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# HYPERSTRUCTURES ASSOCIATED TO A CONNECTED GRAPH

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### Abstract

We are searching to associate, in a natural way, a hyperstructure to a connected graph showing that in many relevant cases it is a join-space (under the convention in [3]).

1. Let  $H$  be a nonempty set and  $\mathcal{P}^*(H) = \{A \mid A \subseteq H, A \neq \emptyset\}$  (where  $\subseteq$  stands for the non-strict inclusion). A *hyperoperation* on  $H$  is a function  $(x, y) \rightarrow xoy$  from  $H \times H$  to  $\mathcal{P}^*(H)$ . We will denote  $A \circ B = \cup\{a \circ b \mid a \in A, b \in B\}$  for  $A, B \in \mathcal{P}^*(H)$ .

A hyperoperation  $\circ$  is called *associative* if

$$(\forall)x, y, z \in H : (x \circ y) \circ z = x \circ (y \circ z).$$

In this case  $(H, \circ)$  will be referred to as a *semi-hypergroup*. The hyperoperation  $\circ$  is called *reproducible*, if  $x \circ H = H \circ x = H$  for each  $x \in H$ .

The semi-hypergroup  $(H, \circ)$  with " $\circ$ " reproducible is called a *hypergroup*. In this case we introduce for each  $a, b \in H$   $a/b = \{c \in H \mid a \in b \circ c\}$ . A commutative hypergroup  $(H, \circ)$  which satisfies:

$$(J) \quad (\forall)(a, b, c, d) \in H^4, a/b \cap c/d \neq \emptyset \implies a \circ d \cap b \circ c \neq \emptyset$$

is called a *join-space* (as in [3]).

If in addition the join-space  $(H, \circ)$  has the property (see [1], pag. 71):

$$x \circ x = \{x\} = x/x \text{ for all } x \in H$$

$(H, \circ)$  is said to be a *geometrical join-space*. Otherwise it is called a non-geometrical join-space.

2. Let  $G=(V, E)$  be a tree (from now on we suppose  $\text{card}(V) \geq 2$ ).

(a)  $\forall x \in V, x \circ x = \{x\}$ ;

(b)  $\forall x, y \in V, x \neq y, x \circ y = [xy] = \{z \in V \mid z \text{ is a vertex on the elementary path that joins } x \text{ and } y\}$ . Note that because  $G$  is a tree, (b) is consistent and

non-contradictory ( $G$  tree  $\implies \forall x, y \in V, x \neq y$ , there exists exactly one path joining  $x$  and  $y$ ).

**Theorem 1.** Let  $G=(V,E)$  be a tree and  $(V,o)$  be its associated hyperstructure by the method above. Then  $(V,o)$  is a non-geometrical join-space.

**Proof.** We notice for the beginning that  $o$  is commutative,  $x \circ x = x/x = \{x\}$  for each  $x \in V$ , and  $\{x, y\} \subseteq x \circ y$ , for any  $x, y \in V$  with  $x \neq y$ .  
To prove the associativity we start by showing that:

$$(V)x, y, z \in V : (x \circ y) \circ z \subseteq x \circ (y \circ z). \tag{1}$$

If  $x = y$  then  $(x \circ y) \circ z = x \circ z, x \circ (y \circ z) = x \circ (x \circ z)$  and since  $x \circ z \subseteq x \circ (x \circ z)$ , (1) follows.

If  $x \neq y$ , denote by  $x = \alpha_1, \alpha_2, \dots, \alpha_n = y$  the path  $[xy]$  that joins  $x$  and  $y$ , and by  $y = \beta_1, \beta_2, \dots, \beta_k = z$  the path  $[yz]$ . Two situations occur:

i)  $x \circ y \cap y \circ z = \{y\}$ ; then one can easily see that

$$(x \circ y) \circ z = x \circ y \cup y \circ z = \{\alpha_1, \alpha_2, \dots, \alpha_n = y = \beta_1, \beta_2, \dots, \beta_k = z\}$$

and also

$$x \circ (y \circ z) = x \circ \{\beta_1, \beta_2, \dots, \beta_k = z\} = \{\alpha_1, \alpha_2, \dots, \alpha_n = y = \beta_1, \beta_2, \dots, \beta_k = z\};$$

so we have equality.

ii)  $x \circ y, y \circ z$  have common points different from  $y$ . As  $G$  is a tree, there exists  $p \in \{1, \dots, n\}, q \in \{1, \dots, k\}$  such that  $q = \max\{i \mid \beta_i \in x \circ y\}$   $\alpha_p = \beta_q$  and

$$\begin{aligned} x \circ y &= \{\alpha_1 = x, \alpha_2, \dots, \alpha_{p+1} = \beta_{q-1}, \dots, \alpha_p = \beta_q, \alpha_n = y = \beta_1\} \\ y \circ z &= \{\beta_1 = y = \alpha_n, \beta_2 = \alpha_{n-1}, \dots, \beta_q = \alpha_p, \beta_{q+1}, \dots, \beta_k = z\}. \end{aligned}$$

As a consequence of this,

$$x \circ z = \{x = \alpha_1, \alpha_2, \dots, \alpha_p = \beta_q, \beta_{q+1}, \dots, \beta_k = z\} \text{ (because } G \text{ has no cycles).}$$

Finally one obtains

$$(x \circ y) \circ z = x \circ (y \circ z) = \{\alpha_1, \dots, \alpha_n, \beta_{q+1}, \dots, \beta_k\}.$$

So far we proved (1), therefore we can write:

$$(x \circ y) \circ z \subseteq x \circ (y \circ z) = (y \circ z) \circ x \subseteq y \circ (z \circ x) = (z \circ x) \circ y \subseteq z \circ (x \circ y) = (x \circ y) \circ z.$$

Now all the signs  $\subseteq$  can be replaced by  $=$  in this last relation; and therefore, the associativity of  $o$  is proved.

We noticed at the beginning that  $(V)x, y \in V : y \in x \circ y$ . So  $(V)y \in V : y \in x \circ y \subseteq x \circ V$  where from  $(V)y \in V : y \in x \circ V = V \circ x$ . Hence  $o$  is reproducible.

To finish the proof we must show that (J) holds. Let  $(a, b, c, d) \in V^4$  such that  $a/b \cap c/d \neq \emptyset$ . This means  $(\exists)\alpha \in V, \alpha \in a/b \cap c/d$  (hence  $a \in b \circ \alpha, c \in d \circ \alpha$ ). We have two cases to analyze:

i)  $b \circ \alpha \cap d \circ \alpha = \{\alpha\}$ . As  $a \circ d \cap b \circ c$  contains at least  $a, c, \alpha$  (J) follows.

ii)  $b \circ \alpha \cap d \circ \alpha \neq \{\alpha\}$ . Let  $b \circ \alpha = \{\alpha_1 = b, \dots, \alpha_i = a, \dots, \alpha_n = \alpha\}, d \circ \alpha = \{\beta_1 = d, \dots, \beta_j = c, \dots, \beta_m = \alpha\}$  and  $\beta_{j_0} = \max\{k \mid \beta_k \in b \circ \alpha\}, \alpha_{i_0} = \beta_{j_0} = e$ .

There are two possibilities for  $a$ :  $a \in b \circ e$  or  $a \in e \circ \alpha$  and likewise for  $c$ :  $c \in d \circ e$  or  $c \in e \circ \alpha$ . As we will see, in all cases (J) is valid:

- a)  $a \in b \circ e, c \in e \circ \alpha; b \circ c \cap a \circ d$  contains  $a, e$ .
- b)  $a \in b \circ e, c \in d \circ e$ ; we find  $a, c, e$  are in  $a \circ c \cap a \circ d$ .
- c)  $\{a \in e \circ \alpha, c \in e \circ \alpha\}$  or  $\{a \in b \circ e, c \in d \circ e\}$ ; then  $e \in b \circ c \cap a \circ d$ .

The proof is now complete.

**Remark 1.** Denote by  $H_G$  the join-space  $(V,o)$  associated to a tree  $G=(V,E)$  like above. If  $\Gamma$  is the set of isomorphism classes of trees and  $G$  the set of isomorphism classes of join-spaces, then the function  $f : \Gamma \rightarrow G$  defined by  $f(\bar{G}) = \overline{H_G}$  is injective (here  $\bar{x}$  is the class of  $x$ ). Indeed, the condition  $x \circ y = \{x, y\}$  in  $H_G$  is equivalent to  $(x, y)$  being an edge in the tree  $G$  and so we can construct in only one way the graph  $G$  starting from  $H_G$ .

3. Let  $G=(V,E)$  be a graph. We say that  $G$  is *2-connected* if  $card(V) > 2$ ,  $G$  is connected and by removing a vertex one obtains a connected graph too. A block in  $G$  is a maximal 2-connected subgraph of  $G$ .

Let  $G=(V,E)$  be a connected graph and denote by  $b_1, \dots, b_k$  its blocks. Construct the graph  $G^* = (V^*, E^*)$  where  $V^* = V \cup \{b_1, \dots, b_k\}$  and  $(x, y) \in E^*$  if and only if one of the following cases occurs:

- i)  $x \in V, y = b_i$  (for some  $i \in \{1, \dots, k\}$ ) and  $x$  is a vertex of  $b_i$  in  $G$  (or vice versa  $y \in V, x = b_i$ .)
- ii)  $(x, y) \in E$  and there is no elementary cycle passing through  $x$  and  $y$ .

**Example 1.** Let  $G$  as below (fig. 4),  $b_1 = \{6, 7, 8, 9\}, b_2 = \{2, 3, 4, 5\}$ . Then  $G^*$  is as in fig. 5.

**Remark 2.**  $G^*$  is a tree. Indeed,  $G$  connected implies  $G^*$  connected, and if in  $G^*$  there exists a cycle it contains at least two blocks  $b_i$  and  $b_j$  of  $G$  as points. Then in  $G^*$  are two paths from  $b_i$  to  $b_j$  and the same will be in  $G$ . But then  $b_i$  and  $b_j$  are contained in the same block of  $G$ , contradiction.

Let  $G=(V,E)$  be a connected graph and  $G^*$  its block-tree. We denote by  $H_G$  the join space  $H_{G^*}$  and we will shall say that  $H_G$  is the join-space associated to  $G$ .

**Remark 3.** i) If  $G$  is a tree then  $G$  is its own block-tree and so the definition above extends the one we gave in 2.

ii) If we denote by  $\Gamma$  the set of isomorphism classes of graphs then  $f^* : \Gamma \rightarrow G$  defined by  $f^*(\bar{G}) = \overline{H_G}$  is (unlike in the case of  $f$ ) not injective.

iii) When  $G$  is 2-connected its block-tree is a star-shaped graph (there exists a point  $b \in V$  such that any edge in  $G^*$  is of a form  $(b, a)$  or  $(a, b)$ ,  $a \in V$ ) and the join-space associated to it

$$(V^*, \circ) : V^* = V \cup \{b\}, x \circ y = \{x, y, b\} \text{ for any } x, y \in V, x \neq y$$

gives no information about the graph-structure of  $G$ . In fact  $H_G$  characterizes the tree-structure of a graph; so it is natural to ask if there exists another hyperstructure construction for 2-connected graphs. An answer is given in Section 5.

4. In Section 3 we associated a join-space to a connected graph using its path-structure. A question arises: given a join-space  $(J, \circ)$  with  $\text{card}(J) < \infty$ , under which conditions upon  $J$  there exists a graph  $G$  (in fact a tree) such that  $J = H_G$ ?

In this section we suppose  $J$  is a finite set having at least two elements and  $\circ$  is a hyperoperation on  $J$ . Let us consider the following conditions:

- (A<sub>1</sub>)  $\circ$  is associative;
- (A<sub>2</sub>)  $\circ$  is commutative;
- (A<sub>3</sub>)  $x \circ J = J \circ x = J$ ,  $\forall x \in J$ ;
- (A<sub>4</sub>)  $x \circ x = x$ ,  $\forall x \in J$ .

**Proposition 1.** *If (A<sub>1</sub>) - (A<sub>4</sub>) and (J) are valid for (J, \circ), then for any  $x, y, z \in J$  such that  $z \in x \circ y$  and  $y \in x \circ z$  one has  $y = z$ .*

**Proof.** Indeed,  $z \in x \circ y \Rightarrow x \in z/y$  and  $y \in x/y$  and  $y \in x \circ z \Rightarrow x \in y/z$ . Then  $z/y \cap y/z \neq \emptyset \Rightarrow y \circ y \cap z \circ z \neq \emptyset \Rightarrow y = z$ .

A join-space for which (A<sub>1</sub>) - (A<sub>4</sub>) and (J) hold, as well as (G)  $\forall x, y \in J$ ,  $x \neq y : \{x, y\} \subseteq x \circ y$  is called a (g) - join space.

**Proposition 2.** *Let (J, \circ) be a (g)-join-space. Then for each  $x \in J$  there exists  $y \in J$  such that  $x \circ y = \{x, y\}$ .*

**Proof.** Let  $y \in J$  be an element of  $J \setminus \{x\}$  for which  $\text{card}(x \circ y)$  is minimal. By hypothesis (J, \circ) is a (g)-join-space; so that  $\text{card}(x \circ y) \geq 2$ . If  $\text{card}(x \circ y) = 2$  than  $(x \circ y) = \{x, y\}$ . If  $\text{card}(x \circ y) \geq 3$  there exists  $z \in x \circ y \setminus \{x, y\}$ . Now,  $z \in x \circ y$  gives  $x \circ z \subseteq x \circ (x \circ y) = (x \circ x) \circ y = x \circ y$ . The case  $x \circ z = x \circ y$  yields  $y \in x \circ z \subseteq x \circ y$  (where  $\subseteq$  stands for the strict inclusion) implies  $\text{card}(x \circ z) < \text{card}(x \circ y)$ , contradiction. This completes the proof.

Let  $(J, \circ)$  be a (g)-join-space. We define a relation  $\sim$  over  $J$  by:

$$x \sim y \leftrightarrow \{x \neq y \text{ and } x \circ y = \{x, y\}\}.$$

0	a	b	c	d	e
a	a	a,b	a,b,c	a,d,e	a,e
b	a,b	b	b,c	b,c,d	a,b,e
c	a,b,c	b,c	c	c,d	c,d,e
d	a,d,e	b,c,d	c,d	d	d,e
e	a,e	a,b,e	c,d,e	d,e	e

One can easily see that  $\sim$  is symmetrical whenever  $\circ$  is commutative. Any (g)-join-space  $(J, \circ)$  which satisfies the condition:

$$(P) \forall k \in N, (\forall) \{x_1, x_2, \dots, x_k\} \subseteq J \text{ with } x_1 \sim x_2, \dots, x_{k-1} \sim x_k \text{ then } \{x_1, \dots, x_k\} \subseteq x_1 \circ x_k$$

is called a path join-space.

**Remark 4.** i) From  $\{x_1, \dots, x_k\}$  being a set it follows  $x_i \neq x_j$  for all  $i, j \in \{1, \dots, k\}$ ,  $i \neq j$ .

ii) The above definition is not trivial. This is shown from the following example: Let  $(J, \circ), J = \{a, b, c, d, e\}$  be a (g)-join-space as below:

Note that  $x \circ y$  is the minimal path that joins  $x$  and  $y$  in the graph from fig. 6. This (g)-join-space is not a path join-space.

**Theorem 2.** *Let (J, \circ) be a path join-space. Then there is a tree  $G = (V, E)$  such that  $J = H_G$ .*

**Proof.** Let  $V = J$ ,  $E = \{(x, y) \in J^2 \mid x \sim y\}$ ,  $G = (V, E)$ . We shall show that  $G$  is a tree and  $J = H_G$ . To prove there is no cycle in  $G$ , suppose by contradiction that there exists  $\{x_1, \dots, x_k\} \subseteq J$  such that  $x_1 \sim x_2, x_2 \sim x_3, \dots, x_{k-1} \sim x_k, x_k \sim x_1$  ( $k \geq 3$ ). Then:

$$x_2 \in x_1/x_k \text{ (because } x_k \sim x_1, x_1 \sim x_2 \Rightarrow x_1 \in x_2 \circ x_k)$$

$$x_2 \in x_k/x_1 \text{ (because } x_1 \sim x_k, x_k \sim x_{k-1}, \dots, x_3 \sim x_2 \Rightarrow x_k \in x_1 \circ x_2).$$

Hence  $x_1/x_k \cap x_k/x_1 \neq \emptyset$  where from  $x_1 \circ x_k \cap x_k \circ x_1 \neq \emptyset$ , which is impossible.

Now suppose that  $G$  is not connected, i.e. there exist at least two connected components in  $G$ . Let  $x_i$  and  $x_j$  be such that  $\text{card}(x_i \circ x_j)$  is minimal with the property that  $x_i$  and  $x_j$  are not in the same component. It follows that  $x_i \not\sim x_j$ ; so  $(\exists) z \in x_i \circ x_j \setminus \{x_i, x_j\}$ . We have seen that  $x_i \circ z \subseteq x_i \circ x_j, z \circ x_j \subseteq x_i \circ x_j$  implies  $\text{card}(x_i \circ z) < \text{card}(x_i \circ x_j), \text{card}(z \circ x_j) < \text{card}(x_i \circ x_j)$ . Since  $x_i, x_j$  are not in the same component at least one of the pairs  $(x_i, z), (x_j, z)$  has the same property, in contradiction with the minimality of  $\text{card}(x_i \circ x_j)$ . So far  $G$

is a tree. Let  $\square$  be the hyperoperation associated to the tree-structure of  $G$ . By condition (P),  $x \square y \subseteq x \circ y$  for each  $x, y \in J$ . Suppose that:

$$(*) \quad c \square y \subset x \circ y \text{ (obviously } x \neq y) \text{ for some } x, y \in J,$$

and take  $(x, y)$  with the property that  $\text{card}(x \circ y)$  has the minimum value among all the pairs which satisfy (\*). Let  $z \in x \circ y \setminus x \square y$  (of course  $z \notin \{x, y\}$ ). Since  $z \in x \circ y$  and  $z \notin \{x, y\}$ ,  $\text{card}(x \circ z) < \text{card}(x \circ y)$ ,  $\text{card}(z \circ y) < \text{card}(x \circ y)$  and, by minimality,  $x \square z = x \circ z$  and  $z \square y = z \circ y$ .

In  $G$ ,  $x \square z$  intersects  $z \square y$  at least in one point (for example  $z$ ). If  $x \square z \cap z \square y = \{z\}$  we obtain  $z \in x \square y$ , contradiction. It follows that  $\exists t \in (x \square z \cap z \square y) \setminus \{z\}$ . Let us choose  $t$  such that  $x \square t \cap y \square t = \{t\}$ . Then  $t \in x \square z \subseteq x \circ z$ , and this yields  $x \in t/z$ . Since  $z \in x \circ y \Rightarrow x \in z/y$  it follows that:

$$z/y \cap t/z \neq \emptyset \Rightarrow z \circ z \cap t \circ y \neq \emptyset \Rightarrow z \in y \circ t.$$

On the other hand we have in  $(J, \square)$   $y \square x = y \square t \cup t \square x$ . This yields  $y \square t \subseteq y \square x$  where from  $z \in y \square x$ , contradiction. Hence  $x \square y = x \circ y$ ,  $\forall x, y \in J$  and consequently,  $H = H_G$ .

**Remark 5.** The result above shows that - supposing we keep the set  $V$  and we make no difference among paths joining the same end-points - the construction in Section 2 is unique and is possible only for trees.

5. Let  $G=(V, E)$  be a 2-connected graph. We have seen in Section 3 that in this case a new construction is needed. This is founded on an equivalent definition of a 2-connected graph according to which for any two different elements  $x, y \in V$  there is a cycle passing through  $x, y$ . Define a hyperoperation  $\circ : V \times V \rightarrow \mathcal{P}^*(V)$  by:

- a.  $x \circ x = \{x\} \forall x \in V$ ;
  - b.  $x \circ y = \{z \in V \mid \text{there is a cycle through } x, y, z\} \forall x, y \in V, x \neq y$ .
- $G$  being 2-connected  $\circ$  is well defined. It is easy to see that  $\circ$  is commutative and

(\*\*) if  $\{x, y, z\} \subseteq V$  then  $x \in y \circ z \Leftrightarrow y \in x \circ z \Leftrightarrow z \in x \circ y$ .  
In general,  $\circ$  is not associative. This is shown by the following

**Example 2.** Let  $H = \{a_1, a_2, b_1, b_2, c_1, c_2, m_1, m_2, n_1, n_2, p_1, p_2\}$  and the graph (fig. 8)

We notice that  $(a_1 \circ a_1) \circ b_1 = a_1 \circ b_1 = H \setminus \{a_2, b_2\}$  and  $a_1 \circ (a_1 \circ b_1) = a_1 \circ H \setminus \{a_2, b_2\} = H$ , so  $\circ$  is not associative.

**Theorem 3.** Let  $G=(V, E)$  be a 2-connected graph and  $\circ$  be the hyperoperation just defined. Then  $\circ$  is associative if and only if  $x \circ y = V$  for each  $x, y \in V, x \neq y$ .

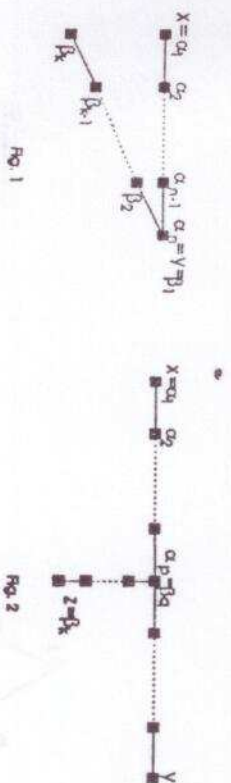
**Proof.** One can easily see that  $\circ$  is reproducible and so the implication  $\Rightarrow$  is obvious. Conversely, assume that  $\circ$  is associative and let  $a, b, c \in V$  with  $b \neq c$ . We will shall prove  $a \in b \circ c$ . Suppose on the contrary  $a \notin b \circ c$ . If there exists  $f \in b \circ c \setminus \{b, c\}$  with  $f \in a \circ b$  then  $a \neq f$  (because  $a \notin b \circ c$ ). But  $f \in a \circ b \Rightarrow a \in b \circ f$ ; hence  $a \in b \circ f \subseteq b \circ (b \circ c) = (b \circ b) \circ c = b \circ c$ , contradiction. It follows  $a \circ b \cap b \circ c = \{b\}$  ( $c \in a \circ b$  would imply  $a \in b \circ c$ ). Similarly, we may show that  $a \circ c \cap b \circ c = \{c\}$ . Suppose now that there exists  $x \in a \circ c \cap a \circ b, x \neq a$ . Of course, if  $x = b$  then  $b \in a \circ c$ , impossible. Hence  $x \neq b$  and in the same way  $x \neq c$ . Now  $x \in a \circ c, x \neq a, c \Rightarrow a \in c \circ x$  and  $x \in a \circ b, x \neq a, b \Rightarrow b \in a \circ x$ . So,  $b \in (c \circ x) \circ x$  where from  $b \in c \circ x$  which in turn gives  $x \in b \circ c$ , in contradiction with  $x \in a \circ c$ . We therefore proved that:

$$a \circ b \cap b \circ c = \{b\}; \quad a \circ b \cap a \circ c = \{a\}; \quad b \circ c \cap a \circ c = \{c\}. \tag{2}$$

In  $a \circ b$  there is a path  $P_{ab}$  joining  $a$  and  $b$ ; let also  $P_{ac}$  and  $P_{cb}$  be paths in  $a \circ c$  and  $b \circ c$  respectively. We obtain from (2) that  $P_{ab} \cup P_{ac} \cup P_{cb}$  is a cycle containing  $a, b, c$ ; hence  $a \in b \circ c$ , contradiction. Summarizing up,  $a \notin b \circ c$  leads to a contradiction; so  $a \in b \circ c$  for any  $b, c \in V, b \neq c$ , and the proof is complete.

**Remark 6.** i) The results above can be restated as:  $\circ$  is associative if and only if for any three points in  $V$  there is a cycle containing them.

ii) An interesting class of 2-connected graphs is the one consisting of all such objects that contain a hamiltonian cycle. In this case  $\circ$  is associative but this property is not sufficient for a 2-connected graph to have a hamiltonian cycle as one can see from the example in fig. 9.



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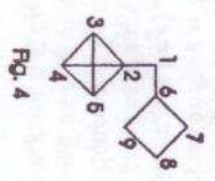


Fig. 4

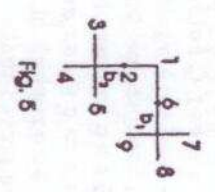


Fig. 5



Fig. 7

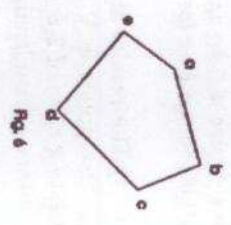


Fig. 6

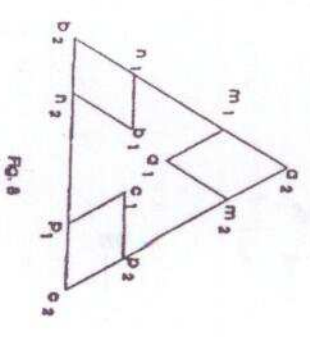


Fig. 8

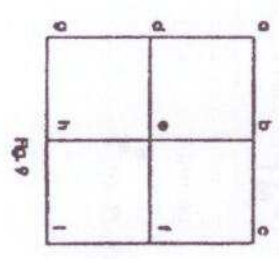


Fig. 9