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RELIABILITY THEORY

COMPUTATIONS OF SIGNATURES OF COHERENT SYSTEMS WITH FIVE COMPONENTS

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Key Words: coherent systems; reliability; signature; minimal signature; exponential distribution.

ABSTRACT

The signatures of coherent systems are useful tools to compute the system reliability functions, the system expected lifetimes and to compare different systems using stochastic orderings. It is well known that there exist 2, 5 and 20 different coherent systems with 2, 3 and 4 components, respectively. The signatures for these systems were given in Shaked and Suarez-Llorens (2003, JASA 98, 693–702). In this paper, we obtain an algorithm to compute all the coherent systems with n components and their signatures. Using this algorithm we show that there exist 180 coherent systems with 5 components and we compute their signatures.

1. INTRODUCTION

The study of coherent systems is a very relevant topic in reliability and survival studies. The concept of signature of a coherent system, introduced by Samaniego (1985), has become a very useful tool to compute the system reliability and expected lifetime and to compare different systems using stochastic orderings when the components are independent and iden-

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tically distributed (IID) with a common continuous distribution. An excellent review of the results based on signatures obtained since 1985 was given in Samaniego (2007). Additional results for systems with IID or exchangeable components can be seen in Navarro and Rychlik (2007), Navarro, Rychlik and Shaked (2007), Bairamov and Arnold (2008), Bhattacharya and Samaniego (2008), Li and Zhang (2008), Navarro, Balakrishnan and Samaniego (2008) and Navarro, Samaniego, Balakrishnan and Bhattacharya (2008).

Recently, Navarro, Ruiz and Sandoval (2007) introduced the concepts of minimal and maximal signatures of coherent systems with exchangeable components. These signatures are also useful in order to study coherent systems. Some applications can be seen in Navarro and Shaked (2006), Navarro, Ruiz and Sandoval (2007), Navarro and Eryilmaz (2007), Navarro and Hernandez (2008) and Eryilmaz (2008).

It is well known that there exist only two coherent systems with two components, the series and the parallel systems. Shaked and Suarez-Llorens (2003) proved that there exist 5 and 20 different coherent systems with 3 and 4 exchangeable components, respectively. They also computed the signatures of these systems and they used them to obtain some ordering properties. The signatures were also used in Kochar, Mukerjee, and Samaniego (1999), Navarro, Ruiz and Sandoval (2005) and Navarro, Samaniego, Balakrishnan and Bhattacharya (2008) to study system comparisons based on stochastic, hazard rate and likelihood ratio orderings. The number of coherent system increases exponentially as the number of components increases (Samaniego 2007, p. 11).

In this paper, we introduce an algorithm to compute coherent systems and we use it to obtain all the different coherent system with 5 components and computing their signatures, minimal signatures and maximal signatures. Moreover, we compute their expected lifetimes (also called in reliability studies Mean Time To Failure, MTTF) when the components are IID with a common exponential distribution. These expected values are used to order the systems given in Table 1 from the worse one (series system) to the best one (parallel system).

The remainder of this paper is organized as follows. In Section 2, we give the basic concepts and the main result of the paper, an algorithm to generate all the coherent sys-

tems with n components. In Section 3, we use this algorithm to compute all the coherent systems with five components, their signatures and their expected lifetimes in the case of IID component lifetimes with a common exponential distribution. In Section 4, we discuss the advantages and disadvantages of the results included in this paper, discussing some open questions and topics for future research. The tables are given in the Appendix.

2. THE MAIN RESULT

Let us consider a system ψ with n components which has two possible states, $\psi = 1$ if the system is functioning and $\psi = 0$ if the system has failed. We assume that the state of the system is determined completely by the states of the components, so that we may write

$$\psi = \psi(x_1, x_2, \ldots, x_n),$$

where $x_i = 1$ if the ith component is functioning and $x_i = 0$ if it has failed. The function ψ is called the *structure function*. A system ψ is a *coherent system* (Barlow and Proschan 1975, p. 6) if ψ is non-decreasing in every component and for every $i = 1, 2, \ldots, n$, ψ is strictly increasing in x_i for some specific values of $x_1, x_2, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$. The first property means that if a system is working and we replace a failed component by a functioning component, then the system must be working. The second property says that every component is *relevant* for the system in some situations, that is, for each i there exists $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \{0, 1\}^n$ such that the system works (*i.e.* $\psi(x_1, x_2, \ldots, x_n) = 1$) if and only if $x_i = 1$. The *dual system* of a coherent system with structure function ψ is the system with structure function

$$\psi_D(x_1, x_2, \ldots, x_n) = 1 - \psi(1 - x_1, 1 - x_2, \ldots, 1 - x_n).$$

A *path set* for a given system ψ is a set of indices P such that if the components in P work, then the system works. A path set P is a *minimal path set* if no proper subset of P is a path set. For example, the minimal path sets of the system with structure function $\psi(x_1, x_2, x_3) = \min(x_1, \max(x_2, x_3))$ are $P_1 = \{1, 2\}$ and $P_2 = \{1, 3\}$. The structure function

of a coherent system can be written in terms of its minimal path sets as

$$\psi(x_1, x_2, \ldots, x_n) = \max_{1 \leq j \leq m} \min_{i \in P_j} x_i, \tag{1}$$

where the sets $P_1, P_2, \ldots, P_m$ are all the *minimal path sets* of the system. A proof of (1) may be found in Barlow and Proschan (1975, p. 12).

The next well known lemma characterizes the minimal path sets of a coherent system (see Exercise 5, Barlow and Proschan, 1975, p. 15). Hence the minimal path sets can be used to obtain all the coherent systems with n components.

Lemma 1. *If $P_1, P_2, \ldots, P_m$ are subsets of $\{1, 2, \ldots, n\}$, then they are the minimal path sets of a coherent system with n components if, and only if, $P_i \not\subseteq P_j$ for all $i \neq j$ and*

$$P_1 \cup P_2 \cup \ldots \cup P_m = \{1, 2, \ldots, n\}.$$

The lifetime T of a coherent system with structure function ψ and component lifetimes $X_1, X_2, \ldots, X_n$ can be written as

$$T = \psi(X_1, X_2, \ldots, X_n) = \max_{1 \leq j \leq m} \min_{i \in P_j} X_i. \tag{2}$$

A proof of (2) may be found in Barlow and Proschan (1975, p. 12). It is easy to prove from (2) using the inclusion-exclusion formula that the reliability function $\overline{F}_T(t) = \Pr(T > t)$ of T can be written as

$$\overline{F}_T(t) = \sum_{1 \leq j \leq m} \overline{F}_{P_j}(t) - \sum_{1 \leq i < j \leq m} \overline{F}_{P_i \cup P_j}(t) + \ldots + (-1)^{m+1} \overline{F}_{P_1 \cup P_2 \cup \ldots \cup P_m}(t), \tag{3}$$

where $\overline{F}_P(t) = \Pr(\min_{i \in P} X_i > t)$ is the reliability function of the lifetime of the series system with components in P.

Samaniego (1985) defined the *signature* vector of a coherent system as $\mathbf{s} = (s_1, s_2, \ldots, s_n)$, where $s_i = \Pr(T = X_{i:n})$ for $i = 1, 2, \ldots, n$ and $X_{1:n}, X_{2:n}, \ldots, X_{n:n}$ are the order statistics associated to the component lifetimes $X_1, X_2, \ldots, X_n$. He proved that if the component lifetimes are IID with a common continuous distribution, then $\mathbf{s}$ only depends on the structure function of the system and the reliability function $\overline{F}_T(t) = \Pr(T > t)$ of T can be written as

$$\overline{F}_T(t) = \sum_{i=1}^{n} s_i \overline{F}_{i:n}(t), \tag{4}$$

where $\overline{F}_{i:n}(t) = \Pr(X_{i:n} > t)$ for $i = 1, 2, \ldots, n$. Navarro and Rychlik (2007) proved that this representation also holds for coherent systems with component lifetimes having a joint exchangeable absolutely continuous distribution, that is, when $(X_1, X_2, \ldots, X_n)$ has a joint probability density function f such that

$$f(x_1, x_2, \ldots, x_n) = f(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(n)})$$

for any permutation σ. Kochar, Mukerjee and Samaniego (1999) proved that if a system ψ has signature $\mathbf{s} = (s_1, s_2, \ldots, s_n)$, then the signature of the dual system ψ_D is

$$\mathbf{s}_D = (s_n, s_{n-1}, \ldots, s_1). \tag{5}$$

Recently, Navarro, Samaniego, Balakrishnan and Bhattacharya (2008) proved that representation (4) also holds for systems with exchangeable components without the assumption of an absolutely continuous joint distribution but, in this case, one must use the signature vector obtained in the absolutely continuous case. Moreover, they showed that if $T = \psi(X_1, X_2, \ldots, X_k)$ is the lifetime of a coherent system with k components and $k < n$, then there exists a vector of coefficients $(s_1, s_2, \ldots, s_n)$ such that (4) holds. This vector was called *signature of order* n. They compute the signatures of order 4 for all the coherent systems with 4 or fewer components and compare them stochastically.

Navarro, Ruiz and Sandoval (2007) proved that the reliability function $\overline{F}_T$ of a coherent system with exchangeable components can also be written as

$$\overline{F}_T(t) = \sum_{i=1}^{n} a_i \overline{F}_{1:i}(t) = \sum_{i=1}^{n} b_i \overline{F}_{i:i}(t), \tag{6}$$

where $\overline{F}_{1:i}(t) = \Pr(X_{1:i} > t)$ and $\overline{F}_{i:i}(t) = \Pr(X_{i:i} > t)$ for $i = 1, 2, \ldots, n$, that is, they are the reliability functions of series and parallel system lifetimes, respectively. The vectors of coefficients $\mathbf{a} = (a_1, a_2, \ldots, a_n)$ and $\mathbf{b} = (b_1, b_2, \ldots, b_n)$ (which only depend on the structure function of the system) were called *minimal signature* and *maximal signature*, respectively. They also proved that the coefficients in $\mathbf{a}$ can be obtained from representation (3). Moreover, they showed that the minimal (maximal) signature of the dual system is equal to the maximal

(minimal) signature of the parent one, that is,

$$\mathbf{a}_D = \mathbf{b} \text{ and } \mathbf{b}_D = \mathbf{a}. \tag{7}$$

From (3) or from the expressions given in David and Nagaraja (2003, p. 46), we have that there exist two triangular (non-singular) matrices A_n and B_n such that

$$\mathbf{s} = \mathbf{a}A_n = \mathbf{b}B_n. \tag{8}$$

Therefore, if we know a signature vector (*i.e.* we know $\mathbf{s}$, $\mathbf{a}$ or $\mathbf{b}$) we can compute the other two vectors.

The aim of the following algorithm is to compute all the coherent systems with n components using the minimal path sets representation (2) and Lemma 1. We use a recurrence method in the number m of minimal path sets. Moreover, we include a method to delete collections of minimal path sets which lead to the same coherent systems. For example the minimal path sets $P_1 = \{1,2\}, P_2 = \{1,3\}$ determine the same coherent system as the minimal path sets $P_1 = \{1,2\}, P_2 = \{2,3\}$. Even in the non-exchangeable case, note that the lifetime T_2 of the system determined by the minimal path sets $P_1 = \{1,2\}, P_2 = \{2,3\}$, $T_2 = \psi_2(X_1, X_2, X_3) = \min(X_2, \min(X_1, X_3))$, can be obtained using the structure function $\psi_1(x_1, x_2, x_3) = \min(x_1, \min(x_2, x_3))$ determined by the minimal path sets $P_1 = \{1,2\}, P_2 = \{1,3\}$ by permuting the component lifetimes in the random vector of component lifetimes, that is, $T_2 = \psi_1(X_2, X_1, X_3)$. So we say $P_1 = \{1,2\}, P_2 = \{1,3\}$ and $P_1 = \{1,2\}, P_2 = \{2,3\}$ determine the same system.

ALGORITHM. Generation of coherent systems with n components.

STEP 1. For $m = 1$ (m is the number of minimal path sets), the unique coherent system is the series system with minimal path set $P_1 = \{1, 2, \ldots, n\}$.

STEP 2. For $m = 2$, we generate the coherent systems by combining all the pairs of subsets obtained from $\{1, 2, \ldots, n\}$ and deleting the combinations which do not determine coherent systems (*i.e.* which do not satisfy the properties given in Lemma 1) or that which

determine coherent systems already determined by a preceding combination. We save all these combinations $\{P_1, P_2\}$ in an increasing cardinality order (*i.e.* $|P_1| \leq |P_2|$).

STEP 3. For $m = 3, 4, \ldots$, we generate the coherent systems recursively in m as follows. If $\{P_1, P_2, \ldots, P_{m-1}\}$ belongs to the list of sets of $m-1$ minimal path sets which determine coherent systems, we combine $P_1, P_2, \ldots, P_{m-2}$ with all the pairs of non-empty subsets obtained from $\{1, 2, \ldots, n\}$. We delete the collections of minimal path sets which do not determine coherent systems or that which determine coherent systems already determined by a preceding combination. We save the list of these collections of minimal path sets $\{P_1, P_2, \ldots, P_m\}$ in an increasing cardinality order (*i.e.* with $|P_1| \leq |P_2| \leq \ldots \leq |P_m|$).

STEP 4. Stop when for m the list of new coherent systems is empty.

Theorem 2. *The preceding algorithm generates all the coherent systems with n components.*

Proof. The combinations generated by Algorithm 1 determine coherent systems since they satisfy the properties of Lemma 1.

Consider a coherent system ψ with minimal path sets $P_1, P_2, \ldots, P_m$. Let us see that ψ is equal to a coherent system generated by the preceding algorithm. Without loss of generality we can assume that $|P_1| \leq |P_2| \leq \ldots \leq |P_m|$. If $m = 1$ or $m = 2$, then the property is true because the algorithm generates all the options.

By induction on m, suppose that the property is true for $m-1$, that is, let us assume that the algorithm generates all the coherent systems with $m-1$ minimal path sets. Then, for a given collection of m minimal path sets $P_1, P_2, \ldots, P_m$ which determine a coherent system, we have two options:

(i) If $\cup_{i=1}^{m-1} P_i = \{1, 2, \ldots, n\}$, then $\{P_1, P_2, \ldots, P_{m-1}\}$ are the minimal path sets of a coherent system ψ^* with $m-1$ minimal path sets. So, by induction, ψ^* is obtained in the step $m-1$. Therefore, ψ is obtained when we add the pair $\{P_{m-1}, P_m\}$ to $\{P_1, P_2, \ldots, P_{m-2}\}$.

(ii) If $\cup_{i=1}^{m-1} P_i \neq \{1, 2, \ldots, n\}$, then $\{P_1, P_2, \ldots, P_{m-1}\}$ does not determine a coherent system. Define

$$Q = P_{m-1} \cup \left(\{1, 2, \ldots, n\} - \cup_{i=1}^{m-2} P_i\right).$$

Then the collection $\{P_1, P_2, \ldots, P_{m-2}, Q\}$ satisfies $P_1 \cup P_2 \cup \cdots \cup P_{m-2} \cup Q = \{1, 2, \ldots, n\}$, $P_i \not\subseteq Q$ and $Q \not\subseteq P_i$ (since $P_i \not\subseteq P_{m-1}$ and $P_{m-1} \not\subseteq P_i$) for $i = 1, 2, \ldots, m-2$. Then, from Lemma 1, it determines a coherent system. By induction, as the system determined by $\{P_1, P_2, \ldots, P_{m-2}, Q\}$ has $m-1$ minimal path sets, it is generated by the algorithm in step $m-1$. Moreover, as $|P_{m-2}| \leq |P_{m-1}| < |Q|$, Q is the last minimal path set in the collection. Therefore, ψ is obtained in step m of the algorithm when we delete Q in the collection $\{P_1, P_2, \ldots, P_{m-2}, Q\}$ and we add the pair $\{P_{m-1}, P_m\}$. □

The main advantage of the preceding algorithm is that it is faster than an exhaustive enumeration of all the options of m subsets from $\{1, 2, \ldots, n\}$ for $m = 1, 2, \ldots$. Note that in the algorithm we only need all the pairs of subsets from $\{1, 2, \ldots, n\}$. This list is computed in Step 2 for $m = 2$ and it is used several times in Step 3 for $m = 3, 4, \ldots$. This advantage allows us to compute all the coherent systems with 5 components (see next section) using a standard PC while this is not possible using an exhaustive enumeration method. Even more, we have computed all the coherent systems with 6 components. There are 180 and 16145 coherent systems with 5 and 6 components, respectively. The coherent systems with 5 components are given in Table 1. A file with the coherent systems with 6 components can be obtained from the authors. However, we must note that the number of coherent systems with n components grows faster than an exponential function of n and hence our algorithm cannot be computed in a short time for any n.

Moreover, the algorithm provides the minimal path sets of the systems. Hence we can compute easily their minimal signatures using (3). Therefore we can compute Samaniego signatures $\mathbf{s}$ and maximal signatures $\mathbf{b}$ using (8) and

$$\mathbf{b} = \mathbf{a} A_n B_n^{-1}.$$

These signatures can be used to compute system reliability functions and moments when the component lifetimes have a specific distribution. This procedure is illustrated in the next section for systems with 5 exchangeable components. They can also be used to obtain

stochastic ordering properties using the results given in Kochar, Mukerjee and Samaniego (1999) and Navarro, Ruiz and Sandoval (2005).

3. COHERENT SYSTEMS WITH 5 COMPONENTS

We have written a code to compute all the coherent systems with 5 components using the algorithm given in the preceding section. Moreover, we have computed their signatures. It is not clear how the systems can be ordered. Hence, we have computed their expected lifetimes assuming that the component lifetimes are IID exponentially distributed with common mean μ. To compute these expected lifetimes we use that, if $\overline{F}(t) = \exp(-t/\mu)$ for $t \geq 0$, then $\overline{F}_{1:i}(t) = \exp(-it/\mu)$ for $t \geq 0$ and $E(X_{1:i}) = \mu/i$ for $i = 1, 2, \ldots, n$. Therefore, from (6), the expected lifetime of a system with minimal signature $\mathbf{a}$ and IID exponential components is given by

$$E(T) = \mu \sum_{i=1}^{n} \frac{a_i}{i}.$$

The minimal path sets of the 180 coherent systems with 5 components are given in Table 1. They are ordered using their expected lifetimes in the IID exponential case. Their structure functions and lifetimes can be obtained from (2).

In Table 2 we give the signatures of the 180 coherent systems with 5 components. Note that different systems can have the same signature. For example, systems numbers 9 and 10 have the same signatures. However, they are different system since their minimal path sets are $P_1 = \{1, 2, 3\}$, $P_2 = \{1, 2, 4\}$ and $P_3 = \{1, 3, 4, 5\}$ (system number 9) and $P_1 = \{1, 2, 3\}$ and $P_2 = \{1, 4, 5\}$ (system number 10). Therefore, from the results given in Navarro, Samaniego, Balakrishnan, and Bhattacharya (2008), their lifetimes will be equal in law (distribution) if they have exchangeable components. However, this property is not necessarily true in the case of non-exchangeable components. To obtain the signatures given in Table 2, we first compute the minimal signatures using minimal path set representation (3)

and then we compute Samaniego's signatures using that

$$\mathbf{s} = \mathbf{a}A_5 = \mathbf{a}\begin{pmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{10} & \frac{1}{5} & \frac{1}{10} & 0 \\ \frac{3}{5} & \frac{3}{10} & \frac{1}{10} & 0 & 0 \\ \frac{4}{5} & \frac{1}{5} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

We do not include maximal signatures since they can be obtained from properties (5) and (7). For example, the dual of system 81 with signature $\mathbf{s} = (0, 3/10, 1/2, 1/5, 0)$ is system 103 with signature $\mathbf{s}_D = (0, 1/5, 1/2, 3/10, 0)$ and hence, the maximal signature of system 81 is $\mathbf{b} = \mathbf{a}_D = (0, 3, -1, -2, 1)$ (*i.e.* the minimal signature of its dual system) and the maximal signature of system number 103 is $\mathbf{b}_D = \mathbf{a} = (0, 2, 1, -3, 1)$. In Table 2 we also include $E(T)$ when the system have IID exponential component lifetimes with common mean $\mu = 1$ (if the mean is μ, then $E(T_\mu) = \mu E(T)$).

The minimal signatures of systems with less than 5 components were given in Tables 1 & 2 of Navarro, Ruiz and Sandoval (2007) and their expected values in the exponential case were given in Tables 2 & 3 of Navarro and Rycklik (2007). Thus, we can compute their signatures of order 5 and compare stochastically systems with 5 or less exchangeable components using the results given in Kochar, Mukerjee, and Samaniego (1999), Navarro, Ruiz and Sandoval (2005) and Navarro, Samaniego, Balakrishnan, and Bhattacharya (2008). In the following example we show how the signatures can be used to compute reliability functions, expected lifetimes and hazard rate functions or to compare stochastically a system with the components on it.

Example 3. *The system number 103 in Table 1 has minimal path sets* $P_1 = \{1, 2\}, P_2 = \{1, 3\}, P_3 = \{1, 4\}, P_4 = \{2, 3, 4\}$ *and* $P_5 = \{2, 3, 5\}$*, lifetime*

$$T = \max(\min(X_1, \max(X_2, X_3, X_4)), \min(X_2, X_3, X_4), \min(X_2, X_3, X_5)),$$

signature $(0, 1/5, 1/2, 3/10, 0)$ *and minimal signature* $(0, 3, -1, -2, 1)$*. If the components are IID with common reliability function* $\overline{F}$ *and probability density function* f*, then the reliability*

$\overline{F}_T$ and hazard rate $h_T = f_T/\overline{F}_T$ functions of the system can be computed using the minimal signature and (6) through

$$\overline{F}_T(t) = 3\overline{F}^2(t) - \overline{F}^3(t) - 2\overline{F}^4(t) + \overline{F}^5(t)$$

and

$$h_T(t) = f(t)\frac{6 - 3\overline{F}(t) - 8\overline{F}^2(t) + 5\overline{F}^3(t)}{3\overline{F}(t) - \overline{F}^2(t) - 2\overline{F}^3(t) + \overline{F}^4(t)},$$

respectively. In particular, if the components are IID exponential with mean 1, then $E(T) = 0.8667$, $\overline{F}_T(t) = 3e^{-2t} - e^{-3t} - 2e^{-4t} + e^{-5t}$ and

$$h_T(t) = \frac{6 - 3e^{-t} - 8e^{-2t} + 5e^{-3t}}{3 - e^{-t} - 2e^{-2t} + e^{-3t}}$$

for $t \geq 0$. By plotting this expression we see that h_T is strictly increasing with $h_T(0) = 0$ and $\lim_{t\to\infty} h_T(t) = 2$, that is, it is asymptotically equivalent to a two component series system (see Navarro and Shaked, 2006). Therefore, as the common hazard rate of the component lifetimes X_i is $h(t) = 1$, then X_i and T are not hazard rate ordered. By plotting $\overline{F}$ and $\overline{F}_T$ we see that X_i and T are not stochastically ordered. The system is more (less) reliable than its components for $t \in (0, 0.8095)$ ($t > 0.8095$). However, if we compare their expected lifetimes (MTTF), we have $E(T) = 0.8667 < E(T) = 1$.

4. SUMMARY

Samaniego (2007, p. 11) noted that the number of coherent systems with n components grows exponentially and that there are more than a billion coherent systems with 30 components. Therefore, finding an efficient algorithm for identifying all the coherent systems of order n is a difficult open problem. However, in this paper we show that some advances can be obtained for small n obtaining an algorithm which allow us to compute all the coherent systems with 5 and 6 components. Moreover, we believe that with some improvements, this algorithm might allow us to compute the systems with 7 (or even 8) components. However, note that even before of the process of deleting duplications of coherent systems, the number of operations in step 3 of our algorithm grows faster than an exponential function of the number n of components. Hence it cannot be computed in a short time for arbitrary n.

The algorithm is based on the minimal path set representation and hence it allows us to compute systems signatures. Therefore we can compute system moments and system reliability functions. This fact allows us to compare systems in the IID case when we know the component reliability function. Moreover, using the results in Kochar, Mukerjee, and Samaniego (1999) and Navarro, Ruiz and Sandoval (2005) we can obtain stochastic ordering properties based on signatures in the IID and exchangeable cases, respectively. Even more, from the results given in Navarro, Samaniego, Balakrishnan, and Bhattacharya (2008), we can compare stochastically systems with different number of components. A complete study of these ordering relations for systems with 5 or less components is left for future research.

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APPENDIX

Table 1: Coherent systems with 5 components

N	Minimal path sets
1	$P_1 = \{1,2,3,4,5\}$
2	$P_1 = \{1,2,3,4\}$, $P_2 = \{1,2,3,5\}$
3	$P_1 = \{1,2,3,4\}$, $P_2 = \{1,2,3,5\}$, $P_3 = \{1,2,4,5\}$
4	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4,5\}$
5	$P_1 = \{1,2,3,4\}$, $P_2 = \{1,2,3,5\}$, $P_3 = \{1,2,4,5\}$, $P_4 = \{1,3,4,5\}$
6	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4,5\}$, $P_3 = \{1,3,4,5\}$
7	$P_1 = \{1,2,3,4\}$, $P_2 = \{1,2,3,5\}$, $P_3 = \{1,2,4,5\}$, $P_4 = \{1,3,4,5\}$, $P_5 = \{2,3,4,5\}$
8	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$
9	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4,5\}$
10	$P_1 = \{1,2,3\}$, $P_2 = \{1,4,5\}$
11	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4,5\}$, $P_3 = \{1,3,4,5\}$, $P_4 = \{2,3,4,5\}$
12	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4,5\}$
13	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$
14	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4,5\}$, $P_4 = \{2,3,4,5\}$
15	$P_1 = \{1,2,3\}$, $P_2 = \{1,4,5\}$, $P_3 = \{2,3,4,5\}$
16	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$
17	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$
18	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4,5\}$, $P_5 = \{2,3,4,5\}$
19	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4,5\}$
20	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4\}$, $P_4 = \{2,3,4,5\}$
21	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{3,4,5\}$
22	$P_1 = \{1,2\}$, $P_2 = \{1,3,4,5\}$
23	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$
24	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4,5\}$
25	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{2,3,4,5\}$
26	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,4,5\}$
27	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4\}$, $P_4 = \{2,3,5\}$
28	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{3,4,5\}$

Continues next page

N	Minimal path sets
29	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$
30	$P_1 = \{1,2\}$, $P_2 = \{1,3,4,5\}$, $P_3 = \{2,3,4,5\}$
31	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$
32	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,3,4,5\}$
33	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,4,5\}$, $P_5 = \{3,4,5\}$
34	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4\}$, $P_4 = \{2,3,5\}$, $P_5 = \{2,4,5\}$
35	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{3,4,5\}$
36	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{2,3,5\}$
37	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{2,3,4\}$
38	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{2,3,4,5\}$
39	$P_1 = \{1,2\}$, $P_2 = \{3,4,5\}$
40	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$, $P_7 = \{2,3,4,5\}$
41	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,3,4\}$, $P_4 = \{2,3,5\}$, $P_5 = \{2,4,5\}$, $P_6 = \{3,4,5\}$
42	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{2,3,5\}$, $P_6 = \{3,4,5\}$
43	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{2,3,4\}$, $P_6 = \{3,4,5\}$
44	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,4,5\}$
45	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,3,4\}$
46	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$
47	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4,5\}$
48	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{3,4,5\}$
49	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{2,3,5\}$
50	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,4,5\}$, $P_7 = \{3,4,5\}$
51	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,3,4\}$, $P_7 = \{2,4,5\}$
52	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,3,4\}$, $P_7 = \{2,3,5\}$
53	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$, $P_7 = \{2,3,4\}$

Continues next page

N	Minimal path sets
54	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4,5\}$
55	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{2,3,5\}$, $P_4 = \{3,4,5\}$
56	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{2,3,4\}$, $P_4 = \{3,4,5\}$
57	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{3,4,5\}$
58	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,4,5\}$
59	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$
60	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$
61	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{2,3,4\}$, $P_7 = \{2,4,5\}$, $P_8 = \{3,4,5\}$
62	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$, $P_7 = \{2,3,4\}$, $P_8 = \{2,3,5\}$
63	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4,5\}$
64	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,4,5\}$, $P_5 = \{3,4,5\}$
65	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{3,4,5\}$
66	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,4,5\}$
67	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$
68	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{3,4,5\}$
69	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$
70	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$, $P_7 = \{2,3,4\}$, $P_8 = \{2,3,5\}$, $P_9 = \{2,4,5\}$
71	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4,5\}$
72	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,4,5\}$
73	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,4,5\}$, $P_6 = \{3,4,5\}$
74	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$, $P_6 = \{3,4,5\}$
75	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$, $P_6 = \{3,4,5\}$
76	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$, $P_6 = \{2,3,5\}$
77	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,4,5\}$, $P_4 = \{3,4,5\}$
78	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4\}$, $P_4 = \{2,4,5\}$
79	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4\}$, $P_4 = \{2,3,5\}$
80	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,4,5\}$
81	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$

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N	Minimal path sets
82	$P_1 = \{1,2\}$, $P_2 = \{3,4\}$, $P_3 = \{1,3,5\}$
83	$P_1 = \{1,2,3\}$, $P_2 = \{1,2,4\}$, $P_3 = \{1,2,5\}$, $P_4 = \{1,3,4\}$, $P_5 = \{1,3,5\}$, $P_6 = \{1,4,5\}$, $P_7 = \{2,3,4\}$, $P_8 = \{2,3,5\}$, $P_9 = \{2,4,5\}$, $P_{10} = \{3,4,5\}$
84	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$, $P_6 = \{2,3,5\}$, $P_7 = \{3,4,5\}$
85	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$, $P_6 = \{2,3,5\}$, $P_7 = \{2,4,5\}$
86	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3,4,5\}$
87	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$
88	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4\}$, $P_4 = \{2,4,5\}$, $P_5 = \{3,4,5\}$
89	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4\}$, $P_4 = \{2,3,5\}$, $P_5 = \{2,4,5\}$
90	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,4,5\}$, $P_5 = \{3,4,5\}$
91	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,4,5\}$
92	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$
93	$P_1 = \{1,2\}$, $P_2 = \{3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{2,4,5\}$
94	$P_1 = \{1,2\}$, $P_2 = \{3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$
95	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3,5\}$
96	$P_1 = \{1,2\}$, $P_2 = \{1,3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,4\}$, $P_6 = \{2,3,5\}$, $P_7 = \{2,4,5\}$, $P_8 = \{3,4,5\}$
97	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3,4\}$, $P_4 = \{2,3,5\}$, $P_5 = \{2,4,5\}$, $P_6 = \{3,4,5\}$
98	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,4,5\}$, $P_6 = \{3,4,5\}$
99	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$, $P_6 = \{2,4,5\}$
100	$P_1 = \{1,2\}$, $P_2 = \{3,4\}$, $P_3 = \{1,3,5\}$, $P_4 = \{1,4,5\}$, $P_5 = \{2,3,5\}$
101	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3,4,5\}$
102	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3,5\}$, $P_5 = \{2,4,5\}$
103	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$
104	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,4\}$, $P_4 = \{3,4,5\}$
105	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,4\}$, $P_4 = \{1,4,5\}$
106	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,3\}$, $P_4 = \{1,4,5\}$
107	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{4,5\}$
108	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4,5\}$, $P_4 = \{2,3,4\}$, $P_5 = \{2,3,5\}$, $P_6 = \{2,4,5\}$,

Continues next page

N	Minimal path sets
	$P_7 = \{3, 4, 5\}$
109	$P_1 = \{1, 2\}$, $P_2 = \{3, 4\}$, $P_3 = \{1, 3, 5\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{2, 3, 5\}$, $P_6 = \{2, 4, 5\}$
110	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3, 4\}$
111	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 3, 5\}$, $P_5 = \{2, 4, 5\}$, $P_6 = \{3, 4, 5\}$
112	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 3, 4\}$, $P_5 = \{2, 3, 5\}$, $P_6 = \{2, 4, 5\}$
113	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{3, 4, 5\}$
114	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{2, 3, 5\}$
115	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 3\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{2, 4, 5\}$
116	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$,$P_3 = \{4, 5\}$, $P_4 = \{2, 3, 4\}$
117	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3, 4\}$, $P_6 = \{2, 3, 5\}$
118	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 5\}$
119	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 3, 4\}$, $P_5 = \{2, 3, 5\}$, $P_6 = \{2, 4, 5\}$, $P_7 = \{3, 4, 5\}$
120	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{2, 3, 5\}$, $P_6 = \{3, 4, 5\}$
121	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 3\}$, $P_4 = \{1, 4, 5\}$, $P_5 = \{2, 4, 5\}$, $P_6 = \{3, 4, 5\}$
122	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{4, 5\}$, $P_4 = \{2, 3, 4\}$, $P_5 = \{2, 3, 5\}$
123	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3, 4\}$, $P_6 = \{2, 3, 5\}$, $P_7 = \{2, 4, 5\}$
124	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 4\}$, $P_5 = \{1, 4, 5\}$
125	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{3, 4, 5\}$
126	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{2, 3, 4\}$
127	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 3\}$, $P_5 = \{2, 4, 5\}$
128	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 5\}$
129	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3\}$
130	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3, 4\}$, $P_6 = \{2, 3, 5\}$, $P_7 = \{2, 4, 5\}$, $P_8 = \{3, 4, 5\}$
131	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 4\}$, $P_5 = \{1, 4, 5\}$, $P_6 = \{2, 3, 5\}$
132	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{2, 3, 4\}$, $P_6 = \{3, 4, 5\}$
133	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{2, 3\}$, $P_5 = \{2, 4, 5\}$, $P_6 = \{3, 4, 5\}$
134	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 5\}$, $P_5 = \{1, 4, 5\}$
135	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{2, 3\}$, $P_4 = \{4, 5\}$

Continues next page

N	Minimal path sets
136	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4,5\}$
137	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,5\}$, $P_5 = \{3,5\}$
138	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,5\}$
139	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4,5\}$, $P_7 = \{3,4,5\}$
140	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,5\}$, $P_5 = \{3,5\}$, $P_6 = \{2,3,4\}$
141	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,5\}$, $P_6 = \{3,4,5\}$
142	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,4\}$, $P_6 = \{3,4,5\}$
143	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{2,4\}$, $P_4 = \{3,5\}$, $P_5 = \{4,5\}$
144	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{4,5\}$
145	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,5\}$, $P_5 = \{3,5\}$, $P_6 = \{4,5\}$
146	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$
147	$P_1 = \{1\}$, $P_2 = \{2,3,4,5\}$
148	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,5\}$, $P_5 = \{3,5\}$, $P_6 = \{4,5\}$, $P_7 = \{2,3,4\}$
149	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{3,4,5\}$
150	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,5\}$, $P_6 = \{4,5\}$
151	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,4\}$, $P_6 = \{3,5\}$
152	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{4,5\}$
153	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{2,5\}$
154	$P_1 = \{1\}$, $P_2 = \{2,3,4\}$, $P_3 = \{2,3,5\}$
155	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{2,5\}$, $P_8 = \{3,4,5\}$
156	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{2,3\}$, $P_5 = \{2,4\}$, $P_6 = \{3,5\}$, $P_7 = \{4,5\}$
157	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{3,5\}$
158	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{3,4\}$
159	$P_1 = \{1\}$, $P_2 = \{2,3,4\}$, $P_3 = \{2,3,5\}$, $P_4 = \{2,4,5\}$
160	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{3,5\}$, $P_8 = \{4,5\}$
161	$P_1 = \{1,2\}$, $P_2 = \{1,3\}$, $P_3 = \{1,4\}$, $P_4 = \{1,5\}$, $P_5 = \{2,3\}$, $P_6 = \{2,4\}$, $P_7 = \{2,5\}$,

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N	Minimal path sets
	$P_8 = \{3, 4\}$
162	$P_1 = \{1\}$, $P_2 = \{2, 3, 4\}$, $P_3 = \{2, 3, 5\}$, $P_4 = \{2, 4, 5\}$, $P_5 = \{3, 4, 5\}$
163	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4, 5\}$
164	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3\}$, $P_6 = \{2, 4\}$, $P_7 = \{2, 5\}$, $P_8 = \{3, 4\}$, $P_9 = \{3, 5\}$
165	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4, 5\}$, $P_4 = \{3, 4, 5\}$
166	$P_1 = \{1, 2\}$, $P_2 = \{1, 3\}$, $P_3 = \{1, 4\}$, $P_4 = \{1, 5\}$, $P_5 = \{2, 3\}$, $P_6 = \{2, 4\}$, $P_7 = \{2, 5\}$, $P_8 = \{3, 4\}$, $P_9 = \{3, 5\}$, $P_{10} = \{4, 5\}$
167	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 4, 5\}$
168	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{4, 5\}$
169	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{2, 5\}$
170	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{3, 4, 5\}$
171	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 5\}$
172	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{3, 5\}$, $P_5 = \{4, 5\}$
173	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{3, 4\}$
174	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{3, 4\}$, $P_6 = \{3, 5\}$
175	$P_1 = \{1\}$, $P_2 = \{2, 3\}$, $P_3 = \{2, 4\}$, $P_4 = \{2, 5\}$, $P_5 = \{3, 4\}$, $P_6 = \{3, 5\}$, $P_7 = \{4, 5\}$
176	$P_1 = \{1\}$, $P_2 = \{2\}$, $P_3 = \{3, 4, 5\}$
177	$P_1 = \{1\}$, $P_2 = \{2\}$, $P_3 = \{3, 4\}$, $P_4 = \{3, 5\}$
178	$P_1 = \{1\}$, $P_2 = \{2\}$, $P_3 = \{3, 4\}$, $P_4 = \{3, 5\}$, $P_5 = \{4, 5\}$
179	$P_1 = \{1\}$, $P_2 = \{2\}$, $P_3 = \{3\}$, $P_4 = \{4, 5\}$
180	$P_1 = \{1\}$, $P_2 = \{2\}$, $P_3 = \{3\}$, $P_4 = \{4\}$, $P_5 = \{5\}$

Table 2: Signatures of coherent systems with 5 exchangeable components. The systems are ordered by their expected lifetimes $E(T)$ in the case of IID exponential components with mean 1.

N	**a**	**s**	$E(T)$
1	$(0,0,0,0,1)$	$(1,0,0,0,0)$	0.2
2	$(0,0,0,2,-1)$	$(\frac{3}{5},\frac{2}{5},0,0,0)$	0.3
3	$(0,0,0,3,-2)$	$(\frac{2}{5},\frac{3}{5},0,0,0)$	0.35
4	$(0,0,1,1,-1)$	$(\frac{2}{5},\frac{1}{2},\frac{1}{10},0,0)$	0.3833
5	$(0,0,0,4,-3)$	$(\frac{1}{5},\frac{4}{5},0,0,0)$	0.4
6	$(0,0,1,2,-2)$	$(\frac{1}{5},\frac{7}{10},\frac{1}{10},0,0)$	0.4333
7	$(0,0,0,5,-4)$	$(0,1,0,0,0)$	0.45
8	$(0,0,3,-3,1)$	$(\frac{2}{5},\frac{3}{10},\frac{3}{10},0,0)$	0.45
9-10	$(0,0,2,0,-1)$	$(\frac{1}{5},\frac{3}{5},\frac{1}{5},0,0)$	0.4667
11	$(0,0,1,3,-3)$	$(0,\frac{9}{10},\frac{1}{10},0,0)$	0.4833
12-13	$(0,0,3,-2,0)$	$(\frac{1}{5},\frac{1}{2},\frac{3}{10},0,0)$	0.5
14-15	$(0,0,2,1,-2)$	$(0,\frac{4}{5},\frac{1}{5},0,0)$	0.5167
16-17	$(0,0,4,-4,1)$	$(\frac{1}{5},\frac{2}{5},\frac{2}{5},0,0)$	0.5333
18-21	$(0,0,3,-1,-1)$	$(0,\frac{7}{10},\frac{3}{10},0,0)$	0.55
22	$(0,1,0,1,-1)$	$(\frac{1}{5},\frac{1}{2},\frac{1}{5},\frac{1}{10},0)$	0.55
23	$(0,0,5,-6,2)$	$(\frac{1}{5},\frac{3}{10},\frac{1}{2},0,0)$	0.5667
24-28	$(0,0,4,-3,0)$	$(0,\frac{3}{5},\frac{2}{5},0,0)$	0.5833
29	$(0,0,6,-8,3)$	$(\frac{1}{5},\frac{1}{5},\frac{3}{5},0,0)$	0.6
30	$(0,1,0,2,-2)$	$(0,\frac{7}{10},\frac{1}{5},\frac{1}{10},0)$	0.6
31	$(0,1,2,-3,1)$	$(\frac{1}{5},\frac{3}{10},\frac{2}{5},\frac{1}{10},0)$	0.6167
32-37	$(0,0,5,-5,1)$	$(0,\frac{1}{2},\frac{1}{2},0,0)$	0.6167
38-39	$(0,1,1,0,-1)$	$(0,\frac{3}{5},\frac{3}{10},\frac{1}{10},0)$	0.6333
40-45	$(0,0,6,-7,2)$	$(0,\frac{2}{5},\frac{3}{5},0,0)$	0.65
46	$(0,1,3,-5,2)$	$(\frac{1}{5},\frac{1}{5},\frac{1}{2},\frac{1}{10},0)$	0.65
47-49	$(0,1,2,-2,0)$	$(0,\frac{1}{2},\frac{2}{5},\frac{1}{10},0)$	0.6667
50-53	$(0,0,7,-9,3)$	$(0,\frac{3}{10},\frac{7}{10},0,0)$	0.6833
54-59	$(0,1,3,-4,1)$	$(0,\frac{2}{5},\frac{1}{2},\frac{1}{10},0)$	0.7

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N	**a**	**s**	$E(T)$
60	$(0,2,0,-2,1)$	$(\frac{1}{5},\frac{1}{5},\frac{2}{5},\frac{1}{5},0)$	0.7
61-62	$(0,0,8,-11,4)$	$(0,\frac{1}{5},\frac{4}{5},0,0)$	0.7167
63	$(0,2,-1,1,-1)$	$(0,\frac{1}{2},\frac{3}{10},\frac{1}{5},0)$	0.7167
64-69	$(0,1,4,-6,2)$	$(0,\frac{3}{10},\frac{3}{5},\frac{1}{10},0)$	0.7333
70	$(0,0,9,-13,5)$	$(0,\frac{1}{10},\frac{9}{10},0,0)$	0.75
71-72	$(0,2,0,-1,0)$	$(0,\frac{2}{5},\frac{2}{5},\frac{1}{5},0)$	0.75
73-76	$(0,1,5,-8,3)$	$(0,\frac{1}{5},\frac{7}{10},\frac{1}{10},0)$	0.7667
77-82	$(0,2,1,-3,1)$	$(0,\frac{3}{10},\frac{1}{2},\frac{1}{5},0)$	0.7833
83	$(0,0,10,-15,6)$	$(0,0,1,0,0)$	0.7833
84-85	$(0,1,6,-10,4)$	$(0,\frac{1}{10},\frac{4}{5},\frac{1}{10},0)$	0.8
86	$(0,3,-3,2,-1)$	$(0,\frac{2}{5},\frac{3}{10},\frac{3}{10},0)$	0.8
87	$(0,4,-6,4,-1)$	$(\frac{1}{5},\frac{1}{5},\frac{1}{5},\frac{2}{5},0)$	0.8
88-94	$(0,2,2,-5,2)$	$(0,\frac{1}{5},\frac{3}{5},\frac{1}{5},0)$	0.8167
95	$(0,3,-2,0,0)$	$(0,\frac{3}{10},\frac{2}{5},\frac{3}{10},0)$	0.8333
96	$(0,1,7,-12,5)$	$(0,0,\frac{9}{10},\frac{1}{10},0)$	0.8333
97-100	$(0,2,3,-7,3)$	$(0,\frac{1}{10},\frac{7}{10},\frac{1}{5},0)$	0.85
101	$(0,4,-6,5,-2)$	$(0,\frac{2}{5},\frac{1}{5},\frac{2}{5},0)$	0.85
102-107	$(0,3,-1,-2,1)$	$(0,\frac{1}{5},\frac{1}{2},\frac{3}{10},0)$	0.8667
108-109	$(0,2,4,-9,4)$	$(0,0,\frac{4}{5},\frac{1}{5},0)$	0.8833
110	$(0,4,-5,3,-1)$	$(0,\frac{3}{10},\frac{3}{10},\frac{2}{5},0)$	0.8833
111-116	$(0,3,0,-4,2)$	$(0,\frac{1}{10},\frac{3}{5},\frac{3}{10},0)$	0.9
117-118	$(0,4,-4,1,0)$	$(0,\frac{1}{5},\frac{2}{5},\frac{2}{5},0)$	0.9167
119-122	$(0,3,1,-6,3)$	$(0,0,\frac{7}{10},\frac{3}{10},0)$	0.9333
123-128	$(0,4,-3,-1,1)$	$(0,\frac{1}{10},\frac{1}{2},\frac{2}{5},0)$	0.95
129	$(0,5,-7,4,-1)$	$(0,\frac{1}{5},\frac{3}{10},\frac{1}{2},0)$	0.9667
130-135	$(0,4,-2,-3,2)$	$(0,0,\frac{3}{5},\frac{2}{5},0)$	0.9833
136-138	$(0,5,-6,2,0)$	$(0,\frac{1}{10},\frac{2}{5},\frac{1}{2},0)$	1
139-144	$(0,5,-5,0,1)$	$(0,0,\frac{1}{2},\frac{1}{2},0)$	1.0333
145-146	$(0,6,-9,5,-1)$	$(0,\frac{1}{10},\frac{3}{10},\frac{3}{5},0)$	1.05
147	$(1,0,0,1,-1)$	$(0,\frac{2}{5},\frac{1}{5},\frac{1}{5},\frac{1}{5})$	1.05
148-152	$(0,6,-8,3,0)$	$(0,0,\frac{2}{5},\frac{3}{5},0)$	1.0833

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N	**a**	**s**	$E(T)$
153	$(0,7,-12,8,-2)$	$(0,\frac{1}{10},\frac{1}{5},\frac{7}{10},0)$	1.1
154	$(1,0,2,-3,1)$	$(0,\frac{1}{5},\frac{2}{5},\frac{1}{5},\frac{1}{5})$	1.1167
155-158	$(0,7,-11,6,-1)$	$(0,0,\frac{3}{10},\frac{7}{10},0)$	1.1333
159	$(1,0,3,-5,2)$	$(0,\frac{1}{10},\frac{1}{2},\frac{1}{5},\frac{1}{5})$	1.15
160-161	$(0,8,-14,9,-2)$	$(0,0,\frac{1}{5},\frac{4}{5},0)$	1.1833
162	$(1,0,4,-7,3)$	$(0,0,\frac{3}{5},\frac{1}{5},\frac{1}{5})$	1.1833
163	$(1,1,0,-2,1)$	$(0,\frac{1}{10},\frac{2}{5},\frac{3}{10},\frac{1}{5})$	1.2
164	$(0,9,-17,12,-3)$	$(0,0,\frac{1}{10},\frac{9}{10},0)$	1.2333
165	$(1,1,1,-4,2)$	$(0,0,\frac{1}{2},\frac{3}{10},\frac{1}{5})$	1.2333
166	$(0,10,-20,15,-4)$	$(0,0,0,1,0)$	1.2833
167-168	$(1,2,-2,-1,1)$	$(0,0,\frac{2}{5},\frac{2}{5},\frac{1}{5})$	1.2833
169	$(1,3,-6,4,-1)$	$(0,\frac{1}{10},\frac{1}{5},\frac{1}{2},\frac{1}{5})$	1.3
170-171	$(1,3,-5,2,0)$	$(0,0,\frac{3}{10},\frac{1}{2},\frac{1}{5})$	1.3333
172-173	$(1,4,-8,5,-1)$	$(0,0,\frac{1}{5},\frac{3}{5},\frac{1}{5})$	1.3833
174	$(1,5,-11,8,-2)$	$(0,0,\frac{1}{10},\frac{7}{10},\frac{1}{5})$	1.4333
175	$(1,6,-14,11,-3)$	$(0,0,0,\frac{4}{5},\frac{1}{5})$	1.4833
176	$(2,-1,1,-2,1)$	$(0,0,\frac{3}{10},\frac{3}{10},\frac{2}{5})$	1.5333
177	$(2,1,-5,4,-1)$	$(0,0,\frac{1}{10},\frac{1}{2},\frac{2}{5})$	1.6333
178	$(2,2,-8,7,-2)$	$(0,0,0,\frac{3}{5},\frac{2}{5})$	1.6833
179	$(3,-2,-2,3,-1)$	$(0,0,0,\frac{2}{5},\frac{3}{5})$	1.8833
180	$(5,-10,10,-5,1)$	$(0,0,0,0,1)$	2.2833