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## A note on the uncertain trend in US real GNP: Evidence from robust unit root test

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Charles Amélie (\*)  
Darné Olivier (\*\*)

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(\*) Audencia Nantes, School of Management  
(\*\*) LEMNA - Université de Nantes – France

# **A note on the uncertain trend in US real GNP: Evidence from robust unit root test**

**Amélie CHARLES\*** and **Olivier DARNÉ†**

## **Abstract**

In this paper, we test the presence of stochastic trend in long series of US real GNP measured by Balke and Gordon (1989) and Romer (1989). This is analyzed from two recent robust unit root tests proposed by Cavaliere and Georgiev (2009) and Lima and Xiao (2010), for which critical values are adapted to the small sample size. The former is improved by selecting optimally GLS detrending parameter to make the test in small samples powerful. We obtain mixed results on the full sample (1869–1993). However, the post-1929 GNP and GNP per capita series reject the unit-root null hypothesis, whereas for the pre-1929 GNP data, i.e. the period where the GNP series have been reconstructed, the unit-root hypothesis is not rejected for GNP series proposed by Balke-Gordon and Romer but this hypothesis is rejected for the same series in per capita form. This difference can be explained by the data-construction procedure employed for the pre-1929 GNP series.

*Keywords:* GNP; robust unit root test.

*JEL Classification:* C22; N1.

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\*Audencia Nantes, School of Management, 8 route de la Jonelière, 44312 Nantes Cedex 3. Email: acharles@audencia.com.

†Corresponding author: LEMNA, University of Nantes, IEMN–IAE, Chemin de la Censive du Tertre, BP 52231, 44322 Nantes, France. Email: olivier.darne@univ-nantes.fr.

# 1 Introduction

In spite of numerous studies, the question of deterministic versus stochastic trend in long-term US GNP remains open. A lot of effort has been dedicated to this question in the macroeconometrics literature. Whether the time series can be modeled as stationary fluctuations around a deterministic trend or as difference stationary process is an important issue for many reasons, mainly for economic forecasts, shock identification and regression analysis. A number of studies have examined the long spans of data on US real GNP over the period 1875-1993 with mixed conclusions (e.g., Diebold and Senhadji, 1996; Cheung and Chinn, 1997; Newbold et al., 2001). This lack of consensus can be explained by infrequent but relevant events, which can be considered as outliers or structural breaks in the data series and can have important effects on the unit root tests (e.g., Franses and Haldrup, 1994; Hoek et al., 1995; Burridge and Taylor, 2006). The mixed conclusion on the trend in real GNP can be also caused by the period of turmoil experienced from 1929 to 1949 due to the Great Depression and World War II (e.g., Newbold et al., 2001; Papell and Prodan, 2004). Indeed, Balke and Fomby (1991), Murray and Nelson (2000) and Darné (2009) showed the presence of outliers for annual GNP series during this period. Therefore, various techniques have been employed to take into account this phenomenon, such as unit root tests based on intervention analysis (Balke and Fomby, 1991), unit root tests with unrestricted (Murray and Nelson, 2000) or restricted (Papell and Prodan, 2004) structural breaks, unit root tests on the outlier-adjusted data (Darné, 2009). However, these techniques have some drawbacks: unit root tests based on intervention analysis are very sensitive to the specification of the alternative model (Montañés et al., 2005); the unit root tests with endogenous structural breaks are sensitive to the number of breaks taken into account, the date of the break (Kim et al., 2000) and the specification of the model (Sen, 2003); the size performance of unit root tests on outlier-corrected data has been investigated but not its power performance. Considering these drawbacks, we propose to use an alternative approach that is based on robust statistics to assess the presence of stochastic trend in long series of US real GNP. To overcome these drawbacks, we apply two recent robust tests: the partially adaptive ADF test proposed by Lima and Xiao (2010) and the quasi maximum likelihood ADF test developed by Cavaliere and Georgiev (2009). For the latter we improve its power in small samples by selecting optimally GLS detrending parameter, as suggested by Broda et al. (2009). We also used critical values adapted to the small sample sizes of the GNP series of interest.

The remainder of this paper is organized as follows: Section 2 describes the two robust unit root tests. Section 3 describes the empirical results on the US GNP series. Section 4 concludes.

## 2 Robust unit root tests

### 2.1 Cavaliere and Georgiev (2009) test

Cavaliere and Georgiev (2009) are interested in estimating parameters and testing the unit root null hypothesis  $H_0 : \alpha = 1$  against local alternatives  $H_c : \alpha = 1 - c/T (C > 0)$  and fixed stable alternatives  $H_s : \alpha = \alpha^* (|\alpha^*| < 1)$  in the following model for the observable variable  $y_t$

$$\begin{aligned} y_t &= \alpha y_{t-1} + u_t & t = 1 - k, \dots, T, \\ u_t &= \sum_{i=1}^k \bar{\gamma}_i u_{t-i} + \varepsilon_t + \delta_t \theta_t & t = 1, \dots, T, \end{aligned} \quad (1)$$

where, for  $k \geq 1$ ,  $(u_0, \dots, u_{1-k}, y_{-k})'$  may be any random vector (for  $k = 0$ ,  $y_0$  may be any random scalar) whose distribution is fixed and independent of  $T$ . Moreover, the authors postulate that:<sup>1</sup>

- the so-called long-run variance of  $u_t$ , hereafter  $\sigma^2 = \sigma_\varepsilon^2 \bar{\Gamma}(1)^{-2}$ , is well defined,
- the term  $\delta_t \theta_t$  is the outlier component of the model, with  $\delta_t$  an unobservable binary random variable indicating the occurrence of an outlier at time  $t$ , and  $\theta_t$  the associated random outlier size,
- the random number of outliers given by  $N_T := \sum_{t=1}^T \delta_t$  is bounded in probability conditionally on  $N_T \geq 1$ ,
- $\theta_t = T^{1/2} \eta_t$ , where  $\{\eta_t\}_{t=1}^T$ , and  $\{\eta_t^{-1}\}_{t=1}^T$  are  $O_p(1)$  sequences as  $T \rightarrow \infty$ ,
- for all  $T$ ,  $\{\delta_t\}_{t=1}^T$  is independent of  $\{\varepsilon_t, \eta_t\}_{t=1}^T$ ,  $y_{-k}$  and, if  $k \geq 1$ , of  $(u_0, \dots, u_{1-k})'$ .

In the analysis of model (1), the following alternative parametrization will be used. Let  $\gamma := (\gamma_1, \dots, \gamma_k)'$  and  $\Gamma = (\pi, \gamma)'$ , where, under  $H_0$  and  $H_{c,\pi} := 0$ ,  $\gamma_i := \bar{\gamma}_i (i = 1, \dots, k)$  whereas under  $H_s$  the new parameters are defined through the identity  $(1 - \alpha z) \bar{\Gamma}(z) = 1 - (\pi + 1)z - \sum_{i=1}^k \gamma_i z^i (1 - z)$ . Then  $\Delta y_t$  has the representation

$$\Delta y_t = \pi y_{t-1} + \gamma' \nabla \mathbf{Y}_{t-1} + e_t = \Gamma' \mathbf{Y}_{t-1} + e_t \quad t = 1, \dots, T,$$

<sup>1</sup>See Cavaliere and Georgiev (2009) for more details.

where  $\nabla \mathbf{Y}_{t-1} := (\Delta y_{t-1}, \dots, \Delta y_{t-k})'$  and  $\mathbf{Y}_{t-1} := (y_{t-1}, \nabla \mathbf{Y}'_{t-1})'$ . Under  $H_0$  and  $H_s$  this is a regression with error term  $e_t = \varepsilon_t + \delta_t \theta_t$ , whereas under  $H_c$  it is an approximate regression whose error term differs from  $\varepsilon_t + \delta_t \theta_t$  infinitesimally. Under  $H_0$  and  $H_s$  the components of  $\nabla \mathbf{Y}_{t-1}$  will be referred to as stable regressors whereas under  $H_c$  the components of  $\mathbf{Y}_{t-1}$  will be referred to as such.

The proposed quasi maximum likelihood (QML) is based on the observation that the innovation term of the model (1) has a mixture distribution, with mixing variable  $\delta_t$  and mixture components  $\varepsilon_t$  (when  $\delta_t = 0$ ) and  $\varepsilon_t + \theta_t$  (when  $\delta_t = 1$ ). Although no parametric hypothesis on the joint process  $\{\varepsilon_t, \theta_t\}$  is made, it is possible to estimate jointly the outlier indicators and the parameters of interest in a QML framework.<sup>2</sup>

Suppose that the QML estimator is based on the following distribution: (a) the innovations  $\varepsilon_t$  are normally distributed; (b) the outlier indicator  $\delta_t$  are Bernoulli random variables with  $P(\delta_t = 1) = \lambda/T, T > \lambda > 0$ ; (c) the outlier magnitudes  $\eta_t$  are Gaussian with mean zero and variance  $\sigma_\eta^2$ ; and (d)  $\varepsilon_t, \delta_t$  and  $\eta_t$  are *i.i.d.* and mutually independent.

Let  $\theta := (\Gamma', \sigma_\varepsilon^2, \sigma_\eta^2, \lambda)'$ . Under (a)-(d) and conditional on the initial values, the QML function is, up to an additive constant, given by

$$\Lambda(\theta) := \sum_{t=1}^T \ln \left( \frac{\lambda}{T} l_t(\theta, 1) + \left(1 - \frac{\lambda}{T}\right) l_t(\theta, 0) \right), \quad (2)$$

where

$$l_t(\theta, i) := \frac{1}{(\sigma_\varepsilon^2 + Ti\sigma_\eta^2)^{1/2}} \exp \left( - \frac{(\Delta y_t - \Gamma' Y_{t-1})^2}{2(\sigma_\varepsilon^2 + Ti\sigma_\eta^2)} \right), \quad i = 0, 1.$$

The weights are defined as follow

$$d_t(\theta) := \frac{\lambda l_t(\theta, 1)}{\lambda l_t(\theta, 1) + (T - \lambda) l_t(\theta, 0)}$$

which, under (a)-(d) correspond to the probability of occurrence of an outlier at time  $t$ , i.e.  $\delta_t$ , conditional on the data.

By equating to zero the derivatives of  $\Lambda(\theta)$ , it is possible to determine the normal equations  $\theta = \Phi(\theta)$ , where  $\Phi := ((\Phi^\Gamma)', \Phi^\varepsilon, \Phi^\eta, \Phi^\lambda) : R^{k+4} \rightarrow R^{k+4}$  is the random map

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<sup>2</sup>Cavaliere and Georgiev (2009) showed in their Monte Carlo simulations that the robust QML approach is more powerful than the robust method proposed by Lucas (1995a, 1995b).

with components:

$$\begin{aligned}\Phi^\Gamma(\theta) &:= \left( \sum_{t=1}^T \omega_t(\theta) (\mathbf{Y}_{t-1} \mathbf{Y}'_{t-1}) \right)^{-1} \sum_{t=1}^T \omega_t(\theta) (\mathbf{Y}_{t-1} \Delta y_t), \\ \Phi^\varepsilon(\theta) &:= \left( \sum_{t=1}^T (1 - d_t(\theta)) \right)^{-1} \sum_{t=1}^T (1 - d_t(\theta)) (\Delta y_t \Gamma' \mathbf{Y}_{t-1})^2, \\ \Phi^\eta(\theta) &:= \left( T \sum_{t=1}^T d_t(\theta) \right)^{-1} \sum_{t=1}^T d_t(\theta) (\Delta y_t - \Gamma' \mathbf{Y}_{t-1})^2 - T^{-1} \sigma_\varepsilon^2, \\ \Phi^\lambda(\theta) &:= \sum_{t=1}^T d_t(\theta),\end{aligned}$$

with  $\omega_t(\theta) := d_t(\theta)/(\sigma_\varepsilon^2 + T\sigma_\eta^2) + (1 - d_t(\theta))/\sigma_\varepsilon^2$ . By iterating the map  $\Phi$ , a QML estimator  $\check{\theta}$  could be computed. The ADF statistics obtained are the following:  $ADF_\alpha^Q := T\check{\pi}/|\Gamma(\check{1})|$  and  $ADF_t^Q := [\sum_{t=1}^T \omega_t(\check{\theta}) (Y_{t-1} Y'_{t-1})]^{-1/2}$ .

The authors proposed a variant of the  $ADF^Q$  tests which display better finite-sample properties when some of outliers are additive. They mimic the practice of dealing with AOs by using  $k + 2$  consecutive dummies as proposed by Perron and Rodriguez (2003). Let  $b_t(\theta) := b_t(\check{\theta})/(\check{\sigma}_\varepsilon^2 + T\check{\sigma}_\eta^2) + (1 - d_t(\check{\theta}))/\check{\sigma}_\varepsilon^2$ . The modified test statistics,  $A\check{D}F^Q$ , are computed as before with  $\check{\Gamma}$  replaced by the following estimator  $\left( \sum_{t=1}^T \check{\omega}_t(\check{\theta}) (Y_{t-1} Y'_{t-1}) \right)^{-1} \sum_{t=1}^T \check{\omega}_t(\check{\theta}) (Y_{t-1} \Delta y_t)$ .

Cavaliere and Georgiev (2009) proposed a sequential procedure for the linear trend case by applying the robust QML approach on the GLS detrended series, as in Elliott and al. (1996), giving the  $ADF\text{-}GLS^Q$  statistic test. The GLS detrending depend on a parameter  $\alpha = 1 - (c/T)$ , where  $c$  fixed and  $T$  is the sample size. Elliott et al. (1996) report that choosing  $c = -13.5$  for the trend linear case leads to tests with asymptotic power curves (asymptotic power envelopes equal to 0.5). Nevertheless, Broda et al. (2009) show that an inappropriate choice of  $c$  can lead to less powerful tests. These authors proposed a procedure which numerically determine values of  $c$  that minimize a weighted power loss criterion for each test and sample size, and it is powerful in small samples.

## 2.2 Lima and Xiao (2010) test

Lima and Xiao (2010) are interested in estimating parameters and testing the unit root null hypothesis  $H_0 : \alpha = 1$  against the alternative  $H_s : \alpha = \alpha^* (|\alpha^*| < 1)$  in the following model for the observable variable  $y_t$

$$y_t = \alpha y_{t-1} + u_t \quad t = 1, \dots, T, \quad (3)$$

with  $u_t$  the residual term supposed to be serially correlated.

Following Dickey and Fuller (1979), we can parameterize  $u_t$  as a stationary AR( $k$ ) process

$$A(L)u_t = \varepsilon_t \quad (4)$$

where  $A(L) = \sum_{i=0}^k a_i L^i$  is a  $k$ -th order polynomial of the lag operator  $L$ ,  $a_0 = 1$ , and  $\varepsilon_t$  is an *i.i.d.* sequence. Combining (3)-(4), we obtain the following regression model

$$\Delta y_t = \rho y_{t-1} + \sum_{j=1}^k \psi_j \Delta y_{t-j} + \varepsilon_t \quad (5)$$

We may include in (5) a deterministic trend component

$$\Delta y_t = \gamma' x_t + \rho y_{t-1} + \sum_{j=1}^k \psi_j \Delta y_{t-j} + \varepsilon_t \quad (6)$$

where  $x_t$  is the deterministic component of known form (a constant term if  $x_t = 1$  or a linear term if  $x_t = (1, t)'$ ) and  $\gamma$  is a vector of unknown parameters.

In the case where  $\varepsilon_t$  is normally distributed, the maximum likelihood estimators of  $\gamma$ ,  $\rho$  and  $\{\psi_j\}_{j=1}^k$  correspond to the ordinary least squares by minimizing the residual sum of squares. In the case where  $\varepsilon_t$  has Student-t distribution, the estimators are biased. One solution is to use robust estimators such as those proposed by Huber (1973). The  $M$  estimators of  $(\gamma, \rho, \{\psi_j\}_{j=1}^k) = \Pi$  solves the following first-order conditions<sup>3</sup>

$$\sum_{t=j+2}^n \left( 1 + \frac{\Theta}{\nu} \left[ \Delta y_t - \gamma' x_t - \rho y_{t-1} - \sum_{j=1}^k \psi_j \Delta y_{t-j} \right]^2 \right)^{-1} \frac{\delta \varepsilon_t}{\delta \Pi} = 0$$

If  $\hat{\rho}$  is  $M$  estimator of  $\rho$ , then the  $t$ -ratio statistic of  $\hat{\rho}$  is

$$t_{\hat{\rho}} = \frac{\hat{\rho}}{s(\hat{\rho})} \quad (7)$$

The limiting distribution is given by (Lucas, 1995a)

$$t_{\hat{\rho}} \implies \sqrt{1 - \lambda^2} N(0, 1) + \lambda \left( \int \underline{W}_1(r^2) dr \right)^{-1/2} \int \underline{W}_1 dW_1 \quad (8)$$

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<sup>3</sup>Hoek et al. (1995), Lucas (1995a, 1995b) and Thompson (2004) also developed robust unit root tests based on  $M$  estimators.



where  $\underline{W}_1$  is a detrended Brownian motion and the weights are defined by  $\lambda$

$$\lambda^2 = \frac{\sigma_{u\phi}^2}{\omega_\phi^2 \omega_u^2} \quad (9)$$

where  $\omega_u^2$  is the long-run variance of  $\{u_t\}$ ,  $\omega_\phi^2$  is the long-run variance of  $\{\phi'(\varepsilon_t)\}$ , and,  $\sigma_{u\phi}(\tau)$  is the long-run covariance of  $\{u_t\}$  and  $\{\phi'(\varepsilon_t)\}$ .

Nevertheless, the Student- $t$  distribution is indexed by the degrees of freedom  $\nu$  which is to be chosen in an arbitrary way and the limiting distribution depends on a nuisance parameter  $\lambda$ .

Lima and Xiao (2010) use a data-dependent procedure to select an appropriate criterion function for the estimation. They consider the partially adaptive approach that tends to give correct critical values because it approximates the true distribution by the data distribution and uses the latter to estimate  $\lambda$  and then the critical values. They use the partially adaptive estimator based on the family of Student- $t$  distributions introduced by Potscher and Prucha (1986). If we denote  $E(|u_t|^k)$  as  $\sigma_k$ , then for  $\nu > 2$ , we have

$$\begin{aligned} \frac{\sigma_2}{\sigma_1^2} &= \frac{\pi}{\nu - 2} \frac{\Gamma[\nu/2]^2}{\Gamma[(\nu - 1)/2]^2} = d(\nu) \\ \Theta &= \frac{1}{\pi} \frac{\nu \Gamma[(\nu - 1)/2]^2}{\sigma_1^2 \Gamma[\nu/2]^2} = q(\nu, \sigma_1) \end{aligned}$$

Potscher and Prucha (1986) showed that  $d(\cdot)$  is analytic and monotonically decreasing on  $(2, \infty)$  with  $d(2) = \infty$  and  $d(\infty) = \pi/2$ . The estimator of  $\theta$  is defined as follows

$$\hat{\theta} = \frac{q(\nu, \sigma_1)}{\hat{\nu}} = \frac{1}{\pi} \frac{\Gamma[(\hat{\nu} - 1)/2]^2}{\hat{\sigma}_1^2 \Gamma[\hat{\nu}/2]^2} \quad (10)$$

For the estimation of  $\sigma_1$  and  $\sigma_2$ , it uses the sample moments

$$\hat{\sigma}_k = \frac{1}{n} \sum_t |\hat{\varepsilon}_t|^k \quad (11)$$

The  $t$ -ratio is the same as in (7). In order to identify the critical values, one needs to estimate  $\lambda^2$ . Lima and Xiao (2010) estimate  $\omega_u^2$ ,  $\omega_\phi^2$  and  $\sigma_{u\phi}$  parametrically.

### 3 US Real GNP

We study the four same annual US real GNP data spanning the period 1869 to 1993 as in Diebold and Senhadji (1996), i.e. GNP-BG, GNP-R, GNP-BGPC and GNP-RPC,

based on whether measures from Balke and Gordon (1989) (BG) or Romer (1989) (R) were employed or whether the GNP was expressed in per capita (PC) form. Diebold and Senhadji (1996) created these real GNP series by splicing the 1869-1929 real GNP series of Balke and Gordon (1989) or Romer (1989) to the 1929-1993 real GNP series reported from the National Income and Product Accounts by the U.S. Department of Commerce, measured in billions of 1987 dollars. The logarithmic transformation is applied on the data.

We apply the procedure of Broda et al. (2009) to determine the appropriate choice of  $c$  depending on the sample size. We obtain  $c = -12.5$  for our full sample size (1869–1993) with  $T = 125$ . For the unit root test of Cavaliere and Georgiev (2009) we use the finite-sample critical values computed by Cook (2006) for various values of  $c$ . For  $c = -12.5$ , the critical values at the 1%, 5% and 10% levels of significance are -3.60, -3.00 and -2.71, respectively.

The results of the robust unit root tests are given in Table 1. In practice, the lag truncation  $k$  is chosen by a data-dependent procedure. We used the Schwarz information criterion (SIC) and the general-to-specific (GS) strategy, which consist in starting with a maximum value of  $k$  chosen *a priori*, deleting lags sequentially until significance of the 0.10 level. Here, we set  $k_{max} = 8$ .

The unit-root null hypothesis is rejected for all the series at the 5% level, except for GNP-R. This result is surprising since we do not have the same conclusion for the two GNP series in non per capita form, i.e. GNP-RPC and GNP-BGPC. An explanation could be the difference in the construction of the series: the post-1929 values are identical for both series, while the pre-1929 values differ because of the differing assumptions underlying their construction. Balke and Gordon (1989) used more indicators than Romer (1989) to backcast GNP, and this procedure tends to accentuate the fluctuations of the output and therefore the series is less smooth for the period 1869-1929.<sup>4</sup>

Therefore, we re-examine the unit-root null hypothesis on two sub-periods: (i) on the pre-1929 period (1869–1928), i.e. the period where the series are differently constructed; and (ii) on the post-1929 period (1929–1993), i.e. the period where the series are identical. For the Cavaliere-Georgiev unit root tests, we obtain  $c = -11.6$  as optimal parameter for the two sub-samples, with  $T = 60$  and 65, respectively. The critical values at the 1%, 5% and 10% levels of significance are -3.58, -2.98 and -2.69,

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<sup>4</sup>Darné (2009) found more shocks in the data sets constructed by Balke and Gordon (1989) than those based on Romer (1989) for the period 1869-1929. Furthermore, Murray and Nelson (2000) suggested measurement errors in the reconstructed series.

respectively. The results displayed in Table 1 show that the post-1929 GNP and GNP per capita series do not reject the null hypothesis, and thus that there is evidence in favor of difference stationarity.<sup>5</sup> For the pre-1929 GNP data, the unit-root hypothesis is not rejected for GNP-BG and GNP-R but this hypothesis is rejected for the same series in per capita form GNP-RPC and GNP-BGPC. Newbold et al. (2001) also found evidence against trend stationarity hypothesis but for the four GNP series. Note that, in some cases, rejection rates differ sharply depending on which lag selection method is used. GS generally chooses a much larger value of  $k$  than SIC, especially for the Romer data. This difference between GNP in per capita form or not can be explained by the fact that use of per capita GNP eliminates a possibility of non-stationarity in GNP time series resulting from inflation and population growth. Another explanation suggested by Cheung and Chinn (1997) is that the trend-stationarity result for the historical annual data is driven by the data-construction procedure. Jaeger (1990) show that segmented linear interpolation may be responsible for the finding of a stochastic trend in prewar US GNP.<sup>6</sup> Furthermore, Stock and Watson (1986) conjecture that linear interpolation may cause the difference between the GNP shock persistence of prewar and postwar series. This finding raises the question about the relevance to use reconstructed data for the econometric analysis and on the conclusions resulting from this.

## 4 Conclusion

This paper tested the presence of stochastic trend in long series of US real GNP measured by Balke and Gordon (1989) and Romer (1989). This was analyzed from two recent robust unit root tests proposed by Cavaliere and Georgiev (2009) and Lima and Xiao (2010), for which critical values are adapted to the small sample size. The former was improved by selecting optimally GLS detrending parameter to make the test in small samples powerful. We obtained mixed results on the full sample (1869–1993). However, the post-1929 GNP and GNP per capita series rejected the unit-root null hypothesis, whereas for the pre-1929 GNP data, the unit-root hypothesis was not rejected for GNP series proposed by Balke–Gordon and Romer but this hypothesis was rejected for the same series in per capita form. This difference can be explained by

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<sup>5</sup>Newbold et al. (2001) found evidence against trend-stationary representation in the post-World War II period (1950–1993). We obtained the same results on this period from the two robust unit root tests.

<sup>6</sup>From Monte Carlo experiments Jaeger (1990) suggests that segmented linear interpolation reduces the size of shock persistence in a difference stationary series. Dezhbakhsh and Levy (1994) also show that the interpolated series may exhibit more shock persistence than the original trend stationary series.

the data-construction procedure employed for the pre-1929 GNP series. This finding raises the question about the relevance to use reconstructed data for the econometric analysis and on the conclusions resulting from this, as suggested by Stock and Watson (1986) and Jaeger (1990).

Table 1: Results of robust unit root test for annual US GNP series – 1869–1993.

Data series	$t$ -stat	1869–1993	$k$	1869–1928	$k$	1929–1993	$k$
GNP-R	$ADF - GLS^Q$	-2.3399	SIC=0	-2.1673	SIC=0	-2.3789	SIC=1
		-2.3399	GS=0	-2.5199***	GS=6	-2.1127	GS=6
	$PADF$	-2.6909	SIC=0	-1.4244	SIC=0	-1.6003	SIC=1
		-2.6909	GS=0	-3.9834*	GS=6	-1.6266	GS=6
GNP-BG	$ADF - GLS^Q$	-4.1244*	SIC=0	-2.3777	SIC=1		
		-3.9512*	GS=2	-2.3777	GS=1		
	$PADF$	-3.5303*	SIC=0	-2.4456	SIC=1		
		-3.3821*	GS=2	-2.4456	GS=1		
GNP-RPC	$ADF - GLS^Q$	-5.3494*	SIC=0	-3.0987**	SIC=0	-2.2781	SIC=1
		-4.9772*	GS=1	-3.6880*	GS=3	-2.2781	GS=1
	$PADF$	-4.2127*	SIC=0	-2.4816	SIC=0	-1.5792	SIC=1
		-3.2296***	GS=1	-3.6039*	GS=3	-1.5792	GS=1
GNP-BGPC	$ADF - GLS^Q$	-7.0314*	SIC=0	-3.2801**	SIC=0		
		-7.0314*	GS=0	-3.2801**	GS=0		
	$PADF$	-4.0917*	SIC=0	-3.6095*	SIC=0		
		-4.0917*	GS=0	-3.6095*	GS=0		

Notes: \*, \*\* and \*\*\* indicate rejection of the unit-root null hypothesis at the 1%, 5% and 10% level of significance, respectively. The variables are the log of Romer's (1989) gross national product (GNP-R), the log of Balke and Gordon's (1989) gross national product (GNP-BG), the log of Romer's (1989) gross national product per capita (GNP-RPC), and the log of Balke and Gordon's (1989) gross national product per capita (GNP-BGPC).

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