aluminum. The gray area corresponds to the frequency range in which a unique branch of negative group velocity takes place. The dashed and dotted lines correspond to the longitudinal and transverse velocities in bulk aluminum, respectively.

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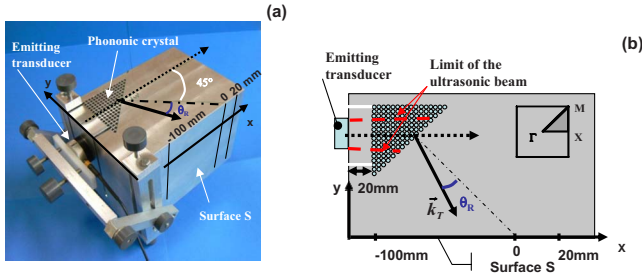


FIG. 2. (Color online) (a): Photograph of the PC. (b): Top view of the experimental sample.

frequency range of interest to 30 000 m/s in the upper part. These values are much higher than the longitudinal and transverse wave velocities in bulk aluminum (respectively, dashed and dotted lines on Fig. 1). In what follows, one uses the values of the wavenumber in the PC,  $k_{pc}$  at frequencies  $F=450$  and  $480$  kHz defined as the radii of the EFS in the PC at these frequencies. These values are, respectively, equal to  $k_{pc}=270.5$  and  $186.08 \text{ m}^{-1}$ .

The PC is realized in an aluminum block ( $170 \times 120 \times 120 \text{ mm}^3$ ). The prismatic PC (15 rows by 15 columns) is presented in Fig. 2. Diameter and hole spacing are defined in the previous section. The height of the holes, equal to 60 mm, is large enough to be considered as infinite compared to the width of the incident ultrasonic beam. Generation of shear waves polarized in the  $y$  direction is performed with a Panametrics V151 transducer (one-inch diameter, 500 kHz central frequency). The incident wave is along  $\Gamma X$  direction (Fig. 2). Two slits are added on both sides of the emitting transducer to avoid diffraction of longitudinal and transverse waves by PC borders. Electrical drive is constituted by ten cycles burst at a given frequency  $F$  and 10 V amplitude. The transverse wave transmitted through the PC is detected on the surface  $S$  (Fig. 2) using a laser vibrometer. The sample is translated along the direction  $Ox$  from  $x=-100$  to  $20$  mm with  $0.2 \text{ mm}$  space step. The position  $x=0$  is located at the intersection of the normal to the exit surface of the PC with the surface  $S$  (Fig. 2). In order to enhance the signal to noise ratio, at each position the registered oscillogram is the mean of 300 successive measurements. Displacements versus position and time are displayed in Fig. 3 in the case of a signal excitation at a frequency  $F=480$  kHz.

The signal corresponding to the direct transmission of the transverse wave through the PC is highlighted in Fig. 3.

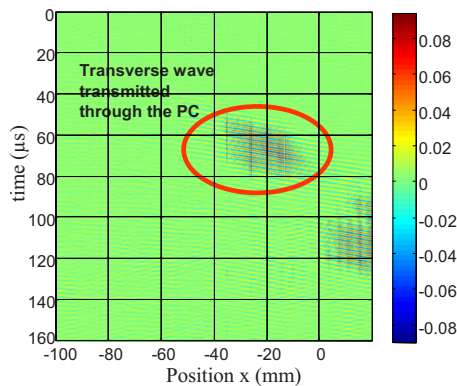


FIG. 3. (Color online) Time-space representation of the signal measured with the laser vibrometer on the surface  $S$  along the  $x$  direction. Color scale is in arbitrary units.

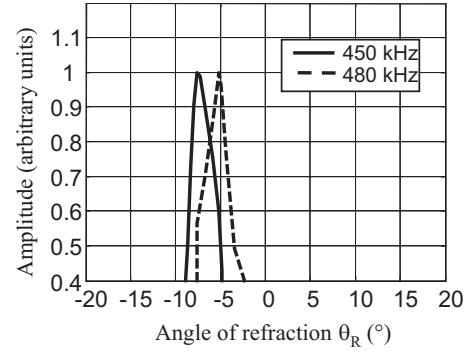


FIG. 4. Amplitude in arbitrary units of the transmitted signal through the PC vs the angle of refraction for two incident transverse waves at frequencies  $F=480$  and  $450$  kHz.

This signal is clearly located in the area where negative refraction is expected (negative values of  $x$ ). A theoretical value of the time of flight for a transverse wave transmitted through the PC can be estimated using group velocity in the PC and transverse wave velocity in aluminum. Due to the prism shape of the PC, a mean distance of propagation (35 mm in PC, 127 mm in aluminum) is considered corresponding to the center of the ultrasonic beam. Theoretical time of arrival equal to  $57 \mu s$  is in good agreement with approximate experimental value of  $52 \mu s$ . This time of arrival is clearly incompatible with the velocity of longitudinal waves whose transmission through the PC is not observed in Fig. 3. Moreover, as mentioned previously, in the frequency range of interest (see Fig. 1) the dispersion curves of the PC show a band with a predominantly transverse behavior that implies a strong attenuation of the longitudinal component of the wave in the PC. Later time signals can be attributed to multiple reflections of transverse wave in the aluminum block.

The experimental value of the angle of refraction  $\theta_R^{exp}$  can be determined accurately from the measured displacements of the negatively refracted wave. Values of the  $x$  component  $k_{Tx}$  of the wave vector  $\vec{k}_T$  are obtained by performing a time and a spatial Fourier transform on time-space signals in Fig. 3. The refraction angles  $\theta_R^{exp}$  are then computed thanks to the relation  $k_{Tx} = (2\pi F / V_{T-al}) \cos(45 + |\theta_R^{exp}|)$ . In Fig. 4 is plotted the amplitude of the transmitted signal through the PC versus the angle of refraction. For an incident transverse wave packet of central frequency  $F=480$  kHz, the resulting peak is centered around  $-5^\circ$ . The wave in the PC reaches the exit interface with an angle of incidence  $\theta_{pc}$  equal to  $45^\circ$  and, assuming that the PC behaves as a bulk homogeneous material having effective properties given in band structure (Fig. 1), the refraction can be described by the Snell law  $[\sin \theta_R = (V_{T-al} / V_{pc}) \sin \theta_{pc}]$ . This leads to a theoretical refraction angle  $\theta_R^{theo} = -7.9^\circ$  higher than the measured angle at  $F=480$  kHz. A second measurement is performed for an incident wave at frequency  $F=450$  kHz, for which the branch with a negative slope is no more unique (Fig. 1). That means that some other modes may propagate in the PC in other directions than  $\Gamma X$ . At  $F=450$  kHz, a negatively refracted wave is measured at the exit of the PC and the resulting values of the refraction angle are plotted in Fig. 4. The corresponding peak is now centered around  $\theta_R^{exp} = -7.6^\circ$ . This value is still lower than the theoretical angle of refraction ( $\theta_R^{theo} = -12.3^\circ$  at  $F=450$  kHz). However, the absolute value of  $\theta_R^{exp}$  increases with decreasing frequency. This is in

accordance with the band structure of the PC where the magnitude of the wave vector decreases with the frequency for the branch of interest in  $\Gamma X$  direction.

The negative refraction of a transverse ultrasonic wave is demonstrated experimentally. A discrepancy between the experimental angle of refraction and the theoretical value computed from band structure and EFS is observed. The discrepancy may result from the difference between finite thickness of the experimental PC and infinite PC size used in dispersion curves computation. As shown in Fig. 2(b), the minimum distance in the PC crossed by the bounded ultrasonic beam is equal to about 20 mm (five rows of hole). This distance is lower than the wavelength of the wave in the PC ( $\lambda=23.2$  mm at  $F=450$  kHz and  $\lambda=33.8$  mm at  $F=480$  kHz). Under these conditions, the use of theoretical properties of the infinite PC to interpret the behavior of the studied experimental PC is critical as already demonstrated in photonic crystals.<sup>14</sup>

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