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OPTIMIZATION OF WEIGHTING FUNCTION SELECTION FOR H_∞ CONTROL OF SEMI-ACTIVE SUSPENSIONS

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ABSTRACT

Multi-objective optimization is a popular problem in engineering design. In semi-active suspension control, comfort and road holding are two essential but conflicting performance objectives. In a previous work, the authors proposed an LPV formulation for semi-active suspension control of a realistic nonlinear suspension model where the nonlinearities (i.e the bi-viscous and the hysteresis) have been taken into account; an H_∞ /LPV controller to handle the comfort and road holding has been also designed. The present paper aims at improving the method of [6] by using Genetic Algorithms (GAs) to select the optimal weighting functions for the H_∞ /LPV synthesis. First, a general procedure for the optimization of weighting function for the H_∞ /LPV synthesis is proposed and then applied to the semi-active suspension control. Thanks to GAs, the comfort and road holding conflicting objectives are handled using a single high level parameter and illustrated via the Pareto optimality. The simulation results performed on a nonlinear vehicle model emphasize the efficiency of the method.

1. INTRODUCTION

The H_∞ control design approach is an efficient tool for improving the performance of a closed-loop system in pre-defined frequency ranges. The key step of the H_∞ control design relies on the selection of weighting functions which depends on the engineer skill and experience. However, this is not always an easy task even with good engineers. In many real applications, the difficulty in choosing the weighting functions still highly increases because the performance specification is not accurately defined i.e it is simply to achieve the best possible performance (optimal design) or to achieve an optimally joint improvement of more than one objective (multi-objectives design). So the optimization of weighting functions to satisfy the desired performance is still an interesting problem.

In the literature, some works are oriented towards this goal e.g [2] [11] ... They propose to consider each system, no matter how complex it is, as a combination of sub-systems of the first and second order, for which it is easy to find the weighting functions. However, there is no explicit method to find these functions in the general case. One popular way to do is to proceed by trial-and-error. Recently, the idea to use an optimization tool was proposed in [1] and [12]. The choice of GAs seems natural because their formulation is well suited for this type of problematic.

This paper aims at using GAs to obtain optimal weighting functions for the H_∞ /LPV control of semi-active suspension systems. First, from the frequency based

industrial performance criteria of suspension systems as defined in [17], an appropriate cost function is formulated. This cost function is in fact a convex combination of two specifications: passenger comfort and road holding, allowing to tune the suspension system using a single high level parameter. Second, the weighting functions are then tuned by GAs in such a way that the cost function is minimized. Finally, the best weighting functions are chosen for the synthesis of H_∞ /LPV controller. For illustration, the performances of the closed-loop semi-active suspension system associated with the controller obtained with GAs will be analyzed to emphasize the efficiency the approach.

It is important to note that this methodology provides a new efficient and generic tool for suspension control design, where the industrial performance objectives are included in the design procedure (and not evaluated a posteriori).

The outline is as follows. In Section 2, a brief introduction about multi-objective optimization using GAs is presented. The procedure for the optimization of weighting functions for H_∞ /LPV problem using GAs is then introduced in Section 3. In Section 4, the H_∞ /LPV control synthesis for semi-active suspensions, presented in [6], is recalled. In Section 5, the application of weighting function selection using GAs on the H_∞ /LPV control problem formulated in Section 4 is presented. The results obtained in simulations with a nonlinear quarter car model are discussed in Section 6. Finally, some conclusions and perspectives are given in Section 7.

2. GENETIC ALGORITHMS AND MULTI-OBJECTIVE OPTIMIZATION

2.1 Genetic algorithms

Genetic algorithms have now grown strongly from the first study of [10], a popular theory-oriented book of [9] and an application-oriented book of [5]. The algorithms are based on the mechanism of the natural selection and have been proven to be very effective in optimization with many real applications such as in finance and investment strategies, robotics, engineering design, telecommunications... They are likely global optimization techniques (despite the high computational expense) (see [13]) using probabilistic, multi-points search, random combination (crossover, mutation) and information of previous iteration to evaluate and improve the population. A great advantage of GAs compared with other searching methods (for example gradient methods) is that they search regardless of the nature of the objective functions and constraints.

GAs initializes with a random population. Through the genetic operation: *selection*, *crossover* and *mutation*, new population will be obtained. By using selection process, the fittest individuals based on their *fitness* values will be chosen; crossover and mutation will be then apply to create the new population. The genetic operation on individuals of population continues until the optimization criterion is satisfied or a certain number of generations is reached.

Fitness function - The fitness of an individual is useful for choosing between “good” and “bad” individuals. An individual with a high fitness value will has a great chance to be selected.

Selection - This step is to sort and copy individuals by order of satisfaction of the fitness function. The higher the value of the fitness, associated to an individual, the

greater the individual's chances to be selected to participate in the next generation. "Proportionate selection" (see [10]) and "tournament selection" (see [15]) are two most popular selection methods.

Crossover - This is the main operation acting on the population of parents. It consists of an exchange of parts of chains between two selected individuals (parents) to form two new individuals (children). This exchange may be due either to a single point or to multiple points. The Fig. 1 is an example for a binary coding crossover.

Mutation - Mutation operates on a single individual by changing randomly a part of it. In the case of binary coding, this is done by reversing one or more bits in a chromosome (see Fig. 1). Other methods can be used as determined by subtracting the mutation on a gene or replacing it with a random value chosen from a subset of values.

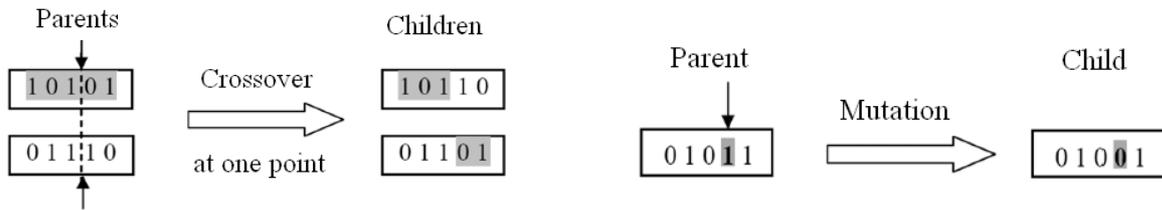


Fig. 1 Crossover and mutation operation

2.2 Multi-objective optimization

One well-known application of the GAs is to find the optimal solution for the multi-objective optimization problem involving multiple and conflicting objectives. This is a very popular problem in practice and can be described as follows

$$\min_{x \in C} F(x) = \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_{n_{obj}}(x) \end{bmatrix}, \quad n_{obj} \geq 2, \quad (1)$$

where x is called the decision vector, C the set of possible decision vectors (or the searching space), and $F(x)$ the objective vector.

The existence of an ideal solution x^* that can minimize simultaneously all objective functions $f_1, f_2, \dots, f_{n_{obj}}$ is in fact rarely feasible. Hence, in this case, the conception of *Pareto-optimal* is usually used to describe the solution of multi-objective optimization problem and it is defined as follows.

Definition 1: Pareto-optimal

Consider two decision vectors $a, b \in C$. Vector a dominates b if and only if:

$$\forall i \in \{1, 2, \dots, n_{obj}\}: f_i(a) \leq f_i(b) \text{ and } \exists j \in \{1, 2, \dots, n_{obj}\}: f_j(a) < f_j(b) \quad (2)$$

All decision vectors are not dominated by any other decision vector are called *non-dominated* or *Pareto-optimal*. The family of non-dominated vectors is denoted as *Pareto-set*. In the Pareto-set, one cannot improve any objective without degrading at least one other objective.

There are many formulations to solve problem (1) like weighted min-max method, weighted global criterion method, goal programming methods... see [13] and references therein. One of the most popular and simple approaches is the weighted sum method which converts the multi-objective problem into a single objective problem. In this paper, a particular case of the weighted sum method, where the multi-objective functions F is replaced by the convex combination of objectives, is used

$$\min J = \sum_{i=1}^{n_{obj}} \alpha_i f_i(x), \quad s.t \quad x \in C, \quad \sum_{i=1}^{n_{obj}} \alpha_i = 1 \quad (3)$$

The vector $\alpha = [\alpha_1, \alpha_2, \dots, \alpha_{n_{obj}}]$ represents the gradient of function J (see Fig. 2).

By using various set of α , one can generate several points in the Pareto set. It is worth noting that the solution of (3) provides a sufficient condition for (1) i.e the minimum of (3) is also Pareto optimal point for (1), see [20] and [8]. However it does not provide a necessary condition for Pareto optimality. If the Pareto set is not convex, there does not exist any α to obtain points which lies in the nonconvex part (see [13]).

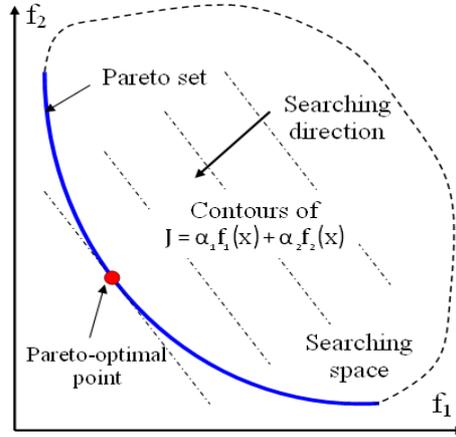


Fig. 2 Pareto optimal set

3. OPTIMIZATION OF WEIGHTING FUNCTIONS FOR H^∞ /LPV SYNTHESIS USING GENETIC ALGORITHM

3.1 General H^∞ /LPV problem

Consider a general LPV system

$$\begin{aligned} \dot{x} &= A(\rho(t))x + B_1(\rho(t))w + B_2(\rho(t))u \\ z &= C_1(\rho(t))x + D_{11}(\rho(t))w + D_{12}(\rho(t))u \\ y &= C_2(\rho(t))x + D_{21}(\rho(t))w + D_{22}(\rho(t))u \end{aligned} \quad (4)$$

where $x \in R^n$, $u \in R^m$, $w \in R^q$, $z \in R^r$ and $y \in R^p$ are the state, the input, the disturbance vectors, the control output and the measured output, respectively. $\rho(t) \in R^k$ is a vector of scheduling parameters and assumed to be known (measured or estimated). From now on, $\rho(t)$ is simply denoted as ρ .

It is well-known that to satisfy some required closed-loop performance, one must select suitable weighting functions. While the weights on disturbance inputs are always defined thanks to the knowledge about their bandwidth and magnitude, the selection of weighting functions on control outputs is more difficult and has a key role in the synthesis procedure. Without loss of generality, it is assumed that there is no

weight on disturbance inputs. The control outputs are weighted as $z_w(s) = W(s)z(s)$ where the weighting function matrix $W(s)$ has a state-space representation as follows

$$\begin{aligned}\dot{x}_w &= A_w x_w + B_w z \\ z_w &= C_w x_w + D_w z\end{aligned}\quad (5)$$

Define ν the vector of the weighting functions parameters which need to be optimized.

From (4) and (5), the augmented system used for controller synthesis is written as

$$\begin{aligned}\dot{\xi} &= A(\rho, \nu)\xi + B_1(\rho, \nu)w + B_2(\nu)u \\ z_w &= C_1(\rho, \nu)\xi + D_{11}(\rho, \nu)w + D_{12}(\nu)u \\ y &= C_2\xi + D_{21}w\end{aligned}\quad (6)$$

Where

$$\begin{aligned}\xi &= \begin{pmatrix} x^T & x_w^T \end{pmatrix}^T \\ A(\rho, \nu) &= \begin{bmatrix} A(\rho) & 0 \\ B_w(\nu)C_1(\rho) & A_w(\nu) \end{bmatrix}, \quad B_1(\rho, \nu) = \begin{bmatrix} B_1(\rho) \\ B_w(\nu)D_{11}(\rho) \end{bmatrix} \\ B_2(\nu) &= \begin{bmatrix} B_2 \\ B_w(\nu)D_{12} \end{bmatrix}, \quad C_1(\rho, \nu) = [D_w(\nu)C_1(\rho) \quad C_w(\nu)] \\ C_2 &= C_2, \quad D_{11}(\rho, \nu) = D_w(\nu)D_{11}(\rho), \quad D_{12}(\nu) = D_w(\nu)D_{12}, \quad D_{21} = D_{21}.\end{aligned}$$

The *LPV* controller is defined as follows

$$K(\rho) : \begin{pmatrix} \dot{x}_c \\ u \end{pmatrix} = \begin{pmatrix} A_c(\rho) & B_c(\rho) \\ C_c(\rho) & D_c(\rho) \end{pmatrix} \begin{pmatrix} x_c \\ y \end{pmatrix}\quad (7)$$

where x_c , y and u are the state, the input and output of the controller, respectively, of the controller associated to the system (6); $A_c \in R^{n \times n}$, $B_c \in R^{n \times n_y}$, $C_c \in R^{n_u \times n}$ and $D_c \in R^{n_u \times n_y}$.

H ∞ /LPV problem - The objective of the synthesis is to find an LPV controller $K(\rho)$ of the form (7) such that the closed-loop system is quadratically stable and that, for a given positive real γ , the L_2 -induced norm of the operator mapping w into z is bounded by γ i.e

$$\frac{\|z_w\|_2}{\|w\|_2} \leq \gamma\quad (8)$$

for all possible trajectories of ρ .

In this paper, the polytopic approach with quadratic Lyapunov function is employed for the synthesis of a *H ∞ /LPV* controller. The following is assumed: $\rho \in \text{conv}(\rho_1, \dots, \rho_k)$, the matrices B_2 , D_{12} , C_2 , D_{21} are parameter independent and D_{22} . For known weighting functions and a suitable pre-defined real positive scalar γ , the sufficient condition that solves the *H ∞ /LPV* problem is giving by Eq. (9)-(10) (see the detail of the solution in [19]). It is worth noting that, unlike the original solution, the

weighting functions parameter are present in the LMIs problem and will be optimized by genetic algorithm

$$\begin{bmatrix} M_{11}(\rho_i, \nu) & * & * & * \\ M_{21}(\rho_i, \nu) & M_{22}(\rho_i, \nu) & * & * \\ M_{31}(\rho_i, \nu) & M_{32}(\rho_i, \nu) & -\gamma I_m & * \\ M_{41}(\rho_i, \nu) & M_{42}(\rho_i, \nu) & M_{43}(\rho_i, \nu) & -\gamma I_p \end{bmatrix} < 0 \quad (9)$$

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} \succ 0 \quad (10)$$

for $i = 1 : k$

where

$$M_{11}(\rho_i, \nu) = A(\rho_i, \nu)X + XA(\rho_i, \nu)^T + B_2\hat{C}(\rho_i) + \hat{C}(\rho_i)^T B_2^T$$

$$M_{21}(\rho_i, \nu) = \hat{A}(\rho_i) + A(\rho_i, \nu)^T + C_2^T \hat{D}(\rho_i)^T B_2^T$$

$$M_{22}(\rho_i, \nu) = YA(\rho_i, \nu) + A(\rho_i, \nu)^T Y + \hat{B}(\rho_i)C_2 + C_2^T \hat{B}(\rho_i)^T$$

$$M_{31}(\rho_i, \nu) = B_1(\rho_i, \nu)^T + D_{21}^T \hat{D}(\rho_i)^T B_2^T$$

$$M_{32}(\rho_i, \nu) = B_1(\rho_i, \nu)^T Y + D_{21}^T \hat{B}(\rho_i)^T$$

$$M_{41}(\rho_i, \nu) = C_1(\rho_i, \nu)X + D_{12} \hat{C}(\rho_i)$$

$$M_{42}(\rho_i, \nu) = C_1(\rho_i, \nu) + D_{12} \hat{D}(\rho_i)C_2$$

$$M_{43}(\rho_i, \nu) = D_{11}(\rho_i, \nu) + D_{12} \hat{D}(\rho_i)D_{21}$$

The controller K_{c_i} at vertex i is then reconstructed as

$$\begin{aligned} D_c(\rho_i) &= \hat{D}(\rho_i) \\ C_c(\rho_i) &= \left(\hat{C}(\rho_i) - D_c(\rho_i)C_2X \right) M^{-T} \\ B_c(\rho_i) &= N^{-1} \left(\hat{B}(\rho_i) - YB_2D_c(\rho_i) \right) \\ A_c(\rho_i) &= N^{-1} \left(\hat{A}(\rho_i) - YA(\rho_i)X - YB_2D_c(\rho_i)C_2X \right) M^{-T} \\ &\quad - B_c(\rho_i)C_2XM^{-T} - N^{-1}YB_2C_c(\rho_i) \end{aligned} \quad (11)$$

where M, N are defined such that $MN^T = I_n - XY$ which can be solved through a singular value decomposition and a Cholesky factorization. The global H_∞/LPV controller is the convex combination of these local controllers (11).

3.2 Optimizing the weighting functions

Decision vector - Certainly ν is one element of the decision vector. By experience, the authors have observed that the real positive scalar γ must be taken into account in the decision vector as well. In an usual H_∞/LPV problem, the attenuation level γ is to be minimized to satisfy the performance objectives (H_∞ optimal control problem).

Thanks to the Genetic Algorithms optimization, the provided methodology will rather allow here to minimize a cost function representing the true performance requirement objectives. Therefore the optimization problem of interest relies on the minimization of this cost function and not on the minimization of γ . We have chosen to let γ as a decision parameter (to be determined by the GAs) in order to add more degrees of freedom, and then to solve a sub-optimal H_∞ control problem.

To avoid the infeasibility of the LMIs (9)-(10) resulting from the bad (i.e too small) value of γ generated by GAs, γ will be decomposed into two positive real elements γ_{min} and γ_{ga} where γ_{min} is the minimal γ satisfying the LMIs (9)-(10), and γ_{ga} is tuned by GAs. Due to the convexity of the LMIs problem, the existence of γ_{min} will ensure the feasibility of LMIs (9)-(10) with $\gamma = \gamma_{min} + \gamma_{ga}$ for all positive real γ_{ga} . The minimal value γ_{min} can be found by using LMIs toolbox like Yalmip & Sedumi.

Objective functions - In this paper, the frequency-based objective functions are considered. With the remarks above, the performance of the closed-loop systems depends on the choice of weighting functions and the real positive scalar γ (and γ_{ga}). Hence, to refer to these implicit dependencies, the objective function can be written as $J = J(\nu, \gamma)$.

Proposed weighting function optimization procedure for H_∞ /LPV synthesis

Step 1: Initiate with random positive weighting functions $\nu = \nu^0$ and random positive real $\gamma_{ga} = \gamma_{ga}^0$.

Step 2: Solve the minimization problem of γ subject to the LMIs (9)-(10) to compute the minimal real scalar γ_{min} . Solve again the LMIs (9)-(10) with the couple (ν, γ) where $\gamma = \gamma_{min} + \gamma_{ga}$. At the end of this step, compute the objective function $J = J(\nu, \gamma)$.

Step 3: Apply selection procedure.

Step 4: Apply crossover and mutation to generate new generation: $\nu = \nu^{new}$ and $\gamma_{ga} = \gamma_{ga}^{new}$.

Step 5: Evaluate the new generation: If the criteria of interest (for example, reaching the limit number of generation) are not satisfied, go to Step 2 with $\nu = \nu^{new}$ and $\gamma_{ga} = \gamma_{ga}^{new}$; Else, stop and save the best individual $\nu^{opt} = \nu^{new}$ and $\gamma_{ga}^{opt} = \gamma_{ga}^{new}$.

4. SEMI-ACTIVE SUSPENSION CONTROL

4.1 System description

Consider a simple quarter vehicle model (see Fig. 3) made up of a sprung mass (m_s) and an unsprung mass (m_{us}). A spring with the stiffness coefficient k_s and a semi-active damper connect these two masses. The wheel tire is modelled by a spring with the stiffness coefficient k_t . In this model, z_s (respectively z_{us}) is the vertical position of m_s (respectively m_{us}) and z_r is the road profile. It is assumed that the wheel-road contact is ensured.

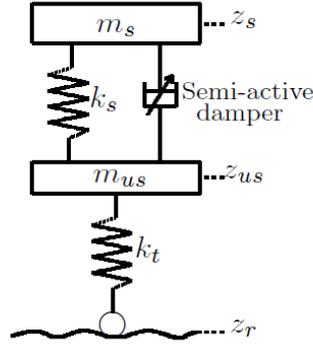


Fig. 3 Model of quarter vehicle with a semi-active damper

The dynamical equations of a quarter vehicle are governed by:

$$\begin{cases} m_s \ddot{z}_s = -F_{spring} - F_{mr} \\ m_{us} \ddot{z}_{us} = F_{spring} + F_{mr} - k_t (z_{us} - z_r) \end{cases} \quad (12)$$

where $F_{spring} = k_s z_{def}$ is the spring force, $z_{def} = z_s - z_{us}$ the damper deflection (assumed to be measured or estimated), and $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$ the deflection velocity (can be directly computed from z_{def}).

In this paper, the behavior of the semi-active suspension is represented using the following nonlinear equation, as in

$$F_{mr} = a_2 \left(\dot{z}_{def} + \frac{v_0}{x_0} z_{def} \right) + a_1 \tanh \left(a_3 \left(\dot{z}_{def} + \frac{v_0}{x_0} z_{def} \right) \right) \quad (13)$$

where a_2 , a_3 , v_0 and x_0 are constant parameters and a_1 is varying according to the electrical current in coil ($0 < a_{1min} < a_1 < a_{1max}$). This model allows fulfilling the dissipativity constraint of the semi-active damper.

In the following, in order to better emphasize the controllable property of the damper, a_1 will be denoted as the control input, and for simplicity, the damping and stiffness parameters are defined respectively by $c_{mr} = a_2$ passive damper damping coefficient, $k_{mr} = a_2 \frac{v_0}{x_0}$ passive damper stiffness coefficient.

The quarter vehicle used in this paper is the ‘‘Renault M3gane Coup3’’ model (see [21]) whose specific parameters are: $m_s = 315 \text{ kg}$, $m_{us} = 37.5 \text{ kg}$, $k_s = 29500 \text{ N/m}$, $k_t = 210000 \text{ N/m}$.

The damper model parameters have been chosen according to the MR damper in [16]: $a_2 = 1500 \text{ Ns/m}$, $a_3 = 129 \text{ s/m}$, $v_0 = 0.788 \times 10^{-3} \text{ m/s}$, $x_0 = 1.195 \times 10^{-3} \text{ m}$, $a_{1min} = 0 \text{ N}$, $a_{1max} = 400 \text{ N}$

4.2 LPV model formulation [6]

A control oriented LPV model with input saturation is obtained with two scheduling parameters ρ_1 , ρ_2

$$\begin{cases} \dot{x} = A(\rho_1^*, \rho_2^*)x + Bu_c + B_1w \\ y = Cx \end{cases} \quad (14)$$

where

$$x = \begin{pmatrix} x_s^T & x_f^T \end{pmatrix}^T, x_s = \begin{pmatrix} z_s, \dot{z}_s, z_{us}, \dot{z}_{us} \end{pmatrix}^T, A(\rho_1, \rho_2) = \begin{pmatrix} A_s + \rho_2 B_{s2} C_{s2} & \rho_1 B_s C_f \\ 0 & A_f \end{pmatrix}, B = \begin{pmatrix} 0 \\ B_f \end{pmatrix},$$

$$B_1 = \begin{pmatrix} B_{s1} \\ 0 \end{pmatrix}, C = \begin{pmatrix} C_s \\ 0 \end{pmatrix}^T, B_{s2} = \begin{pmatrix} 0; -\frac{F_0}{m_s}; 0; \frac{F_0}{m_{us}} \end{pmatrix}^T, C_{s2} = \begin{pmatrix} \frac{a_3 v_0}{x_0}; a_3; -\frac{a_3 v_0}{x_0}; -a_3 \end{pmatrix}^T$$

$$A_s = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s + k_{mr}}{m_s} & -\frac{c_{mr}}{m_s} & \frac{k_s + k_{mr}}{m_s} & \frac{c_{mr}}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s + k_{mr}}{m_{us}} & \frac{c_{mr}}{m_{us}} & -\frac{k_s + k_{mr} + k_t}{m_{us}} & -\frac{c_{mr}}{m_{us}} \end{pmatrix}, B_s = \begin{pmatrix} 0 \\ -\frac{1}{m_s} \\ 0 \\ \frac{1}{m_{us}} \end{pmatrix}, B_{s1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{k_t}{m_{us}} \end{pmatrix}, C_s = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$F_0 = (a_{1min} + a_{1max}) / 2, F_1 = (a_{1max} - a_{1min}) / 2$$

$$\rho_1 = \tanh(C_{s2} x_s) \tanh\left(\frac{C_f x_f}{F_1}\right) \frac{F_1}{C_f x_f}, \rho_2 = \frac{\tanh(C_{s2} x_s)}{C_{s2} x_s}$$

x_f, A_f, B_f, C_f are states and states-space matrices of a strict low-pass filter which is introduced to satisfy the hypothesis for polytopic LPV design.

Notice also that ρ_1, ρ_2 are not independent and the relation between them are presented in Fig. 4. For the polytopic approach, the polytope (P_1, P_2, P_3, P_4) that contain the set of (ρ_1, ρ_2) will be considered. While P_3 and P_4 are related to the real set of (ρ_1, ρ_2), P_1 and P_2 in fact do not belong to the set.

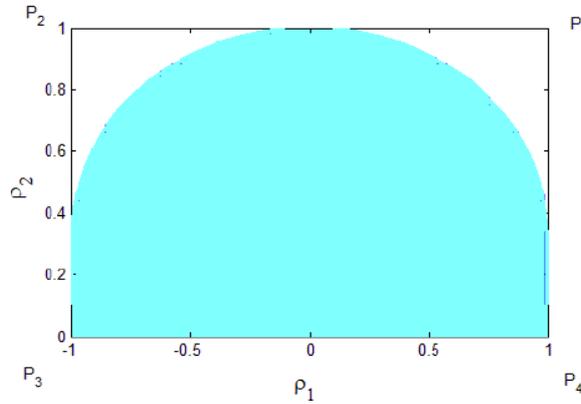


Fig. 4 Set of (ρ_1, ρ_2) (shaded area)

4.3 Suspension Performance Objectives

The performance objectives for comfort and road holding are obtained following ideas from [17].

- Comfort in high frequencies ([3 – 10] Hz): $\min \int_3^{10} \ddot{z}_s / z_r(f) df$.
- Comfort in low frequencies ([0–4] Hz): $\min \bar{\sigma}(z_s / z_r(f))$.

- Road holding ($[0 - 20]$ Hz): $\min \bar{\sigma}(z_{us} / z_r(f))$ (rebound of wheel) and $\min \int_0^{20} (z_{us} - z_r) / z_r(f) df$ (contact force)

5. SEMI-ACTIVE SUSPENSION CONTROL USING GENETIC ALGORITHMS

The procedure for optimizing the weighting functions in Sec. 3 is now applied for the LPV semi-active suspension system (14) with the following specifications.

5.1 Parameterizing weighting functions

Along with the bounded real positive scalar γ , the following weighting functions are used for H_∞/LPV synthesis

$$W_{filter} = \frac{\Omega_f}{s + \Omega_f}, W_{z_s} = k_{z_s} \frac{s^2 + 2\xi_{11}\Omega_{11}s + \Omega_{11}^2}{s^2 + 2\xi_{12}\Omega_{12}s + \Omega_{12}^2}, W_{z_{us}} = k_{z_{us}} \frac{s^2 + 2\xi_{21}\Omega_{21}s + \Omega_{21}^2}{s^2 + 2\xi_{22}\Omega_{22}s + \Omega_{22}^2} \quad (15)$$

It is worth noting that the structures of these weighting functions are the same as in [6].

The parameter encoding is defined as follows

$$\text{Parameter Value} = (a + 0.1b + 0.01c + 0.001d) \times 10^{\frac{e-2}{3}} \quad (16)$$

where $(a, b, c, d, e) \in [0; 9]^5$. This encoding covers the range $[0; 100]$.

5.2 Objective and fitness function

Objective function - In Eq. (3), the value of a weight α_i is significant not only relative to other weights but also relative to its corresponding objective function. To choose accurately the weights, it is better to transform the functions so that they all have similar magnitudes [14]. As a result, two single-objective functions corresponding to the comfort and road holding are normalized and the overall objective function to be minimized is as follows

$$J = \alpha J_{\text{Comfort}} + (1 - \alpha) J_{\text{Road Holding}} \quad (17)$$

J_{Comfort} and $J_{\text{Road Holding}}$ are performance indices corresponding to vertex P_3 (Note that solutions at P_3 and P_4 are identical because of the symmetric property of the polytope (P_1, P_2, P_3, P_4) and defined as follows (according to Sec. 4.3)

$$J_{\text{Comfort}} = \frac{\int_1^{10} \ddot{z}_s / z_r(f) df}{\int_1^{10} (\ddot{z}_s / z_r)_{\text{Passive}}(f) df}, J_{\text{Road Holding}} = \frac{\bar{\sigma}_{f \in [10-20]}(z_{us} / z_r(f))}{\bar{\sigma}_{f \in [10-20]}(z_{us} / z_r(f))_{\text{Passive}}} \quad (18)$$

Remark 1: The feasibility of the new generated elements is not guaranteed because of the possible infeasibility of the LMIs (9)-(10). The problem can be overcome by repeating the crossover or mutation until the feasible solution is obtained. However, a simpler way is to assign a great objective value (for instant $J = \infty$) to these infeasible solutions and as a consequence, they will be eliminated by the selection procedure after some generations.

Remark 2: In many cases, to preserve the performance of closed-loop system with input saturation, a stable stabilizing controller is required. For LTI systems, this problem (usually called *strong stabilization* problem) has been studied by some

authors such as [3], [4],... Similarly, for the H_∞/LPV control of LPV systems, to obtain a stable LPV controller, it suffices to ensure that all local controllers at each vertex of the polytope are stable. In this paper, the theoretical solution for the existence of a stable LPV controller is not given. However, a stable LPV controller can be obtained by eliminating the "unstable solutions" corresponding to at least one unstable local controller during the synthesis. It can be accomplished with GAs by simply choosing $J=\infty$ for "unstable solutions". Due to the "survival of the fittest" property, these "unstable solutions" will disappear after some generations.

To sum up, the objective function is chosen as follows.

Algorithm 1: Objective value assignment

```

if (9)-(10) is feasible then
    if all local controller  $K_{Ci}$  are stable then
        Calculate J using Eq. 17 and Eq. 18
    else
         $J = \infty$ 
    end
else
     $J = \infty$ 
end.

```

Fitness function - Because the working principle of GAs is the maximization of fitness, the following fitness function is used

$$F_{\text{fitness}} = 10 - 10 \frac{1 - 5e^{-12(J-0.6)}}{1 + 5e^{-12(J-0.6)}} \quad (19)$$

As seen in Fig. 5, individuals in improved region have the greatest chance to survive while ones in degraded region have a very small chance to be selected.

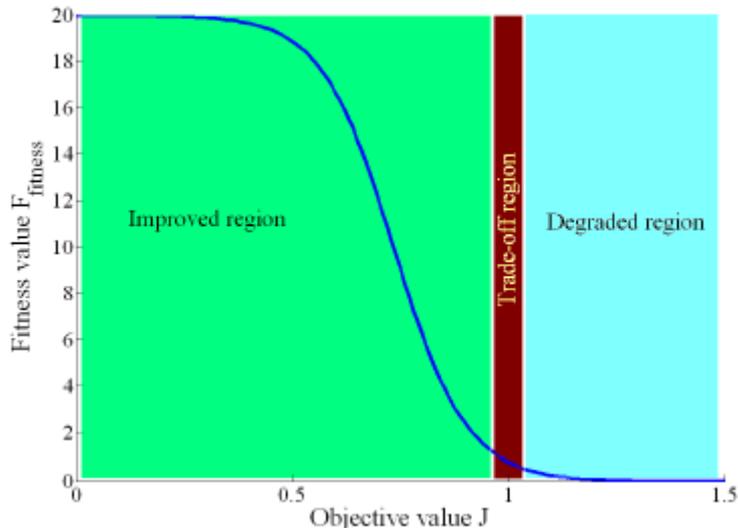


Fig. 5 *Fitness function (blue line).*

5.3 Genetic operation

The *selection* method used in this paper is the *proportionate selection* developed by [10]. This method assigns to each individual a probability of selection proportional to its relative fitness. Proportionate selection can be illustrated by a roulette wheel. The *crossover* happens with a probability of 0.9. And the *mutation* happens with a very small probability 0.095.

In Fig. 6.a, some Pareto-optimal points, computed for the closed-loop linear system at vertex P_3 , are generated by using different values of α in the range of [0.1;0.99]. With $\alpha=0.99$, the smallest J_{Comfort} is obtained; with $\alpha=0.1$, the smallest $J_{\text{Road Holding}}$ is obtained. In Fig. 6.b, these performance indices are computed for a nonlinear quarter car model with the H_∞/LPV controllers corresponding to each value of α . In both figures, a conflicting relation between the comfort (in high frequency) and road holding (rebound of wheel) is illustrated.

Three stable H_∞/LPV controllers C_1 , C_2 , C_3 corresponding to $\alpha=0.99$, $\alpha=0.55$, $\alpha=0.1$ (respectively) synthesized with the optimized weighting functions (whose parameters are presented in Tab. 1) are chosen for the analysis. Their performance will be compared with those of the H_∞/LPV controller obtained in [6], the passive soft MR Damper ($a_I=a_{Imin}=0\text{ N}$) and the passive hard MR Damper ($a_I=a_{Imax}=400\text{ N}$).

The nonlinear frequency responses using the ‘‘variance gain’’ algorithm (see [18]) are presented in Fig. 7 - 10 and the overall evaluation of performance based on the criteria given in Sec. \ref{performance-objective} is depicted in Fig. \ref{fig:Overall-Comparison}.

The comfort oriented controller C_1 is the best one for comfort but not good for road holding.

The road holding (rebound of wheel) oriented controller C_3 is best for preventing the rebound of wheel in high frequency. It can be seen that this controller has the same behavior to that of the hard MR damper. However, it does not guarantee a good contact force (because only z_{us}/z_r is presented in the criterion for road holding $J_{\text{Road Holding}}$) and a good comfort in high frequency.

The intermediate controller C_2 turns out to be the best one for road holding in both rebound of wheel and contact force minimization. Specially, it provides a good comfort improvement in high frequency and a medium comfort improvement in low frequency.

Controllers	bounded real scalar $\gamma_{min} - \gamma_{ga}$	Filter Ω_f	$W_{\ddot{z}_s}$					$W_{z_{us}}$				
			Ω_{11}	Ω_{12}	ξ_{11}	ξ_{12}	$k_{\ddot{z}_s}$	Ω_{21}	Ω_{22}	ξ_{21}	ξ_{22}	$k_{z_{us}}$
ACC 2010	363-0	50	125.7	125.7	20	1	1	69.1	69.1	20	1	1
C_1 ($\alpha = 0.99$)	456-463	51.1	33.29	28.54	52.32	5.26	1.95	1.29	0.072	4.09	0.136	0.152
C_2 ($\alpha = 0.55$)	320-163	8.3	0.48	4.66	37.95	0.76	3.77	0.87	3.3	0.19	1.8	8.1
C_3 ($\alpha = 0.1$)	93.9-0.11	1.4	2.373	0.66	0.222	33.87	0.22	24.81	0.49	7.14	5.75	0.343

Table 1. Parameters for H_∞/LPV semi-active suspension design.

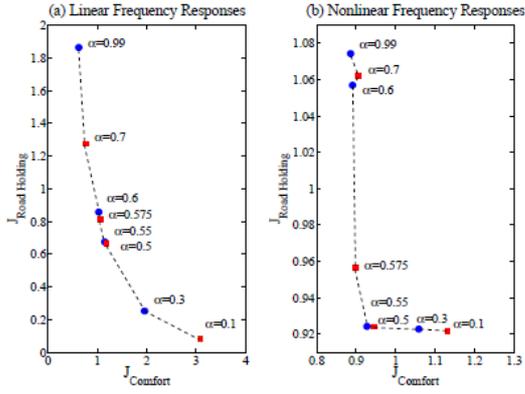


Fig. 6. Linear Design and Nonlinear Simulation.

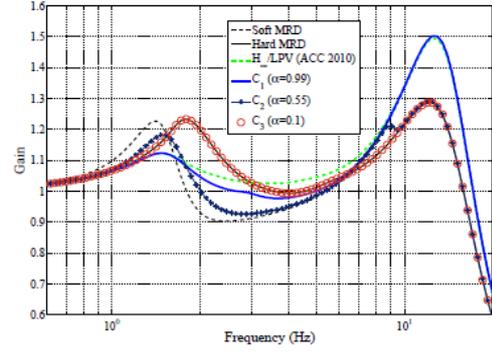


Fig. 9. Road holding z_{us}/z_r .

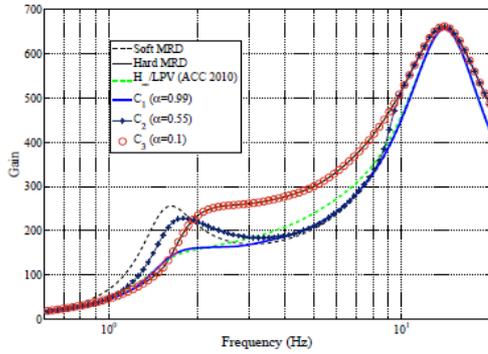


Fig. 7. Comfort in high frequencies z_s/z_r .

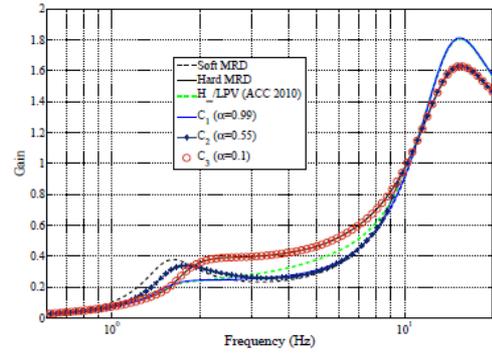


Fig. 10. Road holding $(z_{us} - z_r)/z_r$.

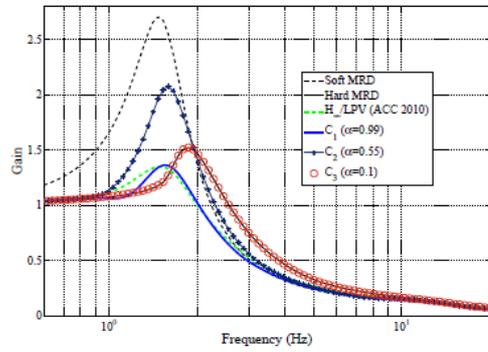


Fig. 8. Comfort in low frequencies z_s/z_r .

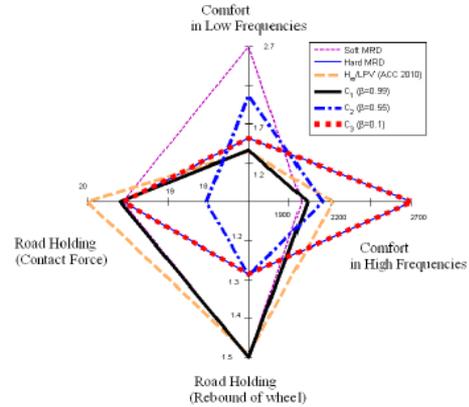


Fig. 11. Overall performance comparison.

6. CONCLUSION

In this paper, a weighting function optimization using GAs for H_∞/LPV semi-active control problem has been proposed. The input used for this optimization procedure is a frequency-based objective function, instead of the disturbance attenuation level γ as in conventional H_∞/LPV design. The simulation results have shown the efficiency of GAs in finding a suitable controller to satisfy some performance objectives. For future work, powerful GAs in multi-objective optimization like SPEA2 or NSGA-II will be considered and compared with the simple weighted sum of objectives method used in this paper.

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