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BACKGROUND MODELING VIA A SUPERVISED SUBSPACE LEARNING

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ABSTRACT

Unsupervised subspace learning methods are widely used in background modeling to be robust to illumination changes. Their main advantage is that it doesn't need to label data during the training and running phase. Recently, White et al. [1] have shown that a supervised approach can improved significantly the robustness in background modeling. Following this idea, we propose to model the background via a supervised subspace learning called Incremental Maximum Margin Criterion (IMMC). The proposed scheme enables to initialize robustly the background and to update incrementally the eigenvectors and eigenvalues. Experimental results made on the Wallflower datasets show the pertinence of the proposed approach.

Index Terms— Background Modeling, Subspace Learning, Maximum Margin Criterion

1. INTRODUCTION

Many background subtraction methods have been developed in video-surveillance to detect moving objects [2][3][4]. These methods have different common steps: background modeling, background initialization, background maintenance and foreground detection. The background modeling describes the kind of model used to represents the background. Once the model has been chosen, the background model is initialized during a learning step by using N frames. Then, a first foreground detection is made and consists in the classification of the pixel as a background or as a foreground pixel. Thus, the foreground mask is applied on the current frame to obtain the moving objects. After this, the background is adapted over time following the changes which have occurred in the scene and so on.

The background modeling is the key choice because it determines how the model will adapt to the critical situations [5]: Noise image due to a poor quality image source, camera jitter, camera automatic adjustments, time of the day, light switch, bootstrapping, camouflage, foreground aperture, moved background objects, inserted background objects,

multimodal background, waking foreground object, sleeping foreground object and shadows. These critical situations have different spatial and temporal properties. The main difficulties come from the illumination changes and dynamic backgrounds.

Background modeling methods can be classified in the following categories: Basic background modeling [6][7][8], statistical background modeling [9][10][11], fuzzy background modeling [12][13][14] and background estimation [15][16][17]. The last decade witnessed very significant contributions [18] in statistical background modeling particularly in background modeling via unsupervised subspace learning due to their robustness to illumination changes. The first approach developed by Oliver et al. [19] consists in applying Principal Component Analysis (PCA) on N images to construct a background model, which is represented by the mean image and the projection matrix comprising the first p significant eigenvectors of PCA. In this way, foreground segmentation is accomplished by computing the difference between the input image and its reconstruction. The main limitation of this method appears for the background maintenance because it is computationally intensive to perform model updating using the batch mode PCA. Moreover without a mechanism of robust analysis, the outliers or foreground objects may be absorbed into the background model. In this context, some authors have proposed different algorithms of incremental PCA. The incremental PCA proposed by Rymel et al. [20] need less computation but the background image is contaminated by the foreground object. To solve this, Li et al. [21] have proposed an incremental PCA which is robust in presence of outliers. However, when keeping the background model updated incrementally, it assigned the same weights to the different frames. Thus, clean frames and frames which contain foreground objects have the same contribution. The consequence is a relative pollution of the background model. To solve this, Skocaj et al. [22] used a weighted incremental and robust. The weights are different following the frame and this method achieved a better background model. However, the weights were applied to the whole frame without considering the contribution of different image parts to building the background model. To achieve a pixel-wise precision for

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the weights, Zhang and Zhuang [23] have proposed an adaptive weighted selection for an incremental PCA. This method performs a better model by assigning a weight to each pixel at each new frame during the update. Wang et al. [24] have used a similar approach using the sequential Karhunen-Loeve algorithm. Recently, Zhang et al. [25] have improved this approach with an adaptive scheme. All these incremental methods avoid the eigen-decomposition of the high dimensional covariance matrix using approximation of it and so a low decomposition is allowed at the maintenance step with less computational load. However, these incremental methods maintain the whole eigenstructure including both the eigenvalues and the exact matrix. To solve it, Li et al. [26] have proposed a fast recursive and robust eigenbackground maintenance avoiding eigen-decomposition. This method achieves similar results than the incremental PCA [21] at better frames rates. In another way, Yamazaki et al. [27] and Tsai et al. [28] have proposed to use another subspace learning method called Independent Component Analysis (ICA) which is a variant of PCA in which the components are assumed to be mutually statistically independent instead of merely uncorrelated. This stronger condition allows remove the rotational invariance of PCA, i.e. ICA provides a meaningful unique bilinear decomposition of two-way data that can be considered as a linear mixture of a number of independent source signals. The ICA model was tested on traffic scenes by Yamazaki et al. [27] and show robustness in changing background like illumination changes. In [28], the algorithm was tested on indoor scenes which present illumination changes too. Recently, Chu et al. [29] have used a Non-negative Matrix Factorization algorithm to model dynamic backgrounds and Bucak et al. [30] have preferred an Incremental version of the Non-negative Matrix Factorization (INMF) which presents similar performance than the incremental PCA [21]. In order to take into account the spatial information, Li et al. [31] have used an incremental Rank-(R1,R2,R3) Tensor (IRT). Results [31] show better robustness to noise. The Table 1 shows an overview of the background modeling based on subspace learning.

However, these different approaches are unsupervised subspace learning methods. Indeed, it doesn't need to label data. Recently, White et al. [1] have proved that the Gaussian Mixture Model (GMM) [32] gives better results when some coefficients are determined in a supervised way. Following this idea, we propose to use a supervised subspace learning for background modeling. Thus, the Maximum Margin Criterion (MMC) offers a nice framework. It was proposed by Li et al. [33] and it can outperform PCA and Linear Discriminant Analysis (LDA) on many classification tasks [34]. MMC search for the projection axes on which the data points of different classes are far from each other meanwhile where data points of the same class are close to each other. As the original PCA and LDA, MMC is a batch algorithm and so it requires that the data must be known in advance and be given

once altogether. Recently, Yan et al. [35] have proposed an incremental version of MMC which is suitable to update online the background model.

The rest of this paper is organized as follows: In the Section 2, we firstly remind the Incremental Maximum Margin Criterion (IMMC). In the Section 3, we present our method using subspace learning via IMMC for background modeling. Then, a comparative evaluation is provided in the Section 4. Finally, the conclusion is given in Section 5.

2. INCREMENTAL MAXIMUM MARGIN CRITERION (IMMC)

This section reminds briefly the principle of IMMC developed in [35]. Suppose the data sample points $u(1), u(2), \dots, u(N)$ are d -dimensional vectors, and U is the sample matrix with $u(i)$ as its i^{th} column. MMC [33] projects the data onto a lower-dimensional vector space such that the ratio of the inter-class distance to the intra-class distance is maximized. The goal is to achieve maximum discrimination and the new low-dimensional vector can be computed as $y = W^T u$ where $W \in \mathbf{R}^{d \times p}$ is the projection matrix from the original space of dimension d to the low dimensional space of dimension p . So, MMC [33] aims to maximize the criterion:

$$J(W) = W^T(S_b - S_w)W \quad (1)$$

where

$$S_b = \sum_{i=1}^c p_i(m_i - m)(m_i - m)^T \quad (2)$$

$$S_w = \sum_{i=1}^c p_i E(u_i - m_i)(u_i - m_i)^T \quad (3)$$

are called respectively the inter-class scatter matrix and the intra-class scatter matrix and c is the number of classes, m is the mean of all samples, m_i is the mean of the samples belonging to class i and p_i is the prior probability for a sample belonging to class i . The projection matrix W can be obtained by solving:

$$(S_b - S_w)w = \lambda w \quad (4)$$

To incrementally maximize the MMC criterion, Yan et al.[35] constraint W to unit vectors, i.e. $W = [w_1, w_2, \dots, w_p]$ and $w_k^T w_k = 1$. Thus the optimization problem of $J(W)$ is transformed to:

$$\max \sum_{k=1}^p w_k^T (S_b - S_w) w_k \quad (5)$$

subject to $w_k^T w_k = 1$ with $k = 1, 2, \dots, p$. W is the first k leading eigenvectors of the matrix $S_b - S_w$ and the column vectors of W are orthogonal to each other. Thus, the problem is learning the p leading eigenvector of $S_b - S_w$ incrementally.

Subspace Learning	Algorithm	Authors - Dates
Principal Components Analysis (PCA)	Batch PCA	Oliver et al. (1999)[19]
	Incremental PCA	Rymel et al. (2004)[20]
	Incremental and Robust PCA	Li et al. (2003)[21]
	Weighted Incremental and Robust PCA	Skocaj et al. (2003)[22]
	Adaptive Weight Selection for Incremental PCA	Zhang and Zhuang (2007)[23]
	Sequential Karhunen-Loeve algorithm	Wang et al. (2006)[24]
	Adaptive Sequential Karhunen-Loeve algorithm	Zhang et al. (2009)[25]
Independent Component Analysis (ICA)	Batch ICA	Yamazaki et al. (2006)[27]
	Incremental ICA	Tsai and Lai (2009)[28]
Non Negative Matrix Factorization (NMF)	Batch NMF	Chu et al. (2010)[29]
	Incremental NMF	Bucak et al. (2007)[30]
Rank-(R1,R2,R3) Tensor (RT)	Incremental RT	Li et al. (2008)[31]

Table 1. Subspace Learning for background modeling: An Overview

2.1. Updating incrementally leading eigenvectors

Let $C = S_b + S_w$ be the covariance matrix, then we have $J(W) = W^T(2S_b - C)W$, $W \in \mathbf{R}^{d \times p}$. Then maximizing $J(W)$ means to find the p leading eigenvectors of $2S_b - C$.

The inter-class scatter matrix of step n after learning from the first n samples can be written as below,

$$S_b(n) = \sum_{j=1}^c p_j(n)(m_j(n))(m_j(n) - m(n))^T \quad (6)$$

and

$$S_b = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n S_b(i) \quad (7)$$

On the other hand,

$$C = E(u(n) - m)(u(n) - m)^T \quad (8)$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n (u(n) - m(n))(u(n) - m(n))^T \quad (9)$$

$2S_b - C$ should have the same eigenvectors as $2S_b - C + \theta I$ where θ is a positive real number and $I \in \mathbf{R}^{d \times d}$. From (7) and (9) we have the following equation:

$$2S_b - C + \theta I = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n A(i) = A \quad (10)$$

where $A(i) = 2S_b(i) - (u(i) - m(i))(u(i) - m(i))^T + \theta I$, $A = 2S_b - C + \theta I$.

The general eigenvector form is $Ax = \lambda x$, where x is the eigenvector of matrix A corresponding to the eigenvalue λ . By replacing matrix A with the MMC matrix at step n , an approximate iterative eigenvector computation formulation is obtained with $\nu = \lambda x$.

$$\nu(n) = \frac{1}{n} \sum_{i=1}^n \left(2 \sum_{j=1}^c p_j(i) \Phi_j(i) \Phi_j(i)^T - (u(i) - m(i))(u(i) - m(i))^T + \theta I \right) x(i) \quad (11)$$

where $\Phi_j(i) = m_j(i) - m(i)$, $\nu(n)$ is the n step estimation of ν and $x(n)$ is the n step estimation of x . Once the estimation of ν is obtained, eigenvector x can be directly computed as $x = \nu / \|\nu\|$. Let $x(i) = \nu(i-1) / \|\nu(i-1)\|$, then the incremental formulation is the following:

$$\nu(n) = \frac{n-1}{n} \nu(n-1) + \frac{1}{n} \left(2 \sum_{j=1}^c p_j(n) \alpha_j(n) \Phi_j(n) - \beta(n)(u(n) - m(n)) + \theta \nu(n-1) \right) / \|\nu(n-1)\| \quad (12)$$

where $\alpha_j(n) = \phi_j(n)^T \nu(n-1)$ and $\beta(n) = (u(n) - m(n))^T \nu(n-1)$, $j = 1, 2, \dots, c$. For initialization, $\nu(0)$ is equal to the first data sample.

2.2. Updating incrementally the other eigenvectors

To compute the $(j+1)^{th}$ eigenvector, its projection is subtracted on the estimated j^{th} eigenvector from the data,

$$u_{1_n}^{j+1}(n) = u_{1_n}^j(n) - (u_{1_n}^j(n))^T \nu^j(n) \nu^j(n) \quad (13)$$

where $u_{1_n}^1(n) = u_{1_n}(n)$. The same method is used to update $m_i^j(n)$ and $m^j(n)$, $i = 1, 2, \dots, c$. Since $m_i^j(n)$ and $m^j(n)$ are linear combinations of $x_{l_i}^j(i)$, where $i = 1, 2, \dots, k$, and $l_i \in \mathbf{1}, \mathbf{2}, \dots, \mathbf{C}$. Φ_i are linear combination of m_i and m , for convenience, only Φ is updated at each iteration step by:

$$\Phi_{1_n}^{j+1}(n) = \Phi_{1_n}^j(n) - (\Phi_{1_n}^j(n))^T \nu^j(n) \nu^j(n) \quad (14)$$

In this way, the time-consuming orthonormalization is avoided and the orthogonal is always enforced when the convergence is reached.

3. APPLICATION TO BACKGROUND MODELING

The Figure 1 shows an overview of the proposed approach. The background modeling framework based on IMMC includes the following stages: (1) Background initialization via MMC using N frames (2) Foreground detection (3) Background maintenance using IMMC. The steps (2) and (3) are executed repeatedly as time progresses.

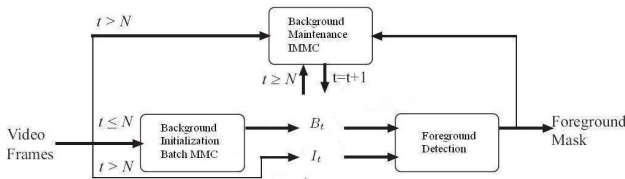


Fig. 1. Overview of the proposed approach

Denote the training video sequences $S = \{I_1, \dots, I_N\}$ where I_t is the frame at time t . Let each pixel (x, y) be characterized by its intensity in the grey scale and assume that we have the ground truth corresponding to this training video sequences, i.e we know for each pixel its class label which can be foreground or background. Thus, we have:

$$S_b = \sum_{i=1}^c p_i (m_i - m)(m_i - m)^T \quad (15)$$

$$S_w = \sum_{i=1}^c p_i E(u_i - m_i)(u_i - m_i)^T \quad (16)$$

where $c = 2$, m is the mean of the intensity of the pixel x, y over the training video and m_i is the mean of samples belonging to class i and p_i is the prior probability for a sample belonging to class i with $i \in \{Background, Foreground\}$. Then, we can apply the batch MMC to obtain the first leading eigenvectors which correspond to the background. The corresponding eigenvalues are contained in the matrix L_M and the leading eigenvectors in the matrix Φ_M . Once the leading eigenbackground images stored in the matrix Φ_M are obtained and the mean μ_B too, the input image I_t can be approximated by the mean background and weighted sum of the leading eigenbackgrounds Φ_M .

So, the coordinate in leading eigenbackground space of input image I_t can be computed as follows:

$$w_t = (I_t - \mu_B)^T \Phi_M \quad (17)$$

When w_t is back projected onto the image space, a reconstructed background image is created as follows:

$$B_t = \Phi_M w_t^T + \mu_B \quad (18)$$

Then, the foreground object detection is made as follows:

$$|I_t - B_t| > T \quad (19)$$

where T is a constant threshold.

Once the first foreground detection is made, we apply the IMMC to update the background model using (12) and (14). The class label for each pixel is obtained using the foreground mask.

Remark: Note that the IMMC can be applied directly at time $t=1$ but its is less robust than to use the batch algorithm on N frames and then apply the IMMC to update the background.

4. EXPERIMENTAL RESULTS

For the performance evaluation, we have compared our supervised approach with the unsupervised subspace learning methods PCA, INMF and IRT using the Wallflower dataset provided by Toyama et al. [5]. This dataset consists in a set of images sequences where each sequence presents a different type of difficulty that a practical task may meet: Moved Object (MO), Time of Day (TD), Light Switch (LS), Waving Trees (WT), Camouflage (C), Bootstrapping (B) and Foreground Aperture (F). The performance is evaluated against hand-segmented ground truth. Three terms are used in evaluation: False Positive (FP) is the number of background pixels that are wrongly marked as foreground; False Negative (FN) is the number of foreground pixels that are wrongly marked as background; Total Error (TE) is the sum of FP and FN. The Table 2 shows the performance in term of FP, FN and TE for each algorithm. The corresponding results are shown in Table 3. As we can see, the IMMC gives the lowest TE followed by the PCA, the INMF and the IRT. Secondly, we have compared our supervised approach with the state-of-art algorithm MOG[10]. As we can see on the Table 2 and Table 3, our algorithm gives better results particularly in the case of illumination changes.

5. CONCLUSION

In this paper, we have proposed to model the background using a supervised subspace learning called Incremental Maximum Criterion. This approach allow to initialize robustly the background and to upate incrementally the eigenvectors and eigenvalues. Experimental results made on the Wallflower datasets show the pertinence of the proposed approach. Indeed, IMMC outperforms the supervised PCA, INMF and IRT.

Algorithm	Error Type	Problem Type							Total Errors (TE)
		MO	TD	LS	WT	C	B	FA	
MOG Stauffer et al.[10]	False neg	0	1008	1633	1323	398	1874	2442	27053
	False pos	0	20	14169	341	3098	217	530	
PCA Oliver et al.[19]	False neg	0	879	962	1027	350	304	2441	17677
	False pos	1065	16	362	2057	1548	6129	537	
INMF Bucak et al.[30]	False neg	0	724	1593	3317	6626	1401	3412	19098
	False pos	0	481	303	652	234	190	165	
IRT Li et al.[31]	False neg	0	1282	2822	4525	1491	1734	2438	17053
	False pos	0	159	389	7	114	2080	12	
IMMC Proposed method	False neg	0	1336	2707	4307	1169	2677	2640	15714
	False pos	0	11	16	6	136	506	203	

Table 2. Performance Evaluation on Wallflower dataset[15]


















































Sequence	MO	TD	LS	WT	C	B	FA
Frame	Frame 985	Frame 1850	Frame 1865	Frame 247	Frame 251	Frame 299	449
Test Image							
Ground Truth							
MOG [10]							
PCA [19]							
INMF [30]							
IRT [31]							
IMMC							

Table 3. Results on Wallflower dataset[15]

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