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NAVIER-STOKES EQUATIONS IN ARBITRARY DOMAINS : THE FUJITA-KATO SCHEME

SYLVIE MONNIAUX

ABSTRACT. Navier-Stokes equations are investigated in a functional setting in 3D open sets Ω , bounded or not, without assuming any regularity of the boundary $\partial\Omega$. The main idea is to find a correct definition of the Stokes operator in a suitable Hilbert space of divergence-free vectors and apply the Fujita-Kato method, a fixed point procedure, to get a local strong solution.

1. INTRODUCTION

Since the pioneering work by Leray [3] in 1934, there have been several studies on solutions of Navier-Stokes equations

$$(NS) \begin{cases} \frac{\partial u}{\partial t} - \Delta u + \nabla \pi + (u \cdot \nabla)u &= 0 \quad \text{in} \quad]0, T[\times \Omega, \\ & \text{div } u &= 0 \quad \text{in} \quad]0, T[\times \Omega, \\ & u &= 0 \quad \text{on} \quad]0, T[\times \partial \Omega, \\ & u(0) &= u_0 \quad \text{in} \quad \Omega. \end{cases}$$

Fujita and Kato [2] in 1964 gave a method to construct so called mild solutions in smooth domains Ω , producing local (in time) smooth solutions of (NS) in a Hilbert space setting. These solutions are global in time if the initial value u_0 is small enough in a certain sense. The case of non smooth domains has been studied by Deuring and von Wahl [1] in 1995 where they considered domains $\Omega \subset \mathbb{R}^3$ with Lipschitz boundary $\partial\Omega$. They found local smooth solutions using results contained in Shen's PhD thesis [4]. Their method does not cover the critical space case as in [2]. One of the difficulty there was to understand the Stokes operator, and in particular its domain of definition.

In Section 2, we give a "universal" definition of the Stokes operator, for any domain $\Omega \subset \mathbb{R}^3$ (Definition 2.3). In Section 3, we construct a mild solution of (NS) with a method similar to Fujita-Kato's [2] (Theorem 3.2) for initial values u_0 in the critical space $D(A^{\frac{1}{4}})$. We show in Section 4 that this mild solution is a strong solution, *i.e.* (NS) is satisfied almost everywhere.

2. The Stokes operator

Let Ω be an open set in \mathbb{R}^3 . The space

$$L^{2}(\Omega)^{3} = \{ u = (u_{1}, u_{2}, u_{3}); u_{i} \in L^{2}(\Omega), i = 1, 2, 3 \}$$

endowed with the scalar product

$$\langle u, v \rangle = \int_{\Omega} u \cdot \overline{v} = \sum_{i=1}^{3} \int_{\Omega} u_i \ \overline{v_i}$$

is a Hilbert space. Define

$$\mathcal{G} = \{\nabla p; p \in L^2_{loc}(\Omega) \text{ and } \nabla p \in L^2(\Omega)^3\};\$$

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the set \mathcal{G} is a closed subspace of $L^2(\Omega)^3$. Let

$$\mathcal{H} = \mathcal{G}^{\perp} = \left\{ u \in L^2(\Omega)^3; \langle u, \nabla p \rangle = 0, \ \forall p \in H^1(\Omega) \right\}$$

The space \mathcal{H} , endowed with the scalar product $\langle \cdot, \cdot \rangle$ is a Hilbert space. We have the following Hodge decomposition

$$L^2(\Omega)^3 = \mathcal{H} \stackrel{\scriptscriptstyle \perp}{\oplus} \mathcal{G}.$$

We denote by \mathbb{P} the projection from $L^2(\Omega)^3$ onto $\mathcal{H} : \mathbb{P}$ is the usual Helmoltz projection. We denote by J the canonical injection $\mathcal{H} \hookrightarrow L^2(\Omega)^3 : J' = \mathbb{P}(J')$ being the adjoint of J and $\mathbb{P}J$ is the identity on \mathcal{H} . Let now $\mathscr{D}(\Omega)^3 = \mathscr{C}^{\infty}_c(\Omega)^3$ and

$$\mathcal{D} = \{ u \in \mathscr{D}(\Omega)^3; \operatorname{div} u = 0 \}.$$

It is clear that \mathcal{D} is a closed subspace of $\mathscr{D}(\Omega)^3$. We denote by $J_0 : \mathcal{D} \hookrightarrow \mathscr{D}(\Omega)^3$ the canonical injection : $J_0 \subset J$. Let \mathbb{P}_1 be the adjoint of $J_0 : \mathbb{P}_1 = J'_0 : \mathscr{D}'(\Omega)^3 \to \mathcal{D}'$. We have $\mathbb{P}_1 \subset \mathbb{P}$. The following theorem characterizes the elements in ker \mathbb{P}_1 .

Theorem 2.1 (de Rahm). Let $T \in \mathscr{D}'(\Omega)^3$ such that $\mathbb{P}_1 T = 0$ in \mathcal{D}' . Then there exists $S \in (\mathscr{C}^{\infty}_c(\Omega))'$ such that $T = \nabla S$. Conversely, if $T = \nabla S$ with $S \in (\mathscr{C}^{\infty}_c(\Omega))'$, then $\mathbb{P}_1 T = 0$ in \mathcal{D}' .

We denote by $H_0^1(\Omega)^3$ the closure of $\mathscr{D}(\Omega)^3$ with respect to the scalar product $(u,v) \mapsto \langle u,v \rangle_1 = \langle u,v \rangle + \sum_{i=1}^3 \langle \partial_i u, \partial_i v \rangle$. By Sobolev embeddings, we have $H_0^1(\Omega)^3 \hookrightarrow L^6(\Omega)^3$. Define

$$\mathcal{V} = \mathcal{H} \cap H^1_0(\Omega)^3.$$

The space \mathcal{V} is a closed subspace of $H_0^1(\Omega)^3$; endowed with the scalar product $\langle \cdot, \cdot \rangle_1$, \mathcal{V} is a Hilbert space. The canonical injection $\tilde{J} : \mathcal{V} \hookrightarrow H_0^1(\Omega)^3$ is the restriction of J to \mathcal{V} . Let $H^{-1}(\Omega)^3 = (H_0^1(\Omega)^3)'$; \mathbb{P}_1 maps $H^{-1}(\Omega)^3$ to \mathcal{V}' : the restriction of \mathbb{P}_1 to $H^{-1}(\Omega)^3$ is $\tilde{\mathbb{P}}$, the adjoint of \tilde{J} . On $\mathcal{V} \times \mathcal{V}$ we define now the form a by $a(u,v) = \sum_{i=1}^3 \langle \partial_i \tilde{J}u, \partial_i \tilde{J}v \rangle$: a is a bilinear, symmetric, $\delta + a$ is a coercive form on $\mathcal{V} \times \mathcal{V}$ for all $\delta > 0$, then defines a bounded self-adjoint operator $A_0 : \mathcal{V} \to \mathcal{V}'$ by $(A_0 u)(v) = a(u,v)$ with $\delta + A_0$ invertible for all $\delta > 0$.

Proposition 2.2. For all $u \in \mathcal{V}$, $A_0 u = \mathbb{P}(-\Delta_D^{\Omega}) \tilde{J} u$, where Δ_D^{Ω} denotes the Dirichlet-Laplacian on $H_0^1(\Omega)^3$.

Proof. For all $u, v \in \mathcal{V}$, we have

$$(A_0 u)(v) \stackrel{(1)}{=} a(u, v) \stackrel{(2)}{=} \sum_{i=1}^{3} \langle \partial_i \tilde{J}u, \partial_i \tilde{J}v \rangle$$
$$\stackrel{(3)}{=} \langle (-\Delta_D^{\Omega}) \tilde{J}u, \tilde{J}v \rangle_{H^{-1}, H_0^1}$$
$$\stackrel{(4)}{=} \langle \tilde{\mathbb{P}}(-\Delta_D^{\Omega}) \tilde{J}u, v \rangle_{\mathcal{V}', \mathcal{V}}.$$

The first two equalities come from the definition of A_0 and a. The third equality comes from the definition of the Dirichlet-Laplacian on $H_0^1(\Omega)^3$ and the fact that for $v \in \mathcal{V}, \ \tilde{J}v = v$. The last equality is due to $\tilde{J}'\varphi = \tilde{\mathbb{P}}\varphi$ in \mathcal{V}' for all $\varphi \in H^{-1}(\Omega)^3$. This shows that A_0u and $\tilde{\mathbb{P}}(-\Delta_{\Omega}^{\Omega})\tilde{J}u$ are two continuous linear forms on \mathcal{V} which coı̈ncide on \mathcal{V} , they are then equal.

Definition 2.3. The operator A defined on its domain $D(A) = \{u \in \mathcal{V}; A_0 u \in \mathcal{H}\}$ by $Au = A_0u$ is called the Stokes operator.

Theorem 2.4. The Stokes operator is self-adjoint in \mathcal{H} , generates an analytic semigroup $(e^{-tA})_{t\geq 0}$, $D(A^{\frac{1}{2}}) = \mathcal{V}$ and satisfies

$$D(A) = \{ u \in \mathcal{V} ; \exists \pi \in (\mathbb{C}^{\infty}_{c}(\Omega))' : \nabla \pi \in H^{-1}(\Omega) \text{ and } -\Delta u + \nabla \pi \in \mathcal{H} \}$$

$$Au = -\Delta u + \nabla \pi.$$

Remark 2.5. Since $H_0^1(\Omega)^3 \hookrightarrow L^6(\Omega)^3$, it is clear by interpolation and dualization that \mathbb{P}_1 maps $L^p(\Omega)^3$ to $D(A^s)'$ for $\frac{6}{5} \leq p \leq 2, \ 0 \leq s \leq \frac{1}{2}$ and $s = -\frac{3}{4} + \frac{3}{2p}$. Since A is self-adjoint, one has $(\delta + A_0)^{-s}D(A^s)' = \{(\delta + A_0)^{-s}u; u \in D(A^s)'\} = \mathcal{H}$. In particular, $(\delta + A_0)^{-\frac{1}{4}}\mathbb{P}_1$ maps $L^{\frac{3}{2}}(\Omega)^3$ into \mathcal{H} .

3. MILD SOLUTION TO THE NAVIER-STOKES SYSTEM

Let T > 0.

Define the following Banach space

$$\mathcal{E}_{T} = \left\{ u \in \mathscr{C}([0,T]; D(A^{\frac{1}{4}}) \cap \mathscr{C}^{1}(]0,T]; D(A^{\frac{1}{4}})) \\ \text{such that} \quad \sup_{0 < s < T} \|s^{\frac{1}{4}}A^{\frac{1}{2}}u(s)\|_{\mathcal{H}} + \sup_{0 < s < T} \|sA^{\frac{1}{4}}u'(s)\|_{\mathcal{H}} < \infty \right\}$$

endowed with the norm

$$\|u\|_{\mathcal{E}_T} = \sup_{0 < s < T} \|A^{\frac{1}{4}}u(s)\|_{\mathcal{H}} + \sup_{0 < s < T} \|s^{\frac{1}{4}}A^{\frac{1}{2}}u(s)\|_{\mathcal{H}} + \sup_{0 < s < T} \|sA^{\frac{1}{4}}u'(s)\|_{\mathcal{H}}.$$

Let α be defined by $\alpha(t) = e^{-tA}u_0$ where $u_0 \in D(A^{\frac{1}{4}})$. Then $\alpha \in \mathcal{E}_T$. Indeed, it is clear that $\alpha \in \mathscr{C}([0,T]; D(A^{\frac{1}{4}}))$. We also have that $t^{\frac{1}{4}}A^{\frac{1}{2}}\alpha(t) = t^{\frac{1}{4}}A^{\frac{1}{4}}e^{-tA}A^{\frac{1}{4}}u_0$ is bounded on (0,T) since $(e^{-tA})_{t\geq 0}$ is an analytic semigroup. Moreover, one has $\alpha'(t) = -Ae^{-tA}u_0$ which yields to $tA^{\frac{1}{4}}\alpha'(t) = -tAe^{-tA}A^{\frac{1}{4}}u_0$ continuous on [0,T], bounded in \mathcal{H} . For $u, v \in \mathcal{E}_T$, we define now

$$\Phi(u,v)(t) = \int_0^t e^{-(t-s)A} (-\frac{1}{2}\mathbb{P}_1)((u(s) \cdot \nabla)v(s) + (v(s) \cdot \nabla)u(s))ds, \quad 0 < t < T.$$

Proposition 3.1. The transform Φ is bilinear, symmetric, continuous from $\mathcal{E}_T \times \mathcal{E}_T$ to \mathcal{E}_T and the norm of Φ is independent of T.

Proof. The fact that Φ is bilinear and symmetric is clear. Moreover, $\Phi(u, v) = e^{-\cdot A} * f$, where f is defined by

$$f(s) = \left(-\frac{1}{2}\mathbb{P}_1\right)\left((u(s) \cdot \nabla)v(s) + (v(s) \cdot \nabla)u(s)\right), \quad s \in [0, T].$$

For $u, v \in \mathcal{E}_T$, it is clear that $(u(s) \cdot \nabla)v(s) + (v(s) \cdot \nabla)u(s) \in L^{\frac{3}{2}}(\Omega)^3$ and therefore $(\delta + A_0)^{-\frac{1}{4}}f(s) \in \mathcal{H}$ with $\sup_{0 < s < T} s^{\frac{1}{2}} \| (\delta + A_0)^{-\frac{1}{4}}f(s) \|_{\mathcal{H}} \le c \|u\|_{\mathcal{E}_T} \|v\|_{\mathcal{E}_T}$. We have then

$$\Phi(u,v) = e^{-\cdot A} * f = (\delta + A)^{\frac{1}{4}} e^{-\cdot A} * ((\delta + A_0)^{-\frac{1}{4}} f)$$

and therefore

$$\begin{split} \|A^{\frac{1}{4}}\Phi(u,v)(t)\|_{\mathcal{H}} &\leq \int_{0}^{t} \|A^{\frac{1}{4}}(\delta+A)^{\frac{1}{4}}e^{-(t-s)A}\|_{\mathscr{L}(\mathcal{H})}\|(\delta+A_{0})^{-\frac{1}{4}}f(s)\|_{\mathcal{H}}ds\\ &\leq c\left(\int_{0}^{t}\frac{1}{\sqrt{t-s}}\frac{1}{\sqrt{s}}\ ds\right)\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}}\\ &\leq c\left(\int_{0}^{1}\frac{1}{\sqrt{1-\sigma}}\frac{1}{\sqrt{\sigma}}\ d\sigma\right)\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}}\\ &\leq c\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}}. \end{split}$$

Continuity with respect to $t \in [0,T]$ of $t \mapsto A^{\frac{1}{4}} \Phi(u,v)(t)$ is clear once we have proved the boundedness. We also have

$$\begin{split} \|A^{\frac{1}{2}}\Phi(u,v)(t)\|_{\mathcal{H}} &\leq \int_{0}^{t} \|A^{\frac{1}{2}}(\delta+A)^{\frac{1}{4}}e^{-(t-s)A}\|_{\mathscr{L}(\mathcal{H})}\|(\delta+A_{0})^{-\frac{1}{4}}f(s)\|_{\mathcal{H}}ds \\ &\leq c\left(\int_{0}^{t}\frac{1}{(t-s)^{\frac{3}{4}}}\frac{1}{\sqrt{s}}\,ds\right)\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}} \\ &\leq ct^{-\frac{1}{4}}\left(\int_{0}^{1}\frac{1}{(1-\sigma)^{\frac{3}{4}}}\frac{1}{\sqrt{\sigma}}\,d\sigma\right)\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}} \\ &\leq ct^{-\frac{1}{4}}\|u\|_{\mathcal{E}_{T}}\|v\|_{\mathcal{E}_{T}}. \end{split}$$

Continuity with respect to $t \in]0, T]$ is clear once we have proved the boundedness. To prove the last part of the norm of $\Phi(u, v)$ in \mathcal{E}_T , we have for $s \in]0, T[$

$$f'(s) = (-\frac{1}{2}\mathbb{P}_1)((u'(s) \cdot \nabla)v(s) + (u(s) \cdot \nabla)v'(s) + (v'(s) \cdot \nabla)u(s) + (v(s) \cdot \nabla)u'(s))$$

and therefore

$$\sup_{0 < s < T} \|s^{\frac{5}{4}} (\delta + A_0)^{-\frac{1}{2}} f'(s)\|_{\mathcal{H}} \le c \|u\|_{\mathcal{E}_T} \|v\|_{\mathcal{E}_T}.$$

We have

$$\Phi(u,v)(t) = \int_0^{\frac{t}{2}} e^{-sA} f(t-s) ds + \int_0^{\frac{t}{2}} e^{-(t-s)A} f(s) ds \quad t \in]0, T[,$$

and therefore

$$\Phi(u,v)'(t) = e^{-\frac{t}{2}A}f(\frac{t}{2}) + \int_0^{\frac{t}{2}} (\delta+A)^{\frac{1}{2}}e^{-sA}(\delta+A_0)^{-\frac{1}{2}}f'(t-s)ds + \int_0^{\frac{t}{2}} -A(\delta+A)^{\frac{1}{4}}e^{-(t-s)A}(\delta+A_0)^{-\frac{1}{4}}f(s)ds,$$

which yields

$$\begin{split} \|A^{\frac{1}{4}}\Phi(u,v)'(t)\|_{\mathcal{H}} &\leq \frac{c}{\sqrt{t}} \left\| (\delta+A_0)^{-\frac{1}{4}} f(\frac{t}{2}) \right\|_{\mathcal{H}} + c \left(\int_0^{\frac{t}{2}} \frac{1}{s^{\frac{1}{2}}} \frac{1}{(t-s)^{\frac{5}{4}}} ds \right) \|u\|_{\mathcal{E}_T} \|v\|_{\mathcal{E}_T} \\ &+ c \left(\int_0^{\frac{t}{2}} \frac{1}{(t-s)^{\frac{5}{4}}} \frac{1}{s^{\frac{1}{2}}} ds \right) \|u\|_{\mathcal{E}_T} \|v\|_{\mathcal{E}_T} \\ &\leq \frac{c}{t} \left(\int_0^{\frac{1}{2}} \frac{d\sigma}{(1-\sigma)^{\frac{5}{4}} \sigma^{\frac{1}{2}}} \right) \|u\|_{\mathcal{E}_T} \|v\|_{\mathcal{E}_T}. \end{split}$$

This last inequality ensures that $\Phi(u, v) \in \mathcal{E}_T$ whenever $u, v \in \mathcal{E}_T$.

Theorem 3.2. For all $u_0 \in D(A^{\frac{1}{4}})$, there exists T > 0 such that there exists a unique $u \in \mathcal{E}_T$ solution of $u = \alpha + \Phi(u, u)$ on [0, T]. This function u is called the mild solution to the Navier-Stokes system.

Proof. Let T > 0. Since $\Phi : \mathcal{E}_T \times \mathcal{E}_T \to \mathcal{E}_T$ is bilinear continuous, it suffices to apply Picard fixed point theorem, as in [2]. The sequence in $\mathcal{E}_T (v_n)_{n \in \mathbb{N}}$ defined by $v_0 = \alpha$ as first term and

$$v_{n+1} = \alpha + \Phi(v_n, v_n), \quad n \in \mathbb{N}$$

converges to the unique solution $u \in \mathcal{E}_T$ of $u = \alpha + \Phi(u, u)$ provided $||A^{\frac{1}{4}}u_0||_{\mathcal{H}}$ is small enough $(||\alpha||_{\mathcal{E}_T} < \frac{1}{4||\Phi||_{\mathscr{L}(\mathcal{E}_T \times \mathcal{E}_T; \mathcal{E}_T)}})$. In the case where $||A^{\frac{1}{4}}u_0||_{\mathcal{H}}$ is not small (that is, if $\|\alpha\|_{\mathcal{E}_T} \geq \frac{1}{4\|\Phi\|_{\mathscr{L}(\mathcal{E}_T \times \mathcal{E}_T; \mathcal{E}_T)}}$) then for $\varepsilon > 0$, there exists $u_{0,\varepsilon} \in D(A)$ such that $\|A^{\frac{1}{4}}(u_0 - u_{0,\varepsilon})\|_{\mathcal{H}} \leq \varepsilon$. If we take as initial value $u_{0,\varepsilon} \in D(A)$, we have

$$\|\alpha_{\varepsilon}\|_{\mathcal{E}_{T}} \leq cT^{\frac{3}{4}} \|Au_{0,\varepsilon}\|_{\mathcal{H}} \xrightarrow[T \to 0]{} 0.$$

Therefore, we can find T > 0 such that $\|\alpha\|_{\mathcal{E}_T} < \frac{1}{4\|\Phi\|_{\mathscr{L}(\mathcal{E}_T \times \mathcal{E}_T; \mathcal{E}_T)}}$.

4. Strong solutions

Let u be the mild solution to the Navier-Stokes system. We show in this section that u in fact satisfies the equations of the Navier-Stokes system in an L^p -sense (for a suitable p). To begin with, we know that $u \in \mathcal{E}_T$ and satisfies

$$u = \alpha + \Phi(u, u) = \alpha + e^{-\cdot A} * \varphi(u),$$

where $\varphi(u) = -\mathbb{P}_1((u \cdot \nabla)u)$ and we have $\|t^{\frac{1}{2}}(u(t) \cdot \nabla)u(t)\|_{\frac{3}{2}} \leq c\|u\|_{\mathcal{E}_T}^2$. Therefore, we get

(4.1)
$$u(0) = \alpha(0) = u_0$$

(4.2)
$$\operatorname{div} u(t) = 0 \text{ in the } L^2 - \text{sense for } t \in]0, T[,$$

and

$$u' + Au = f$$
 in $\mathscr{C}(]0, T[; \mathcal{V}')$

which means that for all $t \in]0, T[$,

$$\mathbb{P}_1(u'(t) - \Delta_D^{\Omega} u(t) + (u(t) \cdot \nabla)u(t)) = 0$$

Then, by Theorem 2.1, there exists $(-\pi)(t) \in (\mathscr{C}_c^{\infty}(\Omega))'$ such that $\nabla \pi(t) \in H^{-1}(\Omega)^3$ and

(4.3)
$$\nabla(-\pi)(t) = u'(t) - \Delta_D^{\Omega} u(t) + (u(t) \cdot \nabla) u(t)$$

and we have for 0 < t < T

$$-\Delta_D^{\Omega} u(t) + \nabla \pi(t) = -u'(t) - (u(t) \cdot \nabla)u(t) \in L^3(\Omega)^3 + L^{\frac{3}{2}}(\Omega)^3.$$

The equation (4.3), together with (4.1) and (4.2), give the usual Navier-Stokes equations which are fulfilled in a strong sense (*a.e.*) where we consider the expression $-\Delta u + \nabla \pi$ undecoupled.

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