Infield logistics planning for crop harvesting operations

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Infield logistics planning for crop harvesting operations

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Abstract

Crop harvesting operations are typically carried out with combine harvesters. The harvested product is transferred to one or more tractors every time the combine's storage capacity is reached. The efficiency of the process can be significantly improved by computing optimal routes and interactions for the harvest vehicles in the field. Furthermore, an automated method for generating itineraries for the harvest vehicles facilitates the planning for autonomous agricultural vehicles. The infield logistics problem is formulated as integer linear programming vehicle routing problem with additional turn penalty constraints, but due to the large number of decision variables, it is not possible to solve cases of realistic field size. The solution time of the infield logistics problem is considerably reduced by reformulating it as a modified minimum cost network flow problem. This specific structure allows the exact solution of intermediate size planning problems in a much shorter time period. The result of solving the infield logistics problem with the proposed modelling approaches is a set of itineraries (tours), covering the entire field. Each tour is characterized by the combine's start and end points and the positions where the combine needs to be unloaded. The planning models minimize non-productivity (i.e., the time when a combine travels in a field without harvesting). The results indicate that the coordination between combines and tractors is also improved.

Keywords: crop harvesting; harvest vehicles; planning of itinerary; vehicle routing; minimum cost network flow.

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1. Introduction

During the last few decades, a substantial advancement in the technological development of agricultural equipment can be observed. The early focus of original equipment manufacturers was on increasing the engine thrust of self-propelled agricultural vehicles (see for example, Hilliard 1972). Gradually the focus shifted towards improving the harvest operations for cutting, threshing and cleaning performed by combines. The recent interest is in improving the utilization of the harvest vehicles by efficiently planning the harvest process (Sørensen 2003) (Foulds et al. 2005).

Crops are harvested by combines (figure 1) following specific patterns to harvest the field. The supporting logistical activities of transferring the grain from the combines and transporting the product from the field to a depot are performed with the help of tractor trailers. Crop harvesting operations, in most real life cases, are planed based on the experience of the farmers operating the vehicles. Delays are often experienced due to bad cooperation between the combines and the tractors, increasing the overall duration of the harvest process.

Figure 1: Crop harvesting with multiple harvest vehicles

The crop harvesting operations require precise routing guidelines for the harvest vehicles. This article proposes a practical planning approach for harvesting a crop field with several capacitated combines and tractors. The goal of the planning is to generate ‘‘minimum cost’’ itineraries that can be followed by one or

several combines available for harvesting. Each tour starts and ends at the unloading positions in the field and takes into account the capacity restrictions of the combines. The article has five sections. Section 2 elaborates on the planning considerations for the crop harvesting process. The research is situated in its environment and relevant literature is mentioned. In Section 3, the modelling efforts are discussed and mathematical programming formulations are presented. The models are tested with a number of crop harvesting cases and the results are discussed. A comparison of the proposed modelling approaches is presented in Section 4 and Section 5 concludes the article.

2. The crop harvesting process

2.1 Operational scheduling

Crop harvesting is often carried out with several combines and tractors. The allocation of combines and tractors to the fields can be performed before the harvesting season by means of a higher-level planning tool. Various planning methodologies have been developed to support the harvest scheduling and the resource allocation decisions. Fokkens and Puylaert (1981) were among the first to formulate a linear programming model for the management of harvest operations at a larger scale grain farm. Their model minimizes the total cost of harvesting, including the operational cost at a single field and the transportation cost between different fields. The outcome of the model provides decision support for the allocation of the harvest vehicles to the fields and the transfer of combines from one field to another. A survey of different mathematical modelling approaches for various farm operations is provided by Glen (1987). His review elaborates on the mathematical models for machinery selection, cropping policies and farm operations scheduling. Sørensen (2003) presents a method to determine the machinery and the workforce requirements for harvesting a field. The method is based on a study of machine performance versus crop condition. The operational model helps to determine the optimal workforce and the machinery size with the objective of reducing the overall operational costs for a field.

Recently, Foulds and Wilson (2005) propose an operational scheduling method for harvesting rapeseed and hay fields. They experienced that the duration of the operations depends upon the interaction and combination of constrained resources allocated to a job. Therefore, the scheduling of harvest operations is truly complex. They present an integer programming model and heuristics to construct operational schedules for harvesting rapeseed and hay fields. These models are based on resource levelling methods and produce significant improvements over the previously used schedules. Basnet et al. (2006) extend this approach to scheduling operations at more than one field. Harvest contractors can use the developed approach to plan harvest operations and determine the sequence in which fields should be visited during the harvesting season.

Although scientific research resulted in good algorithms, it can be observed that, in reality, most infield harvest operations of combines and tractors are still performed without any detailed planning and the efficiency of the process relies heavily on the experience of the workers performing the operations.

2.2 Logistics planning issues

Combines harvesting a crop in a field need to follow a specific route. The path followed by a combine should be optimal with regard to the distance travelled, subject to the operational constraints of the field.

Considerable research has been conducted on various path planning problems, particularly for robot navigation and autonomous agricultural vehicles. Stentz (1994) developed the "D-Algorithm" for the task of path planning for a mobile robot equipped with sensors. The algorithm is designed to find, in real time, the optimal path in a directed graph and allows dynamic planning of the path whenever the robot senses changes in its environment. A fuzzy logic and genetic algorithms based technique is presented by Pratihar et al. (1999) to generate obstacle free paths for mobile robots. Path planning with the objective to completely cover an area resembles the geometric Travelling Salesman Problem (TSP). Several variants of the geometric TSP algorithms have been proposed (Arkin et al. 2000).

Optimal covering tour problems with consideration for turn costs have been investigated by Arkin et al. (2005). This problem is experienced in many actual routing scenarios of automatic inspection, spray painting, milling, lawn mowing, etc. The problem with turn minimization is proved to be NP-complete (based on the well-known hardness of deciding whether a grid graph has a Hamiltonian cycle) and efficient approximation algorithms have been proposed for finding a minimum number of turn tours, particularly for milling and lawn mowing applications (Arkin et al. 2000, 2005).

Path planning, in order to determine feasible paths for agricultural vehicles in a field, has been investigated by Sørensen et al. (2004). In their study, the problem is related to covering tour problems like the Chinese Postman Problem (CPP) and the Rural Postman Problem (RPP). Since these problems are NP-hard in nature, a heuristic is proposed for finding a solution. Bochtis et al. (2007) proposed a multi-travelling salesman problem for planning a fleet of combines operating in a field. Ryerson and Zhang (2007) conducted a feasibility study to determine the applicability of a genetic algorithm for path planning of agricultural vehicles. Although, this methodology did not result in completely optimized paths, the approach achieved a 90% coverage of the field. Recently, Oksanen and Visala (2007) proposed an area coverage planning algorithm for agricultural operations. Their algorithm include procedures for the decomposition of the coverage region in to sub-regions, the selection of sequence of those sub-regions and the generation of a path that covers each sub-region taking into account the desired working direction.

As reported in this section, different path planning and covering tour approaches are developed mainly for robotics and autonomous agricultural vehicles. These approaches do not focus on a specific operation or process performed with these vehicles. Therefore, the existing path planning methods cannot be used without modifications to generate harvest patterns for a field. First of all, combines operating in a field have

a limited bin capacity and need to unload grain to a tractor trailer at regular intervals in order to continue harvesting. Secondly, some additional constraints must be taken into account to deal with the accessibility requirements for the unloading of combines. In addition, sometimes the fieldwork pattern imposes additional constraints that need to be considered in order to make the planning realistic. Thus, a path planning approach should consider the very practical implications of the process under study. A good path plan for crop harvesting ideally results in the identification of a set of itineraries, considering the capacity limits of combines and the requirement that each tour starts and ends at feasible positions.

2.3 Crop harvesting scenarios

There are two main scenarios under which combines and tractors need to operate and coordinate for the crop harvesting operations: continuous and intermittent harvesting.

Continuous harvesting

In continuous harvesting, a tractor trailer approaches the combine in the field to unload grain once a combine reaches its specified bin capacity. For the grain transfer, the combine either stops in the field and unloads the grain to the tractor trailer, or the transfer is achieved by moving the tractor in parallel with the combine, while the combine continues harvesting.

Intermittent harvesting

In this scenario, tractors cannot approach the combines operating in the field. For the grain transfer operation, a combine has to stop the harvest operation and is obliged to travel to the tractor located at a fixed position in the field. After unloading the grain, the combine returns and continues the harvesting operation.

2.4 Logistics planning requirement for crop harvesting

For the crop harvesting process, mainly two issues are important. The first issue is the determination of optimal covering tours for combines operating in the field. The second issue is to identify feasible positions for the grain transfer between the combines and tractors. Both planning issues require routing decisions. The overall aim of the planning is to minimize the total distance travelled by the combines to harvest a crop field, thus minimizing the duration of harvesting process.

3. Modelling the crop harvesting problem

This section explains the development of two integer linear programming (IP) formulations for infield logistics planning for crop harvesting. The infield logistics problem is first reformulated as a modified vehicle routing problem (VRP). Next, the problem is modelled based on a minimum cost network flow problem (MCNFP) to improve the computation times.

3.1 Test cases and problem representation

Both the VRP and the MCNFP based infield logistics planning models are evaluated by means of a number of test cases. The test cases comprise crop fields of different dimensions. Table 1 enlists the different test cases, six without obstacles and two with obstacles in their area. First, in order to verify the planning results of the models, scaled size crop fields are used. Afterwards, fields between one to five hectares are considered. Each problem instance is characterized by the field area, the obstacles in the field and the available combines. The field is represented by a polygonal area defining the boundaries in which the combines can travel. An obstacle in the field is represented by a polygon within the field, enclosing the obstacle. After representing the field and the obstacles, the remaining area is converted into a grid of equally spaced vertices $v \in V$, where each vertex v_i represents the centre point of a cell. The result is a grid graph of a field G (V, E) with vertex set V and arc set E. Each cell approximates the area covered by the combine when standing still. The crop yield from a cell is based on the estimated density of the crop.

Table 1: Test cases for harvest logistics planning models

Test case	Field size	Field vertices	Combine paths to completely cover the field area	Field shape
1	Scaled_field_1	30	2	
2	Scaled_field_2	42	2	
3	1 hectare	167	3	
4	2 hectare	332	6	

5	4 hectare	662	12	
6	5 hectare	827	14	

This particular way of problem representation is used as an input for the path planning algorithms. A path is defined as a sequence of vertex transitions and is considered optimal if the sum of transition costs is minimum across all the possible sequences through the graph**.**

3.2 Modelling as a Vehicle Routing Problem (VRP)

The goals of infield logistics planning for crop harvesting are (i) minimizing the non-productive distance travelled by the combines in the field and (ii) identifying feasible grain transfer positions in the field, taking into account the limited capacity of the combine bin. Harvest logistics planning can thus be divided in a bin packing problem (harvesting with a minimal number of capacitated combines) and a travelling salesperson problem (minimizing the travel distance of the combines). Since the vehicle routing problem (VRP) lies at the intersection of both problems, reformulation of harvest logistics planning problem to a VRP is possible (see for example, Dantzig et al. 1959).

3.2.1 The VRP

The VRP is a well known NP-hard problem. In the VRP, a set of delivery/collection routes are designed for a fleet of vehicles to serve a number of customers from a central depot. The customers are distributed with a travel distance c_{ij} between them. The objective of the VRP is to find a minimum distance tour for each vehicle such that each customer is served exactly once by a vehicle and the route of each vehicle starts and ends at the depot. Various variants of the VRP exist and different solution approaches are proposed. An overview is given by Laporte et al. (2000) and Cordeau et al. (2002). Mazzeo and Loiseau (2004) solve the problem with ant colonization and compare the results with other metaheuristics. An overview of exact approaches is given by Toth and Vigo (2000).

Adaptations to the standard VRP allow to include time windows, site-dependencies, multiple-depots, open routes, etc. The interested reader is referred to Pisinger and Ropke (2007). The capacitated version has the highest relevance for harvest logistics planning. In the capacitated vehicle routing problem (CVRP), each vehicle has a specific capacity and once a vehicle reaches its limit, it returns to the depot.

In the infield logistics problem, the combines operating in the field are the vehicles with capacity constraints. The combines must unload to the tractor every time the bin is filled up with the harvest product. The yield from the crop is spread over the set of vertices V and each field vertex $v_i \in V$ is required to be covered by a combine.

3.2.2 Problem formulation

The infield logistics planning problem for crop harvesting is modelled as a CVRP with additional turn penalty constraints. The following variables are used:

- i, j, h : vertex indices
- k: combine path index
- c_{ij}: travel distance between vertex i and j
- p_i: turn penalty at vertex i
- A_i: yield from vertex i
- C_k: capacity of combine k
- I: set of all vertices
- K: set of all combine paths

The following variables are determined by the model:

- x_{ijk} : binary variable indicating if vertex i is followed by vertex j in a combine path k
- y_{ijk}: binary variable to introduce the turn penalty

The mathematical model becomes:

$$\min \sum_{i=0}^{i=I} \sum_{j=0}^{j=I} \sum_{k=1}^{k=K} c_{ij} x_{ijk} + p \sum_{i=1}^{i=I} \sum_{k=1}^{k=K} y_{ijk} \quad (1)$$

s.t.

$$\sum_{j=0}^{j=I} \sum_{k=1}^{k=K} x_{ijk} = 1 \qquad \forall i \in I \quad (2)$$

$$\sum_{j=1}^{j=I} x_{0jk} = 1 \qquad \forall k \in K \quad (3)$$

$$\sum_{i=1}^{i=I} x_{i0k} = 1 \qquad \forall k \in K \quad (4)$$

$$\sum_{i=0}^{i=I} x_{ihk} - \sum_{j=0}^{j=I} x_{hjk} = 0 \qquad \forall h \in I_0; \forall k \in K; i \neq h; h \neq j \quad (5)$$

$$\sum_{i=0}^{i=I} A_i \sum_{j=1}^{j=I} x_{ijk} \le C_k \qquad \forall k \in K \qquad (6)$$

$$\begin{aligned}
&x_{(i-n,i,k)} + x_{(i,i+1,k)} \le 1 + y_{(i,i+1,k)}\\
&x_{(i-n,i,k)} + x_{(i,i-1,k)} \le 1 + y_{(i,i-1,k)}\\
&x_{(i+n,i,k)} + x_{(i,i+1,k)} \le 1 + y_{(i,i+1,k)}\\
&x_{(i+n,i,k)} + x_{(i,i-1,k)} \le 1 + y_{(i,i-1,k)}\\
&x_{(i-1,i,k)} + x_{(i,i+n,k)} \le 1 + y_{(i,i+n,k)}\\
&x_{(i-1,i,k)} + x_{(i,i-n,k)} \le 1 + y_{(i,i-n,k)}\\
&x_{(i+1,i,k)} + x_{(i,i+n,k)} \le 1 + y_{(i,i+n,k)}\\
&x_{(i+1,i,k)} + x_{(i,i-n,k)} \le 1 + y_{(i,i-n,k)}
\end{aligned} \qquad \forall i \in I; \forall k \in K \qquad (7)$$

$$u_i - u_j + C x_{ijk} + (C - A_i - A_j) x_{ijk} \le C - A_i \qquad \forall i, j \in I_0; \forall k \in K; i \ne j \qquad (8)$$

$$A_i \le u_i - C \qquad \forall i \in I$$

$$x_{ijk} \in \{0,1\} \qquad \forall i, j \in I; \forall k \in K \qquad (9)$$

The objective function (1) minimizes the total distance travelled and the weighted number of turns in the field. Constraints (2) ensure that every vertex in the field is covered by a combine tour. Constraints (3) and (4) make sure that all the available combines are used. The number of combines K (combine paths) can be preset based on the approximate crop yield and the combine bin capacity. Constraints (5) are the flow balance constraints. Capacity constraints (6) guarantee that the combine bin is never filled beyond its capacity limit. As the distance c_{ij} between the adjacent field vertices is constant, turn penalty constraints (7) are used to eliminate unnecessary turns in a field tour. These constraints are evaluated based on the 'if-then' constraints (Winston 2004). They make sure that if a right angle turn is made, i.e., three consecutive vertices (e.g., *i-n, i, i+1*) in a tour form a turn as shown in figure 2, then the penalty cost p is added to the objective value of the model. Thus, the number of turns in a tour is reduced to the bare minimum. The Miller-Tucker-Zemlin subtour elimination constrains (8) prevent generation of subtours and ensure the continuity in a path. (9) limits variables to binary values.

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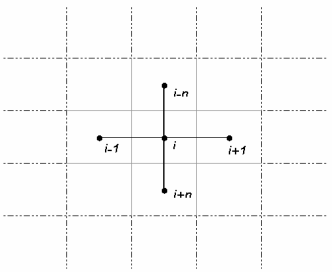

Figure 2: Turn penalty at vertex i

3.2.3 Computational results

The proposed modelling approach is verified with the test cases of the scaled crop fields. Each problem instance is represented on a grid graph and its IP formulation is generated in the C language. The IP models are solved using the ILOG Cplex v10.01 integer linear programming optimizer on a 3.4 GHz Intel work station with 1 GB RAM. Figure 3 shows a result of solving the problem (test case 2) with the CVRP model. The planning model assigns field vertices to combine paths taking into account the crop yield and the combine bin capacity. As can be seen from this figure, each combine path indicates the starting position for a combine, the area to be covered in the field and the position where the combine bin is expected to be filled up. Depending upon the crop harvesting process, the problem structure can be adapted to specify in advance, the preferred end positions for the combine tours. For example, in intermittent harvesting, where the combines operating in the field cannot be approached by the tractors for the grain transfer operation, it is desirable that a combine always ends its tour (with the filled up bin) at the border of the field, near the tractor location. Thus, by identifying the end positions near the border of the field in the problem, the combine tours always end at one of the indicated positions suitable for grain transfer.

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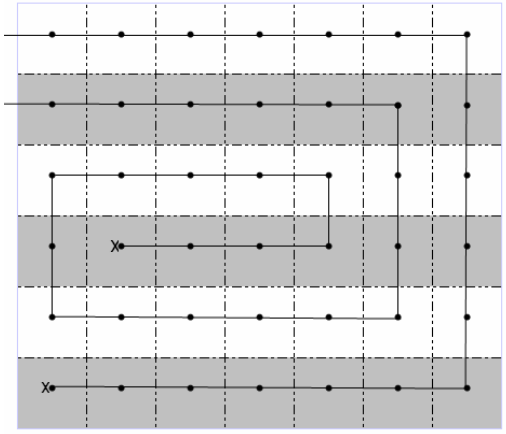

Key: X Grain transfer point

Figure 3: Harvest patterns generated by the VRP model

The planning results for the scaled crop fields provide an indication about the paths to be followed in the real field and the expected grain transfer positions. However, as can be observed from Table 2, computation times increase tremendously with the increase in the field size. With the proposed CVRP approach, problems of realistic sizes cannot be solved exactly. To overcome this limitation, an alternative IP formulation is presented.

Table 2: Computation results of the VRP model

Test case	Field size	Combine paths to completely cover the field area	Solution time CPU (sec.)
1	Scaled_field_1	2	8925.77
2	Scaled_field_2	2	43167.41

3.3 Modelling as a Minimum Cost Network Flow Problem (MCNFP)

The infield logistics problem is reformulated based on a minimum cost network flow problem (MCNFP). The specific structure of MCNFP allows to solve large problem instances in much shorter time and therefore, it can be used to tackle real world decision problems (Frangioni and Manca 2006), (Hamacher et al. 2007).

3.3.1 The MCNFP

The MCNFP involves shipping of a commodity through a single connected network at minimum cost, such that the total flow does not exceed the arc capacities. Given a directed graph G (V, E) with a vertex set V and an arc set E, an upper u_{ij} and a lower bound l_{ij} on the flow through an arc, a nonnegative arc cost c_{ij} and a specification of the net flow b_i generated at vertex v_i , the problem is defined as

$$\min \sum_{all\ arcs} c_{ij} x_{ij} \qquad (1)$$

s.t.

(*for each vertex i in the network*)

$$\sum_j x_{ij} - \sum_k x_{ki} = bi$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \qquad (\textit{for each arc in the network})$$

The problem has many applications ranging from scheduling in the public transport sector to finding an optimal way for routing information through a capacitated communication network (Kamath et al. 1995). The objective in each case is minimization of the total cost of flow from the source to the destination while obeying the capacity constraints defined on the arcs in the network. Numerous, very efficient algorithms have been proposed for solving network flow problems. Frangioni and Manca (2006) present a study comparing the performance of a number of those algorithms.

3.3.2 Problem formulation

The infield logistics problem is modeled as a variant of a MCNFP formulation. Some modifications in the representation of the problem instance (crf. Section 3.1) are required. First of all, each vertex $v_i \in V$, representing a cell in the field G (V, E) is split up into two vertices indexed i' and i". This allows to set a lower bound on the flow through a cell, guaranteeing that all cells in a field are included in the solution. Secondly, the yield from the original vertex $v_i \in V$ is placed on the newly defined arc from vertex i' to i". Thus, the field is represented by a grid network composed of predefined arcs E in a directed graph G (V, E). The flow capacity x_e ($lb \leq x_e \leq ub$), specifying lower and upper bounds respectively and the flow costs c_e are associated with each arc $e_{ij} \in E$ in the network. The set of vertices V is categorized into source vertices, intermediate vertices and sink vertices. The flow starts from a source vertex, travels through intermediate vertices towards a sink vertex, where it is finally absorbed. In harvest logistics planning, the combines are the commodities that need to travel through the capacitated network (representing the field). The total supply is equal to the number of combines K required to completely harvest the field.

The flow capacities of all arcs of type i'-i" in the grid network are set $1 \leq x_e \leq 1$, to ensure that all cells are covered by at least one of the combines. The flows through ‘‘external arcs’’ between different cells in the network are bounded by $0 \leq x_e \leq 1$. The decision whether a particular external arc will be used for the flow is determined by the model, minimizing the total flow cost through the network.

Several combines operating in the field should be able to start their trip from any of the source vertices $S \subset V$ in the field. Similarly, when the bin of the combine is filled up, the combine should be able to finish its trip at one of the sink vertices $F \subset V$ in the field. These requirements are incorporated by introducing a new super source vertex s and a super sink vertex f in the network. An arc (s, v) is then added for each possible source vertex, where $v \in S$. Also for every possible sink vertex $u \in F$, an arc (u, f) is added to the network. This results in a grid network with the set of vertices $V \cup \{s, f\}$. All vertices in V are intermediate vertices with the net flow b_i equal to zero.

In general, routing problems need to be constrained in order to prevent the generation of subtours (i.e., a closed loop that does not start and end at the source and the sink vertices respectively). In this formulation, subtours are avoided by the specific way of defining the problem instance. First, in the field network representation, arcs are defined in alternate directions and no arc points back to its tail as displayed in figure 4. Secondly, higher flow costs c_{ij} are used for arcs between cells in different rows. This way of formulating prevents the generation of subtours in rectangular and L-shaped fields.

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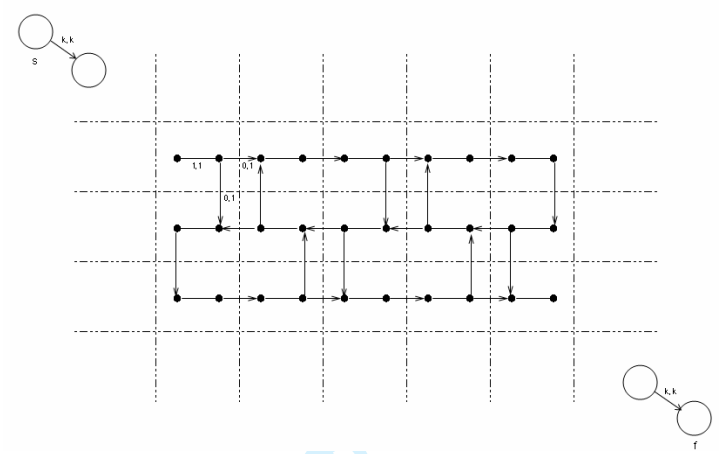


Figure 4: Grid network representation of a field

The variables used are:

- *i,j,h*: arc index
- *k*: combine path index
- c_{ij} : flow cost through an arc
- b_i : net supply at vertex i
- *ub*: upper bound on flow through an arc
- *lb*: lower bound of flow through an arc
- *{s, f}*: source and sink vertices
- *V* : set of all intermediate vertices
- *Ai*: yield from arc (i'-i")
- *E*: set of all arcs
- C_k : bin capacity of combine k
- *K*: set of all combine paths

The variable to be determined by the model is:

- x_{ijk} : binary variable indicating if arc (i,j) is included in the combine path k

The mathematical model is formulated as follows:

$$\min \sum_{(i,j)\in E} \sum_{k=1}^{k=K} c_{ij} x_{ijk} \tag{1}$$

s.t.

$$\sum_{k=1}^{k=K} x_{ijk} \le ub \quad \begin{cases} 1, \forall \text{ arcs between } (i,j) \\ K, \forall \text{ arcs connecting } \{s,f\} \end{cases} \qquad \forall (i,j) \in E \tag{2}$$

$$\sum_{k=1}^{k=K} x_{ijk} \ge lb \quad \begin{cases} 1 \forall \text{ arcs between } (i'-i'') \\ 0, \forall \text{ arcs between } (i,j) \\ K \forall \text{ arcs connecting } \{s,f\} \end{cases} \qquad \forall (i,j) \in E \tag{3}$$

$$\sum_{(h,j)\in E} x_{hjk} - \sum_{(i,h)\in E} x_{ihk} = bi \quad \begin{cases} +K, \forall s \\ -K, \forall f \\ 0, \forall V \end{cases} \qquad \forall k \in K \tag{4}$$

$$A_i \sum_{(i'-i'')\in E} x_{ijk} \le C_k \qquad \forall k \in K \tag{5}$$

$$x_{ijk} \in \{0,1\} \qquad \forall (i,j) \in E; \forall k \in K \tag{6}$$

The objective (1) is to minimize the flow costs of all combines. Constraints (2) set upper bounds on the flow through the arcs in the network. Note that the upper bound is equal to 1 for all the intermediate arcs. Lower bounds on the flow through the arcs are assigned by constraints (3). The lower bound is set equal to 1 for all arcs of type i'-i'' and zero for all external arcs i-j. Constraints (4) are the flow conservation constraints. The left hand side of these constraints indicates the net flow at a vertex. The net flow is equal to zero at all intermediate vertices. The flow entering the network via the super source vertex s is equal to the number of combines K (combine paths) for a field. The flow collected at the super sink f must be equal to the flow leaving the source vertex. The number of combines K required to completely harvest a field can be pre-determined based on the approximate yield of the crop and the capacity of the combine bin. For harvest logistics planning, the MCNFP formulation is modified by two additional constraints. Constraints

(5) are the capacity constraints to make sure that the combine bin is never filled beyond its available capacity and constrains (6) limit the variables to binary values. Notice that formulation (1) – (6) is not a pure MCNFP. To solve the IP to optimality, the branch-and-bound technique is used. Normally branch-and-bound requires large computation times, but, due to the specific MCNFP structure, the search trees are limited and the solution is found in a comparatively short time.

3.3.3 Planning algorithm

The algorithm for the infield logistics planning problem is composed of three phases:

- Field network generation,
- Mixed integer programming (MIP) formulation,
- MIP solution .

The grid network for a field is created following the modelling approach described in Section 3.3.2. In order to solve the planning problems of larger field size, the network generation process is automated. The network grid generator is coded in the C language. After reading the input parameters (i.e., field dimension and arcs costs), the program quickly generates a connected network with the required number of arcs along with their respective flow capacities and costs. The generated network representing the harvest logistics planning problem of a field is then used as an input for the planning model. The linear programming formulation of the problem is generated in C. Additional information about the combine bin capacity and the approximate yield from the crop are user defined. For solving the problem, the ILOG Cplex v10.01 optimizer is used. All computations are performed on a 3.4 GHz Intel work station with 1 GB RAM.

3.3.4 Computational results

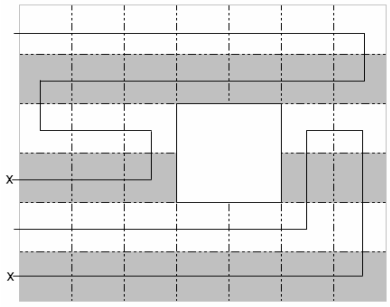
The MCNFP approach is evaluated with a number of test cases. Both intermittent and continuous harvesting are considered. For intermittent harvesting, the sink vertices are located next to the fixed location of the tractor (along the edge of the network). For continuous harvesting, any intermediate vertex can be the sink vertex. For those situations, the grain transfer is performed by approaching the combine in the field. The planning algorithm uses the indicated source and sink vertices as a reference for start and end positions of the combine tours. In many practical situations, it might be necessary to take into account the prior knowledge about field characteristics e.g., field entrances and exits and the fieldwork pattern. These specifications can be included for planning by specifying them in the problem representation of a field. Afterwards, the planning algorithm can be used to find the best solution. Solving the infield logistics problem with the modified MCNFP model gives results very similar to those obtained with the CVRP planning model. The set of itineraries generated covers the entire area of the field with a minimum total distance and a minimum number of turns. Table 3 provides the computation times required to solve the test

cases. Compared to the CVRP model, these computation times are much smaller. This makes it possible to find exact solutions for intermediate size planning problems.

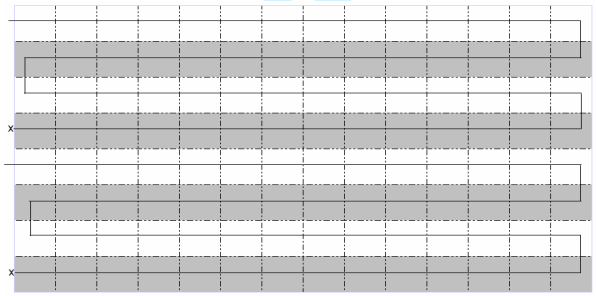
Table 3: Computation results of the MCNFP model

Test Case	Field Size	Combines (combine paths) to completely cover the field area	Solution time CPU(sec.)
1	Scaled_field_1	2	0.03
2	Scaled_field_2	2	0.05
3	1 hectare	3	1.80
4	2 hectare	6	185.61
5	4 hectare	12	1277.03
6	5 hectare	14	36309.9

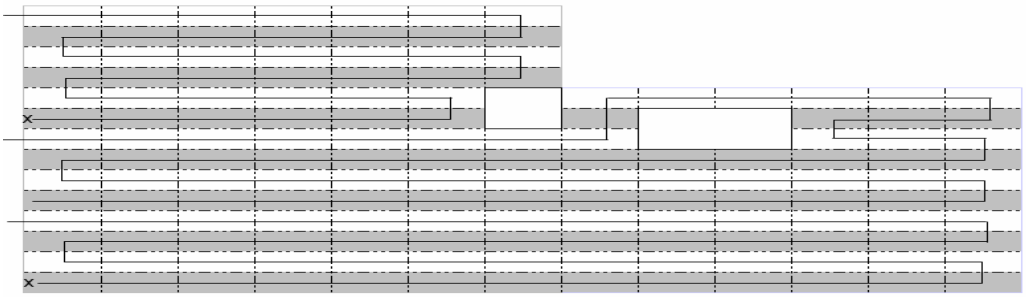
Figure 5 shows the harvest patterns for various infield logistics problems. Each tour indicates the starting vertex for an unloaded combine and a vertex where the harvester bin is expected to be filled with grain. The tour ends at one of the pre-specified end vertices suitable for grain transfer. This prior knowledge about the expected grain transfer positions also improves the cooperation of the tractors with the combines.



(a) Rectangular field with an obstacle



(b) Rectangular field



(c) L-shaped field with obstacles

Key: X Grain transfer point

Figure 5: Crop harvesting patterns generated by the MCNFP model

4. Comparison of the modelling approaches

The CVRP and the MCNFP based planning approaches are evaluated with the selected test cases (crf. Section 3.1). ILOG Cplex v10.01 integer linear programming optimizer is used to obtain the planning results. Careful analysis of the results reveals that the quality of the solution, i.e., the number of turns in a tour and the total distance traveled, remains the same with both planning approaches. However, the solution times required by the modified MCNFP planning model are much smaller than the solution times required for solving the problems with the CVRP model, thus, allowing the cases of realistic field size to be solved exactly. Apparently, branch-and-bound application is limited for the modified MCNFP application. When the CVRP based optimization approach is applied to solve problems of large field size, a very large computation time is required. Interrupting the branch-and-bound process provides a solution, but unfortunately, such intermediate results are not often optimal with respect to the number of turns in the field. Therefore, the planning problems for the crop fields between one to five hectares are solved only with the MCNFP based planning method. This approach generates good results for intermediate size fields. However, the computation times to obtain the exact solutions with the commercial solver still tend to increase for the fields of five and more hectares. On going research is hence focusing on field area planning and segmentation. A large crop field can be split into smaller field segments. Afterwards, the proposed MCNFP planning method can be used to obtain feasible harvest patterns for each segment.

5. Conclusion

Path planning for robotics and agricultural vehicles has been investigated, in order to find the shortest paths and covering tours in an area. However, for crop harvesting process, the existing planning approaches are insufficient to generate the harvest patterns. This is mainly because the combines operating in a field have a limited bin capacity and need to unload grain to the tractor trailer at regular intervals to continue the harvest operation. The infield logistics planning requirements are taken into account and the problem is modeled as

a vehicle routing problem with additional turn penalty constraints. Good results indicating a set of itineraries for the crop field and feasible grain transfer positions for the combines are obtained, but the solution times increase tremendously for larger problem instances. To overcome this, the infield logistics planning is reformulated based on a minimum cost network flow problem. This model allows the planning problems of intermediate field size to be solved exactly. The quality of the solution, that is to say the total distance travelled and the number of turns in the field, remains the same with both planning models. The indication of grain transfer positions also improves the coordination between the combines and the tractors. However, for fields of five or more hectares, the solution times required by the minimum cost network flow approach also tend to increase. The results obtained from both modelling approaches provide useful insight into infield logistics problems. Particularly the minimum cost network flow approach is very useful to generate exact solutions for the problems of intermediate size. Such solutions can serve as benchmark results for evaluating heuristic approaches to the planning problem.

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