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Dynamic modelling for thermal micro-actuators using thermal networks

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Abstract

Thermal actuators are extensively used in microelectromechanical systems (MEMS). Heat transfer through and around these microstructures are very complex. Knowing and controlling them in order to improve the performance of the micro-actuator, is currently a great challenge. This paper deals with this topic and proposes a dynamic thermal modelling of thermal micro-actuators. Thermal problems may be modelled using electrical analogy. However, current equivalent electrical models (thermal networks) are generally obtained considering only heat transfers through the thickness of structures having considerable height and length in relation to width (walls). These models cannot be directly applied to micro-actuators. In fact, micro-actuator configurations are based on 3D beam structures, and heat transfers occur through and around length. New dynamic and static thermal networks

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are then proposed in this paper. The validities of both types of thermal networks have been studied. They are successfully validated by comparison with finite elements simulation and analytical calculations.

Key words:

Micro-actuators, Thermal modelling, Electrical analogy, Thermal network

1. Nomenclature

The notations used in this article are summarized in table 1.

2. Introduction

The thermal actuation is widely present in active microsystems, either by inherent design or combining two or more energy domains such as optical, mechanical or electrical [1, 2, 3].

One of the earliest (1970's) and most commercially successful application of thermal microelectromechanical systems (MEMS) is the ink-jet printer head [4]. Recent applications including microlens [5, 6], microprobes [7, 8], microsensors [9, 10, 11, 12, 13] and micro-actuators, are rapidly gaining importance too. For example, a thermal microlens tunes its focal lens by temperature, which is easily controlled via managing the current input of the heater [6]. An on-wall in-tube flexible thermal sensor is able to measure the flow rate under both developing and fully developed flow conditions. The resistance of the sensor linearly changes with temperature [13].

Thermal micro-actuators are a very popular actuation technology in MEMS. They commonly exploit differential thermomechanical expansion of materials, known as the thermomechanical effect, resulting generally from Joule

Table 1: Nomenclature

| | | | | | |
|-----------|-----------------------------------|--------------------|----------------------|-------------------------------|-------------------|
| a | height | m | T | temperature | K |
| b | width | m | V | voltage | V |
| C_p | specific heat (constant pressure) | J/kg K | x, y, z | Cartesian coordinates | m |
| C_{th0} | thermal capacitor | J/K | Z | thermal impedance | |
| d | characteristic size | m | <i>Greek symbols</i> | | |
| h | heat transfer coefficient | W/m ² K | δ | static validity criterion | |
| i | current | A | ν | dynamic validity criterion | |
| k | thermal conductivity | W/m K | ω | angular frequency | rad/s |
| l | length | | ϕ | heat Laplace transform | W |
| P | lateral perimeter | m | ρ | mass density | kg/m ³ |
| Q_h | heat convection | W | θ | temperature Laplace transform | K |
| Q | heat flux | W | <i>Subscripts</i> | | |
| R_{c0} | conduction thermal resistance | K/W | 1 | lateral face, at $x = 0$ | |
| R_{v0} | convection thermal resistance | K/W | 2 | lateral face, at $x = l$ | |
| S | lateral surface | m | ext | external | |
| t | time | s | lin | linear | |
| | | | st | static | |

19 heating. The common actuation geometries of this kind of micro-actuators
 20 are multimorph [14, 15, 16], U-shape [1, 17, 18, 19, 20, 21] and V-shape
 21 [17, 22, 23, 24, 25]. In all of them, the prevalent geometry is the beam.

22 Properties of thermal micro-actuators strongly depend on their geome-

23 try structure and material properties, as well as the applied current. It is
24 therefore important to model the thermal micro-actuator in order to improve
25 or optimize its design. Thermal micro-actuators are used in microgrippers
26 [16, 19, 25, 26, 27], microheaters [28, 29], micromotors [30], microrobots [31],
27 micromirrors [14], ice gripping [32], etc. Direct applications of these devices
28 are electrics, optics, electronics, mechanics and biomedicine [30, 33]. Ther-
29 mal actuators are usually rather slow, due to thermal time constants typically
30 in the upper millisecond range [34]. In smaller structures, however, it has
31 been shown that substantially higher speeds can be attained, because ther-
32 mal time constants scale linearly with decreasing surface [30, 35]. Compared
33 to their counterparts such as electrostatic or piezoelectric actuators, ther-
34 mal actuation provides larger forces [36]: for typical configurations, thermal
35 actuation provides 4 orders of magnitude higher energy density than electro-
36 static actuation ($450 \mu\text{N}\cdot\text{mm}^{-2}$ for thermal actuation, and $20 \mu\text{N}\cdot\text{mm}^{-2}$ for
37 electrostatic actuation [22]), and 1 to 2 orders of magnitude higher energy
38 density than piezoelectric actuation [1, 30]. Thermal micro-actuators have
39 a high reliability and are also easier to control, compared to shape memory
40 alloy actuators. In addition, they are usually simpler to be fabricated, con-
41 trary to magnetic actuators, for instance, that may require special materials
42 in the fabrication process [22]. Some of the thermal micro-actuators have
43 been made on silicon, polysilicon, and nickel structural components. Metal-
44 based electrothermal micro-actuators provide a larger output displacement
45 with a smaller input voltage. However, they generally suffer from mechanical
46 deficiencies, such as fatigue and aging [33].

47 Fabrication techniques of thermal micro-actuators are well-known mi-

48 crotechnologies such as bulk micromachining, wet and dry etching processes,
49 surface micromachining and LIGA processes, including laser micromachin-
50 ing [1, 2, 6, 14, 24, 37]. Managing thermal phenomena in thermal micro-
51 actuators is a key factor for future progress in their optimization. Modelling
52 heat transfers through the actuator and its surroundings are fundamental for
53 understanding, predicting and controlling the temperature distribution, and
54 consequently the device response characteristics. This paper deals with this
55 problem and proposes a dynamic thermal analysis using electrical analogy.
56 This last is generally used to model heat transfers occurring through the
57 thickness of structures with geometries as walls, where height and length are
58 considerable in relation to width. Here, we develop equivalent electrical mod-
59 els of long structures, which are currently found in thermal micro-actuators.
60 In these thermal structures, heat transfers are present through and around
61 the length.

62 Being the analysis presented in this paper useful for the MEMS com-
63 munity dealing with thermal problems, in Section 3 we present different
64 approaches to model these problems, particularly in micro-electronics and
65 thermal micro-actuators. The beam geometry, currently found in thermal
66 micro-actuators, is analysed dynamically in Section 4 via electrical analogy.
67 In Section 5 we determine the validity of the proposed equivalent electri-
68 cal models, and they are validated by comparing them with finite elements
69 simulations and analytical calculations in Section 6.

70 **3. Thermal modelling in microdevices**

71 Thermal effects have an obvious impact in the performance and relia-
72 bility of electronic systems and thermal devices [38]. Efforts to model the
73 electrothermal actuators and thermal effects in electronic systems have been
74 focused on analytical modelling, finite-element methods, lumped parameters
75 based on electrical analogy, and model order reduction. Several examples of
76 these techniques are presented in this section.

77 *3.1. Analytical modelling*

78 In practice, an analytical solution provide very accurate models, and it
79 may easily consider constraints. Liao et al. theoretically modelled an elec-
80 trothermal micro-actuator for bidirectional motion to forecast the relation-
81 ship between applied voltage and displacement [1]. A one-dimensional con-
82 ductive heat transfer analysis of two types of thermal actuators (U-shaped
83 and V-shaped) are presented by Hickey et al. [17]. Thermal time constants
84 were predicted using this model. Robert et al. predicted the shape of a
85 thermal and electrostatic micro-actuator versus the temperature [3]. An al-
86 gorithm has been also developed in order to evaluate the damping behavior
87 of a microswitch taking the micro-actuator deflection into account. Li and
88 Uttamchandani analysed a modified asymmetric micro-electrothermal actu-
89 ator [18]. The aim of this analysis was to calculate the optimum dimensions
90 of the hot arm of the actuator to maximize the deflection of the actuator
91 before the onset of thermal failure. A theoretical model of the actuation
92 stress of a polymeric (SU-8) thermal micro-actuator with embedded silicon
93 microstructure has been realized by Lau et al. [39]. The analytical model

94 of Boutchich et al. predicted the dependence of the restoring force on the
95 input electrical power and topology of a thermal actuator [15]. A two-step
96 analytical model of a four-hot arm U-shape electrothermal actuator that can
97 achieve bidirectional motion in two axes has been developed by Elbuken et al.
98 [40]. Finally, Mayyas and Stephanou obtained closed-form solutions for the
99 thermal modelling of a general 5 non-homogeneous, lineshape microbeam's
100 actuator using 1-D steady state heat equations under both heat conduction
101 and convection [41].

102 Finite element (FE) based simulations [1, 17, 40, 41] or experimental
103 measurements [3, 15, 17, 18] were usually performed to validate analytical
104 models. Analytic solutions for describing thermal problems are only available
105 for simple geometries. For complex geometries, either numerical methods or
106 approximations may be used.

107 *3.2. Finite element modelling*

108 Electrothermal devices are traditionally simulated with finite element
109 method (FEM) due to the complex coupling of the electrical, thermal and
110 mechanical problem [42]. FEM analysis is principally used to demonstrate
111 the feasibility of the design and to simulate the behaviour; for example, the
112 relationship between the applied voltage and the displacement [1, 37, 43, 44],
113 or the effects of geometrical and material stiffness variations on the perfor-
114 mance [45].

115 The principal drawbacks of implementing analytical and FEM analysis in
116 electrothermal microdevices are regarded to the long simulation time, and the
117 time required to build-up the model in the FEM software. Faster methods
118 become necessary.

119 *3.3. Modelling using electrical analogy*

120 Electrical analogy has been used for the first time by Paschkis and Baker
121 in 1942 to solve the unsteady-state and unidirectional heat conduction equa-
122 tion in a plate [46]. Nevertheless, the principles of the electrical analogy
123 applied to unsteady-state heat conduction problems have been proposed by
124 Jakob in 1949 [47]. Ever since, equivalent electrical models, so-called thermal
125 networks, seem a relatively simple, but sufficiently accurate and powerful tool
126 for simulating real thermal systems and microsystems [48, 49, 50, 51].

127 Dynamic compact thermal networks able to predict the dynamic junction
128 temperature response under any arbitrary set of external cooling conditions
129 have been represented by Christiaens et al. [52]. They have been successfully
130 demonstrated for two types of polymer stud grid array (PSGA) packages, the
131 standard PSGA and the thermally enhanced PSGA. Szekely developed the
132 formulation of a thermal admittance matrix between different ports [53]. Ev-
133 ery element in the matrix leads to a thermal network, and these networks
134 appear coupled by voltage-controlled current sources. An equivalent net-
135 work approach has been applied by Codecasa et al. to the analysis of the
136 electrothermal effects of a metal oxide semiconductor field effect transistor
137 (MOSFET) and bipolar junction transistor (BJT) devices [54]. They have
138 shown how thermal effects, as they are seen at the electrical terminals, can
139 be modeled through an equivalent purely electrical model obtained prop-
140 erly transforming the thermal impedance. The thermal network complexity
141 reduction applied to a vertical power MOSFET device has been also inves-
142 tigated [55]. Few years later, Codecasa proposes passive compact models of
143 dynamic thermal networks with many heat sources [56]. Recently, Li et al.

144 used a nodal analysis method based on circuit analogy for simulating the re-
145 sponses of surface micromachined electrothermal micro-actuators and an out-
146 of-plane beamshaped electrothermal micro-actuator [42, 57]; a lumped ap-
147 proach that decomposes the static and dynamic performances of an U-shape
148 micro-actuator into a series of thermal networks and a mechanical frame
149 has been adopted by Lo et al. [58]; and Szabo and Szekely presented ther-
150 mal networks, defined experimentally or analytically, of a quadratic transfer
151 characteristics (QTC) element of which driving principle is the Seebeck effect
152 [59].

153 Thermal modelling via electrical analogy is often applied to the thermal
154 analysis of microelectronic and thermal microdevices, leading to complex
155 thermal networks. Nevertheless, time set on thermal network simulations are
156 set much shorter than on FEM simulations. Efforts to reduce the complexity
157 of thermal networks are currently done.

158 *3.4. Model order reduction*

159 Simulation at the physical level including all geometrical details and phys-
160 ical interactions is not feasible, not even wished. Consequently, simulation
161 is based on compact models at different levels of abstraction. Whereas nu-
162 merical modeling can provide temperature distributions in the component
163 with enough accuracy, the computing cost to simulate using meshing is ex-
164 tremely high, and it is of the most importance the development of compact
165 dynamic thermal models. Automating the generation of low-dimensional
166 systems of equations by means of mathematical techniques (e.g. Arnoldi,
167 Balanced Truncation Approximation, λ -finder, and Guyan algorithms) has
168 been widely studied [60, 61, 62, 63]. Automatic model order reduction aims

169 at providing reduced models only with minimal intervention by the designer.
170 The goal is to provide a software based on a spatial discretization of the
171 partial differential equations which is capable to return ordinary differen-
172 tial equations with a far lower number of state variables than the previous
173 discretized system without sacrificing too much accuracy. These ordinary dif-
174 ferential equations can then be simulated in acceptable time [60, 61]. Palacin
175 et al. focus their attention to the development of compact thermal models
176 from the analysis of the thermal impedance transients obtained from physi-
177 cal simulation (finite element model) [38]. Dynamic compact thermal models
178 have been obtained for an ultrathin chip stacking technology where several
179 chips can dissipate heat simultaneously. A multiport dynamic model has
180 been obtained and the model has been implemented as a thermal impedance
181 matrix. Different techniques for multiexponential signal analysis have been
182 reviewed and compared (Jansson deconvolution, semiparametric exponential
183 series method, constrained and free nonlinear least squares models).

184

185 Different methods searching modelling thermal problems have been re-
186 viewed. One way to model them is by using electrical analogy. Thermal
187 network approach can provide rapid simulation (the model scale is much
188 smaller than the FE model) with fairly high accuracy [42, 57]. However,
189 electrical analogy is not currently developed for beam geometries, commonly
190 found on thermal micro-actuators, and having heat transfers through and
191 around length. Next section deals with this thermal analysis via electrical
192 analogy.

193 **4. The electrical model**

194 Properties of electrothermal micro-actuators strongly depend on its geo-
195 metric structure and material properties as well as applied voltage or current.
196 It is therefore important to model the electrothermal micro-actuator so that
197 its design may be improved or optimized. Thermal networks seem sufficiently
198 accurate and powerful tool for simulating real thermal systems and microsys-
199 tems. 3D heat transfers on thermal micro-actuators take place through and
200 around long structures. This kind of geometries and thermal transfers has
201 not been yet modelled using electrical analogy. We propose a new model
202 applied to long structures. It is important to note that this study has been
203 developed thinking particularly in thermal micro-actuators, but it can be ap-
204 plied to other thermal microsystems with the same beam geometries, such as
205 microheaters. Table 2 summarizes equivalences between thermal and electri-
206 cal systems.

207

Table 2: Thermal and electrical analogy

| Thermal system | Equivalent electrical system |
|-----------------------------------|-------------------------------------|
| Heat flux Q | Current i |
| Temperature difference ΔT | Voltage difference ΔV |

208 This section is composed as follow: firstly, we analyse analytically the
209 thermal behavior of the beam; secondly, the thermal equation is solved;
210 thirdly, we propose the dynamic thermal network; and fourthly, the static
211 thermal network is presented.

212 4.1. Dynamic thermal analysis

213 In all the configurations of thermal micro-actuators (multimorph, V-
 214 shaped and U-shaped), heat transfers involve a combination of conduction
 215 and convection effects along structures. These structures and their thermal
 216 behavior can be generalized as the beam shown in Fig. 1(a): a heat flux
 217 $Q(x, t)$ through the beam causes a gradient of temperature between the two
 218 lateral faces at a temperatures T_1 and T_2 , respectively; and a heat convection
 219 Q_h takes place from all other surfaces in contact with the external fluid whose
 220 temperature is T_{ext} .

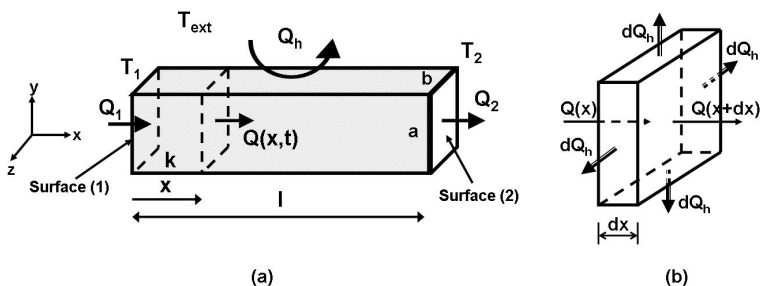


Figure 1: Structure analysed: (a) thermal problem, (b) elementary volume.

221 We consider that the beam has no internal heat sources and the radiation
 222 is neglected. We suppose that convection heat transfer coefficient h is uni-
 223 form on all the surfaces, and the physical properties (thermal conductivity
 224 k , specific heat at constant pressure C_p , density ρ) are constant. The length
 225 l is considered sufficiently larger than the height a and the width b to neglect
 226 the variation of the temperature T along axes y and z . Consequently, the
 227 thermal problem is reduced to a one dimensional problem.

Heat balance for the elementary volume in Fig. 1(b) establishes that dif-
 ference between the incoming heat flux $Q(x)$ and the outgoing heat flux

$Q(x + dx)$ is the sum of the convection heat flux and the stored heat flux which induces a variation of temperature:

$$\frac{\partial Q(x)}{\partial x} dx = Q(x + dx) - Q(x) = -hP dx (T(x) - T_{ext}) + \rho C_p S dx \frac{dT}{dt}, \quad (1)$$

228 where $P = 2a + 2b$ is the lateral perimeter.

The conduction heat flux $Q(x)$ through the element is:

$$Q(x) = -kS \frac{\partial T}{\partial x}, \quad (2)$$

229 where $S = ab$ is the lateral section.

Introducing (2) in (1), the evolution of the temperature $T = T(x, t)$ in the beam satisfies:

$$\alpha \frac{\partial^2 T}{\partial x^2} - \sigma (T - T_{ext}) = \frac{\partial T}{\partial t} \quad (3)$$

$$\text{where } \alpha = \frac{k}{\rho C_p} \quad \text{and} \quad \sigma = \frac{\rho C_p S}{hP}.$$

Moreover, we are considering the following boundary and initial conditions:

$$Q_1 = Q(0, t) = -kS \frac{\partial T}{\partial x}(0, t) \quad (4)$$

$$Q_2 = Q(l, t) = -kS \frac{\partial T}{\partial x}(l, t) \quad (5)$$

$$T(x, 0) = T_{ext} \quad (6)$$

230 Equations (3) to (6) define the thermal behavior of the beam.

231 4.2. Resolution of the thermal equation

The resolution of the thermal problem is done using Laplace transform, where s is the parameter of the Laplace transform. Applying Laplace transform to $T(x, t)$, such as $\theta = \theta(x, s) = \text{Laplace}[T(x, t)]$ and $\theta_{ext} = \text{Laplace}[T_{ext}] =$

$\frac{T_{ext}}{s}$, equation (3) can be written as:

$$\frac{1}{q^2} \frac{d^2\theta}{dx^2} - \theta = -\theta_{ext} \quad (7)$$

$$\text{where } q^2 = \frac{s + \sigma}{\alpha}. \quad (8)$$

In the same way, the Laplace transform is applied to the boundary conditions (4) and (5):

$$\phi_1 = \text{Laplace}[Q_1] = -kS \frac{d\theta}{dx}(0, s) \quad (9)$$

$$\phi_2 = \text{Laplace}[Q_2] = -kS \frac{d\theta}{dx}(l, s) \quad (10)$$

The solution of (7) considering (9) and (10) is thus:

$$\theta(x, s) = \frac{\phi_1 \cosh(ql) - \phi_2}{kSq \cdot \sinh(ql)} \cosh(qx) - \frac{\phi_1}{kSq} \sinh(qx) + \theta_{ext} \quad (11)$$

232 where cosh and sinh are the hyperbolic cosine and the hyperbolic sine, re-
233 spectively.

234 This non-linear equation represents the Laplace transform of the unidi-
235 rectional temperature distribution $T(x, t)$ of any beam.

236 4.3. Dynamic thermal model

To propose the thermal network of the beam, heat flux ϕ_1 and ϕ_2 are first calculate as a function of $\theta_1 = \theta(0, s)$ and $\theta_2 = \theta(l, s)$:

$$\phi_1 = -\frac{\theta_{ext} - \theta_1}{Z_2(q)} + \frac{\theta_1 - \theta_2}{Z_1(q)} \quad (12)$$

$$\phi_2 = -\frac{\theta_2 - \theta_{ext}}{Z_2(q)} + \frac{\theta_1 - \theta_2}{Z_1(q)} \quad (13)$$

where the thermal impedances Z_1 and Z_2 are:

$$Z_1 = \frac{\sinh(ql)}{kSq} \quad (14)$$

$$Z_2 = \frac{\sinh(ql)}{kSq(\cosh(ql) - 1)} \quad (15)$$

To be able to deduce the equivalent electrical model, thermal impedances Z_{1lin} and Z_{2lin} will be found using the infinite series expansion when $(ql) \rightarrow 0$. Based on (8), this limit tends to the consideration of only the slower and predominant dynamics ($s \rightarrow 0$). Considering this limit, equations(14) and (15) tend to:

$$Z_{1lin} = R_{c0} \quad (16)$$

$$Z_{2lin} = \frac{1}{\frac{1}{R_{v0}} + sC_{th0}} \quad (17)$$

where the conduction thermal resistance R_{c0} , the convection thermal resistance R_{v0} and the thermal capacitor C_{th0} are respectively:

$$R_{c0} = \frac{l}{kS} \quad (18)$$

$$R_{v0} = \frac{2}{hPl} \quad (19)$$

$$C_{th0} = \frac{\rho C_p Sl}{2} \quad (20)$$

Inserting (16) and (17) in (12) and (13), and applying the inverse Laplace transform, heat flux are:

$$Q_1 = -\frac{T_{ext} - T_1}{R_{v0}} + C_{th0} \frac{dT_1}{dt} + \frac{T_1 - T_2}{R_{c0}} \quad (21)$$

$$Q_2 = -\frac{T_2 - T_{ext}}{R_{v0}} - C_{th0} \frac{dT_2}{dt} + \frac{T_1 - T_2}{R_{c0}} \quad (22)$$

237 The dynamic thermal network of the beam described in Fig. 1 can be
 238 constructed as schematized in Fig. 2.

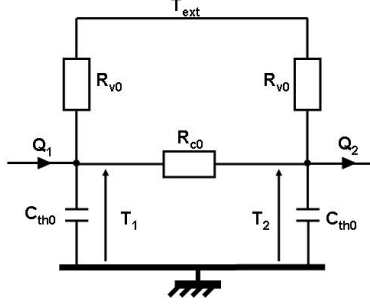


Figure 2: Dynamic thermal network of the beam.

239 4.4. *Static thermal network*

240 Here, we show that the static equivalent electrical model can be obtained
 241 without linearization, contrary to the dynamic model previously described.

Static impedances can be found using the general analytical study above. Impedances Z_1 and Z_2 in (14) and (15) respectively, depends on q , defined in (8). In the static case $s = 0$, static thermal impedances, Z_{1st} and Z_{2st} are:

$$Z_{1st} = \frac{L \sinh(l/L)}{kS} \quad (23)$$

$$Z_{2st} = \frac{L \sinh(l/L)}{kS(1 - \cosh(l/L))} \quad (24)$$

$$\text{where } L^2 = \frac{kS}{hP}$$

Both expressions, (23) and (24) are analytical expressions (not linearized) of static thermal impedances of the beam in Fig. 1. Using them in (12) and (13), analytical expressions of the static heat flux at $x = 0$ and $x = l$ are:

$$Q_{1st} = -\frac{kS}{L \sinh(l/L)} [(T_{ext} - T_{1st})(\cosh(l/L) - 1) + (T_{2st} - T_{1st})] \quad (25)$$

$$Q_{2st} = -\frac{kS}{L \sinh(l/L)} [(T_{2st} - T_{ext})(\cosh(l/L) - 1) + (T_{2st} - T_{1st})] \quad (26)$$

242 Static thermal network is thus deduced as shown in Fig. 3. This model con-
 243 sideres only conduction and convection heat transfers represented respectively
 244 by thermal resistances Z_{1st} and Z_{2st} .

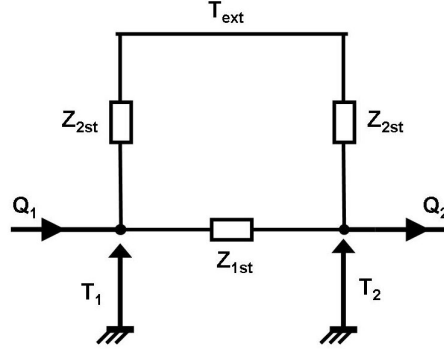


Figure 3: Static thermal network of the beam.

245 Next section deals with the validity of thermal networks obtained from
 246 linearized thermal impedances.

247 5. Validity of thermal networks

248 Thermal impedances Z_{1lin} and Z_{2lin} , and consequently thermal networks,
 249 have a validity condition because they have been obtained thanks to the
 250 linearization of impedances Z_1 and Z_2 . This section deals with the validity
 251 of the impedances, the validity of thermal networks, and the scaling effect of
 252 the validity criteria.

253 5.1. Validity of thermal impedances

Impedances Z_{1lin} and Z_{2lin} are only valid when they are near to the ana-
 lytical impedances Z_1 and Z_2 . Applying a second order Taylor development

to impedances in (14) and (15), they can be written as:

$$Z_1 \approx \frac{l}{kS} \left(1 + \frac{(ql)^2}{6} \right) = Z_{1lin} \left(1 + \frac{(ql)^2}{6} \right) \quad (27)$$

$$Z_2 \approx \frac{2}{kSlq^2} \left(1 + \frac{(ql)^2}{12} \right) = Z_{2lin} \left(1 + \frac{(ql)^2}{12} \right) \quad (28)$$

The dynamic thermal network model is valid if at least:

$$\left| \frac{(ql)^2}{6} \right| \ll 1 \quad (29)$$

In case of an sinusoidal input with an angular frequency $\omega(s = j\omega)$, and remembering (8), condition (29) is expressed by:

$$\omega^2 \ll \omega_v^2 \quad (30)$$

where:

$$\omega_v^2 = \frac{36\alpha^2}{l^4} - \sigma^2 = \frac{36}{l^4} \left(\frac{k}{\rho C_p} \right)^2 - \left(\frac{hP}{\rho C_p S} \right)^2 \quad (31)$$

254 Linear thermal impedances are thus valid only when angular frequency ω is
 255 lower than the characteristic frequency ω_v , which includes all the parameters
 256 of the beam.

257 5.2. Validity of the dynamic thermal network

The objective of this section is to compare the frequency ω_v with the poles of the dynamic thermal network. Based on the dynamic thermal network in Fig. 2, two poles s_1 and s_2 can be calculated:

$$s_1 = -\frac{1}{R_{v0}C_{th0}} \quad (32)$$

$$s_2 = -\frac{R_{c0} + 2R_{v0}}{R_{c0}R_{v0}C_{th0}} \quad (33)$$

The poles are the representation of the characteristic frequencies of the dynamic system, thus considering (30), poles s_1 and s_2 have to respect:

$$|s_1| \ll \omega_v^2, \quad (34)$$

$$\text{and } |s_2| \ll \omega_v^2. \quad (35)$$

Decomposing the terms, conditions (34) and (35) can be defined as a unique condition, which defines the validity of the dynamic thermal network:

$$\nu \ll 1 \quad (36)$$

$$\text{where } \nu = \frac{hPl^2}{(\sqrt{14} - 2)kS} \quad (37)$$

258 Criterion ν establishes the validity zone of the dynamic thermal network.

259 *5.3. Validity of the static results of thermal networks*

The network defined in Fig. 3 which uses Z_{1st} and Z_{2st} is always valid whatever the geometry is. This section deals with the definition of validity domain of the static results obtained with the dynamical network defined in Fig. 2. In this case, the condition of validity (30), knowing that $\omega = 0$, becomes:

$$\delta \ll 1 \quad (38)$$

$$\text{where } \delta = \frac{hPl^2}{6kS} \quad (39)$$

260 Criterion δ establish the validity zone for the static results of the network
261 defined in Fig. 2.

262 *5.4. Scaling effect on validity criteria*

This section is dedicated to the study of the scale effect on this criterion. If d is the characteristic size of the beam, criteria ν and δ are:

$$\delta \sim \nu \sim \frac{hd^3}{kl^2} \sim \frac{h}{k}d \quad (40)$$

Thermal conductivity k is an intrinsic property of the material and is thus independent of scaling effects, so:

$$\delta \sim \nu \sim hd \quad (41)$$

263 In the macroworld, convection coefficient h is considered constant and
264 independent of the scaling effects: criteria ν and δ are proportional to the
265 size d . Consequently, the validity of thermal networks tends to be improved
266 when size reduces. In the microworld, convection coefficient h is considered
267 inversely proportional to size d ($h \sim d^{-1}$), criteria ν and δ are, consequently,
268 independents of size d . This study of the scale effect shows the relevance of
269 our models in the microscale.

270 In the next section, we compare thermal networks to analytical results
271 and FEM simulations.

272 **6. Validation of thermal networks**

273 The objective of this section is to compare our models with analytical
274 and FE models for different values of the criteria δ and ν .

We consider a silicon beam whose physical parameters are defined in Table 3. The external temperature T_{ext} is considered constant at 293 K. The geometrical parameters a and b are constant in all the calculations:

$$a = 200 \text{ }\mu\text{m} \quad \text{and} \quad b = 500 \text{ }\mu\text{m}$$

275 The length l is modified in order to obtain several criteria δ and ν . More-
 276 over, to take into account a large diversity of applications, tests are proposed
 277 in water and air. FEM simulations are done with COMSOL Multiphysics™
 278 3.2. The meshed model consists of triangular elements on the surface and
 279 tetrahedral elements in the volume. The automatic meshing available in
 280 COMSOL Multiphysics has been used to define the meshing. Depending on
 281 the geometry of the beam, the mesh includes 5700 to 8800 elements which
 282 represent between 1300 and 2100 nodes. The thermal network is simulated
 283 using Simulink™ and Matlab™.

Table 3: Physical properties of tested beams.

| Parameter | Value | Unit |
|------------------|--------------|---|
| k | 148 | W.m ⁻¹ .K ⁻¹ [64] |
| h_{air} | 924 | W.m ⁻² .K ⁻¹ [42] |
| h_{water} | 9240 | W.m ⁻² .K ⁻¹ [42, 65] |
| ρ | 2330 | kg.m ⁻³ [64] |
| C_p | 705 | J.kg ⁻¹ .K ⁻¹ [64] |

284 *6.1. Validation of the dynamic thermal network*

In order to validate the thermal network, the heat flux applied $Q_1(t) = Q(0, t)$ and $Q_2(t) = Q(l, t)$ are:

$$Q_1(t) = \begin{cases} 0 & \text{for } t < 0 \\ 0.3 \text{ W} & \text{for } t \geq 0 \end{cases}$$

$$Q_2(t) = \begin{cases} 0 & \text{for } t < 0 \\ -0.1 \text{ W} & \text{for } t \geq 0 \end{cases}$$

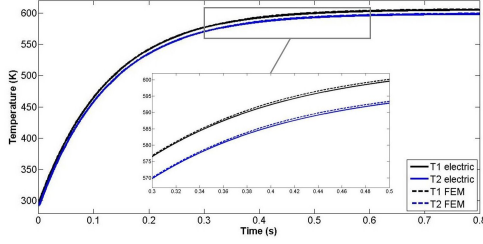
285 The temperatures $T_1(t) = T(0, t)$ and $T_2(t) = T(l, t)$ have been calculated
 286 in air and water using FEM calculations, and compared to the equations
 287 obtained from the thermal network in Fig. 2. Table 4 shows, in air and
 288 water, the criterion ν for two different lengths l , leading to two different
 289 geometries, I and II.

Table 4: Geometries and their validation parameter in air and water.

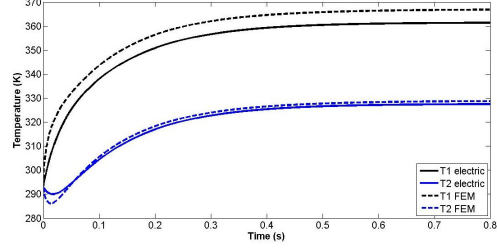
| Geometry | Length (μm) | Criterion ν in air | Criterion ν in water |
|----------|--------------------------|------------------------|--------------------------|
| I | $l = 500$ | 0.01 | 0.1 |
| II | $l = 3000$ | 0.45 | 4.5 |

290 Figures 4(a) and 4(c) present the obtained results with geometry I re-
 291 spectively in both media, air and water, while figures 4(b) and 4(d) illustrate
 292 the results obtained with geometry II respectively in air and water. As the
 293 criterion ν in Fig. 4(a) is better than the criterion ν in Fig. 4(d), thermal
 294 networks curves are closer to FEM curves than in Fig. 4(a).

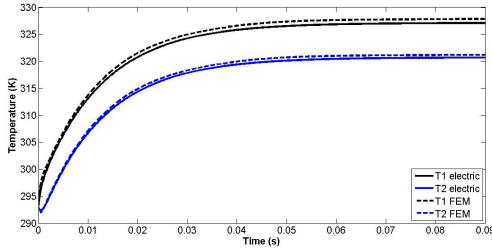
295 These experiments show a good reliability of our model compared to the
 296 FEM calculations and show the relevance of the dynamic criteria ν .



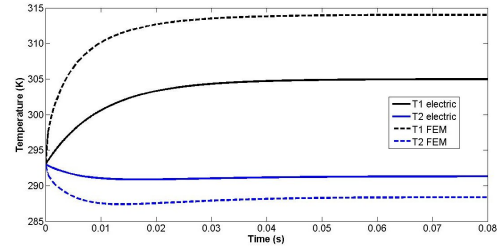
(a) Geometry I in air ($\nu = 0.01$).



(b) Geometry II in air ($\nu = 0.45$).



(c) Geometry I in water ($\nu = 0.1$).



(d) Geometry II in water ($\nu = 4.5$).

Figure 4: T_1 and T_2 for different geometries.

297 *6.2. Dynamic behaviour*

298 The time dependent behaviour is investigated in the angular frequency
 299 (ω) domain. A heat flux $Q_1(t) = \sin(\omega t)$ is imposed, keeping $Q_2(t) = 0$.
 300 Influence of this last, can be deduced by symmetry. The evolution of the
 301 temperatures $T_1(t)$ and $T_2(t)$ obtained by FEM simulations and the thermal
 302 network in Fig. 2 for geometry II in air ($\nu = 0.45$), are plotted in Fig. 5.
 303 These curves confirm the good reliability of the proposed model.

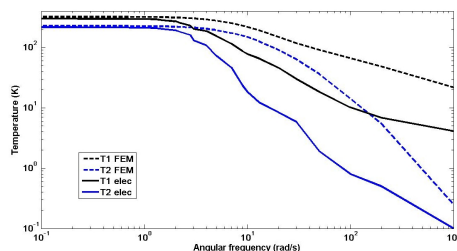


Figure 5: Dynamic behaviour for geometry II in air.

304 *6.3. Validation of the static thermal network*

305 In order to validate the static thermal network, constant temperatures
 306 $T_{1st}=393$ K and $T_{2st}=293$ K are imposed, and heat flux Q_{1st} and Q_{2st} , for
 307 different geometries in water and air, have been calculated using three dif-
 308 ferent models: the static network (SN) in Fig. 3, the static results of the
 309 dynamic network (DN) in Fig. 2, and FEM simulations.

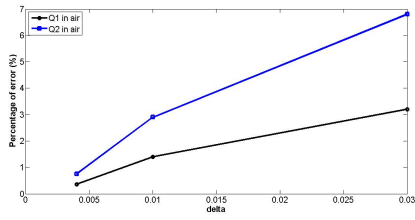
310 The comparison of the values for several cases is presented in Table 5.
 311 Let us note that the criterion δ obtained in (39) defines the validity of the
 312 static results of the dynamic network.

313 First, these results show a good reliability of the static network (SN) com-
 314 pared to FEM simulations whatever the criterion δ is. In fact, the definition
 315 of the impedances Z_{1st} and Z_{2st} has been done without linearization. The
 316 good results obtained comparing either analytical or FEM calculation with
 317 the dynamic or static thermal networks enable us to validate them.

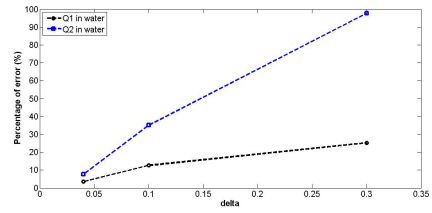
318 Concerning the static results of the dynamic network (DN, as shown in
 319 Fig. 6), the percentage of error is growing when the criterion δ is approaching
 320 to 1. In conclusion, validity criteria δ must also be considered relevant.

Table 5: Comparison of heat flux Q_1 and Q_2 in air and water.

| Length l (μm) | Model | Air | | | Water | | |
|---------------------------------|-------|----------|------------------|------------------|----------|------------------|------------------|
| | | δ | Q_{1st} (W) | Q_{2st} (W) | δ | Q_{1st} (W) | Q_{2st} (W) |
| 500 | SN | | 2.98 | 2.95 | | 3.17 | 2.85 |
| | FEM | | 2.98 | 2.95 | | 3.17 | 2.85 |
| | DN | 0.004 | 2.99 | 2.93 | 0.04 | 3.28 | 2.64 |
| 1000 | SN | | 1.52 | 1.46 | | 1.89 | 1.28 |
| | FEM | | 1.52 | 1.46 | | 1.88 | 1.28 |
| | DN | 0.01 | 1.54 | 1.42 | 0.1 | 2.13 | 0.83 |
| 1500 | SN | | 1.05 | 0.96 | | 1.56 | 0.72 |
| | FEM | | 1.05 | 0.95 | | 1.55 | 0.73 |
| | DN | 0.03 | 1.08 | 0.89 | 0.3 | 1.96 | 0.02 |



(a) In air.



(b) In water.

Figure 6: Percentage of error between the static results of dynamic network and the static network, with respect to δ .

321 **7. Conclusion**

322 Thermal micro-actuators are one of the most promising methods to induce
323 movements in micromechatronics. The different configurations (multimorph,
324 U-shaped and V-shaped) have a common geometry: long structures. As the
325 optimization and control of their heat transfers are a crucial part to perform
326 micro-actuators, we propose to model these systems using electrical analogy.
327 To our knowledge, until now, only classical thermal networks have been de-
328 veloped for thermal systems like walls. In this paper, we develop the dynamic
329 and static equivalent electrical models of long structures. The validity of both
330 thermal networks, dynamic and static, have been also established. Finite ele-
331 ments and analytical results have been compared with results obtained using
332 our thermal networks, in order to validate them. In all the studied cases,
333 they appear sufficiently accurate. Moreover, as the thermal parameters of
334 the proposed thermal networks are defined by the physical properties of the
335 studied structures, constraints and dependences of these properties in rela-
336 tion to other properties could be directly addressed by the thermal networks.
337 The compilation of elementary thermal networks proposed in this paper has
338 been done in order to model a complex thermal microdevice [66].

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