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► **To cite this version:**

Thi Phuong Khanh Nguyen, Bruno Castanier, Thomas G. Yeung. Optimal Maintenance and Replacement Decisions under Technological Change. European Safety and Reliability 2010 (ESREL 2010), Sep 2010, Greece. pp.1430-1437. hal-00538529v1

**HAL Id: hal-00538529**

**<https://hal.science/hal-00538529v1>**

Submitted on 23 Nov 2010 (v1), last revised 1 Dec 2010 (v2)

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# Optimal Maintenance and Replacement Decisions under Technological Change

P.Khanh Nguyen Thi, Thomas G. Yeung & Bruno Castanier

*Ecole des Mines de Nantes/IRCCyN, 44307 Nantes, France*

**ABSTRACT:** Classical spare parts inventory models assume that the same vintage of technology will be utilized throughout the planning horizon. However, replacement often occurs in the form of a new technology. A consequence of such replacements is the rendering of existing spare parts inventories obsolete and the necessity to purchase a new set. This paper aims to study the impact of spare parts inventory on maintenance and replacement decisions under technology change. We model this problem as a Markov decision process where the decisions are made as a function of the state of the asset itself, the technological environment, and the current spare parts inventory. The actions available to the decision maker are to do nothing, imperfect/perfect maintenance, or replace the asset. The replacement decision is complex in that one must decide with which technology available on the market to replace the current asset. Under technological change, the do nothing and repair options have significantly more value as they allow for the appearance of even better technologies in the future.

## 1 INTRODUCTION

Maintenance is a key in ensuring the efficient use of equipment as well as efficient production process. If machines often break down or operate in bad condition, this may cause lower product quality, increasing energy consumption, delivery delay, lost customers, contract penalties, and reduced revenue. Therefore, the maintenance now is not simply overcome the failures, that have become the process to predict, prevent loss at management level. Managers need to analyze all relevant information to assess the profitability of equipment, to give reasonable investment decision, and to consider the possibility of saving charges. In particular, high spare parts inventory is one important factor causing increased maintenance costs. However, lack of inventory increases machine downtime due to waiting time when no spare parts needed in stock. Hence, consideration of spare parts inventory problems is essential task of managers.

There is an intensive research to study the different aspects of spare parts inventory problems such as: management issues (methods of organizing and reducing spares), multi-echelon problems, age-based replacement, repairable spare parts, problems involving obsolescence and special application (Kennedy et al. 2002). As the author's comment, spare parts inventories totally differ from other manufacturing inventories. Its function is to assist a maintenance staff in keeping equipment in operating condition. The close relation between spare parts inventories and maintenance was discussed in several articles. Kabir & Al-Olayan. 1996 study a joint optimization of age based replacement and spare parts inventory

policy (S, s). Consider a block replacement strategy, Vaughan. 2005 utilize a dynamic programming to solve the spare parts ordering problem while Sarker & Haque. 2000, Chelbi & Ait-Kadi. 2001 present management policies for a manufacturing system, aiming to optimize the maintenance strategy with continuous review spare items inventory jointly. Sheu & Chien. 2004 and Chien. 2009 extend the problem by studying also minimal repair at minor failure while Destombes et al. 2006 consider repair capacity of degraded/ failed units after they was replaced by spare parts. However, all the above models are constructed on assumption that the same vintage of technology will be utilized throughout the planning horizon. They do not allow us to take into account the appearance of new technology with lower operating and maintenance costs, smaller failure rates, higher quality and output rates. This information is important for managers to decide the replacement investment plan. On the other side, the models devoted to the spare parts inventory problem involving obsolescence are generally based on the observations of the economical performance of the maintenance process

These reasons motivate us to develop an appropriate model to meet the management's requirements: optimization of maintenance cost, simultaneously, updating information on the technological development, and consider the impact of spare parts inventory levels to make reasonable investment decisions.

This paper is structured as follows: In Section 2, a related literature is presented to motivate the present work. Section 3 is devoted to the mathematical for-

mulation. In Section 4, the performance of our model is discussed through numerical examples. Finally, a conclusion and future work are discussed in Section 5.

We formulate a discrete time non-stationary Markov Decision Process to determine the optimal action policy. The set of decisions that made as a function of the state of the asset itself, the technological environment, and the current spare parts inventory are: to do nothing, imperfectly/ perfectly repair, or replace the asset. The similar model with non-stationary technological appearance's probability in time was proposed by Nair (1995, 1997). However, in these papers, the author only considers replacement problem, and not examining the impact of inventory levels as our model. On the other hand, unlike Hopp & Nair (1994) reviewing unique challenger, we study a technological sequence.

To model sequential technological evolution, we combine the geometric model and uncertain apparition model of technology. The geometric technological evolution model is presented by Borgonovo et al. (2000), Smith et al. (2003), Karsak & Tolga (1998), Hritoneko & Yatsenko (2007, 2008a, b), Bethuynne (2002). But except Borgonovo et al. (2000), the rest study the problem without parametric degradation. They utilize the geometric model to form the cost functions in vintage equipment or in time. Unlike these articles, we present technology change by the improvement of the expected deterioration rate. Moreover, our profit or maintenance cost functions are only dependent on degradation state. As the expected degradation rate of equipments is improved over technology generation, accrued profit and maintenance cost will be dependent on technology generation.

In addition, we also consider non-stationary likelihood of new technology's apparition over time. Thereby, we overcome the disadvantages of the geometric model proposed by Borgonovo et al. (2000). In that article, the failure rate decreases exponentially over time, i.e. at any time, a machine can be replaced by new one which operates better with its reliability parameters determined at that time. In reality, this assumption is unreasonable because technical characteristics of the equipment cannot always be changed over time. It changes only at the concurrent instant of a new technological generation. Recall that Nair (1995, 1997) also considers the non-stationary probability of the appearance of new technologies. But in his model, Nair focuses on the problem of capital investment decisions due to technological change rather than physical deterioration of equipment. To simplify its exposition, he also does not consider salvage values while we establish the reasonable salvage value function which depends on its mean residual life and the purchase price of identical technology at this time.

## 2 MODEL FORMULATION

### 3.1 Problem generalization

Consider a repairable machine which is accompanied by a cargo of  $n_0$  spare parts. They are utilized for the maintenance process of the machine and are not sold separately in the market, i.e, we cannot replenish the spare parts store when it is empty. This is common assumption for special spare parts of machine because it can be difficult and costly to find original and compatible spares to replace degraded units. Spare parts inventory level is a function of the choice of maintenance action: imperfect and perfect maintenance.

The machine that operates continuously from the new state,  $X = 0$  is characterized by its expected deterioration rate. In the failure state, denoted  $m$ , the machine continues to operate but unprofitably. To reveal the deterioration and the spare parts inventory level, periodic inspections are performed. The inter-inspection interval,  $\tau$ , defines the decision epochs.

We assume that only one new technology can appear in a decision interval,  $\tau$ . We introduce  $p_{i+1}^{k+1}$ , the non stationary probability that technology  $k+1$  appears in the interval  $\tau$  given the latest available technology at decision epoch  $i$  is  $k$ . The difference in the technological generations is modeled by an improvement factors on the expected instantaneous deterioration rates and the accrued profit within a decision period.

Let  $(x, k, j, s)$  be the system state at the beginning of the  $i^{\text{th}}$  decision epoch with  $s$  spare parts stocked for maintenance process of the machine having deterioration level  $x$ . That machine belongs among the technological generation  $j$  while the latest available technology in the market is  $k$ ,  $k \geq j$ . Then, the maintenance decisions are restricted to:

1) Do nothing (*DN*): The machine continues to deteriorate until next decision epoch and generates a profit  $g_j(x)$ . Note that  $g_j(x)$  is the expected profit within a period when the deterioration state at the beginning of that period is  $x$  and the utilized technology is  $j$ . The spare parts inventory level is not change, so the holding cost within this period is  $sc_s$  with  $c_s$  is the holding cost per spare part unit in a decision period.

2) Imperfect maintenance (*IM*) which allows to restore the machine in a given deterioration level,  $\max(0, x-d)$  where  $d$  models the maintenance efficiency. An imperfect maintenance cost,  $c_M$  is incurred and a spare part is utilized to replace the degraded unit, so the spare part inventory level is reduce by one unit:  $s - 1$ . As we assume that the maintenance time is negligible, then, in the next decision interval, the machine deteriorates from the level  $\max(0, x-d)$  and generates a profit  $g(\max(0, x-d))$ .

3) Perfect maintenance (*PM*) which allows to restore the machine in the initial deterioration level  $X = 0$ , so the expected profit within next decision interval is  $g(0)$ . This action requires  $n(x)$  spare parts that depend on the deterioration state of the machine. Simultaneously, a perfect maintenance cost  $c_{PM}$  is incurred ( $c_{PM} > c_M$ ).

Note that as we assume the spare parts are just supplied when we buy a new machine, hence we can only execute maintenance actions if and only if there are sufficient numbers of spare parts in store.

4) Replace (*R*) the machine by the available technology  $h$  in the market ( $j \leq h \leq k$ ). The replacement time is also negligible. A cargo of  $n_0$  spare parts is supplied with the new machine. We assume the spares are only compatible with the same generation machine, hence, after replacement, the spare parts inventory level is  $n_0$  if we decide to replace by new-generation machine and equals  $n_0 + s_t$  in the case of replacement by same-generation machine. The cost of such a replacement is given by the difference between the purchase price of the new machine  $c_{i,h}$  and the salvage value  $b_{i,j,k}(x)$ . Note that the purchase price of new technological machine can be estimated as the deterioration rates. This is realistic in case where the technical parameters and specifications of future designs may be know beforehand. We use an increasing function of technology and decreasing over time to define the purchase price. The salvage value is proportional to the purchase price of technology  $j$  at this time, decreasing in the remaining lifetime and greatly reduces in the obsolete case. In the  $i^{th}$  decision period after the replacement, the new machine generates a profit  $g_h(0)$ .

In case of failure, the do nothing action is still allowed but the profit in the next decision epoch is assumed to be negative  $g(m) < 0$ .

## 2.2 Decision criteria formulation

In this paper, we use a non-stationary MDP formulation to find the optimal maintenance-replacement policy to maximize the expected discounted value-to-go over the finite horizon time denoted by  $V^\pi(w)$  with  $w$  is the state of system. If the last decision period is  $N$ , at decision epoch  $N+1$ , we do not make any decision and the maximum expected discounted value-to-go from the decision epoch  $N+1$  over the infinite horizon is  $V_{N+1}(w) = 0$  ( $\forall s \in S$ : state space of system).

Let  $V_i(x, k, j, s)$  denote the maximum expected discounted value from the decision epoch  $i$ , ( $k \leq i$ ) to the last epoch  $N$ . Then,  $V_1(w) = V^\pi(w)$ .

$$V_i(x, k, j, s) = \max \{ DN_i(x, k, j, s), IM_i(x, k, j, s), PM_i(x, k, j, s), R_i^h(x, k, j, s) \} \quad (1)$$

where  $DN_i$ ,  $IM_i$ ,  $PM_i$ ,  $R_i$  are alternately choice to do nothing, to imperfectly maintain, to perfectly maintain and to replace at decision epoch  $i^{th}$ . We have:

$$DN_i(x, k, j, s) = g_j(x) - sc_s + \lambda \left[ \sum_{\forall k' \in \{k, k+1\}} \sum_{\forall x' \in [x, \zeta]} p_i^{k'} p_j(x'|x) V_{i+1}(x', k', j, s) \right] \quad (2)$$

$$IM_i(x, k, j, s) = \left( -c_M + DN_i(\max\{0, x-d\}, k, j, s-1) \right) \mathbf{1}_{\{s \neq 0\}} \quad (3)$$

$$PM_i(x, k, j, s) = (-c_{PM} + DN_i(0, k, j, s - n(x))) \mathbf{1}_{\{s \geq n(x)\}} \quad (4)$$

$$R_i^h(x, k, j, s) = b_{i,j,k}(x) - c_{i,h} + DN_i(0, k, j, s + n_0) \mathbf{1}_{\{h=j\}} + DN_i(0, k, h, n_0) \mathbf{1}_{\{h \neq j\}} \quad (5)$$

$\lambda$ : discount factor;  $\lambda \in [0, 1]$ .

## 2.3 Transition probabilities

To compute the transition probabilities, we propose to discretize the deterioration state of machine as follows. Let  $z$  denote the discrete deterioration state at the beginning of the current decision period.  $z$  is the first value of  $N_X$  discrete intervals of length  $l$  on  $[0, \zeta]$  (which  $\zeta$  is the failure threshold of the machine). That is to say, if the deterioration state ( $x$ ) at the beginning of current decision period belongs to the intervals  $([0, l], [l, 2l], [2l, 3l], \dots, [(N_X-1)l, \zeta])$ , we approximate  $x$  by  $z \in \{0, l, 2l, 3l \dots (N_X-1)l\}$  and when the deterioration state ( $x$ ) at the beginning of current decision period is exceed the failure threshold ( $x \geq \zeta$ ), we use  $m$  to present failure state of the machine. The deterioration state of the machine ( $x$ ) is approximated by  $z, z \in \{0, l, 2l, 3l \dots (N_X-1)l, m\}$ . Then, after preventive maintenance, the deterioration state is reduced by a determined amount of deterioration units  $d$ . The transition probability is:

$$p_i(w'|w) = p_j(x'|x) p_{i+1}^{k'} \quad (6)$$

where  $x' \in \{z, z+l, \dots, (N_X-1)l, \zeta\}$ ;  $k' \in \{k, k+1\}$

Recall that the deterioration state of the machine at the next decision epoch depends only on its deterioration state at the current epoch decision and the technological generation of this machine, denoted by  $p_j(x'|x)$ ; and  $p_{i+1}^{k'}$  is the appearance probability of the

next technological generation ( $k+1$ ) at the next decision epoch ( $i+1$ ) with  $k' = k+1$  or inversely, it is the non appearing probability with  $k' = k$ .

$$p_j(x' | x) = \int_{z'-z}^{z'-z+l} f_j(y) dy \quad (7)$$

with  $z, z' \in \{0, l, 2l, 3l \dots (N_X-1)l\}$  and  $f_j(y)$  is the probability density function of the deterioration process of the machine's generation  $j^{\text{th}}$  within the decision period  $\tau$ . Similarly,

$$p_j(m | x) = \int_{\zeta-z}^{\infty} f_j(y) dy \quad (8)$$

### 3 NUMERICAL EXAMPLES

In this section, we present numerical examples to illustrate the performance of our model.

#### 3.1 Input parameters

##### 4.1.1 The appearance probability of new technology

We define the appearance probability of new technology  $k+1$  at decision epoch  $i+1$ , given the latest available technology at decision epoch  $i$  is  $k$ , as a time increasing function:

$$p_{i+1}^{k+1} = (1 - \delta \varepsilon)^{i-k} \quad (8)$$

$\delta$  is the factor that reflects the non-appearance probability of next generation ( $k+1$ ) at next decision epoch ( $i+1$ ) when the latest available technology at the current epoch  $i$  is  $k$  and  $k \equiv i$ . The smaller  $\delta$  is, the greater appearance probability is. And  $\varepsilon$  is the factor characterized the increasing rate of the appearance probability of new technology over time; ie. if the technological generation  $k+1$  is not appear at decision epoch  $i+1$ , then it can appear at the next decision epoch ( $i+2$ ) with probability  $1 - \delta \varepsilon$  given the appearance probability of ( $k+1$ ) at ( $i+1$ ) is  $1 - \delta$ . We have:  $\delta, \varepsilon \in [0, 1]$ .

##### 4.1.2 Deterioration process

We consider the machine whose degradation process is modeled by the Gamma distribution: Gamma processes are often used to model the equipment's degradation (Van der Weide et al. 2007, Van Noordwijk. 2009).

In any decision period, the increments of deterioration  $X_i(i+1) - X_i(i)$  are independent, identical, and follow the stationary Gamma distribution with shape parameter  $\alpha_j \tau$  (recall that  $\tau$  is length of a decision period) and scale parameter  $\beta$ . The probability density function of the deterioration process of the machine's generation  $j$  in decision period  $\tau$  is:

$$f_j(x) = \frac{\beta^{\alpha_j \tau}}{\Gamma(\alpha_j \tau)} x^{\alpha_j \tau - 1} e^{-\beta x} \quad (9)$$

where  $\beta$  is a constant and a discussion for the improvement of  $\alpha_j$  is given in the next paragraph.

##### 4.1.3 Impacts of the technological evolution

As we have assumed, the technological evolution aims to improve degradation characteristics, and specifically the expected degradation rate. In case of stationary gamma processes, this expected degradation rate is directly proportional to the shape parameter  $\alpha_j$ . We model the impact of the technological evolution with the following decreasing exponential geometric function:

$$\alpha_j = a e^{-\kappa(j-1)} + b \quad (10)$$

where  $\kappa, a, b$  are constants;  $j \geq 1$ .

Due to technological development, the deterioration rate of the machine is improved. It is convergent to the critical value,  $b$ , but the deterioration could not be excluded. We choose arbitrarily  $\kappa, a, b$  such as values in Table 1.

Additionally, under technological evolution, the purchase price of a new machine is assumed to be decreasing over time and normally increasing over technological generation:

$$c_{i,k} = c_{1,1} v^{i-1} u^{k-1} \quad (11)$$

where  $c_{1,1}$  is the purchase price of the first of technological generation at the first decision epoch;  $v$  is a constant, characterizing the decrease of purchase price over time ( $v \leq 1$ ) and  $u$  is constant, characterizing the change of the purchase price over technological generation. We choose arbitrarily  $c_{1,1}, v, u$  such as values in Table 1, then  $c_{i,k}$  is given in Table 2.

We assume the salvage value is a function of the current purchase price of this technology at this decision epoch, and the Mean Residual Lifetime (*MRL*). According to the degradation assumptions, if  $x$  is the observed state, we define the *MRL*( $x$ ) as the expected number of decision epoch from the current decision epoch until the failure. In case of stationary gamma processes, the mean deterioration rate on a decision epoch is constant and equals to  $\alpha_j \tau / \beta$ . Hence, we have:

$$MRL(x) = \frac{\beta}{\alpha_j \tau} (\zeta - x). \quad (12)$$

Then, we propose the following function for the salvage value,  $\forall x \in [0, \zeta]$

$$b_{i,j}(x) = h c_{i,j} [1 - \exp(-r MRL(x))]$$

$$= hc_{i,j} [1 - \exp(-r \frac{\beta}{\alpha_j \tau} (\zeta - x))] \quad (13)$$

$h, r$  are constant.

#### 4.1.4 The profit and maintenance cost function

We know that the machine will operate less efficiently when its deterioration state is greater. Therefore, the expected accrued profit function in a decision period  $\tau$  is decreasing by deterioration state and the greater the deterioration state is, the faster the decreases of the profit function is. To reflect this nature, we use a decreasing concave function of deterioration state  $x$  to characterize the accrued profit.

$$g(x) = g_0 - r_g \exp(r \frac{\beta}{\alpha_1 \tau} x) \quad (14)$$

$\forall x \in [0, \zeta]$ ;  $g_0, r_g$  are constant.

On the contrary, the greater the deterioration state is, the faster the increase of the maintenance cost function is. Therefore, we use an increasing convex maintenance cost function.

$$c_M(x) = c_0 + r_c \exp(r \frac{\beta}{\alpha_1 \tau} x) \quad (15)$$

$\forall x \in [0, \zeta]$ ;  $c_0, r_c$  are constant.

Table 1. The input parameters for the Example 1

Appearance probability	$\delta$	$\varepsilon$			
	0.8	0.96			
Profit & Discount factor	$g_0$	$r_g$	$\lambda$		
	213.2	1.2	0.8		
Maintenance & Failure threshold	$d$	$c_0$	$r_c$	$\zeta$	
	1.4	3.322	0.178	20	
Deterioration process	$\beta$	$a$	$b$	$\kappa$	$N_x$
	2.22	3	0.4	0.4	100
Salvage value & Purchase price	$h$	$r$	$c_{i,j}$	$v$	$u$
	0.8	0.4	100	0.98	1.05

Table 2. The purchase price for the Example 1 of the technology  $k$  at decision epoch  $i$ ,  $N = 5$ .

$i$	$c_{i,1}$	$c_{i,2}$	$c_{i,3}$	$c_{i,4}$	$c_{i,5}$
1	100				
2	98	102.9			
3	96.04	100.84	105.88		
4	94.12	98.83	103.77	108.95	
5	92.24	96.85	101.69	106.78	112.11

### 3.2 Analysis from numerical experiments

#### 4.2.1 Basic properties of the optimal policy.

The optimal policy for the Example 1 is given in Table 3. For each decision epoch  $i$ , with the used technology  $j$ , given the generation  $k$  is the latest available technology, the decision matrix defines the optimal

maintenance decision according to the current deterioration  $x$ : Do nothing if  $x \in [x_1, x_2)$ , maintain if  $x \in [x_2, x_3)$  and replace with the new technology if  $x \in [x_3, x_4]$ .

Table 3. The optimal policy in Example 1,  $N = 5$

$i$	$k, j$	Do Nothing	Maintenance	Replacement
		$[x_1, x_2)$	$[x_2, x_3)$	$[x_3, x_4]$
1	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
2	1, 1	[0, 2.3)	[2.3, 7.7)	[7.7, 20]
	2, 1	[0, 2.3)	[2.3, 8.3)	[8.3, 20]
3	2, 2	[0, 2.9)	[2.9, 7.7)	[7.7, 20]
	1, 1	[0, 3.7)	[3.7, 8.3)	[8.3, 20]
	2, 1	[0, 3.7)	[3.7, 9.1)	[9.1, 20]
	2, 2	[0, 4.3)	[4.3, 8.3)	[8.3, 20]
	3, 1	[0, 3.7)	[3.7, 9.9)	[9.9, 20]
4	3, 2	[0, 4.3)	[4.3, 9.1)	[9.1, 20]
	3, 3	[0, 4.5)	[4.5, 8.5)	[8.5, 20]
	1, 1	[0, 5.9)	[5.9, 9.5)	[9.5, 20]
5	2, 1	[0, 5.9)	[5.9, 10.5)	[10.5, 20]
	2, 2	[0, 6.1)	[6.1, 9.3)	[9.3, 20]
	3, 1	[0, 5.9)	[5.9, 11.3)	[11.3, 20]
	3, 2	[0, 6.1)	[6.1, 10.3)	[10.3, 20]
	3, 3	[0, 6.3)	[6.3, 9.3)	[9.3, 20]
	4, 1	[0, 5.9)	[5.9, 11.9)	[11.9, 20]
	4, 2	[0, 6.1)	[6.1, 11.1)	[11.1, 20]
	4, 3	[0, 6.3)	[6.3, 10.3)	[10.3, 20]
	4, 4	[0, 6.5)	[6.5, 9.7)	[9.7, 20]
	1, 1	[0, 10.1)	[10.1, 12.9)	[12.9, 20]
5	2, 1	[0, 10.1)	[10.1, 13.7)	[13.7, 20]
	2, 2	[0, 10.1)	[10.1, 11.7)	[11.7, 20]
	3, 1	[0, 10.1)	[10.1, 14.5)	[14.5, 20]
	3, 2	[0, 10.1)	[10.1, 12.9)	[12.9, 20]
	3, 3	[0, 10.1)	[10.1, 11.5)	[11.5, 20]
	4, 1	[0, 10.1)	[10.1, 15.3)	[15.3, 20]
	4, 2	[0, 10.1)	[10.1, 13.7)	[13.7, 20]
	4, 3	[0, 10.1)	[10.1, 12.5)	[12.5, 20]
	4, 4	[0, 10.1)	[10.1, 11.5)	[11.5, 20]
	5, 1	[0, 10.1)	[10.1, 15.9)	[15.9, 20]
	5, 2	[0, 10.1)	[10.1, 14.5)	[14.5, 20]
	5, 3	[0, 10.1)	[10.1, 13.3)	[13.3, 20]
	5, 4	[0, 10.1)	[10.1, 12.5)	[12.5, 20]
	5, 5	[0, 10.1)	[10.1, 11.7)	[11.7, 20]

We find that the optimal policy for Example 1, given in Table 3 has some basic properties:

- 1) The maintenance threshold ( $x_2$ ), i.e. the first time where the optimal policy prescribes to maintain across deterioration state  $x$ , depends only on the technological generation of the used machine  $j$ . Consider, for example, at decision epoch  $i = 3$ , the used technology  $j = 1$ , for which the optimal policy prescribes maintenance from the deterioration state  $x_2 = 3.7$  despite the latest available technology  $k = 1, 2$ , or 3. Moreover, the greater the used technology is, the higher the threshold is,

because deterioration rate is improved under technological development. For example, at  $i = 3$ ,  $k = 3$ , this threshold is:  $x_2 = 3.7; 4.3; 4.5$  for the used technology  $j = 1, 2, 3$  respectively.

2) The replacement threshold ( $x_3$ ) is non-decreasing in the difference between the latest available technology and the used technology because the purchase price is increasing over technological generation. For example, at decision epoch  $i = 3$ , when the latest available technology is  $k = 3$ , the replacement threshold is 9.9; 9.1; 8.5 for the technology used is  $j = 1, 2, 3$  respectively. Certainly, this threshold depends also on used technological generation ( $j$ ). It is non-decreasing in the used technology  $j$ . For example, at the decision epoch  $i = 3$ , when the used technology ( $j$ ) is also the latest technology available ( $k$ ), the replacement threshold is 8.3; 8.3; 8.5 for  $j = k = 1, 2, 3$  respectively.

These properties are maintained even if the finite horizon  $N$  is large enough. Consider, Example 2 with the input parameters as Example 1: the optimal policy for the first three decision epochs in planning horizon  $N = 20$  is given in Table 4.

Table 4. The optimal policy for the first three decision epochs in planning horizon  $N = 20$ .

$i$	$k, j$	Do Nothing [ $x_1, x_2$ )	Maintenance [ $x_2, x_3$ )	Replacement [ $x_3, x_4$ ]
1	1, 1	[0, 1.5)	[1.5, 7.3)	[7.3, 20]
2	1, 1	[0, 1.5)	[1.5, 7.1)	[7.1, 20]
	2, 1	[0, 1.5)	[1.5, 7.3)	[7.3, 20]
	2, 2	[0, 1.7)	[1.7, 7.1)	[7.1, 20]
3	1, 1	[0, 1.5)	[1.5, 7.1)	[7.1, 20]
	2, 1	[0, 1.5)	[1.5, 7.1)	[7.1, 20]
	2, 2	[0, 1.7)	[1.7, 7.1)	[7.1, 20]
	3, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
	3, 2	[0, 1.7)	[1.7, 7.5)	[7.5, 20]
	3, 3	[0, 1.9)	[1.9, 7.3)	[7.3, 20]

#### 4.2.2 Influence of the technological improvement parameter on the optimal policy

Recall that technological development is characterized by the improvement of the deterioration rate and the change of the purchase price  $c_{i,k}$ . Now, we consider the influence of these parameters on the optimal maintenance-replacement policy.

Note that the characterization of purchase price is represented by equation:  $c_{i,k} = c_{1,1} v^{i-1} u^{k-1}$  where  $u$  is parameter that reflects directly the change of purchase price under technological development. When  $u > 1$ , the purchase price is increasing in technological generation, inversely,  $u < 1$  this is the case where the technological improvement contributes to reduce the purchase price, and  $u = 1$  is the case where the technological change does not influence on the purchase price.

As illustrated by the numerical examples in planning horizon  $N = 20$ , with  $u = 0.95, 1$  and  $1.05$ , consider the first three decision epochs (Table 5), we find that at the first decision epoch, the smaller  $u$  is, the higher the replacement threshold ( $x_3$ ) is.  $x_3 = 7.7; 7.5; 7.3$  respectively. The firm tends to keep the machine used for waiting the appearing of new technology. In the case where the new technology was available on the market, the firm tends to replace earlier when  $u$  is smaller. For example, at decision epoch  $i = 2$ , given  $j = 1$  and  $k = 2$ , the replacement threshold is 7.3, 6.9, 6.5 for  $u = 1.05, 1, 0.95$  respectively.

Specially, in the obsolete case, when the firms decide to replace early, the optimal policy can be non-monotone with respect to the DN and DM across deterioration  $x$  for given  $k, j$ . For example, with  $u = 0.95$ , at decision epoch  $i = 2$ , for  $k = 2, j = 1$ , the optimal policy prescribes do nothing until  $x = 2.5$ , maintain from  $x = 2.5$  to 4.7 and then do nothing again at  $x = 4.7$  until 6.5 (Table 6).

Table 5. The replacement threshold for the first three decision epochs in planning horizon  $N = 20$  with  $u = 0.95; 1; 1.05$

$i$	$k, j$	$u = 0.95$	$u = 1$	$u = 1.05$
1	1, 1	7.7	7.5	7.3
2	1, 1	7.7	7.5	7.1
	2, 1	6.5	6.9	7.3
	2, 2	7.1	7.1	7.1
3	1, 1	7.7	7.5	7.1
	2, 1	6.5	6.9	7.1
	2, 2	7.1	7.1	7.1
	3, 1	5.5	6.5	7.5
	3, 2	6.1	6.5	7.5
	3, 3	6.7	6.9	7.3

Table 6. The optimal policy for the first three decision epochs in the planning horizon  $N = 20$  with  $u = 0.95$

$i$	$k, j$	Do Nothing	Maintenance	Replacement
1	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
2	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
	2, 1	[0, 2.5)	[2.5, 4.7)	[6.5, 20]
		[4.7, 6.5)		
	2, 2	[0, 1.9)	[1.9, 7.1)	[7.1, 20]
3	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
	2, 1	[0, 3.1)	[3.1, 4.1)	[6.5, 20]
		[4.1, 6.5)		
	2, 2	[0, 1.9)	[1.9, 6.9)	[7.1, 20]
		[6.9, 7.1)		
	3, 1	[0, 5.5)	-----	[5.5, 20]
	3, 2	[0, 6.1)	-----	[6.1, 20]
	3, 3	[0, 2.1)	[2.1, 6.7)	[6.7, 20]

Now, we will consider how the improvement of the deterioration rate influences the optimal policy. Recall that the shape parameter of stationary Gamma

function of deterioration process is represented by the decreasing exponential geometric function (equation 10); where  $\kappa$  characterizes directly the improvement of the deterioration rate.

We implement numerical examples in planning horizon  $N = 20$  with  $\kappa = 0.4, 1, 1.5$  and obtain the results in Table 7. We find that when  $j = k$ , the replacement threshold is non decreasing in  $\kappa$ . Specially, at the first decision epoch, the replacement threshold is increasing in  $\kappa$ , because the firms tend to replace later for waiting the new technology when the improvement of deterioration rate is more efficient ( $\kappa$  is increasing). Consider, at  $i = 1$ , the shape parameter of the first technological generation is the same as in the case where  $\kappa = 0.4, 1, 1.5$ , then, the replacement threshold is 7.3, 7.5, 7.7, respectively. The case where the obsolete problem appears ( $j < k$ ) is more complex. The replacement threshold is non-monotonic in  $\kappa$ . For example, at  $i = 3$ , the replacement threshold is decreasing in  $\kappa$ , for  $(k = 2, j = 1)$  or for  $(k = 3, j = 1)$ , but it is increasing in  $\kappa$  for  $(k = 3, j = 2)$ .

Moreover, this parameter ( $\kappa$ ) influences also on the maintenance policy such as: the increase of the maintenance threshold ( $x_2$ ) in  $j > 1$  and the appearing of the non-monotone property with respect to the DN and DM across deterioration  $x$  for given  $k, j$ . As illustrated by Table 8, with  $\kappa = 1.5$ , at decision epoch  $i = 2$ , for  $k = 2, j = 1$ , the optimal policy prescribes do nothing until  $x_2 = 1.5$ , maintain from  $x = 1.5$  to 5.9 and then do nothing again from  $x = 5.9$  to 6.3.

Table 7. The dependence of the replacement threshold on  $\kappa$  in planning horizon  $N = 20$

$i$	$k, j$	$\kappa = 0.4$	$\kappa = 1$	$\kappa = 1.5$
1	1, 1	7.3	7.5	7.7
2	1, 1	7.1	7.5	7.5
	2, 1	7.3	6.5	6.3
	2, 2	7.1	7.3	7.3
3	1, 1	7.1	7.3	7.5
	2, 1	7.1	6.5	6.3
	2, 2	7.1	7.1	7.3
	3, 1	7.5	6.9	6.7
	3, 2	7.5	7.7	7.9
	3, 3	7.3	7.5	7.5

Table 8. The optimal policy for the first three decision epochs in the planning horizon  $N = 20$  with  $\kappa = 1.5$

$i$	$k, j$	Do Nothing	Maintenance	Replacement
1	1, 1	[0, 1.5)	[1.5, 7.7)	[7.7, 20]
2	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]
	2, 1	[0, 1.5)	[1.5, 5.9)	
		[5.9, 6.3)		[6.3, 20]
	2, 2	[0, 2.1)	[2.1, 7.3)	[7.3, 20]
3	1, 1	[0, 1.5)	[1.5, 7.5)	[7.5, 20]

2, 1	[0, 1.5)	[1.5, 5.9)	
	[5.9, 6.3)		[6.3, 20]
2, 2	[0, 2.1)	[2.1, 7.3)	[7.3, 20]
3, 1	[0, 1.5)	[1.5, 6.7)	[6.7, 20]
3, 2	[0, 2.1)	[2.1, 7.9)	[7.9, 20]
3, 3	[0, 2.1)	[2.1, 7.5)	[7.5, 20]

## 4 CONCLUSION

In this paper, we proposed a model that allows us to consider both the investment and the maintenance problem of the stochastic deterioration system under the technological development. It determines the maintenance strategy from the operator's point of view, based on parametric performance of system. In addition, it allows the manager to take into account the necessary information of technology change to decide the best time for replacement investment of equipment as well as to consider the impact of technological evolution on the maintenance strategies.

We have considered a lot of assumptions and parameters in our model to tackle the complexity of the decision environment for the maintenance managers. We have assumed that technological evolution is stochastic and the impact of a new technology can be measured not only from an economical point of view but also on the system deterioration performance. This high number of parameters is also due to our choice of integrating a quite well-advanced maintenance strategy (condition-based repair and replacement policy) to ensure the "local" optimality, i.e. independently on the technical change opportunity, in the strategic decision context.

We then used stochastic dynamic programming (i.e., discrete non-stationary Markov decision process) to solve for the optimal maintenance and replacement policy of the equipment as a function of performance and cost. And finally, we presented numerical examples to illustrate the performance of our model and to consider the influence of the parameters characterized the technological development on the optimal maintenance-replacement policy.

Some proposed assumptions can be seen as limitations of our model. The uncertainty in the technological evolution, e.g., is just considered in the time of appearance of a new generation but the associated purchase cost and the deterioration improvement are deterministic. In fact, these can be stochastic and difficult to capture.

The future work could reflect the stochastic characterization of these parameters. Furthermore, the stochastic efficiency of the imperfect maintenance action could also be included in our model or the conception of technology horizon  $N$  such that the initial optimal decision would be invariant even if more than  $N$  technologies wear to appear in future, could be consider .



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