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A class of perturbed cell-transmission models to account for traffic variability

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Abstract

We introduce a general class of traffic models derived as perturbations of cell-transmission type models. These models use different dynamics in free-flow and in congestion phases. They can be viewed as extensions to cell transmission type models by considering the velocity to be a function not only of the density but also of a second state variable describing perturbations. We present the models in their discretized form under a new formulation similar to the classical supply demand formulation used by the seminal *Cell-Transmission Model*. We then show their equivalence to hydrodynamic models. We detail the properties of these so-called perturbed cell-transmission models and illustrate their modeling capabilities on a simple benchmark case. It is shown that they encompass several well-known phenomena not captured by classical models, such as forward moving disturbances occurring inside congestion phases. An implementation method is outlined which enables to extend the implementation of a cell transmission model to a perturbed cell transmission model.

1 Introduction

Classical macroscopic models of traffic. The modeling of highway traffic at a macroscopic level is a well established field in the transportation engineering community, which goes back to the seminal work of Lighthill, Whitham [17] and Richards [23]. Their work introduced to the traffic community the kinematic wave theory which enables one to reconstruct fundamental macroscopic features of traffic flow on highways such as queues propagation. The so-called *LWR model*, based on conservation of vehicles, encompasses most of the non linear phenomena observed on highways in a computationally tractable framework.

In order to close the model, one needs to assume a relation between the velocity and density of vehicles. Greenshields [13] empirically measured a relation between the density and the flow of vehicles, now known as the *fundamental diagram*, which led to the formulation of the LWR problem as a single unknown state variable problem, which could be solved by discretization techniques.

A way to approach the resolution of the discretization of the mass conservation equation in a tractable manner was later proposed by Lebacque [15]. It was shown that a discrete solution of the LWR equation could be constructed by considering the local *supply demand* framework. In the case of concave fluxes, this solution is equivalent to the one obtained using a classical numerical method in conservation laws, the Godunov scheme [12].

The triangular model. Newell [18, 19, 20] introduced the *triangular fundamental diagram*, which is to this date one of the most standard models for queuing phenomena observed at bottlenecks, and for highway traffic modeling in general. Daganzo [7, 8] derived a discrete equivalent of the LWR equation in the case of the triangular fundamental diagram. This model known as the *Cell-Transmission Model* provided the transportation community with a meaningful modeling tool for highway traffic. One of the main assumptions of all the classical models is that the speed of vehicles is a single-valued function of the density.

Second order and perturbed models. Following hydrodynamic theory, attempts at modeling highway traffic with a second conservation equation and a second state variable to augment the mass conservation equation led to the development of so-called *second order models*, such as the Payne [22] and Whitham [25] model. Unfortunately, this model exhibited flaws pointed out by Daganzo [9], Del Castillo [10] and Papageorgiou [21], including the possibility for vehicles to drive backwards along the highway. These flaws were corrected in a new generation of second order models proposed for instance by Aw and Rascle [3], Lebacque [16] and Zhang [28, 29]. By considering a second state variable, these models offer additional capabilities with respect to classical models and for example enable the possibility to include velocity measurements such as the ones obtained from GPS cell phones [26].

The phase transition model Colombo [6] developed a phase transition traffic model with different dynamics for congestion and free-flow, to model fundamental diagrams observed in practice [1]. Like the work of Newell, this approach was motivated by the fundamentally different features of traffic in free-flow and in congestion [24]. In particular, this model includes a set-valued congested part of the fundamental di-

45 agram and a single-valued free-flow part of the fundamental diagram. The set-valued
46 congestion phase enables one to account for much more measurements in the con-
47 gestion phase than the classical fundamental diagram does. Indeed, in the classical
48 setting, a measurement falling outside of the fundamental diagram has to be discarded
49 or approximated. Thus for any tasks involving real data, information is lost at the
50 data processing step. In the setting proposed by the phase transition model and the
51 subsequent perturbed cell transmission model, a whole cloud of measurements can be
52 considered valid.

53 In part due to the complexity of practical implementation, Colombo's model was
54 extended in [5], leading to a new class of models taking in account the perturbation
55 around the classical fundamental diagram known to exist in practice. Similar to the
56 work of Zhang [28], an assumption is made that a classical fundamental diagram can
57 be viewed as an equilibrium (or average) of the highway traffic state in the perturbed
58 model. In this article, we describe the physical approach developed in [5] and present
59 simple and meaningful local rules to implement a class of discrete perturbed models.
60 We also provide a set of simple steps which can be followed to extend the well-known
61 implementation of the cell-transmission model to an implementation of a perturbed
62 cell transmission model.

63 **Outline.** The outline of this work is as follows. In Section 2, we recall the classical
64 framework for discrete macroscopic models, and introduce the discrete formulation of
65 a class of phase transition models relying on physical consideration about traffic flow
66 properties. In particular we show that these models reduce to a set-valued version
67 of the cell-transmission model in the case of a triangular flux. Section 3 provides
68 some examples of the modeling abilities of the class of perturbed models derived, and
69 illustrates the better performances of the class of perturbed models. Section 4 gives a
70 guidebook for perturbed model deployment. Conclusions and future research tracks
71 are outlined in Section 5.

72 **2 Discrete formulation of macroscopic traffic flow** 73 **models**

74 We consider the representation of a stretch of highway by N space cells C_s , $0 \leq s \leq N$
75 of size Δx and assume that representing time evolution by a discrete sequence of times
76 with a Δt step size yields a correct approximation for traffic flow modeling.

77 We make the usual assumption that there is no ramp on the link of interest, and
78 assume by considering a one-dimensional representation of the traffic conditions that
79 even on a multi-lanes highway, traffic phenomena can be accurately modeled as one
80 lane highway. The results presented here can easily be generalized to networks, for
81 example using the framework developed by Piccoli [11].

82 The following section presents the fundamental macroscopic traffic modeling equa-
83 tion, i.e. the mass conservation.

84 2.1 Classical models

85 2.1.1 Mass conservation equation

We call k_s^t the density of vehicles in the space cell C_s at time t , and $Q_{s\text{-up}}^t$ (respectively $Q_{s\text{-down}}^t$) the flux upstream (respectively downstream) of cell s between time t and time $t + 1$. The absence of ramp in cell s allows us to write the following conservation equation for the density of vehicles in cell s :

$$k_s^{t+1} \Delta x - k_s^t \Delta x = Q_{s\text{-up}}^t \Delta t - Q_{s\text{-down}}^t \Delta t \quad (1)$$

86 which states that between two consecutive times the variation of the number of vehi-
87 cles cell C_s is exactly equal to the difference between the number of vehicles having
88 entered the cell from upstream and the number of vehicles having exited the cell from
89 downstream.

90 Equation (1) which is the mass conservation from fluid dynamics (in a discrete
91 setting) is widely used among the transportation engineering community and consid-
92 ered as one of the most meaningful ways to model traffic flow on highways. Defining
93 the fluxes $Q_{s\text{-down}}$, $Q_{s\text{-up}}$ between two cells is a more complex problem, which can be
94 approached by considering a supply demand formulation.

95 2.1.2 The supply demand approach

96 The supply demand approach [15] states that the flow of cars that can travel from
97 an upstream cell to the next downstream cell depends on both the upstream density
98 and the downstream density. If we define the demand function $\Delta(\cdot)$ as a continu-
99 ous increasing function of the density and the supply function $\Sigma(\cdot)$ as a continuous
100 decreasing function of the density, then the flux between two cells is given by the
101 minimum of the *upstream demand* and the *downstream supply*. The supply and de-
102 mand function are bounded above on each cell by the flow capacity of the cell. Using
103 the notations introduced above, the supply demand formulation reads:

$$\begin{aligned} Q_{s\text{-up}}^t &= \min(\Delta(k_{s-1}^t), \Sigma(k_s^t)) \\ Q_{s\text{-down}}^t &= \min(\Delta(k_s^t), \Sigma(k_{s+1}^t)). \end{aligned} \quad (2)$$

104 The demand and supply functions are related to the fundamental diagram as follows.
105 In free-flow the supply $\Sigma(\cdot)$ is simply limited by the capacity of the cell whereas
106 in congestion, the supply is limited by current traffic conditions. In free-flow, the
107 demand $\Delta(\cdot)$ is limited by current traffic conditions whereas in congestion the demand
108 is constrained by the capacity of the cell. Given a fundamental diagram $Q(\cdot)$, with a
109 unique maximum at the critical density k_c , the supply $\Sigma(\cdot)$ and demand $\Delta(\cdot)$ functions
110 can thus be defined as:

$$\Delta(k) = \begin{cases} Q(k) & \text{if } k \leq k_c \\ Q(k_c) & \text{otherwise} \end{cases} \quad \text{and} \quad \Sigma(k) = \begin{cases} Q(k_c) & \text{if } k \leq k_c \\ Q(k) & \text{otherwise} \end{cases}$$

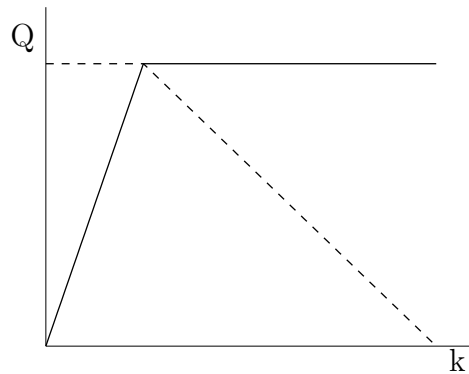


Figure 1: **Supply demand.** The supply curve (bold line) is an increasing function of density and the demand curve (dashed line) is a decreasing function of density.

111 When the fundamental diagram is triangular, the demand and supply functions are
 112 piecewise affine as illustrated on Figure 1, and the supply demand approach is exactly
 113 the cell-transmission model [7].

114 The supply demand approach enables one to define two types of traffic conditions;
 115 free-flow and congestion, which have fundamentally different features.

116 2.2 Two traffic phases

117 The behavior of traffic depends on the relative values of supply and demand. When
 118 the supply is higher than the demand, traffic flow is said to be in *free-flow*, the flux is
 119 defined by the number of cars that can be sent from upstream (upstream demand).
 120 On the opposite, when the demand is higher than the supply, the traffic is said to
 121 be in *congestion* because the flux is defined by the number of cars that the road can
 122 accept downstream (downstream supply).

123 These two dynamics exhibit at least one capital difference:

- 124 • In free-flow the flux is defined from upstream and information is moving forward,
 125 whereas in congestion the flux is defined from downstream and information is
 126 moving backwards.

127 One may note that the seminal Cell-Transmission Model considers this property as a
 128 required model feature, and thus can be viewed as a phase transition model. Figure 2
 129 illustrates two typical sets of experimental measurements. Two distinct phases appear
 130 characterized by:

- 131 • In free-flow, the speed is constant and the flux is uniquely determined by the
 132 density of cars (straight line through the origin for low densities in Figure 2).
 133 The knowledge of density or count seems to provide enough information to
 134 represent the traffic state.

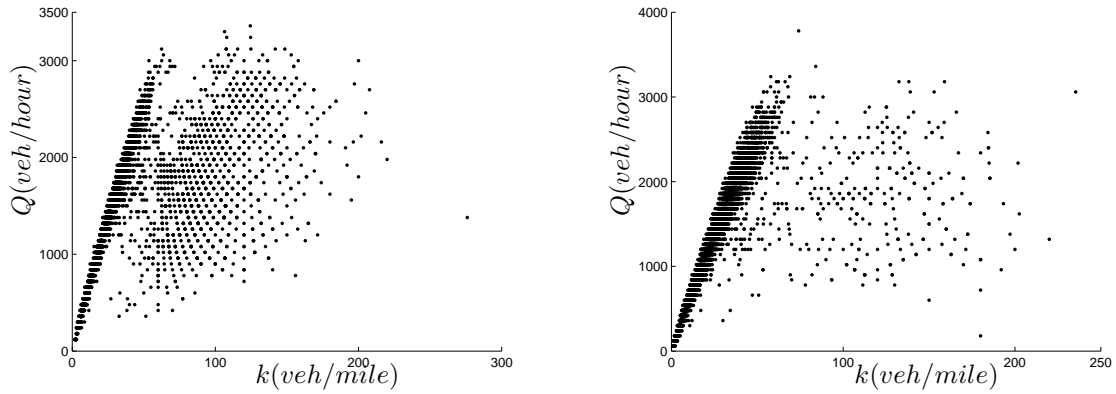


Figure 2: Experimental flow-density relations over a one week-period at two locations on a highway in Roma. Flow was directly measured and density was computed from the measured flow and the measured speed. In free-flow the speed is constant. The shape of the congestion phase changes for different locations.

- 135 • In congestion, a given density does not correspond to a unique speed, i.e. the
 136 fundamental diagram is set-valued. A second variable must be introduced to
 137 model the traffic state.

138 The first observation is taken in account by the triangular model whereas the second
 139 observation motivates the use of a phase transition model [5, 6] using different dynam-
 140 ics for free-flow and congestion, and justify the introduction of a perturbed model in
 141 congestion to define the dynamics of two variables necessary to model the congested
 142 traffic state [24, 27]. We introduce in the following section a class of perturbed cell
 143 transmission type models directly derived from classical models.

144 2.3 Perturbation of cell-transmission type models

145 2.3.1 A perturbed fundamental diagram

146 We propose to describe traffic state on a link of highway by using a perturbed phase
 147 transition model. Assuming that the highway link is composed of the cells C_s for
 148 $s = 1, \dots, N$, we define the speed of traffic in each cell as follows:

$$v_s = \begin{cases} V_{\text{ff}} & \text{if } C_s \text{ is in free-flow} \\ V(k_s)(1 + q_s) & \text{if } C_s \text{ is in congestion} \end{cases} \quad (3)$$

149 where V_{ff} is the free-flow speed and $V(\cdot)$ is the velocity function of a classical model.

150 **Application to the cell transmission model** The velocity function for the
 151 classical cell transmission model reads $V(k_s) = w(1 - k_j/k_s)$ where w is the backwards
 152 speed propagation and k_j is the jam density. Thus the perturbed speed reads:

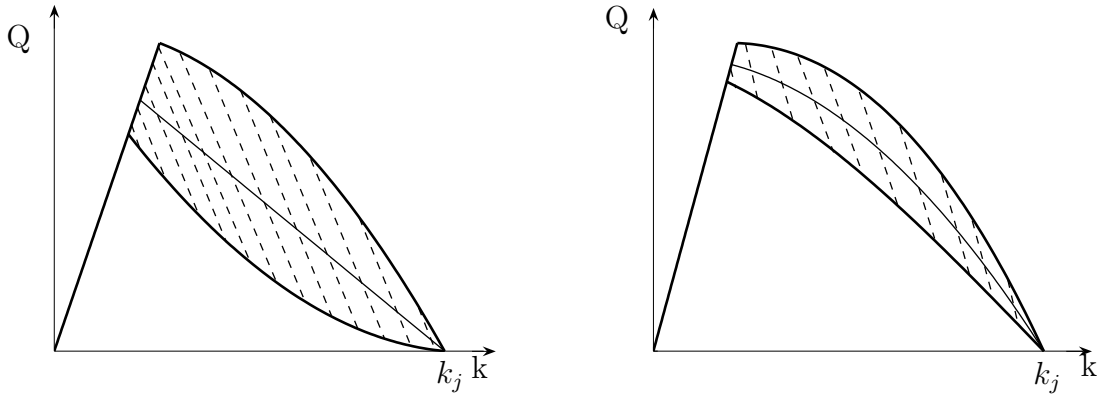


Figure 3: **Left:** Perturbed triangular fundamental diagram (the equilibrium flux function is linear decreasing in congestion). **Right:** Perturbed Greenshields fundamental diagram (the equilibrium flux function is parabolic decreasing in congestion). One can note that the free-flow speed is constant in both models and the flux is set-valued in congestion, i.e. to one density corresponds several values of the flux.

$$v_s = V(k_s) (1 + q_s) = w \left(1 - \frac{k_j}{k_s}\right) (1 + q_s)$$

153 and yields the fundamental diagram from Figure 3 left.

154 In free-flow, we describe the speed to be constant as per the triangular model,
 155 whereas in congestion we introduce a second variable q_s , modeling the fact that for
 156 a given density k_s the speed of cars is not uniquely determined by the density. The
 157 multiplicative factor $1 + q_s$ means that q_s can be viewed as a perturbation around
 158 the reference state of traffic which is given by the classical fundamental diagram. In
 159 the following we call *equilibrium speed* the value of the speed for $q_s = 0$ (which is
 160 the speed of the classical model according to equation (3)). The state of traffic is
 161 described by:

$$\begin{cases} k_s & \text{if } C_s \text{ is in free-flow} \\ (k_s, q_s) & \text{if } C_s \text{ is in congestion.} \end{cases}$$

162 In free-flow the density k_s completely describes the traffic state and the speed of
 163 vehicles is constant equal to V_{ff} . The flux of vehicles in the cell is the product of
 164 the density of vehicles and their speed $k_s V_{ff}$. In congestion, the state of traffic is
 165 described by the two variables density k_s and perturbation q_s . According to the
 166 expression outlined in (3), the speed of vehicles is $V(k_s) (1 + q_s)$. The flux of vehicles
 167 is the product of the density and the speed and is given by $k_s V(k_s) (1 + q_s)$.

168 **Remark 1.** *In the following, we assume that the equilibrium speed function in conges-*
 169 *tion is continuous, decreasing, vanishes at the maximal density, equals the free-flow*
 170 *speed at the critical density, and that the equilibrium flux is concave.*

171 **Remark 2.** For the sake of mathematical and physical consistency, the size of the
 172 perturbation q_s cannot be chosen arbitrarily and must satisfy the following constraints:

- 173 • The perturbed speed must be positive, i.e. $q_s \geq -1$.
- 174 • The curves on which q_s/k_s is constant (see section 2.3.3 for a physical inter-
 175 pretation of these curves) have a concavity with constant sign. This yields a
 176 bound on the perturbation which can be analytically computed by writing that
 177 the second derivative of the flux $k_s V(k_s) (1 + q_s)$ with respect to the density k_s
 178 has a constant sign for a given value of q_s/k_s .

179 2.3.2 Conservation equations for traffic states

180 Having defined the state of traffic in congestion and in free-flow, we define the dy-
 181 namics of these quantities as follows. The density k_s is assumed to satisfy the mass
 182 conservation given by equation (1). We assume that the macroscopic perturbation
 183 $q_s \Delta x$ is also conserved, and thus that q_s satisfies the perturbation conserva-
 184 tion:

$$q_s^{t+1} \Delta x - q_s^t \Delta x = R_{s\text{-up}}^t \Delta t - R_{s\text{-down}}^t \Delta t \quad (4)$$

185 where $R_{s\text{-up}}^t$ (respectively $R_{s\text{-down}}^t$) is the flow of macroscopic perturbation entering
 186 the cell C_s from upstream (respectively exiting from downstream). The dynamics
 187 satisfied by the traffic states is:

$$\begin{cases} k_s^{t+1} \Delta x - k_s^t \Delta x = Q_{s\text{-up}}^t \Delta t - Q_{s\text{-down}}^t \Delta t & \text{in free-flow} \\ \begin{cases} k_s^{t+1} \Delta x - k_s^t \Delta x = Q_{s\text{-up}}^t \Delta t - Q_{s\text{-down}}^t \Delta t \\ q_s^{t+1} \Delta x - q_s^t \Delta x = R_{s\text{-up}}^t \Delta t - R_{s\text{-down}}^t \Delta t \end{cases} & \text{in congestion} \end{cases} \quad (5)$$

188 One must be careful that at any location, the flux of mass $Q_{s\text{-up}}$ and the flux of
 189 perturbation $R_{s\text{-up}}$ are coupled by the relation (3) defining the speed and thus can not
 190 be defined independently by two uncoupled supply demand relations similar to (2).
 191 A coherent approach to the definition of the cell boundary fluxes is to consider the
 192 microscopic meaning of the state variable q_s .

193 2.3.3 From a macroscopic perturbed model to a behavioral driver model

194 Equation (4) expresses the conservation of the macroscopic perturbation $q_s \Delta x$. The
 195 usual classical fundamental diagram corresponds to the equilibrium velocity function
 196 (i.e. at $q_s = 0$), and for a given density this velocity function can take values above
 197 or below the equilibrium velocity function depending on the sign of q_s .

198 This variation of the velocity function around its equilibrium value leads us to
 199 consider the state variable q_s as characterizing the propension of an element of traffic
 200 to move forward, in a very similar way to the *driver's ride impulse* from [2]. Indeed,
 201 in a cell C_s with a density of vehicles k_s , high values of q_s model aggressive drivers
 202 who are eager to move forward and adopt high speed. Low values of q_s model passive
 203 drivers who adopt low values of speed.

204 The speed v_s of drivers and their average aggressiveness defined by the quantity
 205 q_s/k_s will play a decisive role in the definition of the boundary fluxes.

206 **Remark 3.** *One may note that it is not possible to measure the aggressiveness level of*
 207 *drivers. According to the definition of our class of model, this quantity is completely*
 208 *determined by the knowledge of the speed and density. Thus measures of counts or*
 209 *speeds can be combined with measures of density in order to compute values of the*
 210 *aggressiveness level.*

211 2.3.4 Traffic rules defining flow between cells

212 The supply demand formulation does not yield a simple formalism for perturbed
 213 models. We choose to define the fluxes from equation (5) by other equivalent physical
 214 considerations. We propose two different sets of rules depending on whether the traffic
 215 state in the upstream cell is in free-flow or in congestion.

216 Congested upstream cell

217 We consider two neighboring cells C_{s-1} and C_s with traffic states (k_{s-1}^t, q_{s-1}^t) and
 218 (k_s^t, q_s^t) such that the upstream cell is in a congested state. We define the following
 219 two rules who will define the flux between these two cells between times t and $t + 1$:

- 220 • To enter the downstream cell, the vehicles from the upstream cell must modify
 221 their speed from v_{s-1}^t to the speed of the vehicles from the downstream cell v_s^t .
- 222 • The vehicle from the upstream cell modify their speed according to their average
 223 driving aggressiveness q_s/k_s .

224 These two rules imply that the vehicles which will exit the upstream cell C_{s-1} to enter
 225 the downstream cell C_s will have speed v_s and will have an average aggressiveness
 226 q_s/k_s . Thus the flux between cell C_{s-1} and cell C_s correspond to a new traffic state
 227 $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ which can be defined by the system of equations:

$$\frac{q_{s-1/2}^{t+1/2}}{k_{s-1/2}^{t+1/2}} = \frac{q_{s-1}}{k_{s-1}} \quad \text{and} \quad v_{s-1/2}^{t+1/2} = v_s \quad (6)$$

228 where the second equation can be rewritten as an equation in $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ us-
 229 ing the expression from (3). This yields a system of two independent equations in
 230 $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$. The corresponding speed $v_{s-1/2}^{t+1/2}$ can be computed from the expression
 231 of $k_{s-1/2}^{t+1/2}$ and $q_{s-1/2}^{t+1/2}$ using equation (3). The mass flux and perturbation flux can be
 232 then defined as:

$$Q_{s\text{-up}}^t = k_{s-1/2}^{t+1/2} v_{s-1/2}^{t+1/2} \quad \text{and} \quad R_{s\text{-up}}^t = q_{s-1/2}^{t+1/2} v_{s-1/2}^{t+1/2}$$

233 **Free-flowing upstream cell**

234 We consider two neighboring cells C_{s-1} and C_s with traffic states k_{s-1}^t (free-flow) and
 235 (k_s^t, q_s^t) (congestion). The boundary flux of vehicles between the upstream cell C_{s-1}
 236 and the downstream cell C_s falls into one of these two cases:

- 237 • If the upstream flow is lower than the downstream flow then traffic conditions
 238 are imposed from upstream and the boundary flow is the upstream flow. This
 239 leads to the boundary flow:

$$Q_{s\text{-up}}^t = k_{s-1}^t V \quad \text{and} \quad R_{s\text{-up}}^t = q_{s-1/2}^{t+1/2} V$$

240 where $q_{s-1/2}^{t+1/2}$ is the perturbation defined by $V(k_{s-1}^t)(1 + q_{s-1/2}^{t+1/2}) = V$.

- 241 • If the upstream flow is higher than the downstream flow then traffic conditions
 242 are imposed from downstream and we obtain similar conditions to the case of
 243 two congested cells. Incoming vehicles will adapt their speed to the downstream
 244 speed and adopt the lowest corresponding average level of aggressiveness allow-
 245 able by the fundamental diagram. These two conditions yield the equations:

$$\frac{q_{s-1/2}^{t+1/2}}{k_{s-1/2}^{t+1/2}} = \frac{q_{\min}}{k_j} \quad \text{and} \quad v_{s-1/2}^{t+1/2} = v_s \quad (7)$$

246 where q_{\min}, k_j are the minimal density of perturbation and jam density (maximal
 247 density). If we note $(k_{s-1/2}^{t+1/2}, q_{s-1/2}^{t+1/2})$ the solution of (7), the boundary fluxes are
 248 given by:

$$Q_{s\text{-up}}^t = k_{s-1/2}^{t+1/2} v_{s-1/2}^{t+1/2} \quad \text{and} \quad R_{s\text{-up}}^t = q_{s-1/2}^{t+1/2} v_{s-1/2}^{t+1/2}$$

249 **3 Benchmark cases**

250 **3.1 Encounter of two flows with different properties**

251 **3.1.1 Perturbed model features**

252 We consider the situation of two cells with congested flows. In the upstream cell the
 253 traffic state is (k_A, q_A) with high density and low speed and in the downstream cell
 254 the state is (k_B, q_B) with low density and high speed. These two traffic states are
 255 represented by the points A and B on Figure 4 (right).

256 According to the rules described in section 2.3.4, the cars from the upstream cell
 257 will increase their speed while keeping the same average aggressiveness level q_A/k_A .
 258 Physically this means that the drivers from the traffic state A which is slower and
 259 denser increase their speed when they reach the front end of the flow A , but do not
 260 change their behavior.

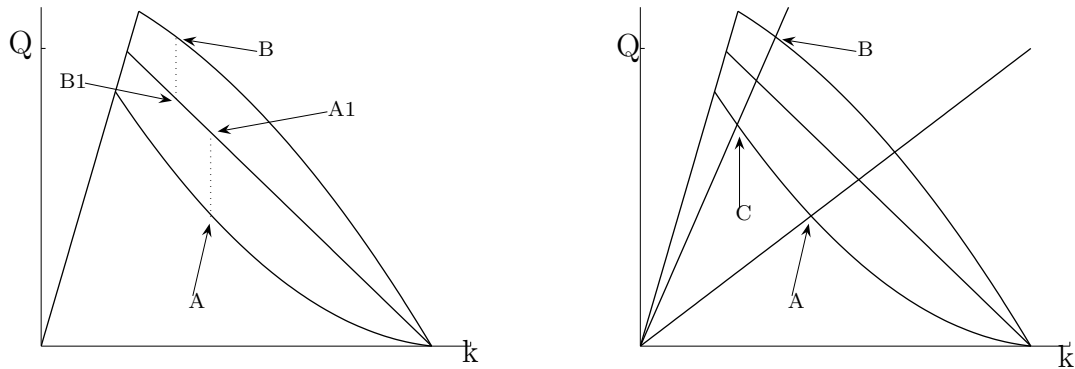


Figure 4: **Left: Classical model.** A and B fall outside of the classical fundamental diagram and are viewed as $A1$ and $B1$; the resulting steady state is $B1$. **Right: Perturbed model.** A and B fall in the perturbed fundamental diagram; the resulting steady state is C .

261 Thus the flow of cars moving from the upstream cell to the downstream cell will
 262 be in state C , defined by the intersection of two curves. The first curve is the straight
 263 line defined by the speed being the speed of B , namely $v_C = V(k_B, q_B)$ according to
 264 expression (3). The second curve is defined by the average aggressiveness of drivers
 265 being the average aggressiveness of drivers from state A , namely $q_C/k_C = q_A/k_A$.
 266 One can note that this set of two equations is the one introduced at (6).

267 3.1.2 Comparison of perturbed and classical model

268 We compare the evolution predicted by a classical model and by its associate per-
 269 turbed model, for the two flows described in previous section. The evolution given
 270 by the perturbed model was described in previous section.

271 The classical model can not take in account the states A and B as such because
 272 they fall outside of the classical fundamental diagram. Joint measurements of speed
 273 and density returning traffic states A and B would have to be approximated. They
 274 could be understood as states $A1$ and $B1$ if the density measurement were more
 275 reliable.

276 The interaction of states $A1$ and $B1$ is described by the cell-transmission model as
 277 producing the steady state $B1$. One can note that this state is significantly different
 278 from the steady state C predicted by the perturbed model.

279 3.2 Homogeneous in speed states

280 Traffic flows composed of various densities in which all the vehicles drive at the same
 281 speed are commonly observed but cannot be accounted for by classical models which
 282 assume that for one given density, only one speed can occur.

283 Perturbed models allow traffic states with different densities to have the same

284 speed, and can model the homogeneous in speed states observed by Kerner [14]. For
285 instance, if we consider the encounter of two traffic flows with the same speed and
286 different densities such as the state B and C from Figure 4, the model predicts that
287 the difference in flows and densities between the two traffic states is such that the
288 discontinuity propagates downstream at exactly the same speed. It is the similar
289 situation that is observed in free-flow for the triangular model. Indeed one could
290 imagine that the straight line of constant speed defined by $v = v_C$ is the free-flow
291 part of a classical triangular fundamental diagram, in which case the same type of
292 propagation of the two states B and C would be predicted by the cell-transmission
293 model.

294 **4 Implementing a perturbed cell-transmission** 295 **model**

296 In this section we propose to give a brief outline of the way to implement a perturbed
297 cell-transmission model.

298 1 Define a classical fundamental diagram which fits the dataset best. Depending on
299 the implementation constraints, this can be done in a variety of methods, from a
300 visual agreement to an optimization routine [4]. In particular, identify the free-flow
301 speed V_{ff} , the jam density k_j and the critical density k_c . This corresponds to the
302 classical implementation method for the CTM.

303 2 Compute bounds on the perturbation according to the limitations expressed in
304 remark 2. This requires to compute the maximum and minimum of the second
305 derivative of the flux function along a curve of constant aggressiveness level.

306 3 Given a traffic condition, i.e. a point (ρ, q) , check that all the discrete congested
307 states fall into the fundamental diagram, otherwise use an approximation method
308 to map it back to the fundamental diagram, similarly to the case of the classical
309 fundamental diagram.

310 4 Evolve the model in time using the rules proposed in section 2.3.4.

311 This shows that implementing a perturbed cell-transmission model is almost as
312 simple as implementing the classical cell-transmission model. We illustrated in sec-
313 tion 3 the added value of these models.

314 **5 Conclusion**

315 In this article we propose a class of perturbed models which match empirical features
316 of highway traffic more closely than classical models by incorporating a set-valued
317 fundamental diagram in congestion. We show that by considering a second state
318 variable in congestion, this class of models has greater modeling capabilities.

319 We follow the principles of the cell-transmission model which assumes that the two
320 phases of traffic, free-flow and congestion, have fundamentally different behaviors. We
321 consider that the speed of traffic is constant in free-flow whereas in congestion it has a
322 perturbed value around the equilibrium speed. The class of models introduced is cus-
323 tomizable in the sense that traffic engineers can select the most appropriate classical
324 fundamental diagram and perturb it according to experimental measurements.

325 We make the assumption that the state variable introduced satisfies a conserva-
326 tion equation, which is motivated by its physical interpretation. At the macroscopic
327 level, it can be considered as a perturbation of the traffic state around the classical
328 fundamental diagram. At a microscopic level, this variable models the behavior of
329 drivers, who make different speed choices for the same observed density. We provide
330 simple meaningful rules to march the model forward in time.

331 Finally, we provide a simple way to implement this perturbed class of traffic
332 models in the framework currently used by traffic engineers. We show that these
333 models which result from an extension of usual cell-transmission type models can be
334 derived in a straightforward manner.

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