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Replanning working time under annualised working hours^Y

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Abstract

Using annualised hours (i.e. distributing working hours irregularly over a year) grants a company the flexibility needed to meet the seasonal nature of demand. Since annualised hours can lead to a worsening of the staff's working conditions, many laws and collective bargaining agreements contain constraints that affect the distribution of working time. In the past few years, efforts have been made to develop methods for optimally solving planning problems involving annualised working hours. However, to our knowledge, the problem of replanning work time with annualised working hours has not been addressed in the literature. Herein we explore different ways of achieving said replanning, using mixed-integer linear programming models. Two main objectives are considered: the cost of the new plan and the stability of the scheduling of workers' working time. Solving the models for various scenarios yields the quantitative information that is needed to replan an annualised hours system.

Keywords: personnel management, manpower planning, annualised hours

1. Introduction

Annualising working hours consists in hiring workers for a certain number of hours per year and distributing these hours irregularly over the year to accommodate fluctuations in demand, i.e. whilst observing the restrictions imposed by laws and collective bargaining agreements, workers can be called in for the hours they are needed. It allows for better use of potential capacity, as capacity can be better adapted to demand over time. This flexibility in the use of human resources is useful in services and in manufacturing organisations. However, use of annualised hours (AH) often implies a worsening of the staff's working conditions and requires solving complex working time planning problems.

A number of papers deal with production and working time planning problems and consider some kind of flexibility (see, for example, Wild and Schneeweiss, 1993). However, the subject of annualised hours had remained largely unexplored until just a few years ago. Indeed, some authors—including Hung (1999b), Grabot and Letouzey (2000), and Azmat and Widmer (2004)—have mentioned that the concept of annualised hours was surprisingly absent from the literature on planning and scheduling. Oke (2000) outlined various actions aimed at providing flexibility in the use of human resources. The author indicated that approximately 40% of companies consider the use of AH to be one of the most desirable options, mainly because of its low cost, although at the time it was still uncommon (approximately 10% of companies used it).

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3 During the past few years, efforts have been made to develop methods for optimally
4 solving planning problems involving annualised working hours. Corominas *et al.*
5 (2004a) discussed the characteristics of the annualised working hours planning problem
6 and introduced a classification scheme that yields thousands of different cases. The
7 authors, as well as many others, have solved some of these types of problems (see, for
8 example, Hung, 1997, 1999a and 1999b; Gonçalves and Marcola, 2001; Corominas *et al.*
9 *et al.*, 2002, 2004b, 2005, 2007a, 2007b and 2007c; Azmat and Widmer, 2004; Azmat *et al.*
10 *et al.*, 2004; and Lusa *et al.*, 2007). **Pinker and Larson (2003) dealt with a related problem,
11 with uncertainty in the demand for labour, in which workers are guaranteed a minimum
12 number of working days per month and are called in or not each day depending upon
13 demand.**

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17 As Hur *et al.* (2004) mentioned, companies, namely those operating in the services
18 sector, typically invest a substantial amount of resources in estimating and planning the
19 number of workers they need to meet their slated workload. The literature includes the
20 examples of health care organisations (Cayirli and Veral, 2003), postal services
21 (Malhotra and Ritzman, 1994), call centres (Green *et al.*, 2003) and emergency services
22 (Mabert, 1983). However, although planning adjustments for working hours are clearly
23 commonplace in many service organisations, they have been the subject of very little
24 academic research. This is especially true of replanning of working time with
25 annualised working hours.
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29 **Replanning is a way to deal with uncertain demand. Ideally, in such environment, the
30 problem should be stated as a dynamic programming (as in Pinker and Larson, 2003).
31 However, this approach may be impractical, owing to the number and the complexity of
32 the constraints that solutions have to fulfil or because there is not enough available
33 information about the probability distribution of the demand.**

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36 In fact, although a rolling horizon is considered an essential characteristic of aggregated
37 planning, introductory texts generally do not state much on the implications of carrying
38 out periodic replanning. Nonetheless, it is evident that a new plan cannot generally be
39 executed without taking into consideration the previous one. In any case, some of the
40 ideas described in this work that relate to replanning in the context of generalised hours
41 may be useful for aggregate planning in general.
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45 Let us assume there is a plan beginning at time t_0 that determines the weekly working
46 hours of each employee; some time after t_0 the planner may update the information and
47 draw up a new plan to begin at $t_0 + \eta$ (where η is the replanning period). From the
48 outset, the objective is to minimise the cost of the new plan; however, it is desirable too
49 that the replanning have as little impact as possible on the expected scheduled workload
50 of the workers in order to avoid, for instance, excessive rotation. This can be achieved
51 by delimiting the changes made from one plan to the next (via restrictions), or by
52 minimising a discrepancy measurement between the two plans. Clearly, in either case,
53 the objective of maximising the benefit or minimising the cost must remain present in
54 one form or another. In the context of annualised hours, the appropriate value for the
55 replanning period η may depend on several factors, which may be specific to the firm.
56 However, a value of two or three months may be convenient in many circumstances.
57 Planning and replanning can be dealt with via mixed-integer linear programming
58 (MILP) models. Solving MILP models for different settings provides the quantitative
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3 information that management needs when replanning under AH; this information is in
4 practice very relevant.
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7 The main objective of the paper is to propose a method for replanning, taking into
8 account the cost and the stability of the workers' planned working time. The use of this
9 method will allow the manager: i) to know the cost of the new plan and evaluate the
10 increase in that cost when constraints to limit the changes between two consecutive
11 plans are added and ii) to calculate the initial value of a measure of discrepancy between
12 plans and evaluate the decrease when a specific increase in cost is allowed. To introduce
13 the method we use an AH planning model obtained from the literature.
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16 The remainder of this paper is organised as follows: Section 2 sets out a planning
17 problem under annualised hours (which is based on the problem introduced in
18 Corominas *et al.*, 2002) and its subsequent modelling via mathematical programming,
19 Section 3 discusses replanning in aggregated planning and presents various options for
20 replanning working time with annualised working hours and modelling said options via
21 mathematical models, Section 4 describes the computational experiment and Section 5
22 contains the conclusions.
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24

25 26 **2. A planning problem under annualised hours**

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28 As mentioned previously, AH enables managers to irregularly distribute a given number
29 of working hours throughout the year, which provides flexibility. However, for AH to
30 be implemented many obstacles must be overcome, since annualising involves, from the
31 outset, a worsening of staff working conditions. Therefore, it must be negotiated and
32 accompanied by some kind of compensation or incentive. Of course, as in other
33 contexts, the diversity of production systems means that a wide variety of models are
34 needed to deal with planning under AH. Herein, we consider a production process
35 obtained from Corominas *et al.* (2002) that is performed on an individual basis. We first
36 explain the planning problem and then describe (in the Appendix) a model that is
37 sufficiently detailed enough to ensure the proposed solutions are suited to replanning.
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41 The planning problem consists in determining the weekly number of working hours for
42 each member of staff for all non-holiday weeks (which are previously agreed upon) in
43 the planning horizon (a year), with the objective of optimising a utility function. This
44 situation is common in the services industry. Here, we also assume that the product
45 (presumably a service) is not storable.
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48 In Corominas *et al.* (2002) it is assumed that the production capacity in any week must
49 be greater than or equal to that which is needed to satisfy the expected demand and that
50 if this capacity is not entirely provided by the staff temporary workers will be hired for
51 the required number of hours at a given cost. Hence, the objective function is the cost of
52 overtime plus the cost of employing temporary workers.
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55 The conditions to be fulfilled by the solution may come from a legal resolution, such as
56 the French 35-hour workweek law (MES, 2006), or from a collective bargaining
57 agreement between management and workers. Due to the impossibility of establishing
58 an exhaustive list of conditions *a priori*, in order to build a basic model for the problem
59 the most common conditions are considered; some of these were found in collective
60 bargaining agreements, whereas others were determined from real cases. For any

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3 specific case, the model to be used could be obtained by adding or deleting certain
4 constraints.
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7 The French law requires that the weekly number of working hours must fall within an
8 interval defined by lower and upper bounds; that the total annual working hours be
9 upper bounded; and that the average of weekly working hours, for any set of twelve
10 consecutive weeks, be upper bounded (it is assumed that this condition only applies to
11 sets of twelve consecutive non-holiday weeks). Moreover, as mentioned above,
12 overtime is admitted.
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15 These rules can be complemented by other rules. To consider the working conditions of
16 the staff, the collective agreement may also state that a working week be considered
17 'weak' or 'strong' depending on the number of working hours (for example, no more
18 than 30 hours/week or more than 44 hours/week, respectively). Thus, the number of
19 annual 'weak' working weeks is lower bounded (e.g. 10 weeks), whereas the annual
20 number of 'strong' working weeks is upper bounded (e.g. 15 weeks).
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23 Finally, we assume that there are different sets of worker categories as well as different
24 types of tasks to perform. The workers are cross-trained (Nembhard, 2007): certain
25 categories of workers can perform different types of tasks, with different efficiencies for
26 each type of task and for each category. To model this, we consider a relative efficiency
27 for each type of task and for each category: a value of 0.9 signifies that a worker in a
28 given category needs to work $1/0.9$ hours to meet a demand that a worker with a relative
29 efficiency equal to one would meet in one hour. Moreover, as in Corominas *et al.*
30 (2002), it is assumed that there is a penalty matrix, whose elements correspond to the
31 penalty associated with an hour of work in a task of a specific type of a staff member of
32 a certain category. The suitability of task assignment to individual employees arises in
33 several types of companies. For instance, the authors know of a case in a department
34 store in which a section manager is capable of performing all of the required tasks:
35 supervising, processing sales, replacing items and even cleaning. However, it is
36 preferable for this employee to perform the supervision tasks corresponding to his/her
37 position.
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42 The characteristics of this planning problem under annualised hours are summarised
43 below:
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- 45 • The annual number of ordinary working hours for each staff member is fixed.
46 Additional hours are considered as overtime.
- 47 • Annual overtime is upper bounded.
- 48 • There are different categories of workers and different types of tasks. Certain
49 categories of workers can perform different types of tasks and thus their associated
50 relative efficiencies are different.
- 51 • The weekly number of working hours is lower and upper bounded.
- 52 • The average number of working hours for a group of L consecutive non-holiday
53 weeks cannot be greater than h_L hours per week.
- 54 • The annual number of 'weak' working weeks is lower bounded (a week is
55 considered 'weak' if the number of working hours is no greater than h_w hours).
- 56 • The annual number of 'strong' working weeks is upper bounded (a week is
57 considered 'strong' if the number of working hours is greater than h_s hours).
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- Each worker has two consecutive holiday weeks in winter and four consecutive holiday weeks in summer. The holiday weeks for each worker are previously agreed upon.
- The product (presumably a service) is not storable.
- Hiring temporary workers is possible.
- The objective function includes the cost of overtime plus that of the external workers and the (weighted) penalties associated with the assignment of functions to the types of employees on the staff.

These conditions can be modelled as linear constraints, thus giving place to an MILP model whose resolution provides an initial plan under annualised hours (the Appendix includes an MILP model that is based on that presented in Corominas *et al.*, 2002).

3. Replanning

Business environments are dynamic; hence, unforeseen events can make initially proposed plans unfeasible or ineffective (Méndez and Cerdá, 2003). This makes replanning as important as the original planning problem itself (Vin and Ierapetritou, 2000). Any event that forces replanning or rescheduling is generally labelled a *disruption*.

Replanning of aggregated planning is mentioned in many articles and textbooks. It is based on starting from a planning horizon (which is divided into various time intervals), as well as on a replanning interval (which indicates the time between one replanning and another) and a period of rigidity (during which any decisions made in the last plan cannot be modified in the new plan). The extent to which the predicted plan is altered depends on the replanning.

In terms of manpower, there are two main reasons that replanning may be required. Firstly, the actual work that is performed may not coincide with the work that had been planned. This could stem from many factors, including more or less hours having been worked, employee absences, shift changes, staff changes (e.g. hiring or termination) and employees taking unplanned holiday days or weeks. Secondly, after some time has passed since initial planning, a company may have access to more accurate information on the near future, enabling it to update its original plans (e.g. those for required working hours for tasks) and forecasts of the availability of resources (Hur *et al.*, 2004).

As previously mentioned, plans lack stability; in the literature, this feature is dubbed *nervousness* (de Kok and Inderfurth, 1997). Despite the numerous publications that refer to nervousness (e.g. Carlson *et al.*, 1979; Blackburn *et al.*, 1986 and 1987; Sridharan *et al.*, 1987; Vollmann *et al.*, 1992; and Kadipasaoglu and Sridharan, 1995), de Kok and Inderfurth (1997) state that '*little work has been done in developing and defining generally applicable measures of nervousness*' (p. 56).

Of course, the concept of nervousness may apply to plans of various degrees of aggregation. The majority of publications refer to plans—typically known as master production schedules—whose elements are work orders or purchase orders such as those obtained with a material requirement planning (MRP) system. Hence, nervousness is measured according to the changes in orders from one plan to the next, for example,

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3 according to the number of changes in the quantity of orders (de Kok and Inderfurth,
4 1997); the number of times that the orders are reprogrammed, regardless of whether the
5 corresponding quantities have also been modified (Heisig, 1997); or the length in
6 periods of a gap between having and not having orders, or vice versa (van Donselaar *et*
7 *al.*, 2000).
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10 Other researches have referred to changes in scheduling in various sectors (Vieira *et al.*,
11 2003). These cases include aircraft rescheduling (Thengvall *et al.*, 2000), crew
12 rescheduling in European airlines (Nissen and Haase, 2006), rail crew rescheduling
13 Huisman, 2007) and disruption in resource-constrained project scheduling (Zhu *et al.*,
14 2005). Analytical models are proposed for estimating performance measures such as
15 average flow time and machine utilization when a single-machine system is rescheduled
16 (Vieira *et al.*, 2000a) and when the system consists of parallel machines (Vieira *et al.*,
17 2000b). Yuan and Mu (2007) describe an algorithm for rescheduling jobs on a single
18 machine to minimise makespan under a limit on maximum disruption, considering time
19 and sequence disruptions. Qi *et al.* (2006) pointed out that in most scheduling research,
20 the new schedule has to optimise the original objective in the new environment; the
21 authors proposed the alternative idea of taking into account the deviations from the
22 original schedule.
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27 Regarding the revision of an aggregated plan, some authors have only considered
28 maximising the benefit or minimising the cost of the new plan. In any case, the new
29 plan is typically designed to resemble the initial plan as much as possible—for example,
30 by minimising the number of workers whose shifts are changed (Hur *et al.*, 2004). The
31 replanning should have as little impact as possible on the programming that is under
32 way. With this aim, several authors have defined a discrepancy between the plan that is
33 under way and the new plan, which is incorporated into (and penalised in) the objective
34 function (Méndez and Cerdá, 2003; Carlson *et al.*, 1979; Tsuborne and Furuta, 1996;
35 and Vin and Ierapetritou, 2000).
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39 Some authors limit the number of permissible changes between plans (Méndez and
40 Cerdá, 2003). Others define, in addition to the rigidity period, periods of semi-rigidity in
41 which decisions (e.g. those dealing with production) between the initial plan and the
42 replanned one are allowed to vary by a certain percentage above or below the value in
43 the former. Carlson *et al.* (1979) presented a time-dependent function of the costs
44 associated with the changes between plans. It encompasses an initial period, in which
45 the cost is infinite (which coincides with the period of rigidity, since no changes can be
46 made); intermediate periods, in which the cost is a non-increasing, time-dependent
47 function; and final periods, in which the cost is zero. A similar idea can be found in the
48 work of Kadipasaoglu and Sridharan (1997).
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52 All of the aforementioned characteristics can be applied to a working-time management
53 system based on annualised working hours. This can confer the system with greater
54 flexibility, but may lead to a worsening of the working conditions. As Hur *et al.* (2004)
55 stated, '*Proactive and complete modification of work schedules may improve customer*
56 *service and, in turn, profitability of the organisation. However, such active adjustment*
57 *can result in worker dissatisfaction and increases managerial complexity.'* (p. 325).
58 This in turn requires models with sufficient resolution at different settings to provide the
59 essential quantitative information needed to manage an AH system taking into account,
60 for instance, cost and schedule stability. As Hur *et al.* (2004) stated, '*(...) it is interesting*

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3 to estimate the degree of profit reduction when employee and managerial convenience
4 is emphasised over operational efficiency.' (p. 325).

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7 Hur *et al.* (2004) presented a case of real-time work schedule adjustment decisions at a
8 fast-food restaurant chain. According to the authors, practising managers consider
9 multiple goals for schedule adjustments—relating to customer service, cost control,
10 worker utilisation and schedule stability—but do not necessarily agree amongst
11 themselves on which goal is most important. Interview data revealed that profit
12 maximisation and schedule stability are the major operational goals at the restaurants.
13 Managers assume that profit maximisation is equivalent to the achievement of the target
14 customer service level with minimum labour cost. Schedule stability (to avoid shift
15 changes and task rotations) is also thought to be important because excessive schedule
16 modification increases managerial burden and complexity and worker dissatisfaction.

20 3.1. Replanning working time with annualised working hours

21
22 In this section, we present various options for replanning working time with annualised
23 working hours and then describe methods for applying and managing the process.

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25 Given that the use of AH consists in hiring workers for a certain number of hours per
26 year and then the employer distributing the hours irregularly over the year, the
27 replanning horizon comprises whatever remains of the year, which is divided into time
28 periods of one week. The replanning period might be four weeks, and a period of
29 rigidity might be eight weeks, for example. In any case, the values should be negotiated
30 between the workers and the company, since the degree of flexibility conferred to the
31 system is a function of the value of these and other elements; at any event, an
32 excessively short replanning period (one or two weeks, for instance) would hardly be
33 operative in most cases.

34
35 Replanning starts from the initial plan, the execution of the already completed part of
36 said plan, and new information available from the work environment (i.e. updated
37 forecasts of the working hours for tasks and of the availability of resources). Replanning
38 consists in determining the number of working hours for every worker for each week in
39 the replanning horizon (as previously stated, what remains of the year).

40
41 To minimise the negative effects of replanning, every effort should be made to ensure
42 that it has the least possible impact on the programming that is under way (i.e. that the
43 new plan resembles the initial plan as much as possible). There are two ways of
44 accomplishing this: by adding restrictions that limit the changes to be made, and/or by
45 including a discrepancy measurement between the two plans (and penalising it) in the
46 objective function.

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48 The restrictions to be added should limit the number or magnitude of any changes made.
49 For the AH problem, we consider the following:

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51 C.1. Number of weeks in which the working hours for each worker (or for the entire
52 staff) are changed. This value is upper bounded. Given x'_{it} and x_{it} , which are the
53 working hours for worker i in week t in the initial plan and in the new plan,
54 respectively, it is considered that the working hours do not change when
55 $x'_{it} - \varepsilon_i \leq x_{it} \leq x'_{it} + \varepsilon_i$ (e.g. $\varepsilon_i = 0.5$ hours).

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- C.2. Number of weeks in which for each worker (or for the entire staff) a week stops being (or becomes) ‘weak’ (or ‘strong’). This value is upper bounded. Clearly, when a ‘normal’ week becomes ‘weak’ or a ‘strong’ week becomes either ‘normal’ or ‘weak’, the worker can be considered to benefit from this. Similarly, when a ‘normal’ week becomes ‘strong’, or a ‘weak’ week becomes either ‘normal’ or ‘strong’, the worker stands to lose. Lastly, if a ‘normal’ week becomes ‘weak’, and another ‘weak’ week becomes ‘normal’, the outcome is still negative (i.e. beneficial changes should have less weight than detrimental ones). For example, a beneficial change could be assigned a weight of 0.7 times that of a detrimental change.
- C.3. Definition of periods of semi-rigidity in which the difference in the planned hours for a worker between the two plans in a given week is upper and lower bounded. The further away the replanned week is, the greater the values that these bounds can take.
- C.4. Average number of weekly hours for each worker (or for the entire staff) that are switched from one week to another (i.e. that are planned above or below the previous plan). This value is upper bounded.
- C.5. Difference in resulting annual overtime for each worker (or for the entire staff) between the two plans. This value is upper and/or lower bounded.
- C.6. Difference in the total penalty associated with the assignment of functions to types of employees between the two plans. This value is upper bounded.
- C.7. Increase in cost (or decrease in benefit) between replanning that does not minimise any discrepancy measure between plans, and replanning that does. This value is upper bounded. Hence, if the objective function (OF) of cost is provided and the replanning problem is solved without any discrepancy measure, then a value (z^*) is obtained. The problem can then be solved again by minimising the discrepancy measure and adding the cost restriction $OF \leq \theta \cdot z^*$ (where $\theta > 1$).

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The discrepancy measures quantify the differences between plans. For the AH problem, we consider the following:

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- D.1. Number of weeks in which the number of work hours ‘changes’ for staff members.
- D.2. Number of weeks that stop being (or become) ‘weak’ (or ‘strong’) for staff members. As previously mentioned, the benefits or losses corresponding to the changes must be weighted differently.
- D.3. Number of hours that are switched from one week to another for members of staff. In other words, for each worker, there is a weekly plan whose number of hours is less than or greater than those in the previous plan.

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3.2. A method for managing the replanning of working time

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Described below is a method for managing and implementing the replanning of working time with annualised working hours. The application of the method for different scenarios provides the essential quantitative information required for the management of replanning under AH, considering cost and schedule stability. This management tool would allow the manager: i) to know the cost of the new plan and evaluate the increase (or the decrease in benefit) when constraints are added to limit the changes, and ii) to calculate the initial value of a measure of discrepancy between plans (which measures

the stability of the schedule) and evaluate its decrease as a function of the permitted increase in cost.

The process comprises the following four steps:

Step 1 Calculate the cost of the new plan (z^* , the value of the original objective function OF) without including any limiting elements (i.e. restrictions and/or discrepancy measures) on changes.

Step 2 Evaluate the addition of restrictions that limit the changes. When a new restriction is considered, a new value can be calculated for OF ($z^{*'}).$ Hence, the decision making process can be based on quantifiable elements: the increase in cost (or loss in benefit) is $\Delta = z^{*'} - z^*$, where $z^{*'}$ is the value of OF once all the restrictions have been considered.

Step 3 Calculate the initial value of the selected discrepancy measure in question (ϕ). This is accomplished in replanning by minimising ϕ and adding the constraint $OF \leq z^{*'}$ (to avoid problems with numerical accuracy, what is actually used is $OF \leq (1 + \varepsilon\varepsilon) \cdot z^{*'}$, where $\varepsilon\varepsilon > 0$).

Step 4 Evaluate the decrease in ϕ as a function of the permitted increase in cost (or decrease in benefit). To obtain the ratio between the increase in the cost and the decrease in the discrepancy value, the problem in Step 3 is solved with the addition of the constraint $OF \leq \theta \cdot z^{*'}$, for different values of θ (where $\theta > 1$).

3.3. Mathematical models for replanning

In this subsection, we describe the modelling of the general case, which incorporates the replanning elements defined above. These elements can make solving the mathematical model a highly complex task, since they involve using additional binary variables. The annualisation case used here is the same as the one described in the Appendix. The restrictions and discrepancy functions considered for the replanning are C2, C4, C7 and D1.

To make the model easier to understand, we assume here that no workers have left the staff nor have any new workers been hired, and also that the weekly hours worked up to $t_0 + \eta$ are those resulting from the previous plan.

The following additional data are used:

- μ Initial week of the replanning horizon, at which replanning begins (considering the time of replanning and the period of rigidity). Hence, the weeks of the replanning horizon are $t = \mu, \dots, T$.
- x'_{it} Hours worked by worker i in week t ($\forall i \in E; t = 1, \dots, \mu - 1 \wedge t \in A_i$).
- x''_{it} Working hours planned in the previous plan for worker i in week t ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).

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- w'_{it} This indicates whether week t of worker i had been planned as a ‘weak’ week ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).
- N'_{iw} Number of ‘weak’ weeks already worked by worker i ($\forall i \in E$).
- s'_{it} This indicates whether week t of worker i had been planned as a ‘strong’ week ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).
- N'_{is} Number of ‘strong’ weeks already worked by worker i ($\forall i \in E$).
- \hat{r}_{ik} Number of working hours required for tasks of type k in week t ($t = \mu, \dots, T; k = 1, \dots, |F|$).
- ω_i Weighting factor used to calculate the relative impact ($\omega_i < 1$) of positive changes for worker i , whereby the impact of a positive change in one week is equal to ω_i times that of the impact of a negative change in another week.
- UB_i^{C2} Upper bound on the number of weeks that stop being (or become) ‘weak’ (or ‘strong’) for worker i ($\forall i \in E$).
- UB_i^{C4} Upper bound on the average number of hours or weeks that are switched from one week to another for worker i ($\forall i \in E$).
- ε_i Value which, if $x'_{it} - \varepsilon_i \leq x_{it} \leq x'_{it} + \varepsilon_i$, means that the working hours of worker i in week t have not changed with respect to the initial plan ($\forall i \in E$).
- $\theta > 1$ Parameter corresponding to Step 4 in the Method, which enables the discrepancy measure considered as a function of the permitted increase in cost (or decrease in benefit) to be reduced.

36 The variables used here comprise the ones defined in the Appendix—although those
37 that carry the sub-index t are only defined for $t = \mu, \dots, T$ —as well as the following
38 additional variables:
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- $ca_{it} \in \{0,1\}$ This indicates whether the working hours of worker i in week t change with respect to those planned in the previous plan ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).
- $so_{it} \in \{0,1\}$ This indicates whether week t of worker i that had been planned as a ‘strong’ week is now planned as a ‘normal’ or ‘weak’ week ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$). If no change has occurred, then it takes a value of 0.
- $os_{it} \in \{0,1\}$ This indicates whether week t of worker i that had been planned as a non-‘strong’ week is now planned as a ‘strong’ week ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$). If no change has occurred, then it takes a value of 0.
- $wo_{it} \in \{0,1\}$ This indicates whether week t of worker i that had been planned as a ‘weak’ week is now planned as a ‘normal’ or ‘strong’ week ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$). If no change has occurred, then it takes a value of 0.

$ow_{it} \in \{0,1\}$ This indicates whether week t of worker i that had been planned as a non-
‘weak’ week is now planned as a ‘weak’ week
($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$). If no change has occurred, then it takes a
value of 0.

au_{it}^+ Number of hours worked by worker i in week t in the current plan below
those planned in the previous plan ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).

au_{it}^- Number of hours worked by worker i in week t in the current plan above
those planned in the previous plan ($\forall i \in E; t = \mu, \dots, T \wedge t \in A_i$).

Model for Step 1 of the Method (MM.E1): solution of the initial model in week μ

$$[\text{MIN}] z_2 = \sum_{i \in E} \beta_i \cdot v_i + \sum_{k \in F} \gamma_k \sum_{t=\mu}^T d_{tk} + \lambda \cdot \sum_{t=\mu}^T \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{ijk} \quad (1')$$

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} x_{it} = H_i + v_i - \sum_{t=1, \dots, \mu-1 \wedge t \in A_i} x'_{it} \quad \forall i \in E \quad (2')$$

$$v_i \leq \alpha \cdot H_i \quad \forall i \in E \quad (3)$$

$$\sum_{(i \in \hat{E}_j) \wedge (t \in A_i)} x_{it} = \sum_{k \in \hat{F}_j} y_{ijk} \quad t = \mu, \dots, T; \forall j \in C \quad (4')$$

$$\sum_{j \in \hat{C}_k} \rho_{jk} \cdot y_{ijk} + d_{tk} \geq \hat{r}_{tk} \quad t = \mu, \dots, T; \forall k \in F \quad (5')$$

$$\sum_{t=\tau-L+1}^{\mu-1} x'_{it} + \sum_{t=\max(\mu, \tau-L+1)}^{\tau} x_{it} \leq L \cdot h_L \quad \forall i \in E; (\tau = \max(L, \mu), \dots, T) \wedge (t \in A_i, \tau-L+1 \leq t \leq \tau) \quad (6')$$

$$x_{it} \leq h_S + (h_M - h_S) \cdot s_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (7')$$

$$x_{it} \leq h_M - (h_M - h_W) \cdot w_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (8')$$

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} s_{it} \leq N_S - N'_{iS} \quad \forall i \in E \quad (9')$$

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} w_{it} \geq N_W - N'_{iW} \quad \forall i \in E \quad (10')$$

$$h_m \leq x_{it} \leq h_M \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (11')$$

$$s_{it}, w_{it} \in \{0,1\} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (12')$$

$$v_i \geq 0 \quad \forall i \in E \quad (13)$$

$$y_{ijk} \geq 0 \quad t = \mu, \dots, T; \forall k \in F; \forall j \in \hat{C}_k \quad (14')$$

$$d_{tk} \geq 0 \quad t = \mu, \dots, T; \forall k \in F \quad (15')$$

Model for Step 2 of the Method (MM.E2): inclusion of restrictions that limit the changes

Restriction C2 is modelled via (16), (17) and (18). For worker i , it is assumed that the impact of a positive change in a week is equal to ω_i ($\omega_i < 1$) times that of the impact of a negative change in another week.

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} (wo_{it} - \omega_i \cdot ow_{it} + os_{it} - \omega_i \cdot so_{it}) \leq UB_i^{C2} \quad \forall i \in E \quad (16)$$

$$w'_{it} - w_{it} = wo_{it} - ow_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (17)$$

$$s'_{it} - s_{it} = so_{it} - os_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (18)$$

$$wo_{it}, ow_{it}, so_{it}, os_{it} \in \{0, 1\} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (19)$$

Restriction C4 is modelled via (20), which upon linearisation yields (21), (22) and (23).

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} |x''_{it} - x_{it}| \leq UB_i^{C4} \cdot \sum_{t=\mu, \dots, T \wedge t \in A_i} 1 \quad \forall i \in E \quad (20)$$

$$x''_{it} - x_{it} = au_{it}^+ - au_{it}^- \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (21)$$

$$\sum_{t=\mu, \dots, T \wedge t \in A_i} (au_{it}^+ + au_{it}^-) \leq UB_i^{C4} \cdot \sum_{t=\mu, \dots, T \wedge t \in A_i} 1 \quad \forall i \in E \quad (22)$$

$$au_{it}^+, au_{it}^- \geq 0 \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (23)$$

Model for Step 3 of the Method (MM.E3): discrepancy measure under consideration

The discrepancy measure D1 is modelled via (1''), (24), (25) and (26). (1') does not have to be taken into consideration, nor do the restrictions (11'), which are redundant:

$$[\text{MIN}] z_3 = \sum_{i \in E} \sum_{t=\mu, \dots, T \wedge t \in A_i} ca_{it} \quad (1'')$$

$$x_{it} \leq x'_{it} + \varepsilon_i + (h_M - x'_{it} - \varepsilon_i) \cdot ca_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (24)$$

$$x_{it} \geq x'_{it} - \varepsilon_i + (h_m - x'_{it} + \varepsilon_i) \cdot ca_{it} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (25)$$

$$ca_{it} \in \{0, 1\} \quad \forall i \in E; t = \mu, \dots, T \wedge t \in A_i \quad (26)$$

Let z_2^* be the value of (1') once the agreed-upon restrictions have been considered (in this case, C2 and C4). Constraint (27) must be considered.

$$\sum_{i \in E} \beta_i \cdot v_i + \sum_{k \in F} \gamma_k \sum_{t=\mu}^T d_{tk} + \lambda \cdot \sum_{t=\mu}^T \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{ijk} \leq (1 + \varepsilon\varepsilon) \cdot z_2^* \quad (27)$$

Model for Step 4 of the Method (MM.E4)

Once z_3^* (the value of [1'']) is known, the decrease in the value of z_3 with respect to z_3^* can be evaluated in accordance with the increase in z_2 with respect to z_2^* (controlled with a value of $\theta > 1$). Restriction (27) must be replaced by (27'):

$$\sum_{i \in E} \beta_i \cdot v_i + \sum_{k \in F} \gamma_k \sum_{t=\mu}^T d_{tk} + \lambda \cdot \sum_{t=\mu}^T \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{ijk} \leq \theta \cdot z_2^* \quad (27')$$

4. Computational experiment

We tested the efficacy of our proposed replanning method using a computational experiment. Solving the MILP models under different settings provides the essential quantitative information for the management of an AH system. Hence, it is necessary to evaluate the difficulty of solving MILP models.

The basic data used for the experiment are divided into those required for planning and those required for replanning. The basic data used for planning comprise:

- $|E| = 25, 50$ and 100 staff workers.
- $T = 52$ (considering 46 working weeks and six holiday weeks).
- The number of holiday weeks for each worker is constrained to be equal to six, distributed into two non-interrupted parts including two and four weeks, respectively. The temporary location of holidays was fixed, for each worker, at random (the two weeks of the first part in winter and the four weeks of the second part in summer). This way of generating holiday weeks might be not very realistic, but it does not in any way help to solve the models.
- There are three categories ($|C| = 3$): 50% of workers belong to category 1, 30% belong to category 2, and 20% belong to category 3.
- There are three types of task ($|F| = 3$).
- There are three patterns of penalty matrix (P). Table 1 shows the penalty values associated with each pattern.

Insert Table 1

- There are three patterns of relative efficiency (ρ_{jk}). Table 2 shows the relative efficiency values for each pattern.

Insert Table 2

- There are three different patterns for the required capacity (in working hours) throughout the year (r_{ik}). Type 1 demand corresponds to a non-seasonal capacity pattern (i.e. the required capacity is the same for all weeks in the planning horizon) with noise. Type 2 demand corresponds to a seasonality pattern with one peak, with noise. Type 3 demand corresponds to a seasonality pattern with two peaks, with noise. In Types 2 and 3, the peaks correspond to 125% of the average required capacity. In all three cases, noise corresponds to a random increase or decrease of up to 5% of the required weekly capacity, according to a uniform distribution. In each case, the total demand (before introducing noise) is equal to the total capacity (it is supposed that the number of workers has been fixed according to annual demand). The shape of each pattern is shown in Figure 1.

Insert Figure 1

- Other values have been fixed as follows: $H_i = 1,760$ ($\forall i \in E$); $\alpha = 0.05$; $h_m = 20$ and $h_M = 50$; $L = 12$ and $h_L = 45$; $N_w = 10$ and $h_w = 28$; $N_s = 10$ and $h_s = 48$; $\beta_i = 1$ ($\forall i \in E$); $\gamma_k = 1.5$ ($\forall k \in F$); and $\lambda = 0.001$.

For each combination of E , type of required capacity and pattern of relative efficiency (and of penalty matrix), 25 instances were generated (varying demand noise and holiday weeks at random), giving a total of 675 instances.

Despite the large size of some of the planning models, all of them were solved to optimality with the ILOG CPLEX 10.0 optimiser and a Pentium IV PC (3.0 GHz and 1 Gb of RAM). The relative MIP gap tolerance was set to 0.01. Each of the examples was solved via models $M1$ and $M2$ (introduced in the Appendix). As Corominas *et al.* (2002) concluded, the computational times required to solve $M1$ are very short. Table 3 lists the computing times (in seconds); model sizes; minimum computing times (t_{\min}); average computing times (\bar{t}); and maximum computing times (t_{\max}). Model $M2$ is always solved very quickly since it is a linear program (the average and maximum time taken to solve one of the 675 examples is 0.299 and 0.953 seconds, respectively).

Insert Table 3

The basic data used for replanning comprise:

- Replanning period: $\eta = 4$. This value enables one to determine in which weeks μ replanning will occur: $t = \mu, \dots, T$.
- Rigidity period: $\pi = 8$.
- We assume here that the weekly hours worked up to the replanning (x'_{it}) are those resulting from the previous plan.
- There are four different patterns of change in the required capacity, generated by shifting the required capacity patterns near (two weeks) or far (six weeks) to the left or right: $2L$, $2R$, $6L$ and $6R$. In each of the 2,700 cases, noise is recreated. The patterns provide \hat{r}_{tk} , the required number of working hours for tasks of type k in the week t ($t = \mu, \dots, T; k = 1, \dots, |F|$). These patterns of change could happen, for instance, when there is advancement or delay in a sales campaign of a product. Obviously, many other patterns are possible, since the modifications may be due to diverse factors. What is important here is not the pattern of change, but its significance, since the objective is to verify the capacity of the proposed method for managing the replanning of the working time.
- Other values were fixed as follows: $\omega_i = 0.7$ ($\forall i \in E$); $UB_i^{C2} = 10$ ($\forall i \in E$); $UB_i^{C4} = 5$ ($\forall i \in E$); $\varepsilon_i = 0.5$ ($\forall i \in E$); $\varepsilon\varepsilon = 1.005$; and $\theta = 1.05$.
- The MILP models $MM.E1$, $MM.E2$, $MM.E3$ and $MM.E4$ are solved using hardware and software used for models $M1$ and $M2$, with a maximum calculation time of 3,600 seconds per model.

For all 675 examples generated, and for the four designed patterns of change, the method proposed is applied only to the first replanning. The first replanning leads to the

largest MILPSs. Hence, if the Method proves effective for the first replanning, then it will also be so for successive replanning.

Table 4 lists the number of proven optimal solutions (*Opt*) and the number of feasible solutions (*Fea*) for the instances of each pattern of change in the required capacity (*2L*, *2R*, *6L* and *6R*) and for each of the four MILP models (*MM.E1*, *MM.E2*, *MM.E3* and *MM.E4*) to be solved in the method presented. The Table reveals that a feasible solution can be found for all 2,700 cases: models *MM.E1* and *MM.E2* always guarantee an optimal solution; model *MM.E4* guarantees an optimal solution approximately 40% of the time; and model *MM.E3* involves the greatest difficulty in guaranteeing an optimised solution. It should be noted that in three of the 675 cases, a feasible solution could not be found using model *MM.E4*. Nevertheless, these three cases were satisfactorily solved using the solution found for model *MM.E3* as the initial solution for model *MM.E4*.

Insert Table 4

Table 5 shows the average computing times (in seconds) for each MILP model, according to the number of staff workers $|E|$.

Insert Table 5

The results can be considered satisfactory and relevant for practice, since the calculation time required for companies to solve cases involving several scenarios is acceptable given the scope of the problem. Therefore, the method presented has proven to be an effective tool for managing working time with annualised working hours.

By way of example, different results obtained for a case involving a staff of 50 workers, Pattern 1 of penalty and relative efficiency, and a two-peak required capacity pattern, are described below. Figure 2 shows the total required capacity and the total obtained capacity in each period of time in the planning horizon, derived from the planning model used (*M2*). The lack of capacity observed between Periods 5 and 8, as well as between Periods 25 and 27, is because the majority of workers take their winter and summer vacations, respectively, during these times.

Insert Figure 2

To perform the replanning, a value of $\mu = 9$ is used, since the periods of replanning and of rigidity are $\eta = 4$ and $\pi = 8$, respectively. Moreover, Pattern *2R* of the change in required capacity (obtained by shifting the initial required capacity two weeks to the right) is used; the peaks were assigned a value of 137.5% of the average required capacity; and lastly, random noise of 10% was generated in the required capacity. Figure 3 shows the total required capacity in the replanning, the total obtained capacity resulting from model *MM.E1* (i.e. without any restrictions on changes in the workers' shifts), and the total obtained capacity resulting from planning model *M2*. As observed, when there are no limitations on changes, the new obtained capacity covers the new required capacity perfectly, but the new obtained capacity is very far from the

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3 obtained capacity in the planning (which can imply a high degree of *nervousness* or lack
4 of stability). The value of the objective function is 2,557.
5
6

7 8 **Insert Figure 3**

9
10 The changes between plans are limited by performing Step 2 of the Method (running
11 model *MM.E2* with the following values: $\omega_i = 0.7$ ($\forall i \in E$), $UB_i^{C2} = 10$ ($\forall i \in E$) and
12 $UB_i^{C4} = 2$ ($\forall i \in E$)). Figure 4 shows the total required capacity in the replanning, the
13 total obtained capacity resulting from model *MM.E2*, and the total obtained capacity
14 resulting from planning model *M2*. As observed in this case, when there are limits on
15 the changes, the new obtained capacity approaches the new required capacity, but
16 without straying too far from the obtained capacity in the planning. The value of the
17 objective function is 4,728.
18
19

20 21 **Insert Figure 4**

22
23 The presented method could be used to evaluate different situations under a flexible
24 working time scheme. Table 6 shows the value of the objective function of the MILP
25 model *MM.E2*, for different values of the parameters that limit the changes, UB_i^{C2} and
26 UB_i^{C4} . As observed, in this case the replanning is sensitive to the value of the upper
27 bound on the average number of hours or weeks that carry over from one week to
28 another (UB_i^{C4}), but not to that of the upper bound on the number of weeks in which
29 said weeks stop being (or become) 'weak' (or 'strong') (UB_i^{C2}). By solving the model
30 for different values of the parameters that limit the changes, the information that is
31 required to consider different conditions of the replanning system is obtained.
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38 39 **Insert Table 6**

40
41 Steps 3 and 4 of the Method constitute an additional way of minimising the changes
42 between plans. In this case the aim is to minimise the number of weeks in which the
43 working hours of each member of staff *change* with respect to the initial plan. Once the
44 model *MM.E3* has been solved, the model *MM.E4* can be solved for different values
45 of the parameter θ , thereby providing the value of the objective function that must be
46 minimised as a function of the cost. Figure 5 shows the value of the objective function
47 to be minimised according to the value of the cost-control parameter θ .
48
49

50 51 **Insert Figure 5**

52 53 54 **5. Conclusions and prospects for future research**

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56 Using annualised hours for human resources provides the flexibility needed to meet the
57 seasonal nature of demand. Since annual hours can lead to a worsening of the staff's
58 working conditions, many laws and collective bargaining agreements contain
59 constraints that affect the distribution of working time.
60

In the past few years, much effort has been spent on developing methods for optimally solving annualised working hours planning problems. However, to our knowledge the problem of replanning work time with annualised working hours has not been addressed in the literature.

In this paper we have proposed a method that uses mixed-integer linear programming models. Solving the models for different settings yields the quantitative information required to manage the AH system when replanning is called for. The results can be considered satisfactory, since the calculation time required for companies to solve cases involving several scenarios is acceptable, given the scope of the problem. Therefore, the method presented is indeed an effective, practical tool for managing working time under annualised working hours.

Appendix

In this appendix, we describe an MILP model (based on that presented in Corominas *et al.*, 2002), whose solution provides an initial plan under annualised hours.

Data

T	Weeks in the planning horizon ($t = 1, \dots, T$).
E	Set of members of staff.
C	Set of categories of workers ($j = 1, \dots, C $).
F	Set of types of tasks ($k = 1, \dots, F $).
P	Penalty matrix. Each of its finite elements, p_{jk} , corresponds to the penalty associated with an hour of work for a task of type k of a staff member of category j . An infinite value for an element p_{jk} indicates that workers of category j cannot perform tasks of type k .
ρ_{jk}	Relative efficiency associated with staff members of category j in the accomplishment of tasks of type k ($\forall j \in C; \forall k \in F; 0 \leq \rho_{jk} \leq 1$); $\rho_{jk} = 0$ indicates that workers of category j are not able to perform tasks of type k .
\hat{C}_k	Set of categories of workers that can be assigned to tasks of type k ($k = 1, \dots, F $).
\hat{F}_j	Set of types of tasks that can be performed by workers of category j ($j = 1, \dots, C $).
\hat{E}_j	Set of workers of category j ($j = 1, \dots, C $).
r_{tk}	Required working hours for tasks of type k in the week t ($t = 1, \dots, T; k = 1, \dots, F $).
A_i	Set of available (i.e. non-holiday) weeks for worker i ($\forall i \in E$).
H_i	Stipulated ordinary annual working hours of worker i ($\forall i \in E$).
α	Maximum ratio of overtime to ordinary working hours.

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4 h_m, h_M Lower and upper bounds on the number of working hours per week ($h_M > h_m$).
- 5
6 L, h_L L is the maximum number of consecutive non-holiday weeks in which the
7 weekly average of working hours cannot be greater than h_L ($< h_M$).
- 8
9 N_w, h_w N_w is the minimum number of 'weak' weeks (i.e. those having a number of
10 working hours no greater than h_w).
- 11
12 N_s, h_s N_s is the maximum number of 'strong' weeks (i.e. those having a number of
13 working hours greater than h_s).
- 14
15 β_i Cost of one hour of overtime for worker i ($\forall i \in E$).
- 16
17 γ_k Cost of one hour corresponding to tasks of type k performed by a worker who
18 is not a member of staff ($\gamma_k > \beta_i, \forall i \in \hat{C}_k$).
- 19
20 λ Parameter for weighting the penalties in order to establish the trade-off
21 between them and their corresponding monetary costs.

Variables

- 22
23
24
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26 x_{it} Working hours for worker i in week t ($\forall i \in E; \forall t \in A_i$).
- 27
28 y_{ijk} Working hours for employees of category j dedicated to tasks of type k in
29 week t ($\forall k \in F; \forall j \in \hat{C}_k; t = 1, \dots, T$).
- 30
31 v_i Annual overtime for employee i ($\forall i \in E$).
- 32
33 d_{tk} Working hours corresponding to tasks of type k to be supplied in week t
34 by workers who are not members of staff ($\forall k \in F; t = 1, \dots, T$).
- 35
36 $w_{it} \in \{0,1\}$ This indicates whether employee i has a planned number of working hours
37 less than or equal to h_w hours in week t ($\forall i \in E; \forall t \in A_i$).
- 38
39 $s_{it} \in \{0,1\}$ This indicates whether employee i has a planned number of working hours
40 greater than h_s hours in week t ($\forall i \in E; \forall t \in A_i$).

Model

$$45 \quad [\text{MIN}] z = \sum_{i \in E} \beta_i \cdot v_i + \sum_{k \in F} \gamma_k \sum_{t=1}^T d_{tk} + \lambda \cdot \sum_{t=1}^T \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{ijk} \quad (1)$$

$$46 \quad \sum_{t \in A_i} x_{it} = H_i + v_i \quad \forall i \in E \quad (2)$$

$$47 \quad v_i \leq \alpha \cdot H_i \quad \forall i \in E \quad (3)$$

$$48 \quad \sum_{(i \in \hat{E}_j) \wedge (t \in A_i)} x_{it} = \sum_{k \in \hat{F}_j} y_{ijk} \quad t = 1, \dots, T; \forall j \in C \quad (4)$$

$$49 \quad \sum_{j \in \hat{C}_k} \rho_{jk} \cdot y_{ijk} + d_{tk} \geq r_{tk} \quad t = 1, \dots, T; \forall k \in F \quad (5)$$

$$50 \quad \sum_{t=\tau-L+1}^{\tau} x_{it} \leq L \cdot h_L \quad \forall i \in E; (\tau = L, \dots, T) \wedge (t \in A_i, \tau - L + 1 \leq t \leq \tau) \quad (6)$$

$$x_{it} \leq h_S + (h_M - h_S) \cdot s_{it} \quad \forall i \in E; \forall t \in A_i \quad (7)$$

$$x_{it} \leq h_M - (h_M - h_W) \cdot w_{it} \quad \forall i \in E; \forall t \in A_i \quad (8)$$

$$\sum_{t \in A_i} s_{it} \leq N_S \quad \forall i \in E \quad (9)$$

$$\sum_{t \in A_i} w_{it} \geq N_W \quad \forall i \in E \quad (10)$$

$$h_m \leq x_{it} \leq h_M \quad \forall i \in E; \forall t \in A_i \quad (11)$$

$$s_{it}, w_{it} \in \{0, 1\} \quad \forall i \in E; \forall t \in A_i \quad (12)$$

$$v_i \geq 0 \quad \forall i \in E \quad (13)$$

$$y_{ijk} \geq 0 \quad t = 1, \dots, T; \forall k \in F; \forall j \in \hat{C}_k \quad (14)$$

$$d_{ik} \geq 0 \quad t = 1, \dots, T; \forall k \in F \quad (15)$$

(1) is the objective function, which includes the cost of overtime plus the cost of the temporary workers, as well as the (weighted) penalties for assigning tasks to the types of employees of the staff. (2) imposes that the total number of worked hours be equal to the stipulated ordinary annual hours plus any applicable overtime. (3) imposes that the annual overtime does not exceed its upper bound. (4) is the balance between the available hours from a specific category of staff workers and those assigned to the different types of tasks. (5) expresses that the total number of hours assigned to a type of task, whether for members of staff or temporary workers, must not be less than the required hours. (6) imposes the upper bound on the average of weekly working hours for any subset of L consecutive non-holiday weeks. (7) imposes that if the number of working hours is greater than h_S , then the variable s_{it} is equal to 1. (8) imposes that if the number of working hours is greater than h_W , then the variable w_{it} is equal to 0. (9) and (10) indicate that the number of ‘strong’ or ‘weak’ weeks, respectively, cannot be greater than N_S or less than N_W , respectively. (11) imposes the lower and upper bounds on the number of weekly working hours. (12) expresses the binary character of the corresponding variables. Lastly, (13) to (15) express the non-negative character of the remaining non-binary variables.

As explained in Corominas *et al.* (2002), the aforementioned model ($M1$) usually provides infinitely many optimal solutions—in the one provided by an optimiser, the number of weekly working hours for an employee over a year is usually highly irregular. To smooth the profile of working hours throughout the year (i.e. to obtain, among all the solutions of minimum cost, the most regular one possible), Corominas *et al.* (2002) used a second model ($M2$). Its formalisation may be obtained starting from that of the model $M1$, keeping in mind the following considerations:

- (i) Once the model $M1$ is solved, the values of its binary variables and those corresponding to the overtime for each worker are fixed.
- (ii) A constraint that imposes that the cost of the solution of the model $M2$ cannot exceed the optimum value obtained via model $M1$ is added.
- (iii) The variables x_{it} are eliminated using the expression:

$$x_{it} = \bar{x}_i + \delta_{it}^+ - \delta_{it}^-$$

where \bar{x}_i is the average of the number of weekly working hours corresponding to worker i , and δ_{it}^+ and δ_{it}^- are the positive and negative discrepancies, respectively, relating to the average number of working hours for worker i in week t .

- (iv) An analogous expression is used in order to eliminate the variables d_{ik} .
- (v) The objective function (1) is replaced with a new function that has two weighted components. The first component is the sum of the discrepancies of the working hours of the staff members, and the second, the sum of the discrepancies of the working hours provided by workers who are not members of staff.

Thus, solving models $M1$ and $M2$ successively provides an initial plan under annualised hours.

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	PM-Pattern 1			PM-Pattern 2			PM-Pattern 3		
	Task ₁	Task ₂	Task ₃	Task ₁	Task ₂	Task ₃	Task ₁	Task ₂	Task ₃
Category ₁	0	5	∞	0	5	10	0	∞	∞
Category ₂	∞	0	2	∞	0	2	2	0	∞
Category ₃	∞	∞	0	∞	∞	0	5	∞	0

Table 1. Penalty values for each pattern

	RE-Pattern 1			RE-Pattern 2			RE-Pattern 3		
	Task ₁	Task ₂	Task ₃	Task ₁	Task ₂	Task ₃	Task ₁	Task ₂	Task ₃
Category ₁	1	0.9	0	1	0.9	0.8	1	0	0
Category ₂	0	1	0.9	0	1	0.9	0.9	1	0
Category ₃	0	0	1	0	0	1	0.8	0	1

Table 2. Relative efficiency values for each pattern

$ E $	t_{\min}	\bar{t}	t_{\max}	Number of binary variables (average)	Number of real variables (average)	Number of constraints (average)
25	0.218	0.355	1.953	2,300	1,609	3,162
50	0.468	0.701	1.235	4,600	2,784	6,012
100	1.031	1.617	2.766	9,200	5,134	11,713

Table 3. Computing times (in seconds) and model sizes for model $M1$

	<i>MM.E1</i>		<i>MM.E2</i>		<i>MM.E3</i>		<i>MM.E4</i>	
	<i>Opt</i>	<i>Fea</i>	<i>Opt</i>	<i>Fea</i>	<i>Opt</i>	<i>Fea</i>	<i>Opt</i>	<i>Fea</i>
<i>2L</i>	675	0	675	0	46	629	279	396
<i>2R</i>	675	0	675	0	63	612	272	403
<i>6L</i>	675	0	675	0	52	623	271	404
<i>6R</i>	675	0	675	0	48	627	241	434

Table 4. Number of proven optimal (*Opt*) and feasible (*Fea*) solutions for models *MM.E1*, *MM.E2*, *MM.E3* and *MM.E4*

	$ E $		
	25	50	100
<i>MM.E1</i>	0.52	0.70	1.37
<i>MM.E2</i>	1.81	4.31	9.55
<i>MM.E3</i>	1,011.62	1,399.99	1,555.61
<i>MM.E4</i>	470,13	699,62	1,072,45

Table 5. Average computing times (in seconds) for each MILP model, according to $|E|$

		UB_i^{c2}						
		∞	5	4	3	2	1	0
UB_i^{c4}	∞	2,557	2,557	2,557	2,557	2,557	2,557	2,557
	5	2,557	2,557	2,557	2,557	2,557	2,557	2,557
	4	2,691	2,691	2,691	2,691	2,691	2,691	2,691
	3	3,449	3,449	3,449	3,449	3,449	3,449	3,449
	2	4,728	4,728	4,728	4,728	4,728	4,728	4,728
	1	6,107	6,107	6,107	6,107	6,107	6,107	6,107
	0	7,547	7,547	7,547	7,547	7,547	7,547	7,547

Table 6. Value of the objective function of the model *MM.E2*, for different values of UB_i^{c2} and UB_i^{c4}

FIGURE CAPTION

Figure 1. Patterns of demand

Figure 2. Total required capacity vs. total obtained capacity

Figure 3. Total required capacity, the total obtained capacity (which coincides with the required one) from *MM.E1*, and the total obtained capacity from *M2*

Figure 4. Total required capacity, the total obtained capacity from *MM.E2*, and the total obtained capacity from *M2*

Figure 5. Value of the objective function to be minimised according to the value of θ

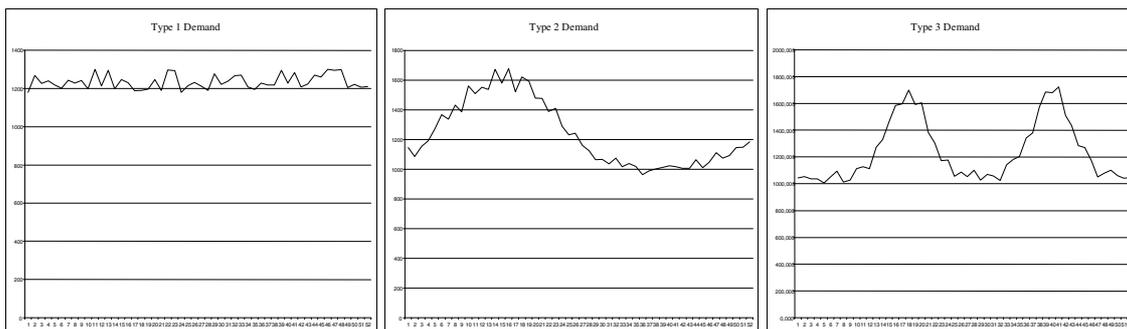


Figure 1. Patterns of demand

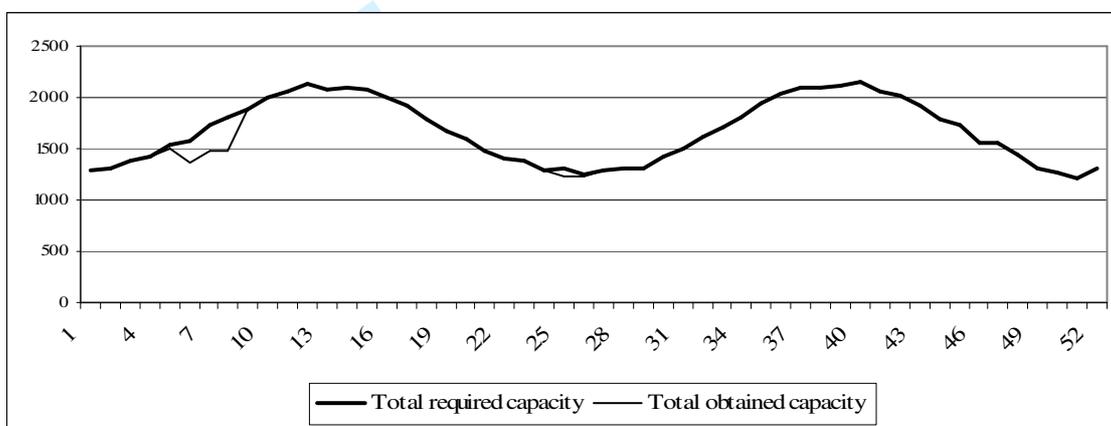


Figure 2. Total required capacity vs. total obtained capacity

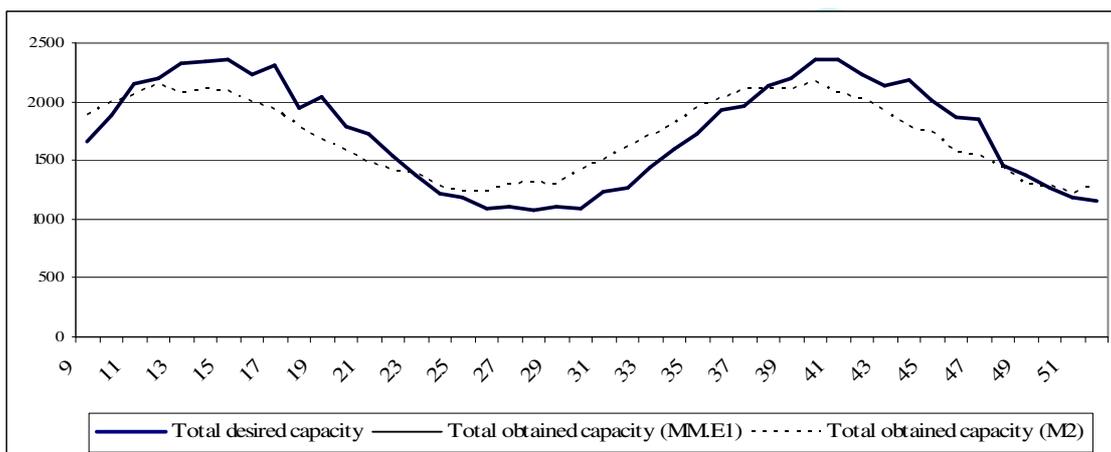


Figure 3. Total required capacity, the total obtained capacity (which coincides with the required one) from *MM.E1*, and the total obtained capacity from *M2*

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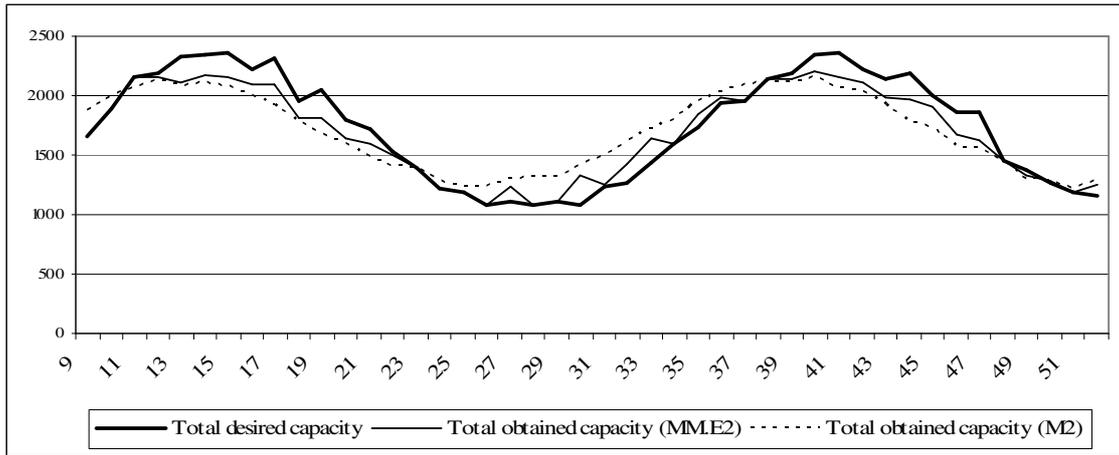


Figure 4. Total required capacity, the total obtained capacity from *MM.E2* , and the total obtained capacity from *M2*

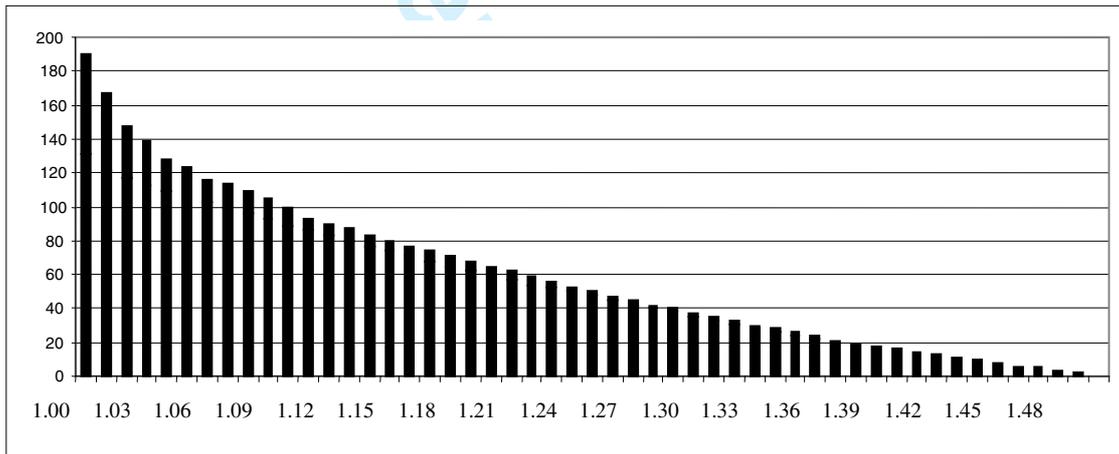


Figure 5. Value of the objective function to be minimised according to the value of θ