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Calculation of distortion generation on a loaded transducer using electric circuits.

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Nonlinear methods are now widespread in medical and industrial applications of ultrasound. The harmonic generation within piezoelectric transducers needs to be quantified. This is particularly the case in nonlinear NDT where the nonlinearity of the medium only has to be identified. High precision measurements are then required that would be spoiled by nonlinearity coming from the transducer itself. In the present work, quadratic nonlinear equations of a piezoelectric element are derived by the perturbation theory in one-dimensional case. It is demonstrated that the nonlinear response can be represented by controlled sources of voltage and current in equivalent electric circuits. This analysis allows the nonlinear response of a loaded piezoelectric transducer to be efficiently predicted. It is shown that the influence of these acoustic loads on the nonlinear effects such as distortion depends phenomenologically on the type of nonlinearity that dominates in the piezoelectric element, i.e. mechanical or electromechanical. A generalisation of nonlinear equivalent electric circuits to cases including propagation media and passive layers is presented, opening the way to the study of the nonlinear response of complex structures.

1 Introduction

In the field of Non-Destructive Evaluation (NDE), it has been shown that nonlinear parameters associated to harmonic generation can provide additional information of interest on the status of these materials, such as state of fatigue [1], [2]. Several NDE measurement methods are based on the spectral modification of an acoustic wave propagating through a sample under test. However, as nonlinear effects in solids are very low compared to linear phenomena, the measurements of these nonlinearities can be severely affected by the existence of external sources of nonlinearity which are the electronics and the ultrasonic transducer. If the effect of the electronics can be reduced by the use of an adapted filtering stage, the transducers can be a recurrent problem. Thus, it has been shown [1] that, in the range of intensities (strain and stress in the transducer respectively of the order of 10^{-5} and 10^6 MPa) commonly used for NDE applications, these nonlinearities are competing with those produced by the propagation medium. In this study, equivalent electrical circuits are extended, from a classical model [3], to the modelling of systems including nonlinear acoustic, electric and piezoelectric elements. The length-extensional mode is presented in order for realistic nonlinear parameters extracted from experimental results concerning PZT ceramics [4] to be introduced.

2 Theoretical background

Assuming a weak nonlinearity, the constitutive equations of the piezoelectric element will be developed up to quadratic terms only. In the case of the length-extensional geometry, the useful relations, in the first nonlinear approximation, between the displacement u along the z -axis of a piezoelectric rod, the first Piola-Kirchhoff stress T , the electrical field E and the electrical displacement D are formally [5] :

$$\begin{aligned}\frac{\partial u}{\partial z} &= sT + gD + \frac{s'}{2}T^2 - \alpha DT - \frac{\gamma}{2}D^2 \\ E &= \beta D - gT + \frac{\alpha}{2}T^2 + \frac{\beta'}{2}D^2 + \gamma DT\end{aligned}\quad (1)$$

where s , g and β are respectively the compliance, piezoelectric and inverse of permittivity second-order constants. The third-order constants s' and β' express respectively second-order elastic and dielectric effects whereas α and γ are related to an electromechanical nonlinearity. According to [6], second-order dielectric effects can be ignored. Besides, as D does not depend on space in the quasi-static approximation of Maxwell's law :

$$\frac{\partial D}{\partial z} = 0 \quad (2)$$

the constant γ plays no role in the dynamic law :

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z} T \quad (3)$$

where ρ is the reference mass density. The constant γ can then also be ignored in a first approach. Thus, the usefull constitutive relationships in this geometry can be reduced to :

$$\begin{aligned} \frac{\partial u}{\partial z} &= sT + gD + \frac{s'}{2}T^2 - \alpha DT \\ E &= \beta D - gT + \frac{\alpha}{2}T^2 \end{aligned} \quad (4)$$

The hypothesis of a weak nonlinearity allows nonlinearity to be considered as a perturbation of linear fields. Each quantity u , E , T and D will be searched as respective sums :

$u = \sum_{i=0}^{\infty} u_i$, $E = \sum_{i=0}^{\infty} E_i$, $T = \sum_{i=0}^{\infty} T_i$ and $D = \sum_{i=0}^{\infty} D_i$ where the subscript $i = 0$ represents the solution of the linear problem and the subscript $i \geq 1$ represents the solution calculated at the $i \geq 1$ order of nonlinearity by successive approximations [7].

3 Linear and nonlinear equivalent circuits

In linear case, the constitutive equations are :

$$\begin{aligned} \frac{\partial u_0}{\partial z} &= sT_0 + gD_0 \\ E_0 &= \beta D_0 - gT_0 \end{aligned} \quad (5)$$

The combination of (Eq 5) with the dynamic law (Eq 3) and the Maxwell-law (Eq 2) leads to the homogeneous wave equation :

$$\rho \frac{\partial^2 u_0}{\partial t^2} - \frac{1}{s} \frac{\partial^2 u_0}{\partial z^2} = 0 \quad (6)$$

The piezoelectric rod is defined by its length $2a$ and a cross-section area $\Sigma = hd$ with $h, d \ll a$ (Fig 1).

It is supposed to be driven by a voltage applied at its ends $z = \pm a$. The relations between the particle velocity $v_0(\pm a, t)$, the stress $T_0(\pm a, t)$ at each face, the electrical current I_0 - defined by $I_0(t) = \Sigma \frac{dD_0}{dt}$ - and the applied voltage $V_0(t)$ can be formally described by an hexapole H_0 with two acoustical ports and an electrical port. In case of a sinusoidal electrical excitation at an angular frequency ω at the connections $z = \pm a$, $V_0(t) = V_0 \exp(j\omega t)$, the particle velocity in the piezoelectric medium can be expressed as the sum of two waves respectively in $j(\omega t + kz)$ and $j(\omega t - kz)$:

$$v_0(z, t) = [A_+ e^{jkz} + A_- e^{-jkz}] e^{j\omega t} \quad (7)$$

where $k = \omega c$ is the wave number, $c = \sqrt{\frac{1}{\rho s}}$ being the longitudinal wave velocity in length-extensional mode.

Introducing the acoustical impedance $Z = \rho c$ of the piezoelectric layer and the phase term $\theta = ka$, one obtains that :

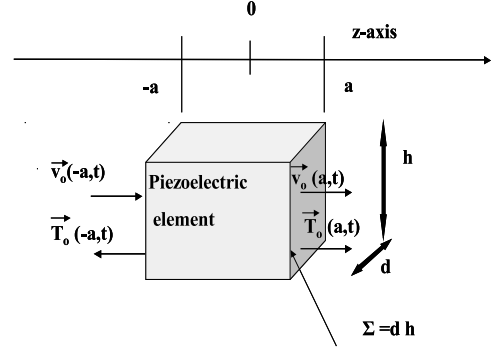


FIG. 1: Scheme of the piezoelectric element. $\vec{T}_0(\pm a, t)$ and $\vec{v}_0(\pm a, t)$ are respectively the stress vector and the particle velocities at each face $z = \pm a$ of the piezoelectric element.

$$\begin{pmatrix} T_0(a) \\ T_0(-a) \\ V_0 \end{pmatrix} = M_{H_0, \omega} \begin{pmatrix} v_0(a) \\ v_0(-a) \\ I_0 \end{pmatrix} \quad (8)$$

where $M_{H_0, \omega}$ is the impedance matrix of the hexapole [8] defined as :

$$M_{H_0, \omega} = -j \begin{pmatrix} Z \cot(2\theta) & -\frac{Z}{\sin(2\theta)} & -\frac{g}{s\Sigma\omega} \\ \frac{Z}{\sin(2\theta)} & -Z \cot(2\theta) & -\frac{g}{s\Sigma\omega} \\ \frac{g}{s\omega} & -\frac{g}{s\omega} & -\frac{\beta s + g^2}{s\Sigma\omega} \end{pmatrix} \quad (9)$$

The use of matrix allows efficient calculations of linear solutions as a function of the boundary conditions. A instrumentation chain can be easily represents by equivalent electric circuits. It is shown by following that such circuits can be extended to the nonlinear problem.

The constitutive relations between the first-order physical quantities and the zero-order ones are :

$$\begin{aligned} \frac{\partial u_1}{\partial z} &= sT_1 + gD_1 + \frac{s'}{2}T_0^2 - \alpha D_0 T_0 \\ E_1 &= \beta D_1 - gT_1 + \frac{\alpha}{2}T_0^2 \end{aligned} \quad (10)$$

According to the expression of the stress $T_0(z, t)$, the previous equation leads to the inhomogeneous wave equation :

$$\rho \frac{\partial^2 u_1}{\partial t^2} - \frac{1}{s} \frac{\partial^2 u_1}{\partial z^2} = \quad (11)$$

$$-\frac{s'}{2s^3} \frac{\partial}{\partial z} \left(\frac{\partial u_0}{\partial z} \right)^2 + \frac{\alpha s + gs'}{s^3} D_0 \frac{\partial^2 u_0}{\partial z^2}$$

The solution of the wave equation (Eq 11) is the sum of the general solution $u_1^g(z, t)$ of the homogeneous wave equation and a particular solution $u_1^p(z, t)$ created by source-terms : $-\frac{s'}{2s^3} \frac{\partial}{\partial z} \left(\frac{\partial u_0}{\partial z} \right)^2$ and $\frac{\alpha s + gs'}{s^3} D_0 \frac{\partial^2 u_0}{\partial z^2}$.

Thus, the particle velocity in the piezoelectric rod $v_1(z, t)$ can be split into $v_1^g(z, t)$ and $v_1^p(z, t)$ corresponding respectively to $u_1^g(z, t)$ and $u_1^p(z, t)$. One can then formally write at $z = \pm a$:

$$v_1(\pm a, t) = v_1^g(\pm a, t) + v_1^p(\pm a, t) \quad (12)$$

Similar decompositions can be performed for the second-order stress T_1 , the second-order potential V_1 and the second-order current I_1 . The relations between $T_1^g(\pm a, t)$, $v_1^g(\pm a, t)$, $V_1^g(t)$ and $I_1^g(t)$ can be described by an hexapole H_1 (Fig 2) whose impedance matrix $M_{H_1}^g$ is similar to $M_{H_0, \omega}$. More precisely, $M_{H_1}^g$ is exactly $M_{H_0, 2\omega}$.

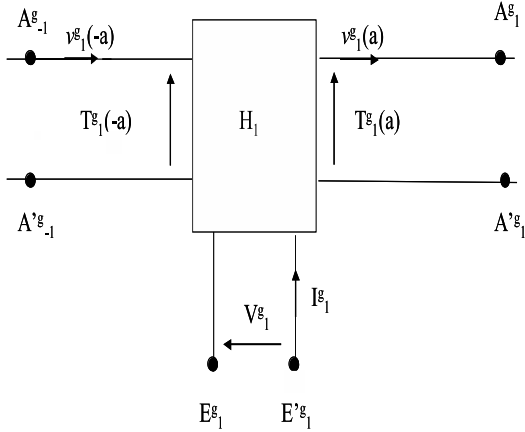


FIG. 2: Electrical equivalent circuit (hexapole) representing relations between quantities T_1^g , V_1^g , v_1^g and I_1^g . Connections at points A_{-1}^g and A'_{-1}^g , respectively A_1^g and A'_{-1}^g , represent the rear acoustic port and the front acoustic port whereas connections at E_1^g and E'_{-1}^g represent the electrical port.

The couples of particular solutions ($T_1^p(-a)$, $v_1^p(-a)$) and ($T_1^p(a)$, $v_1^p(a)$) can be represented either by "T-shaped" quadrupoles or by "II-shaped" quadrupoles placed at each acoustic port of the hexapole H_1 , so can the couple of particular solutions (V_1^p , I_1^p) at the electrical port. The "T-shaped" quadrupoles have been chosen in this work. The quadrupoles placed at each acoustic port $-z = \pm a$ consist of two equivalent controlled voltage sources, $T_1^{SL}(\pm a)$ and $T_1^{SR}(\pm a)$, and one equivalent controlled current source $v_1^S(\pm a)$. The quadrupole placed at the electrical port consists of two controlled voltage sources, respectively V_1^{SL} and V_1^{SR} , and one controlled current source I_1^S (Fig 3). In each of these quadrupoles, the voltage sources placed furthest from the hexapole H_1 depend on the boundary conditions.

In the following, expressions of $v_1^p(\pm a)$, $T_1^p(\pm a)$, V_1^p and I_1^p are explicitied. Due to the fact that the

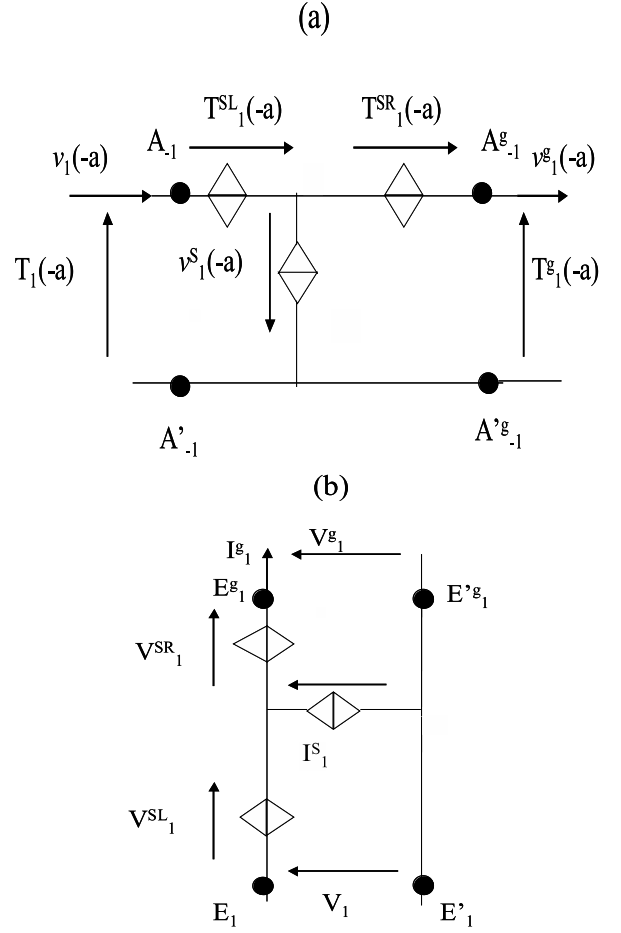


FIG. 3: Electrical equivalent schemes representing the controlled sources (diamonds) located at : (a) the rear acoustic port and (b) : the electrical port. These sources corresponds to couples of particular solutions in the first approximation of the nonlinear problem.

electrical displacement D_0 does not depend on space, the resolution of (Eq 11) imposes that two different kinds of secondary waves $v_1^p(z, t)$ are present in the piezoelectric material. The first ones, related only to the square of $\frac{\partial u_0}{\partial z}$, express a pure mechanical nonlinearity and are propagated at the speed of fundamental waves. They impose cumulative terms in the second-order mechanical response. The second ones are linked to cross-coupled nonlinearity, expressed by $D_0 \frac{\partial^2 u_0}{\partial z^2}$, and are propagated at twice the speed of fundamental waves. These last ones do not impose cumulative terms. As a consequence of the existence of two different kinds of nonlinearities, i.e : cumulative and non-cumulative at twice the speed of fundamental waves, $v_1^p(z, t)$ must be searched in the form :

$$v_1^p(z, t) = z[A_{1+}e^{2j(\omega t + kz)} + A_{1-}e^{2j(\omega t - kz)}] + B_{1+}e^{j(2\omega t + kz)} + B_{1-}e^{j(2\omega t - kz)} \quad (13)$$

The coefficients $A_{1\pm}$ and $B_{1\pm}$ are obtained by identification after reintroducing them in (11) and developing $\frac{\partial}{\partial z} \left(\frac{\partial u_0}{\partial z} \right)^2$ and $D_0 \frac{\partial^2 u_0}{\partial z^2}$. These coefficients are finally :

$$A_{1\pm} = j \frac{s'}{2} Z^2 \omega A_{\pm}^2 \quad (14)$$

$$B_{1\pm} = -2j \frac{\alpha s + g s'}{3 \Sigma k s} I_0 Z A_{\pm}^2$$

The continuity of second-order stress and second-order particle velocity at interfaces $z = \pm a$ with passive layers (Fig 1) impose, in case of coupling to two semi-infinite media :

$$T_1(-a) = Z_r [v_1^S(-a) + v_1^g(-a)] \quad (15)$$

$$T_1(a) = -Z_f [v_1^S(a) + v_1^g(a)]$$

where Z_r and Z_f are respectively the acoustic impedances of the rear and front elastic layers. The terms $Z_r v_1^S(-a)$ and $-Z_f v_1^S(a)$ are voltage sources, depending on boundary conditions in the acoustical port. They correspond respectively to the left side, $T_1^{SL}(-a)$, of the rear source quadrupole and to the right side, $T_1^{SR}(a)$, of the front source quadrupole. On the contrary, the sources $T_1^{SR}(-a)$ and $T_1^{SL}(a)$ do not depend on boundary conditions. They are obtained from (Eq 10) after eliminating all quantities corresponding to the general solution. One finally obtains :

$$T_1^{SR}(\pm a) = -\frac{s'}{2s} T_0^2(\pm a) + \frac{\alpha}{s} D_0 T_0(\pm a) \quad (16)$$

$$+ \frac{1}{2j\omega s} [A_{1+}(1 \pm 2j\theta)e^{\pm 2j\theta} + A_{1-}(1 \mp 2j\theta)e^{\mp 2j\theta}]$$

$$+ \frac{k}{2\omega s} [B_{1+}e^{\pm j\theta} - B_{1-}e^{\mp j\theta}]$$

As no dielectric nonlinearity has been taken into account in the model $-\beta' = 0$ in (Eq 1)-, no second-order current source exists : $I_1 = I_1^g$. As a consequence, for the second-order potential, the source is a single source V_1^S , with :

$$V_1^S = - \int_{-a}^a \left[\frac{\alpha s + g s'}{2s} T_0^2(z) - \frac{g\alpha}{s} D_0 T_0(z) \right] dz \quad (17)$$

$$+ \frac{g}{j\omega s} [(A_{1+} + A_{1-})\cos(2\theta) + j(B_{1+} - B_{1-})\sin(\theta)]$$

Nonlinear instrumentation chain.

From the previous analysis, the modelling of a nonlinear instrumentation chain is possible by connecting together nonlinear equivalent schemes associated to passive or active elements. In order to illustrate the method, the electrical equivalent circuits of a nonlinear active layer coupled at its rear face to a linear semi-infinite passive medium and at its front face to a nonlinear passive layer is given in (Fig 4).

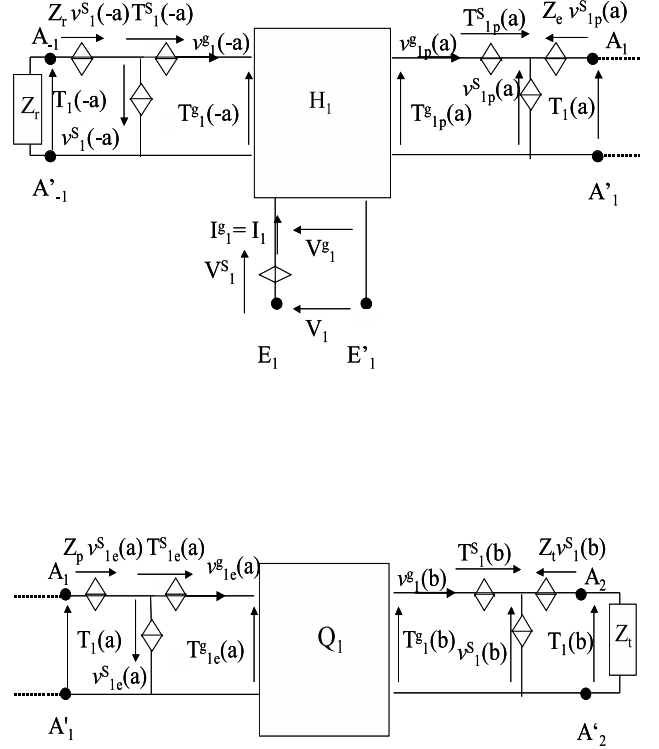


FIG. 4: Equivalent circuits of a piezoelectric layer coupled at rear face to a linear semi-infinite elastic medium and at front face to a nonlinear passive layer. Connections at A_{-1} and A'_{-1} , and at A_1 and A'_1 represent the acoustical ports of the nonlinear active layer whereas connections at A_1 and A'_1 , and at A_2 and A'_2 are also the acoustical ports of the nonlinear passive layer. Z_p and Z_e are respectively the acoustical impedances of the piezoelectric and the passive layer. Z_r and Z_t are respectively the acoustical impedances of the rear and the terminal semi-infinite media. Diamonds represent the controlled voltage sources and the controlled current sources introduced by the quadratic nonlinearity.

4 Applications.

4.1 Influence of the nonlinearity of the transducer on NDE measurements.

In length extensional mode, the parameter $B = \frac{s'}{2s^2}$ is a characteristic of the nonlinearity of an elastic medium. When no source of nonlinearity exists apart from the one in this medium, it can be deduced from (Eq 14) that B is directly related to the ratio of amplitudes of the second harmonic component of particle velocity to the square of the fundamental component at the f frequency by :

$$B = \frac{c^2 |v_1(2f)|}{\pi f z |v_0(f)|^2} \quad (18)$$

where c is the acoustic wave velocity and z the distance of propagation in the medium. (Eq 18) gives then

a practical mean to determine the nonlinear parameter B of a medium by measuring the fundamental and second harmonic amplitudes.

The influence of sources of nonlinearity in the transducer on estimation of B parameter is studied. In order to be realistic, the nonlinear parameters of the piezoelectric material, a PZT ceramic, are those measured in a previous study [4]. In order to model a complete transducer, a backing -considered as a semi-infinite medium- and a quarter wave length matching layer L are added. The semi-infinite medium M in front face is defined by its density, acoustic wave velocity and nonlinear parameter B_0 . A high value of B_0 (compared to the value in quartz [9] in the order of 1) is voluntarily taken. The density ρ , the velocity of sound c and nonlinear parameter B of both passive media are given in (Tab 1).

TAB. 1: Linear and nonlinear constants of the matching layer L , backing and medium M .

constant	L	Backing	M
ρ ($kg.m^{-3}$)	2400	2500	7000
c ($m.s^{-1}$)	5300	4000	3500
B	0	0	50

The amplitudes of fundamental and second harmonic components at different propagation distances z' in medium M have been calculated and the nonlinear parameter B deduced directly from (Eq 18). Results of the estimations versus the distance are given in (Fig 5). The case of a nonlinear piezoelectric rod can be compared to the linear case. As expected, the estimated value of B , in this last case, is a constant equal to the parameter B_0 . One can see that the perturbation introduced by the nonlinearity of the piezoelectric source on the estimation of B decreases with distance, due to cumulative effects of nonlinearity in the semi-infinite elastic medium. However, it must be noted that this evolution is not necessarily monotonous. Thus, one can see that the perturbation introduced by the nonlinearity of the piezoelectric element can lead either to an over-estimation or to an under-estimation of the nonlinear parameter B . This section demonstrates the interest of taking into account external sources of nonlinearity even if the nonlinear parameter is relatively high. However, the problem is to quantify precisely the distortion rate at the output of the transducer, which can depend on the load.

4.2 Influence of a linear load on the nonlinear behavior of the transducer.

The influence of a linear load on the second harmonic distortion rate at the output of the transducer is studied in this paragraph. In order to simplify the problem, the backing and the matching layer will be omitted and the piezoelectric element is directly loaded by the semi-infinite medium defined by its acoustical impedance Z_e compared the acoustical impedance Z_p of the piezoelectric element. The evolution of the quadratic distortion parameter :

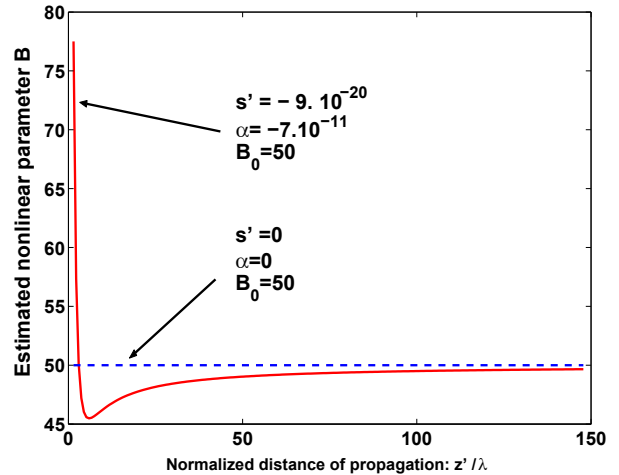


FIG. 5: Estimated nonlinear parameter B versus distance of propagation in the material. This distance is normalized to the wave length λ corresponding to the fundamental frequency f for different cases of nonlinearity of the source of acoustical waves.

$$-d(f) = u(2f)/u^2(f)$$

at the loaded face of the piezoelectric element is studied in the vicinity of the resonance frequency f_0 as a function of the load. The influence of the load on the quadratic distortion parameter d is given in (Fig 6-7) in repetitive cases of a electromechanical nonlinearity ($\alpha \neq 0; s' = 0$) and a mechanical nonlinearity ($\alpha = 0; s' \neq 0$), the linear parameters of the piezoelectric element being the same as used in the last section.

One can see in (Fig 6) that, below the resonant frequency f_0 , the parameter d decreases with the load. However, inflexions in the curves appear in the vicinity of f_0 and respective positions of the curves change. As a result, the distortion parameter d at f_0 for Z_e/Z_p equal to one reaches two times -6 dB- the value corresponding to the unloaded case. Similar phenomena appear in the case of a mechanical nonlinearity (Fig 7). However, in this case, the changes in the curves appear systematically for $f \approx 0.85f_0$. As a consequence, in this case, any load will enhance the second harmonic generation while exciting at the resonant frequency f_0 . Thus, in this case, the substitution of the material by a lower-impedance linear load will automatically lead to under-estimate the rate of external distortion generated while performing tests.

5 Conclusion.

It has been demonstrated in this study that equivalent circuits based on the hexapole formalism can be extended to solve the complex problem of nonlinear propagation in an instrumentation chain with a large number of nonlinear media. The advantage of this decomposition is the possibility of using matrix computations, based on the chain or the transfer matrix, allowing efficient and quick calculations. A first application of this method has been presented, showing the influence of the nonlinearity of the transducer on the evaluation of the nonlinear parameter of an elastic medium. Results have

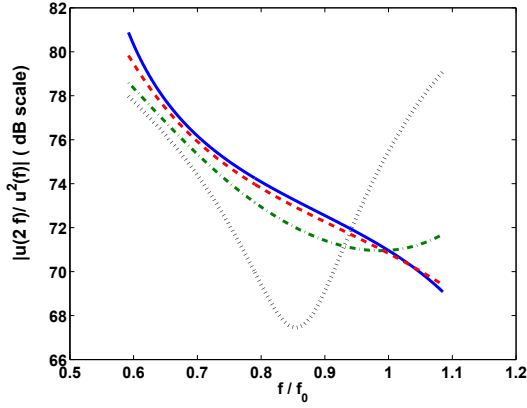


FIG. 6: Modulus of the quadratic distortion parameter $d(f)$ at the loaded face versus frequency for different cases of acoustic loads in case of electromechanical nonlinearity. A logarithmic scale is used where the 0 dB reference is $1m^{-1}$. The ratio of acoustic impedances Z_e/Z_p is respectively equal to : 0 (solid line), 0.25 (dashed line), 0.5 (dash-dotted line) and 1 (dotted line).

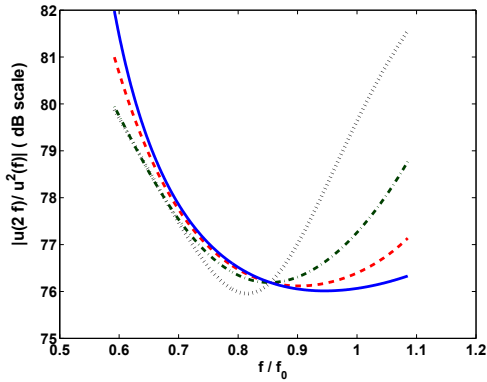


FIG. 7: Modulus of the quadratic distortion parameter $d(f)$ at the loaded face versus frequency for different cases of acoustic loads in case of mechanical nonlinearity. A logarithmic scale is used where the 0 dB reference is $1m^{-1}$. The ratio of acoustic impedances Z_e/Z_p is respectively equal to : 0 (solid line), 0.25 (dashed line), 0.5 (dash-dotted line) and 1 (dotted line).

shown the need to take into account the nonlinear effects of the transducer. The problem is to quantify the distortion rate at the output of the transducer and results have shown the great influence of the load on the generated distortion. Moreover, the complexity of interferences between the secondary waves created by each source of nonlinearity in a propagation line, as revealed in this study, demonstrates the interest of disposing of an efficient method to study the interactions between nonlinear phenomena.

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