



**HAL**  
open science

## Formulation for less master production schedule instability under rolling horizon

Carlos Herrera, André Thomas

► **To cite this version:**

Carlos Herrera, André Thomas. Formulation for less master production schedule instability under rolling horizon. International Conference on Industrial Engineering and Systems Management, IESM 2009, May 2009, Montréal, Canada. pp.CDROM. hal-00546710

**HAL Id: hal-00546710**

**<https://hal.science/hal-00546710>**

Submitted on 14 Dec 2010

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Formulation for Less Master Production Schedule Instability under Rolling Horizon

Carlos HERRERA <sup>a</sup>, André THOMAS <sup>a</sup>,

<sup>a</sup>*Centre de Recherche en Automatique de Nancy (CRAN), CNRS UMR 7039/Nancy Université  
Campus Sciences - BP 70239 - 54506 - Vandœuvre-lès-Nancy Cedex, France.*

---

## Abstract

In Manufacturing Planning and Control Systems, the Master Production Schedule (MPS) makes a link between tactical and operational levels, taking into account information provided by end items, demand forecast as well as Sales and Operations Planning (S&OP) suggestions. Therefore, MPS plays an important role to maintain an adequate customers service level and an efficient production system. In a rolling planning horizon, MPS is periodically computed over whole operational horizon. The differences between scheduled quantities obtained by this process are related to MPS instability. This feature of MPS has negative effects, both, at tactical level and also at operational one. In this paper, a Mixed Integer Programming model is proposed for MPS instability. The proposed model considers instability minimization in addition to inventory costs and set up. Simulation is used to take into account stochastic demand. Computation experiments are presented in order to show the efficiency of this approach by finding stable MPS without a considerably increase in the total cost.

*Key words:* Production, Manufacturing, Integer Programming, Instability, Nervousness.

---

## 1 Introduction

In Manufacturing Planning and Control Systems, the *Master Production Schedule* (MPS) is the link between tactical and operational levels taking into account information provided by demand forecast as well as Sales and Operations Planning (S&OP) suggestions [3,14]. Moreover, the MPS considers availabilities of critical resources, management policies and goals. The main objective of the MPS is to obtain production quantities in each period, minimizing the cost and maximizing bottleneck utilization. Therefore, the MPS is important to maintain an adequate customers service level and an efficient production system through well managed constraints.

The most common way to compute a MPS is to consider a rolling planning horizon. This means, MPS is constantly computed with a specific periodicity  $\Delta t$  (cycles), over whole operational horizon. Hence, this methodology regularly reschedules the production quantities by periods.

The differences in the planned quantities obtained by this rescheduling process, are related to MPS stability. If these differences are not significant, MPS is said to be stable, on the contrary, whenever these differences are more significant, the MPS becomes more unstable. This results of MPS instability,

---

\* This paper was not presented at any other revue. Corresponding author C. Herrera. Tel. +33-03-83684450. Fax +33-03-83684459.

*Email addresses:* [carlos.herrera@cran.uhp-nancy.fr](mailto:carlos.herrera@cran.uhp-nancy.fr) (Carlos HERRERA),  
[andre.thomas@cran.uhp-nancy.fr](mailto:andre.thomas@cran.uhp-nancy.fr) (André THOMAS).

leads to negative effects at both, tactical and operational levels. Some effects at tactical level are: high transaction costs, temporary employee subcontracting and unexpected changes in outsourcing contracts. At operational level it is mainly related to MPS nervousness [2].

MPS nervousness refers to, minor changes in the MPS produce significant changes in MRP plans. This changes in the MPS are caused mainly by: end-item forecast, lot-sizing rules and scheduled receipts changes [18].

When a stability criterion is considered for a MPS, it is necessary to find a trade-off between stability and cost performance [7], i.e., when the MPS is computed excluding stability, the plan can present huge differences between scheduled quantities in successive cycles. Moreover, this situation makes difficult to apply the plan in practice and leads to huge work in process (WIP), and also, more difficult scheduling (shop floor level).

The most commonly used methods to obtain stable plans are: *frozen intervals*, and *safety stock*. Basically, frozen intervals consist in fixing periods in the planning horizon, in which, changes hoped for later reschedules are not allowed. On the other hand, safety stock fits inventory levels to absorb uncertainty demand. The ideas behind these methods are very simple and they have demonstrated good results for specific cases [18]. Moreover, the simplicity of these methods, makes them very useful in practice.

Nevertheless, these methods have a strong static component, because they are based on the idea of fixing some variables. On the contrary, the approach presented in this paper, searches to reduce the instability by a dynamic process. For this, a mixed integer programming (MIP) model is proposed to compute the MPS, which considers the instability minimization directly in its formulation.

Moreover, the nervousness minimization is considered as an implicit target. We consider for that the following proposition: *the minimization of the instability reduces the nervousness*.

The rest of this paper is organized as follows. In section 2, we describe the problem and present a literature review of some contributions related to MPS stabilization. Section 3 defines formally the proposed formulation. In section 4, a simulation to show the efficiency of this approach is provided. Finally, section 5 presents conclusions and future directions.

## 2 Problem description and literature review

### 2.1 Problem description

In a rolling horizon, MPS computes quantities to produce for a given planning horizon ( $n$ ), with a specific periodicity  $\Delta t$  (cycles). Table 1 shows an example of scheduled quantities given by a MPS, where quantity  $q_{kt}$  represents the scheduled quantity of one item, for period  $t$  obtained by the MPS computed in the cycle  $k$ . In this example the parameters are:  $n = 4$  and  $\Delta t = 1$ .

Table 1  
Example of MPS in a rolling horizon.

| k / t | 1        | 2                     | 3                     | 4                     | 5                     | 6        | 7        | 8        |
|-------|----------|-----------------------|-----------------------|-----------------------|-----------------------|----------|----------|----------|
| 1     | $q_{11}$ | $q_{12}$              | $q_{13}$              | $q_{14}$              |                       |          |          |          |
| 2     |          | <b>q<sub>22</sub></b> | <b>q<sub>23</sub></b> | <b>q<sub>24</sub></b> | <b>q<sub>25</sub></b> |          |          |          |
| 3     |          |                       | $q_{33}$              | $q_{34}$              | <b>q<sub>35</sub></b> | $q_{36}$ |          |          |
| 4     |          |                       |                       | $q_{44}$              | <b>q<sub>45</sub></b> | $q_{46}$ | $q_{47}$ |          |
| 5     |          |                       |                       |                       | <b>q<sub>55</sub></b> | $q_{56}$ | $q_{57}$ | $q_{58}$ |

To analyze the stability, we make the difference between MPS instability and nervousness. We define MPS instability, as the differences between quantities scheduled by a MPS in a cycle (e.g., in Table 1 for the cycle  $k = 2$ , differences between **q<sub>22</sub>**, **q<sub>23</sub>**, **q<sub>24</sub>**, **q<sub>25</sub>**). On the other hand, we define nervousness as the difference between quantities scheduled by the MPS in different cycles, for a given period of the planning horizon (i.e. in Table 1, for the period  $n = 5$ , differences between **q<sub>25</sub>**, **q<sub>35</sub>**, **q<sub>45</sub>**, **q<sub>55</sub>**).

For MPS instability we define two measures, Mean Instability (MEI) and Maximum Instability (MAI). Mean Instability is defined as:

$$MEI_k^i := \frac{2}{n(n-1)} \sum_{t=k}^{k+n-1} \sum_{t'=k+1}^{k+n} |x_t^i - x_{t'}^i|, \quad \forall i, \forall k. \quad (1)$$

This measure represents, the *average of the differences* of scheduled quantities, between each period and the next periods, by item  $i$  and by cycle  $k$ .

Maximum Instability can be defined as:

$$MAI_k^i := \max_t \left\{ \frac{1}{n-t} \sum_{t'=k+t}^{k+n} |x_{k+t-1}^i - x_{t'}^i|, t = 1 \dots, n-1 \right\}, \quad \forall i, \forall k. \quad (2)$$

This measure represents the *maximum of the differences* of scheduled quantities, between each period and the next periods, by item  $i$  and by cycle  $k$ .

Now, let  $\Omega_{kt}$  the set of overlap periods in the cycle  $k$  for the period  $t$  (this means, all scheduled quantities for period  $t$ , obtained in the precedent cycles). And let,

$$\Omega_k = \sum_{t=k}^{n+k} |\Omega_{kt}|, \quad \forall k, \quad (3)$$

the set of all overlap periods for the MPS computed in the cycle  $k$ , where  $|\Omega|$  is the cardinality of the set  $\Omega$ . Hence,

$$NA_k^i = \frac{1}{|\Omega_k|} \sum_{t=k}^{k+n-1} \sum_{h \in \Omega_{kt}} |x_t^i - x_h^i|, \quad \forall i, \forall k, \quad (4)$$

represents the nervousness computed as the average of the differences of scheduled quantities, between all periods in the cycle  $k$  ( $t = k, k+1, \dots, k+n-1$ ) and all its quantities scheduled in precedent cycles (the last period are not overlap periods), by item  $i$  and by cycle  $k$ . This measure will be referred as Nervousness All Periods (NA).

Another measure defined, is the nervousness for the first period. This measure represents the nervousness computed of similar form as above, but, only considering the first period. This is considered because finally, if  $\Delta t = 1$ , the implemented period is only the first period. This measure is defined as:

$$NF_k^i = \frac{1}{|\Omega_{k,f}|} \sum_{h \in \Omega_{k,f}} |x_k^i - x_h^i|, \quad \forall i, \forall k, \quad (5)$$

where  $\Omega_{k,f}$  represent the set of overlap periods in the cycle  $k$  only for the first period in the MPS. This measure will be referred as Nervousness First Period (NF).

MPS instability can be viewed as a static effect, since it refers to the differences between the quantities computed for a fixed horizon. This can affect for example, at tactical level, outsourcing, hiring/lay-off or part-time contracts. On the other hand, nervousness has a dynamic character, because it takes place when the differences of the computed quantities in different cycles are obtained. These successive adjustments can produce huge changes at the operational level.

## 2.2 Literature review

Many works study MPS instability and nervousness effects and their impact on production planning and control systems. In the following, some recent contributions related to MPS stabilization are presented.

In [4], the author studies the nervousness effect for stochastic inventory control. For that, he defines a measure of nervousness taking into account exclusively the setup variable. The results show that,

nervousness is affected by the control rules. Following this work, the instability measures defined in the previous subsection, can be classified as *long-term stability* and *quantity-oriented*. This means that, we consider the whole planning horizon for the computations of the differences between production quantities.

In the work presented in [5], the following strategies to reduce nervousness are studied: freezing the MPS, end-item safety stock and lot-for-lot lotsizing rule for components. Some factors considered are: item cost structure, items structure, level of demand uncertainty and lot-sizing method. Decision variables are: cost, instability, and customer service level. The main result is that, frozen periods strategy, presents the best performance under stochastic demand multilevel environment.

In [18], the effects on the stability of the plans, produced by the interaction between several lot-sizing rules and frozen interval selection are studied. The study is performed with a simulation and a completely randomized full factorial design, in order to test the different hypotheses. The results show a strong dependency between MPS instability and lot-sizing rules as well as frozen interval selection. Some works make a similar study for different lot-sizing problems [17,16].

From the perspective of production planning models, [7] explains the impact of the stability in three problems of production planning. Also, some approaches are proposed to measure stability, applying them to the MPS. The effects for different cost structures are simulated and analyzed. Finally, an iterative method is proposed to solve MPS.

In [6], three methods are proposed to reduce MPS instability. The first two methods correspond to modified versions of classic Wagner-Within and Silver-Meal methods. Finally, a method based on MIP, that takes in consideration previous schedules periods to solve MPS is proposed. The results are analyzed by simulation, where, the model based on mixed integer programming, obtains good stability results in some cases.

For a scheduling problem, in [11], the tackled problem considers jointly the optimization of efficiency and stability measures. In the first part, the inclusion of the stability as a variable to be optimized is justified. To solve the problem, an approach of genetic algorithm is proposed. Finally, the results are analyzed by statistical methods, concluding that, the stability does not affect drastically the production system efficiency.

Analyzing the effects of the system structure on the stability, in the context of the supply chain, [8] studies a model for lot-sizing multi-level multi-item problem and considers structural variables as: setup cost, relationships between components and capacity. The statistical results show a strong relation between the considered variables and stability. The results of this work highlight that, it is possible to take emphasize in the following fact: “fewer production orders and capacity increase, have stabilizing effects on the plans”. Following this line, [15] studies the impact of material coordination concepts in supply chain stability, with a more practical approach. Its results, identify three variables that affect strongly the stability, these are: lot-sizes, level of uncertainty in demand and items structure. Its results are very similar to the results obtained by [8].

The works in [12,13], express the importance of considering stability in MRP systems, in the context of remanufacturing process and MPS frozen interval selection respectively.

Finally, in [14], a mathematical programming method to obtain a stable MPS is proposed. This approach is developed with a two steps model at tactical level. In the aggregated plan (S&OP), is proposed a procedure called *reference plan*, that considers compromises between successive planning periods to reduce changes. At detailed plans level (MPS), a heuristic procedure is proposed to obtain a stable MPS. The main result of this work is to improve the stability of the detailed plan.

### 3 Less Master Production Schedule instability formulation

In the first part of this section, we present the classic MIP formulation for a MPS ( $\mathcal{F}_1$ ) and our proposed formulation ( $\mathcal{F}_2$ ). In the second part, we provide an algorithm to compute the solution of  $\mathcal{F}_2$ .

### 3.1 Formulation

Formally, a mixed integer programming formulation for a MPS, considering  $m$  items,  $n$  periods and  $r$  resources, can be stated as follows:

#### 3.1.1 Variables

- $x_t^i$  : production of item  $i$  in period  $t$ .
- $s_t^i$  : inventory of item  $i$  in period  $t$ .
- $y_t^i$  : setup of item  $i$  in period  $t$ , where
- $y_t^i = \begin{cases} 1 & \text{if } x_t^i > 0 \\ 0 & \text{if } x_t^i = 0 \end{cases}$

#### 3.1.2 Parameters

- $d_t^i$  : demand of item  $i$  in period  $t$ .
- $\bar{p}_t^i$  : production cost of item  $i$  in period  $t$ .
- $h_t^i$  : inventory cost of item  $i$  in period  $t$ .
- $q_t^i$  : setup cost of item  $i$  in period  $t$ .
- $L_t^r$  : available capacity of resource  $r$  in period  $t$ .
- $\alpha^{ir}$  : unitary consumption of resource  $r$  by production of item  $i$ .
- $\beta^{ir}$  : setup time for item  $i$  on resource  $r$ .
- $M_t^i = \sum_{t=1}^n d_t^i$ ,  $\forall i$ , upper bound of production of item  $i$  in period  $t$ .

#### 3.1.3 Formulation $\mathcal{F}_1$

$$\min \sum_{i=1}^m \sum_{t=1}^n (\bar{p}_t^i x_t^i + h_t^i s_t^i + q_t^i y_t^i) \quad (6)$$

$$s_{t-1}^i + x_t^i = d_t^i + s_t^i, \quad \forall i, t \quad (7)$$

$$x_t^i \leq M_t^i y_t^i, \quad \forall i, t \quad (8)$$

$$\sum_{i=1}^m (\alpha^{ir} x_t^i + \beta^{ir} y_t^i) \leq L_t^r, \quad \forall r, t \quad (9)$$

$$x \in \mathbb{R}_+^{mn}, s \in \mathbb{R}_+^{m(n+1)}, y \in \{0, 1\}^{mn}. \quad (10)$$

The objective function (6), searches to minimize the costs of production, inventory and setup. Constraint (7), represents the inventory balance and constraint (8), the relationship between production and setup. The constraint (9), represents the available capacity of resources by period. This formulation can be found in [9]. This formulation will be referred as  $\mathcal{F}_1$ .

#### 3.1.4 Formulation $\mathcal{F}_2$

Without loss of generality, we assume  $s_0 = 0$ , and using (11),

$$s_n^i = \sum_{t=1}^n (x_t^i - d_t^i), \quad \forall i, \quad (11)$$

it is possible to eliminate the inventory variable in the formulation. In [1], this formulation is showed for the single-item lot-sizing problem (SILSP). In our case, the above formulation is applied to capacitate multi-item lot-sizing problem (CMILSP). A new formulation, without inventory variables, considering stabilization of the MPS, can be expressed as follows:

$$\min \sum_{i=1}^m \sum_{t=1}^n (p_t^i x_t^i + q_t^i y_t^i) + \sum_{i=1}^m \sum_{t=1}^{n-1} (\lambda^i \omega_t^i) \quad (12)$$

$$\sum_{j=1}^t x_j^i \geq D_t^i, \quad \forall i, \forall t, \quad (\text{auxiliary index } j = 1, \dots, t) \quad (13)$$

$$x_t^i - x_{t+1}^i \leq \omega_t^i, \quad \forall i \quad \text{and} \quad 1 \leq t \leq n-1 \quad (14)$$

$$x_{t+1}^i - x_t^i \leq \omega_t^i, \quad \forall i \quad \text{and} \quad 1 \leq t \leq n-1 \quad (15)$$

$$x_t^i \leq M_t^i y_t^i, \quad \forall i, \forall t \quad (16)$$

$$\sum_{i=1}^m (\alpha^{ir} x_t^i + \beta^{ir} y_t^i) \leq L_t^r, \quad \forall r, t \quad (17)$$

$$x \in \mathbb{R}_+^{mn}, \omega \in \mathbb{R}_+^{m(n-1)}, y \in \{0, 1\}^{mn}, \quad (18)$$

where,

$$p_t^i = \bar{p}_t^i + \sum_{j=t}^n h_j^i, \quad \forall t, i. \quad (19)$$

The objective function (12), searches to minimize the costs of production, inventory (of implicit way) and setup, and also, the differences between consecutive production quantities. In this formulation constraint (13) express that, cumulative production in any period for each item, must be equal or greater than the cumulative demand  $D_t^i$ , in the same period, backlog is not allowed. A constant term  $-\sum_{i,t} h_t^i \bar{d}_t^i$  has been eliminated of the objective function. This formulation will be referred as  $\mathcal{F}_2$ . The value of the original objective function (6), can be obtained, computing the  $s$  variable from (11) substituting  $\mathbf{x}^*$ , and later evaluating in (6). The matrix variable  $\mathbf{x}^*$ , is a  $m \times n$  matrix of optimal solutions computed by  $\mathcal{F}_2$ , with elements  $x_t^i$ .

The variable  $\omega_t^i$  represents an auxiliary variable to minimize the quantity differences between consecutive periods in whole planning horizon  $n$ . The parameter  $\lambda^i$ , represent a control parameter for instability minimization by item. This formulation can be viewed as equivalent to minimize  $\lambda^i |x_{t+1}^i - x_t^i|$ ,  $\forall i$  and  $1 \leq t \leq n-1$ .

Note that, in the measures (1) and (2) it is necessary to compute the differences between all periods in a specific cycle. This is equivalent to consider  ${}_n C_2$  combinations. Nevertheless, considering the high computational cost of this, the minimization is performed only in one step, i.e., for a given production quantity, the model searches to minimize the difference only between its antecessor and successor values.

Moreover, no explicit measure for nervousness minimization have been considered in the model. The reduction of nervousness is considered to be implicit in the instability reduction, under the proposition expressed in the section 1.

In the next subsection a simple algorithm is proposed to find a suitable value to  $\lambda^i$ .

### 3.2 Finding $\lambda$ parameter

For a sake of simplicity, we consider the same  $\lambda$ -value for all items  $i = 1, 2, \dots, m$ . Furthermore, we suppose that, for all measures considered  $ME = \{MEI, MAI, NA, NF\}$ :

$$ME_j(\lambda_0) \leq ME_j(\lambda_1) \leq \dots ME_j(\lambda_K), \quad \forall j \in ME, \quad (20)$$

where,  $\lambda_l = l$ ,  $l = 0, 1, \dots, K$ ,  $K \in \mathbb{N}$ .

Also, we suppose that, the allowed total cost growth (or equivalent, sub-optimal solution allowed) is user-defined by a percentage deviation ( $\delta$ ) of the optimal total cost. Therefore, we are interested in to find the maximum feasible  $\lambda$ -value, such that:

$$z_{\mathcal{F}_1} = z_{\mathcal{F}_2}(\lambda_0) \leq z_{\mathcal{F}_2}(\lambda) \leq (1 + \delta)z_{\mathcal{F}_2}(\lambda_0) = (1 + \delta)z_{\mathcal{F}_1}, \quad (21)$$

where,  $z_{\mathcal{F}_2}(\lambda)$  is defined as the objective value (total cost) of the original problem  $\mathcal{F}_1$ , but, computed with  $\mathbf{x}^*$  (note that, the optimal value of setup variable ( $y_t^i$ ) is implicit in  $\mathbf{x}^*$ ).

For that, we propose a simple algorithm (Algorithm 1) to find the best  $\lambda$ -value, such that  $\lambda \in \mathbb{N}$ . The variables  $\lambda_f$  and  $\lambda_e$  represent the first and the end value of the interval  $[\lambda_f, \lambda_e]$ , where  $\lambda_f \leq \lambda \leq \lambda_e$ .

**Algorithm 1.** Find  $\lambda$

**Require:**  $step$ ,  $\lambda_f = 0$

**Ensure:**  $\lambda$

  solve  $\mathcal{F}_2(\lambda_f)$

$z_{ub} \leftarrow (1 + \delta)z_{\mathcal{F}_2}(\lambda_f)$

$\lambda_e \leftarrow step$

**while**  $z_{\mathcal{F}_2}(\lambda_e) \leq z_{ub}$  **do**

    solve  $\mathcal{F}_2(\lambda_e)$

$\lambda_e \leftarrow \lambda_e + step$

**end while**

$\lambda_e \leftarrow (\lambda_e - step)$

**while**  $(\lambda_e - \lambda_f) \geq 1$  **do**

$\lambda_s \leftarrow \lfloor (\lambda_f + \lambda_e) / 2 \rfloor$

    solve  $\mathcal{F}_2(\lambda_s)$

**if**  $z_{\mathcal{F}_2}(\lambda_s) > z_{ub}$  **then**

$\lambda_e \leftarrow \lambda_s$

**else**

$\lambda_f \leftarrow \lambda_s$

$\lambda \leftarrow \lambda_f$

**end if**

**end while**

The above algorithm first solves  $\mathcal{F}_2(\lambda_0)$ , that is similar to solve  $\mathcal{F}_1$ . Later, it solves  $\mathcal{F}_2$  for some  $\lambda$ -value defined by  $step$ . The parameter  $step$ , can be adjusted from some initial executions of  $\mathcal{F}_2$ . If the cost and demand parameters have not significant differences between periods, parameter  $step$  can be fixed for all operational horizon. The next step in the algorithm is to find the maximum feasible  $\lambda$ -value, such that  $z_{\mathcal{F}_2}(\lambda) \leq (1 + \delta)z_{\mathcal{F}_2}(\lambda_0)$ . For this, the algorithm searches  $\lambda$  by a classic *Golden Section Search*.

## 4 Simulation

### 4.1 Experimental design

The simulation results are obtained for a simulation horizon of  $H = 52$  weeks, with rescheduling interval  $\Delta t = 1$  and for a planning horizon  $n = 8$ .

The parameters  $p_t^i, h_t^i, q_t^i$  are randomly generated and uniformly distributed. The demand for each period is randomly generated as follows:

$$d_{1j}^i \sim U(df^i, dl^i), \quad \forall i \quad \text{and} \quad 1 \leq j \leq n. \quad (22)$$

This represents the demand for the first cycle, where  $U(df^i, dl^i)$  is the uniform distribution between parameters  $df^i$  and  $dl^i$ , for each item  $i$ . The demand quantities for the following planning cycles are obtained as,

$$d_{kj}^i = d_{(k-1)j}^i + \epsilon_j, \quad \forall i \quad \text{and} \quad k \leq j \leq k + n - 1, \quad (23)$$



where,

$$\epsilon_j \sim U[0, cj], \quad k \leq j \leq k + n - 1, \quad c \in \mathbb{N}. \quad (24)$$

Parameter  $\epsilon_j^i$  represents a random error for period  $j$  and item  $i$ , and  $c$  is used to control the error variance. The demand quantities for the new last periods are generated as:

$$d_{k(k+n)}^i \sim U(df^i, dl^i). \quad (25)$$

The capacity parameter  $L_t^r$  must be consistent with constraint (7) and (13) for the formulations  $\mathcal{F}_1$  and  $\mathcal{F}_2$  respectively, to insure feasibility (for the single-item case see [10]). Hence this parameter is randomly generated such that the following condition holds,

$$\sum_{i=1}^m (\alpha^{ir} d_t^i + \beta^{ir}) \leq L_t^r, \quad \forall r, t. \quad (26)$$

Furthermore,  $r = 1$  is considered, supposing this resource as a fixed bottleneck of the system. Table 2 summarize the simulation parameters.

Table 2  
Simulation parameters.

| Parameter           | Value            |
|---------------------|------------------|
| Operational horizon | 52               |
| Planning horizon    | 8                |
| Demand (d)          | U[100,140]       |
| Production (p)      | U([95,105])      |
| Inventory (h)       | U([95,105])      |
| Setup (q)           | U([10000,20000]) |
| $r$                 | 1                |
| $\alpha$            | U[0.01,0.02]     |
| $\beta$             | U[2,3]           |
| $c$                 | 1                |

Simulation code was developed in Python 2.5.<sup>1</sup>. The MIP programs were solved with Glpk 4.29<sup>2</sup>.

#### 4.2 Results

The figures 1,2 and 3 show the difference between formulations  $\mathcal{F}_1$  and  $\mathcal{F}_2$  in terms of Total Cost, Maximum Instability (MAI) and Nervousness First Period (NF) respectively. These differences are computed as a relative value of the measures obtained with  $\mathcal{F}_2$  respect to  $\mathcal{F}_1$  (continuous line). For example, if  $MAI_{\mathcal{F}}$  represents the value of MAI obtained by the formulation  $\mathcal{F}$ , the continuous line in Fig. 2 shows the value of  $((MAI_{\mathcal{F}_2} - MAI_{\mathcal{F}_1})/MAI_{\mathcal{F}_1})$ . The dotted line represents, the mean value of the respectively serie. Finally, the results are presented, only for the cycles with the same number of overlaps periods, i.e., from the cycle  $k = n = 8$ . This last consideration is made, in order to compare the plans in the same conditions.

In the figure 1, Total Cost remains approximately in the interval [0.033, 0.049], with a mean value near to 0.043. This means that in average,  $\mathcal{F}_2$  is approximately 4.3% more expensive that  $\mathcal{F}_1$ . The sudden changes in the series, are as a result of changes in setup variable, and its huge associated costs.

Figure 2 shows the difference of MAI. We can see that in the worst case, the instability of  $\mathcal{F}_2$  never exceeds the instability of  $\mathcal{F}_1$ , and its mean of instability reduction is near to 60%.

<sup>1</sup> <http://www.python.org>

<sup>2</sup> <http://www.gnu.org/software/glpk/>

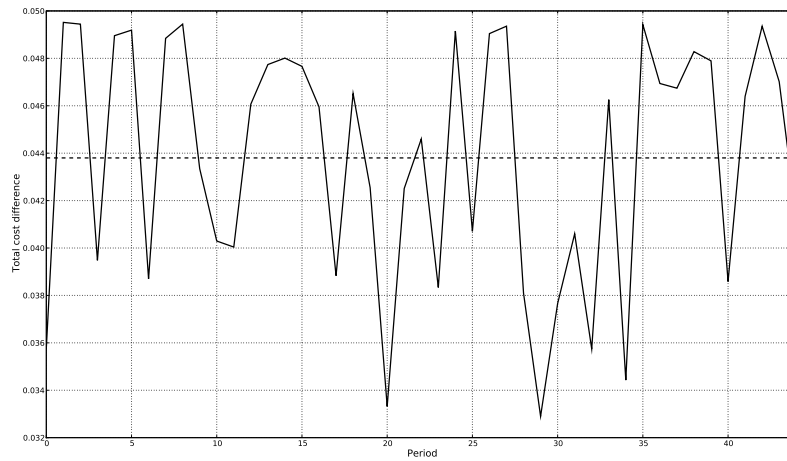


Fig. 1. Difference of Total Cost between  $\mathcal{F}_1$  and  $\mathcal{F}_2$ .

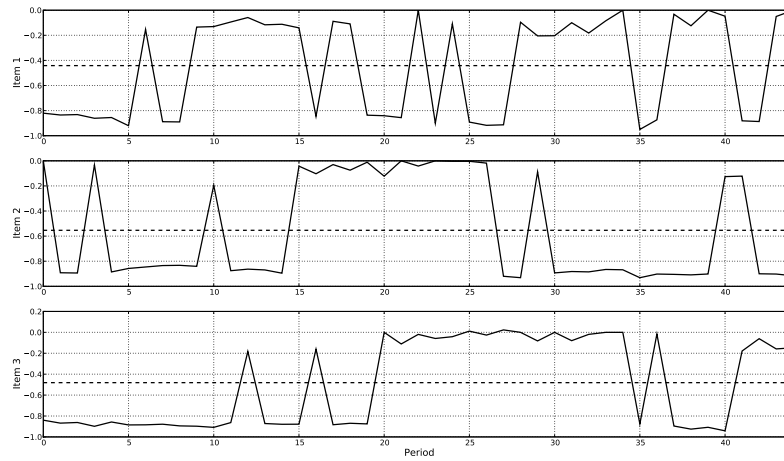


Fig. 2. Difference of Maximum Instability (MAI) between  $\mathcal{F}_1$  and  $\mathcal{F}_2$ , for each item.

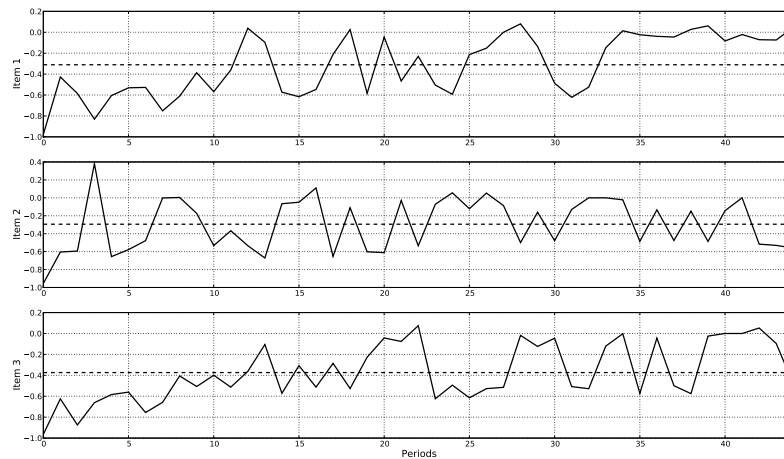


Fig. 3. Difference of Nervousness First Period (NF) between  $\mathcal{F}_1$  and  $\mathcal{F}_2$ , for each item.

Although, NF increases in some periods, in Figure 3 we can see that  $\mathcal{F}_2$  leads to a mean of nervousness reduction near to 40%. The importance of this, is the substantial reduction in terms of nervousness reached for  $\mathcal{F}_2$ , and that, will increase considerably the stability of the plan.

## 5 Conclusions

In this paper a mixed integer programming model for *Master Production Schedule* is proposed, taking into account directly the minimization of MPS instability and implicitly the minimization of nervousness. Simulation results show that, this formulation leads to reduce MPS instability and nervousness without a great difference in terms of total cost, compared with a classical formulation.

These results show that, direct inclusion of the minimization of the MPS instability can be considered to be an alternative to the more extensively recognized methods as frozen periods or safety stock.

In future works, others measures to integrate directly stabilization criterion in the models will be studied, leading to more robust solutions, decreasing instability and muffing nervousness, without a high impact in the optimal objective function value and its computational complexity.

In this work, our target have been to validate the proposed formulation in terms of its effectiveness, with no regard its computational efficiency. Nevertheless, the proposed algorithm solves in average 7 MIP by plan, being near to 15 the entire number of possible evaluations. In a future work, we will tackle in detail this aspect of the formulation.

## References

- [1] N. Brahimi, S. Dauzère-Pérès, N.M. Najid, and A. Nordli. Single item lot sizing problems. *European Journal of Operations Research*, 168(1):1–16, 2006.
- [2] T. de Kok and K. Inderfurth. Nervousness in inventory management: comparison of basic control rules. *European Journal of Operations Research*, 1(16):55–82, 1997.
- [3] P. Genin, S. Lamouri, and A. Thomas. Sales and operations planning optimisation. In A. Dolgui, J. Soldek, and O. Zaïkin, editors, *Supply Chain Optimisation*, volume 94, pages 191–204. Springer US, Boston, 1st edition, 2005.
- [4] K. Inderfurth. Nervousness in inventory control: analytical results. *OR Spektrum*, 16:113–123, 1994.
- [5] S. Kadipasaoglu and V. Sridharan. Alternative approaches for reducing schedule instability in multistage manufacturing under demand uncertainty. *Journal of Operations Management*, 13:193–211, 1995.
- [6] O. Kazan, R. Nagi, and C.M. Rump. New lot-sizing formulations for less nervous production schedules. *Computers & Operations Research*, 27(13):1325–1345, 2000.
- [7] A. Kimms. Stability measures for rolling schedules with applications to capacity expansion planning, master production scheduling, and lot sizing. *Omega*, 26(3):355–366, 1998.
- [8] M. Meixell. The impact of setup costs, commonality, and capacity on schedule stability: An exploratory study. *International Journal of Production Economics*, 95(1):95–107, 2005.
- [9] Y. Pochet and L.A. Wolsey. *Production planning by mixed integer programming*. Springer New York, New York, 2006.
- [10] Y. Pochet and L.A. Wolsey. Single item lot-sizing with non-decreasing capacities. *Mathematical Programming Serie A*, 2008.
- [11] R. Rangsaritratamee, W.G. Ferrell Jr., and M. Beth Kurz. Dynamic rescheduling that simultaneously considers efficiency and stability. *Computers & Industrial Engineering*, 46(1):1–15, 2004.
- [12] K. Richter and M. Sombrutzki. Remanufacturing planning for the reverse wagner/whitin models. *European Journal of Operations Research*, 121(2):304–315, 2000.
- [13] O. Tang and R.W. Grubbstrm. Planning and replanning the master production schedule under demand uncertainty. *International Journal of Production Economics*, 78(3):323–334, 2002.
- [14] A. Thomas, P. Genin, and S. Lamouri. Mathematical programming approaches for stable tactical and operational planning in supply chain and aps context. *Journal of Decision Systems*, 17:425–455, 2008.
- [15] K. van Donselaar, J. van den Nieuwenhof, and J. Visschers. The impact of material coordination concepts on planning stability in supply chains. *International Journal of Production Economics*, 68(1):169–176, 2000.
- [16] J. Xie, T.S. Lee, and X. Zhao. Impact of forecasting error on the performance of capacitated multi-item production systems. *Computers & Industrial Engineering*, 46(2):205–219, 2004.
- [17] J. Xie, X. Zhao, and T.S. Lee. Freezing the master production schedule under single resource constraint and demand uncertainty. *International Journal of Production Economics*, 83(1):65–84, 2003.
- [18] X. Zhao and K. Lam. Lot-sizing rules and freezing the master production schedule in material requirements planning systems. *International Journal of Production Economics*, 53(3):281–305, 1997.