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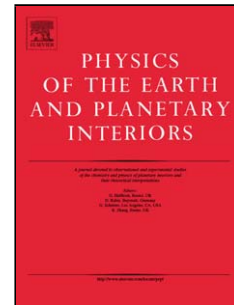
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An empirical comparison among aftershock decay models

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Abstract

We compare the ability of three aftershock decay models proposed in the literature to reproduce the behavior of 24 real aftershock sequences of Southern California and Italy. In particular, we consider the Modified Omori Model (MOM), the Modified Stretched Exponential model (MSE) and the band Limited Power Law (LPL). We show that, if the background rate is modeled properly, the MSE or the LPL reproduce the aftershock rate decay generally better than the MOM and are preferable, on the basis of the Akaike and Bayesian information criteria, for about one half of the sequences. In particular the LPL, which is usually preferable with respect to the MSE and fits well the data of most sequences, might represent a valid alternative to the MOM in real-time forecasts of aftershock probabilities. We also show that the LPL generally fits the data better than a purely empirical formula equivalent to the aftershock rate equation predicted by the rate- and state-

27 dependent friction model. This indicates that the emergence of a negative exponential decay at long
28 times is a general property of many aftershock sequences but also that the process of aftershock
29 generation is not fully described by current physical models.

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30 **Introduction**

31

32 The most commonly used formula to reproduce the decay of aftershock rate after a mainshock,
 33 also adopted in procedures for the real-time forecast of aftershock probabilities in California
 34 (Gerstenberger et al., 2007), is the Modified Omori Model (MOM, Utsu, 1961)

$$35 \quad {}_{MOM} t = \frac{K}{t - c}^p \quad [1]$$

36 where ${}_{MOM} t$ is the intensity (the rate) of a non stationary Poisson process, and p , c and K are free
 37 parameters. The MOM is empirical in nature but it was found to be compatible with the rate- and
 38 state-dependent friction model proposed by Dieterich (1994).

39 A characteristic of the MOM is to predict an infinite number of possible future aftershocks (that is
 40 an infinite number of potential faults) if the power law exponent p is lower than or equal to 1. Since
 41 such p values are often observed for real sequences, the MOM might appear physically unrealistic.
 42 Few alternative formulations, proposed in the last decades, overcome this limitation of the MOM.
 43 We mainly consider here two of them: the Modified Stretched Exponential Model (MSE,
 44 Kisslinger, 1993; Gross and Kisslinger, 1994) and the Band Limited Power Law (LPL, Narteau et
 45 al., 2002; 2003). Both MSE and LPL assume that a negative exponential decay emerges at long
 46 times, hence they predict a finite number of aftershocks (and faults), independently of the value of
 47 the power law exponent. The intensity of the MSE can be written as

$$48 \quad {}_{MSE} t = (1 - r) N^* \exp\left(-\frac{d}{t_0}\right)^{1-r} \exp\left(-\frac{t-d}{t_0} r\right) \exp\left(-\frac{t}{t_0}\right)^r \quad [2]$$

49 where N^* is the total number of potential shocks at the time of the mainshock ($t=0$), t_0 is the
 50 relaxation time of the negative exponential decay process, d a delay time (corresponding to
 51 parameter c of the MOM) and $0 < r \leq 1$ the power-law exponent.

52 The intensity of the LPL is given by

$$53 \quad {}_{LPL} t = B \frac{q_b t^{q_b} + q_a t^{q_a}}{t^q} \quad [3]$$

54 where q is the power-law exponent, B is a normalizing constant (similar to K of the MOM),
 55 $a > b$ are two parameters (having the physical dimensions of rates) that controls the behavior at
 56 long and short times respectively, and γ indicates the incomplete Gamma function

$$57 \quad \Gamma(q, x) = \int_0^x z^{q-1} e^{-z} dz \quad [4]$$

58 When $b \ll a$ (as it may be assumed usually) the behavior of the LPL can be described as the
 59 superposition of three regimes that control the rate at different times: an initial linear decay, which
 60 is followed by a power-law and, at large times, by a negative exponential. Narteau et al (2003)
 61 suggested to considering two times t_1 and t_2 that correspond to the transition between the linear
 62 and the power-law and between the power-law and the exponential decays respectively. They are
 63 defined as the times at which the ratio between aftershock rates predicted by LPL and by a pure
 64 power-law is χ . Narteau et al. (2003) report the values assumed by t_1 and t_2 , for values of
 65 the ratio χ ranging from 0.8 to 0.99 while Lolli et al. (2009) proposed to use $t_b = t_1 2^{-q}$ and
 66 $t_a = t_2 1/e$ (where e is the base of natural logarithms) as they corresponds approximately to c of
 67 the MOM (or d of MSE) and t_0 of the MSE respectively. We will adopt such derived parameters t_b ,
 68 and t_a in the following references to the LPL.

69 Both MSE and LPL are based on reasonable physical assumptions but Lolli and Gasperini (2006)
 70 showed that MSE and LPL are preferable with respect to the MOM for about one fourth of the real
 71 aftershock sequences of Southern California and Italy only. They hypothesized that the limited
 72 duration of the observing time interval they choose (one year) might penalize the MSE and the LPL
 73 with respect to the MOM when the exponential decay emerges later than the end of such interval. In
 74 this work we will test such hypothesis by considering a longer observing interval of four years. We
 75 also consider here the possibility that the background rate (not modeled by Lolli and Gasperini,
 76 2006) might play a role in assigning the preference to the MOM in some cases. The background
 77 seismicity rate is accounted simply by a constant rate μ (to be determined together with the other
 78 parameters of the various decay models) added to the rate equations [1] [2] and [3].

79

80 **Data sources and sequence detection**

81

82 We use essentially the same datasets analyzed by Lolli and Gasperini (2006) but we extend the
83 analysis to a longer time interval of four years after the mainshock and consider the catalogs of
84 southern California and Italy up to July 2008 and May 2008 respectively (instead of December
85 2004). For California we use the revised catalog from 1932 to 2008 available from the Southern
86 California Earthquake Center (SCEC) site (<http://www.scecdc.scec.org/>). For Italy, we merged
87 several catalogs of Italian instrumental earthquakes covering the time interval from 1960 to May
88 2008. From 1960 to 1980, we used the catalog of the Progetto Finalizzato Geodinamica (Postpischl,
89 1985) with magnitudes corrected according to Lolli and Gasperini (2003); from 1981 to 1996, we
90 used the *Catalogo Strumentale dei Terremoti Italiani* dal 1981 a 1996 Version 1.1 (CSTI Working
91 Group, 2004); from 1997 to 2002, we used the Catalogo della Sismicità Italiana 1.1 (Castello et al.,
92 2005); finally, from 2003 to 2008, the data are taken from the instrumental bulletin of the *Istituto*
93 *Nazionale di Geofisica e Vulcanologia* (INGV) available from site
94 <http://www.ingv.it/~roma/reti/rms/bollettino>. Following Ouillon and Sornette (2005), we assumed
95 the completeness of the southern California catalog for $M_L > 3.0$ in 1932 and later years, for $M_L > 2.5$
96 in 1975 and later years, $M_L > 2.0$ in 1992 and later years, and $M_L > 1.5$ in 1994 and later years. For
97 Italy we assumed the completeness for $M_L > 2.5$ for 1984 and later (Lolli and Gasperini, 2003), and
98 for $M_L > 3.0$ before 1984.

99 In a first step we use the same sequence detection algorithm adopted by Lolli and Gasperini (2006)
100 that defines the influence zone of any shock as a circular area centered in the epicenter and assumes
101 as mainshocks (triggering the sequences) all earthquakes with magnitude not lower than 5.0 that are
102 not included in the influence zone of a larger shock. The time window is fixed to four years after
103 the mainshock while the radius R of the influence zone is chosen as a function of magnitude as
104 $\text{Log}_{10}(R) = 0.1238M + 0.983$ (that closely corresponds to Table 1 of Gardner and Knopoff, 1974).

105 Only the shocks shallower than 40 km and with magnitude above completeness threshold are
106 included in sequences. To reduce the possible incompleteness in the first times after the mainshock
107 we only consider the aftershocks with magnitude not lower than mainshock magnitude M_m minus
108 3.5.

109 As the Gardner and Knopoff (1974) radius is likely to overestimate the size of the mainshock
110 influence zone, in a second step we performed an analysis of correlation between the shock rates
111 observed at different distances from the mainshock during a time interval of 200 days after the
112 mainshock. In particular, for distances r varying from 0 to R , we correlate the sequence of rates
113 observed (over 5 days bins) inside the circle with radius r and inside the circular ring with
114 minimum and maximum radius r and R respectively. For each sequence we assumed as influence
115 distance (reported in Table 1 as R_i) the largest r for which the correlation between the sequences of
116 rates is significant at the 0.05 level.

117 To grant a reliable determination of model parameters we consider for the analysis only the
118 sequences including 100 shocks at least within the four years time interval following the
119 mainshock. Moreover, since all the simple decay models we consider are not suitable to reproduce
120 complex sequences with strong secondary clustering we excluded from our dataset, by a visual
121 analysis of the plot of the rate over 5 days bins, the sequences showing at later times one or more
122 peaks of the shock rate with amplitude of the same order of magnitude of the peak following the
123 mainshock.

124 The detected sequences are listed in Table 1. The longer time window (four years instead of one)
125 and the different completeness thresholds and selection criteria here adopted reduces the number of
126 sequences (from 37 to 18 for California and from 10 to 6 for Italy) with respect to those detected by
127 Lolli and Gasperini (2006).

128

129 **Analysis**

130

131 We estimated the parameters of each decay model by the maximum likelihood method (Ogata,
 132 1988). To maximize the likelihood we use an algorithm (Lolli et al., 2009) that combines a random
 133 search over a reasonable interval of the parameters space and Newton-like optimizations (Dennis
 134 and Schnabel, 1983) of the best random solutions. We estimate the parameters of our sequences
 135 both with and without the inclusion of the constant background term \square and by considering different
 136 lengths of the observing interval of 3, 6, 12, 24 and 48 months.

137 We compare the goodness-of-fit of the different decay models by three criteria: the corrected
 138 Akaike Information Criteria (AIC_c , Akaike, 1974; Hurvich and Tsai, 1989), the corrected Bayesian
 139 Information Criteria (BIC , Schwartz, 1978; Draper, 1995) and the simple maximum log-likelihood
 140 function l_{\max} . For the AIC_c and BIC we adopt (consistently with Lolli and Gasperini, 2006) the
 141 following scores

142
$$\frac{2k}{n-k} \ln \left(\frac{2\pi}{n-k} \right) \quad [5]$$

143
$$\frac{2k}{n-k} \ln \left(\frac{2\pi}{n-k} \right) \quad [6]$$

144 where k is the number of free parameters (3 for the MOM, 4 for MSE and LPL and one more for all
 145 models when the background rate \square is considered), and n is the number of data (the number of
 146 aftershocks in each sequence). With these formulations, which differ from the usual ones for the
 147 sign and for a factor of 2, the best model is the one giving the highest score.

148 In the following comparisons, we will also consider l_{\max} because we might hypothesize that the
 149 additional parameter of the MSE and LPL, which models the exponential decay, might not be able
 150 improve significantly the fit (and increase correspondingly the log-likelihood function) when the
 151 length of the observing time interval (the assumed duration of the sequence) is short with respect to
 152 the relaxation time (t_0 for the MSE and about t_a for the LPL). In these cases the penalty terms
 153 assigned by AIC_c and BIC to the additional parameter might be oversized. Moreover, the model
 154 with the highest log-likelihood, whatever the number of parameters, is the one that best reproduce

155 the behavior of the rate. Hence it is the most suitable for real-time forecast of aftershock
156 probabilities, when the peculiar properties of the active sequence are not known well.

157

158 **Results and discussion**

159

160 The parameters of the various models and the relevant goodness-of-fit estimators without and with
161 background are listed in Table 2, for the longest time interval of 48 months (1460 days). For the
162 computations not considering the background rate (Table 2, left), the maximum likelihood estimates
163 of parameter p_3 (corresponding to the exponential decay characteristic time t_0 for the MSE and t_a for
164 the LPL) are in most cases definitely larger than the duration of the observing time interval
165 (highlighted with bold type) and often coincides with the upper limit (Up lim.) of 10^7 days we
166 imposed in likelihood maximization. Conversely, when the background rate is included in
167 computations (Table 2, right), the estimates of p_3 are in most cases shorter than the observing
168 interval (on the order of some weeks to some months). We can also note in Table 2 a general
169 increase of the estimated power law exponent (p_1) for all of the models when the background term
170 is considered. For some sequences (*e.g.* cal08, cal11, cal12, cal16) such increase is particularly
171 relevant for the MOM (from about 1 to 1.5 and more). As shown by Gasperini and Lolli (2008) by
172 simulation of synthetic sequences, such high p values for the MOM might be the symptom of an
173 early startup of the exponential decay. Parameter p_2 (the initial delay time) is not affected instead
174 very much by the inclusion of background. It only tends to slightly increase as a consequence of the
175 increase of the power law exponent, being the two parameters correlated to each other (Gasperini
176 and Lolli, 2006).

177 The inclusion of the background has also the effect to improve the fit of the three models as shown
178 by the increase of maximum log-likelihoods (l_{\max}) for almost all sequences and models. The AIC_c
179 and BIC scores are also higher for most sequences. The sequences showing lower AIC_c scores
180 (cal02, cal03, cal06, cal07, cal10, cal17, ita01, ita06) and BIC (cal01, cal02, cal03, cal06, cal07,

181 cal10, cal17, ita01, ita06) for the best model (highlighted with bold type) are characterized by
182 relatively low background rates. In these cases the slight improvement of the likelihood function
183 induced by the additional parameter (the background rate) is not large enough to compensate the
184 penalty terms added by the information criteria.

185 In the following we will evaluate the relative efficiency of the different models by counting the
186 number of sequences for which each model is the best among the alternatives, according to the
187 three criteria. The results are represented as line plots of such counts as a function of the considered
188 length (of 3, 6, 12, 24 and 48 months) of the time window.

189 Fig. 1 concerns sequences of Southern California when the background rate is neglected. We note
190 (in Fig. 1a) that the MOM is preferable with respect to the other models according to the AIC_c for
191 more than 2/3 of the sequences. Such prevalence is clearer for BIC (Fig. 1b), for which the MOM is
192 preferable with respect to MSE and LPL for more than 3/4 of the sequences. These results are very
193 similar to those obtained by Lolli and Gasperini (2006) on a different set of sequences. We can note
194 that the number of sequences that are better fitted by the MOM increases slightly with the
195 increasing the duration of the time window. Fig. 1c shows that, at the maximum duration of 48
196 months, the MOM has the highest maximum log-likelihood l_{\max} for 7 of the 18 sequences.

197 In Fig. 2 we report the same computations of Fig. 1 when the background rate is included into the
198 rate equations as an additive constant term μ . It is evident how the MSE and the LPL definitely
199 improve their performances with respect to the MOM. For AIC_c (Fig. 1a), the LPL and the MSE are
200 preferred with respect to the MOM for more than one half of the sequences while for BIC (Fig. 1b)
201 the preference goes to MSE or to the LPL for 8 sequences over 18 (Table 3). For the longer time
202 window (48 months), the maximum log-likelihood (Fig. 1c) of the MOM is higher than those of the
203 two alternative models for only one sequence (cal12). We can also note that the numbers of
204 preferences for the various models are weakly dependent on the duration of the observing interval.

205 A similar behavior was shown by the Italian sequences. Even in this cases the values of p_3
206 decreases while p_1 and p_2 increases when the background is included in computations (Table 2). In

207 Fig. 3 we report the number of preferences for the various model only for the AIC_c , which represent
208 an intermediate weighting of the additional parameters between the simple maximum log-likelihood
209 (zero weight) and the BIC (highest weight). We can see that the LPL definitely improves its
210 performance with respect to both the MOM and the MSE when the background is considered. For
211 the longer time window of 48 months, only two Italian sequences (ita02, ita03) show larger scores
212 for the MOM.

213 These evidences indicate that the emergence of exponential decay is a general characteristic of most
214 sequences both in Southern California and Italy and that the background rate (if not appropriately
215 modeled) has the effect to hide such emergence in many cases.

216 In summary (Table 3), for the longer time window of 48 months the MSE or the LPL perform better
217 than the MOM for 15 sequences over 24 for AIC_c , 11 for BIC and 23 for l_{max} , when the
218 background is properly modeled. The MSE and the LPL individually have both a larger l_{max} than
219 the MOM for 18 sequences over 24 but a larger AIC_c scores for 8 and 14 sequences respectively and
220 larger BIC scores for 8 and 10 sequences respectively.

221 The direct comparison between LPL and MSE shows that the former is preferable with respect to
222 the latter for 16 sequences versus 8 for all of the scores (as the two models have the same number of
223 free parameters). We can also note from Table 2 that for the six sequences (cal03, cal07, cal09,
224 cal12, cal 13 and cal17) for which the MOM has a higher log-likelihood score than the LPL the log-
225 likelihood difference is on the order of a few units at most, indicating that the two models show a
226 very similar fit. Hence, even though the exponential decay, on the basis of information criteria,
227 might be not necessary to reproduce some sequences, we can assert that the LPL represents the
228 most suitable model when the actual properties are not known well, as in real-time forecasting of
229 aftershock probabilities (Gerstenberger et al., 2007) of an active sequence.

230

231 **Comparing the LPL with the Dieterich (1995) rate equation**

232

233 To better understand the physical implication of the preference given to the LPL we will attempt an
 234 empirical comparison between such model and the rate equation implied by the Dieterich (1994)
 235 rate- and state-dependent friction model

$$236 \quad \lambda(t) = \frac{\mu_r \dot{\tau} / \dot{\tau}_r}{\left[\frac{\dot{\tau}}{\dot{\tau}_r} \exp\left(\frac{-\Delta\tau}{A\sigma}\right) - 1 \right] \exp\left[\frac{-t}{t_c}\right] + 1} \quad [7]$$

237 where $\dot{\tau}_r$ is the background rate before the mainshock, $\dot{\tau}_r$ and $\dot{\tau}$ are the shear stress rates prior to
 238 and following the shear stress step $\Delta\tau$ induced by the mainshock, A is a fault constitutive
 239 parameter, σ the normal stress and $t_c = A\sigma / \dot{\tau}$ a characteristic relaxation time. When the stress
 240 after the mainshock is about constant ($\dot{\tau} \approx 0$) eq. [7] becomes equivalent to the Omori's law [1]
 241 (with $\lambda = \dot{\tau}_r$). For $\dot{\tau} \neq 0$, eq. [7] also gives the Omori's law at short times ($t/t_c \ll 1$) but merges to
 242 the steady state background rate at long times ($t/t_c \gg 1$).

243 The formulation of eq. [7] is not particularly suitable to empirically fitting real sequences, because
 244 the maximum likelihood method is not able to constrain independently the 6 unknown parameters
 245 ($\dot{\tau}_r$, $\dot{\tau}$, μ_r , μ , A , σ) from sequence data due to their mutual correlation. Although some
 246 assumptions could be made on the values of some parameters we will adopt here a purely empirical
 247 approach where the 6 free parameters of eq. [7] are combined into 3

$$248 \quad \lambda_{DRL}(t) = \frac{\mu}{[C - 1] \exp\left[\frac{-t}{t_c}\right] + 1} \quad [8]$$

249 where $\mu = \mu_r \dot{\tau} / \dot{\tau}_r$ (the steady state background rate after the mainshock), $C = \frac{\dot{\tau}}{\dot{\tau}_r} \exp\left(\frac{-\Delta\tau}{A\sigma}\right)$ and t_c
 250 are empirical parameters to be determined by maximum likelihood estimation. For the sake of a
 251 comparison with the LPL, t_c has a meaning comparable t_a , and the product $C t_c$ roughly
 252 corresponds (see eq. [16] in Dieterich, 1994) to c of the MOM and then to t_b of the LPL. We must
 253 note that such form [8] of the Dieterich rate law (DRL) maximizes the ability to fit the data because
 254 it does not imply any physical constraint on the parameter values. So its performance might be

255 slightly better than those of the original formula (eq. [7]) when a physically consistent value is
256 assigned, for example, to .

257 In Table 4 we compare the estimated values of parameters μ , Ct_c (p2) and t_c (p3) as well as the
258 goodness-of-fit scores of the DRL model with the parameters and the scores of the LPL (including
259 background), for the same set of sequences analyzed previously (Table1) and the maximum
260 duration of 48 months. The values of as well as of Ct_c with t_b (p2 in Table 4) appear reasonably
261 consistent for most sequences. On the contrary t_c is usually larger than t_a (p3 in Table 4) in most
262 cases.

263 For all but four sequences (cal03, cal07, cal09, cal17), the maximum log-likelihood l_{\max} is larger for
264 the LPL than for the DRL. The AIC_c and BIC scores are slightly less favorable to the LPL, due to
265 the lower number of free parameters of DRL ($k=3$) with respect to the LPL ($k=5$ including the
266 background rate). In Fig. 4 we plot the behavior of the number of preferences of AIC_c , BIC and l_{\max}
267 scores for the two models as a function of the duration of the time window over which the
268 sequences are observed. For short durations (3 and 6 months) the, LPL appears preferable with
269 respect to the DRL for about 2/3 of the 24 sequences. The preferences for the LPL tend to increase
270 for increasing durations up to 24 months. Then the performance of the LPL worsens slightly and for
271 48 months (Table 3) the preferences for the LPL and DRL become respectively 17 versus 7 for
272 AIC_c , 15 versus 8 for BIC , and 20 versus 4 for l_{\max} .

273 This behavior can be explained by the interplay between the log-likelihood differences and the
274 penalty terms of the AIC_c and BIC scores: the likelihood difference between the LPL and the DRL
275 tends increase (Fig 4c), for increasing durations from 3 to 12 months, while for larger durations the
276 tendency reverses and even the penalty terms tend to favor more the DRL, due to the increasing
277 number of data.

278 In summary, the LPL performs generally better than the DRL and is definitely the most suitable
279 model when the aim is to reproduce well the behavior of the aftershock rate. However the DRL,

280 which with only three free parameters shows to explain well a significant portion of the sequences,
 281 appears to pick much of the physics of the process of aftershock generation (albeit not all).

282

283 **Visual comparison of rate decay models**

284

285 To better describe the different performance of various models, we plot in Fig. 5 the behavior with
 286 the time elapsed after the mainshock of the observed (symbols) and predicted (lines) rates for two
 287 sequences (cal16 and ita02) that show the clear emergence of the exponential decay at relatively
 288 long times when the background is modeled. We can see how the LPL (blue) and the MSE (green)
 289 models, are able to reproduce better than the MOM (red) the transition of the rate to the background
 290 level at times on the order of 10^2 days. Moreover they also seem both to describe better than the
 291 MOM the rate evolution in the first times after the mainshock (< 0.1 days). In both cases the DRL
 292 (black) appears to be too “rigid” to follow well the behavior of aftershock rate decay. It tends to
 293 overestimate the rate at short and long times and to underestimate at intermediate times (0.1 to 10
 294 days). We can argue that the assumed functional form of the transition to the background rate at
 295 long times and the power law exponent fixed to 1 at intermediate times prevent a good fit even at
 296 short times.

297 In Fig 6 we report, for the same two sequences, the differences between the observed $N_{\text{observed}}(t)$ and
 298 predicted $N_{\text{model}}(t)$ cumulative number of aftershocks with time. The predicted numbers are
 299 computed, for various models, as time integrals of the rate functions from the time t_1 of the first
 300 aftershock to the time t_i of each i -th aftershock

301

$$\boxed{\times}$$

[9]

302 while the observed cumulative number is simply $N_{\text{observed}}(t_i)=i$. These plots confirm that the MOM
 303 has generally larger differences than MSE and LPL both at short and long times, while the DRL
 304 largely underestimates (positive difference) the cumulative number of shocks at times between few

305 days and 200 days. For sequence cal16 we can note a marked negative difference for all models at
306 times between 1 and 3 years that rapidly converge to 0 at larger times. This might indicate that the
307 decay of the main sequence is probably slightly faster than that predicted by all of the models but
308 the occurrence of a burst of shock (maybe due a small sequence) at about 3 years after mainshock
309 prevents a more accurate modeling of the behavior at long times. These late shocks do not affect
310 much the fit at times shorter than three years because the maximum likelihood estimation of
311 parameters is controlled by the more numerous shocks occurring in the first part of the sequence.
312 In Fig. 7 we show the behavior with time of the rate (a) and of the cumulative number difference (b)
313 for a sequence (cal08) characterized by a high power-law exponent ($p=1.86$ for the MOM and
314 $q=1.54$ for the LPL), when the background is included in computations. We can see how the MOM
315 (red) and the LPL (blue) curves are almost superimposed among each other. In fact, although the
316 LPL has higher l_{\max} and AIC_c , the MOM is preferable according to BIC (see Table 2). Such high
317 value of the power-law exponent prevents a good fit by the MSE (green), for which r is limited in
318 the interval $]0,1[$ and by the DRL (black), which assumes a power law exponent equal to 1. This
319 suggests that Dieterich (1994) model might neglect some unknown physics properties of the
320 aftershock generation process.

321

322 **Control experiments**

323

324 As the behavior of aftershock sequences in the first times after the mainshock might not be due to
325 true physical processes but rather to the incompleteness of the seismic catalog (Narteau et al., 2002;
326 Lolli and Gasperini, 2006), we tested the stability of our computations as a function of the starting
327 time of the interval over which the analysis is performed. We rerun our computations, for the
328 longest time interval of 48 months, but removing from the datasets the first 10 minutes (0.007
329 days), the first hour (0.042 days) or the first day. In the first two cases (Table 5), the results of the
330 comparison between MOM, MSE and LPL are similar to those obtained using the entire sequences.

331 We can note only a slight increase of the preferences to the MOM with respect to the alternative
332 models MSE and LPL (2 sequences more for AICc and 1 for BIC and l_{\max}) and an increase of
333 preferences to LPL with respect to MSE (3 sequences more). The direct comparison between LPL
334 and DRL also gives similar results with a slight increase of preferences to LPL.

335 When instead the first day is not considered in the analysis, the preferences to the MOM with
336 respect to MSE and LPL increase further by a couple of sequences for l_{\max} but more clearly for the
337 other criteria so that about 20 of the 24 sequences give a preferences to the MOM according to
338 AICc and BIC. In this case we also have a dramatic increase of p above 1.5 (up to 4) and of c above
339 1 day (up to 31 days), for many sequences (Table 6). Such values of MOM parameters are rarely
340 reported in the literature and can be considered unrealistic and not justified physically. In fact,
341 many authors have argued that c should be zero in principle and that non-zero values are due to the
342 incompleteness of the catalog (Narteau et al., 2002; Lolli and Gasperini, 2006) or to physical
343 processes occurring at very short times after the mainshock (Nanjo et al., 2007). When the first part
344 of the sequence is not considered in computations, such incompleteness or such processes cannot
345 influence significantly the estimated parameters, hence the high values of p and c estimated when
346 $T_s=1$ should be explained otherwise.

347 Lolli et al. (2009) found that high p and c values are estimated when fitting (by a MOM) sequences
348 simulated according to a MSE or a LPL with an early onset of the exponential decay. We could
349 argue that a similar phenomenon occurs in this case. We might say that, as c is not useful to
350 reproduce the decay at short times because such times are excluded from computations, the MOM
351 ‘uses’ c to reproduce the deviation of rate function from power law (due to the onset of the
352 exponential decay). The better AICc and BIC scores of the MOM with respect to the MSE and the
353 LPL can be explained as well by the fact that the latter models pay a penalty for a parameter (d for
354 MSE and t_b for LPL) that is not useful to improve the fit at short times while the MOM can
355 ‘recycle’ parameter c to improve the fit at long times. Hence we might consider the behavior

356 observed when $T_s=1$ as a further evidence of the emergence of the exponential decay for many
357 sequences and of the inadequacy of the MOM to reproduce consistently their behavior.

358 When $T_s=1$ even the direct comparison between LPL and DRL (Table 5) shows significant
359 variations with a reduction of the preferences to LPL for both AICc (2 sequences less) and BIC (4
360 sequences less). Conversely, the maximum log likelihood l_{\max} of the LPL becomes the largest for all
361 of the sequences. This means that the DRL is able to describe as well as the LPL the behavior at
362 long times while it is less appropriate at short times where the earthquake catalog might be
363 incomplete. We can argue that the DRL captures much of the true physical properties but it is not
364 particularly suited in general to reproduce the empirical behavior of real sequences.

365

366 **Conclusions**

367

368 We verified that if background rate is modeled properly, the most of a set of 24 real sequences in
369 Italy (6) and Southern California (18) show the emergence of a negative exponential decay of
370 aftershock rate after the initial time interval where the power-law dominates. In fact, two decay
371 models that predict such exponential decay – the Modified Stretched Exponential (MSE, Gross and
372 Kisslinger, 1994) and the band Limited Power Law (LPL, Narteau et al, 2002) – have a higher
373 maximum log-likelihood l_{\max} than the Modified Omori Model (MOM, Utsu, 1971) for 23 sequences
374 over 24. The MSE and the LPL are to be preferred with respect to the MOM for about one half of
375 the sequences, on the basis of the corrected Akaike Information Criterion (AIC_c) and the Bayesian
376 Information Criterion (BIC). In particular, the LPL alone performs better than the MOM for 18
377 sequences over 24 according to l_{\max} , 14 to AIC_c and 10 to BIC . In most cases, the estimated
378 characteristic times of the exponential decay are on the order of some weeks to some months.

379 The inclusion of the background in the rate equation is necessary because, when neglected, the
380 emergence of the exponential decay is somehow hidden for many sequences. The inclusion of the
381 background term has also the effect to reduce the estimates of the characteristic time of the

382 exponential decay (for the MSE and LPL) and to increase the estimates of the power-law exponent
383 and, to a minor extent, of the initial delay time for all of the models and particularly for the MOM.

384 As the LPL is generally preferable with respect to the MSE and is able to reproduce well the
385 effective rate decay of real sequences in most cases, it is reasonable to adopt it (with background
386 included) in future analyses of aftershock decay and particularly in real-time forecasts of aftershock
387 probabilities (Gerstenberger et al., 2007) where the actual properties of the sequences are not
388 known well.

389 We also found that an empirical rate formula equivalent to that predicted by the Dieterich (1994)
390 rate- and state-dependent friction model, with only three free parameters, is able to explain quite
391 well a significant portion of the sequences but performs generally worse than the LPL. This
392 indicates that the Dieterich (1994) rate equation is able to describe well much of the physics of the
393 process of aftershock generation but also that some further developments are needed to make it
394 suitable for best reproducing the observed behavior of aftershock rates.

395

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- 452

452

Table captions

453 Table 1. List of sequences from Southern California (calxx) and Italy (itaxx) analyzed in this work.
 454 Dates and geographical coordinates refer to mainshock origin time and epicenter. M_m is the
 455 mainshock magnitude, M_{\min} is the minimum magnitude of aftershocks, N_{ev} is the number of
 456 aftershocks (above M_{\min}) within the influence zone of the mainshock, D is the effective duration of
 457 the sequence (the time difference between the mainshock and the last aftershock), and R_i the radius
 458 of the influence zone of the mainshock (see text).

459

460 Table 2. Parameters values and goodness-of-fit scores of the different decay models not including
 461 (left) and including (right) the background rate (μ , in shocks/day), for the maximum sequence
 462 duration of 48 months. p_1 is the power law exponent (p for the MOM, r for the MSE and q for the
 463 LPL), p_2 is the initial delay time (c for the MOM, d for the MSE, t_b for the LPL) in days, and p_3 the
 464 characteristic time of the negative exponential decay (not defined for the MOM, t_0 for the MSE, t_b
 465 for the LPL) in days. p_3 estimates definitely longer than the duration of the sequence (1460 days)
 466 are highlighted with bold type and those at the lower (1 day) and upper ($1 \cdot 10^7$ days) limit assumed
 467 in maximum likelihood estimation are denoted by “Low lim.” and “Up lim.” respectively. AIC_c ,
 468 BIC , and l_{\max} are the goodness-of-fits scores for each decay model (without and with background).
 469 For each sequence, the higher score (highlighted with bold type) indicates the model preferred
 470 according to each criterion.

471

472 Table 3. Counts of sequences for which the different criteria assign the preference to various
 473 models, for the maximum sequence duration of 48 months.

474

475 Table 4. Parameters values and goodness-of-fit scores of the LPL (with background) and RDL
 476 decay models for a sequence duration of 48 months. p_1 is the power law exponent for the LPL, p_2 is

477 the initial delay time (t_b for the LPL, Ct_c for the DRL) in days, and p_3 the characteristic time of the
478 negative exponential decay (t_b for the LPL and t_c for the DRL) in days. p_3 estimates definitely
479 longer than the duration of the sequence (1460 days) are highlighted with bold type and those at the
480 lower (1 day) and upper ($1 \cdot 10^7$ days) limit are denoted by “Low lim.” and “Up lim.” respectively.
481 AIC_c , BIC , and l_{\max} are the goodness-of-fits scores for each decay model. For each sequence, the
482 higher score (highlighted with bold type) indicates the model preferred according to each criterion.

483

484 Table 5. Counts of sequences, from California and Italy, for which the different criteria assign the
485 preference to various models, for the maximum sequence duration of 48 months and different
486 starting times T_s (in days) of the observing time interval.

487

488 Table 6. Parameters values of the MOM for different starting times T_s (in days) of the observing
489 time interval. Values of $p > 1.5$ and $c > 1$ day are highlighted with bold type.

490

490

Table 1

491

Detected sequences

Seq.	Year	Mo	Day	Lat (North)	Lon (East)	M_m	M_{min}	Nev	D (days)	R_i (km)
cal01	1933	3	11	33.638	-117.973	6.4	3.0	269	1451	42
cal02	1946	3	15	35.702	-117.944	6.3	3.0	154	1364	30
cal03	1947	4	10	34.983	-116.531	6.5	3.0	124	1455	27
cal04	1954	3	19	33.298	-116.081	6.4	3.0	136	1452	32
cal05	1968	4	9	33.167	-116.087	6.6	3.1	162	1445	52
cal06	1971	2	9	34.416	-118.370	6.6	3.1	291	1408	27
cal07	1979	3	15	34.327	-116.445	5.3	2.5	176	1451	13
cal08	1981	4	26	33.096	-115.624	5.8	2.5	186	1425	16
cal09	1986	7	8	33.999	-116.608	5.7	2.5	868	1452	44
cal10	1986	7	13	32.971	-117.874	5.5	2.5	1686	1457	22
cal11	1987	2	7	32.388	-115.305	5.4	2.5	225	1454	40
cal12	1987	11	24	33.015	-115.852	6.6	3.1	216	1446	41
cal13	1994	1	17	34.213	-118.537	6.7	3.2	344	1455	25
cal14	1999	10	16	34.594	-116.271	7.1	3.6	151	1367	52
cal15	2001	7	17	36.016	-117.874	5.2	1.7	2009	1457	28
cal16	2002	2	22	32.319	-115.322	5.7	2.2	785	1457	27
cal17	2003	12	22	35.709	-121.104	6.5	3.0	129	1444	32
cal18	2004	9	28	35.812	-120.379	6.0	2.5	147	1377	48
ita01	1980	11	23	40.800	15.367	6.5	3.0	105	1407	41
ita02	1984	4	29	43.204	12.585	5.2	2.5	130	1459	41
ita03	1990	5	5	40.650	15.882	5.6	2.5	109	1453	41
ita04	1997	9	26	43.015	12.854	5.8	2.5	774	1354	46
ita05	2002	9	6	38.381	13.654	5.6	2.5	181	1414	26
ita06	2002	10	31	41.717	14.893	5.4	2.5	163	1280	26

492

493

Table 2

494

Parameters values and goodness-of-fit scores

Seq.	Model	Without background						With background						
		p1	p2	p3	AICc	BIC		μ	p1	p2	p3	AICc	BIC	
cal01	MOM	1.090	0.061		210.778	208.188	213.823	0.012	1.160	0.082		211.467	208.029	215.542
	MSE	0.886	0.029	Low Lim.	206.087	202.649	210.163	0.011	0.837	0.030	Low Lim.	205.811	201.533	210.926
	LPL	1.080	0.055	Up. Lim.	211.975	208.538	216.051	0.011	1.130	0.073	9.1E+06	212.352	208.074	217.467
cal02	MOM	1.180	1.360		-139.495	-141.213	-136.415	0.008	1.290	1.780		-139.746	-142.010	-135.611
	MSE	0.847	0.861	Low Lim.	-142.022	-144.286	-137.888	0.010	0.795	1.030	Low Lim.	-142.147	-144.942	-136.944
	LPL	1.130	0.868	Up. Lim.	-137.854	-140.118	-133.720	0.006	1.190	1.150	5.8E+06	-138.479	-141.274	-133.276
cal03	MOM	0.974	0.008		-37.758	-39.132	-34.658	0.004	1.000	0.012		-38.578	-40.374	-34.409
	MSE	0.948	0.008	Up. Lim.	-38.828	-40.624	-34.660	0.011	0.797	0.003	5	-39.249	-41.451	-33.995
	LPL	0.955	0.005	8.7E6	-39.322	-41.119	-35.154	0.014	0.933	0.004	373	-40.007	-42.209	-34.753
cal04	MOM	1.050	0.005		116.559	115.038	119.650	0.010	1.130	0.010		119.506	117.509	123.659
	MSE	0.950	0.003	Low Lim.	114.436	112.439	118.588	0.014	0.742	0.001	Low Lim.	119.142	116.686	124.373
	LPL	1.040	0.005	Up. Lim.	115.787	113.790	119.939	0.016	1.010	0.005	94	121.385	118.928	126.615
cal05	MOM	0.850	0.008		-212.914	-214.713	-209.838	0.045	1.210	0.108		-196.063	-198.435	-191.935
	MSE	0.831	0.007	Up. Lim.	-214.668	-217.040	-210.541	0.051	0.725	0.031	Low Lim.	-196.443	-199.375	-191.251
	LPL	0.849	0.011	Up. Lim.	-213.667	-216.040	-209.540	0.052	0.979	0.036	59	-196.518	-199.450	-191.326
cal06	MOM	1.050	0.002		650.169	647.457	653.210	0.003	1.060	0.002		649.272	645.671	653.342
	MSE	0.872	0.001	Low Lim.	648.102	644.501	652.172	0.013	0.807	0.001	Low Lim.	648.499	644.015	653.604
	LPL	1.030	0.001	Up. Lim.	648.319	644.718	652.389	0.022	0.992	0.001	190	650.093	645.609	655.198
cal07	MOM	0.983	0.042		-79.017	-80.946	-75.947	0.002	0.993	0.046		-80.042	-82.590	-75.925
	MSE	0.869	0.026	102	-79.962	-82.511	-75.845	0.017	0.697	0.002	12	-80.096	-83.251	-74.920
	LPL	0.973	0.040	Up. Lim.	-80.996	-83.544	-76.879	0.025	0.799	3.4E-07	104	-81.636	-84.791	-76.459
cal08	MOM	1.080	0.018		134.924	132.908	137.990	0.036	1.860	0.229		187.611	184.946	191.722
	MSE	0.999	0.003	Low Lim.	130.356	127.690	134.466	0.036	0.501	1.0E-06	Low Lim.	175.349	172.046	180.516
	LPL	1.090	0.028	Up. Lim.	136.149	133.483	140.259	0.036	1.540	0.139	46	187.865	184.562	193.031
cal09	MOM	0.802	0.002		100.959	96.580	103.973	0.253	1.070	0.035		171.333	165.500	175.357
	MSE	0.788	0.001	Up. Lim.	96.460	90.626	100.483	0.280	0.809	0.018	Low Lim.	170.929	163.643	175.964
	LPL	0.803	0.003	Up. Lim.	100.924	95.090	104.947	0.274	1.030	0.024	773	168.214	160.928	173.249

495

Table 2 (continued)

Seq.	Model	Without background						With background						
		p1	p2	p3	AICc	BIC		μ_M	p1	p2	p3	AICc	BIC	
cal10	MOM	0.725	0.031		183.112	177.731	186.119	1.2E-05	0.725	0.031		182.106	174.934	186.118
	MSE	0.598	1.0E-06	1260	193.267	186.095	197.279	1.2E-05	0.598	1.0E-06	1260	192.261	183.298	197.279
	LPL	0.630	4.1E-06	1790	194.930	187.757	198.942	1.2E-05	0.630	3.2E-06	1790	193.924	184.961	198.942
cal11	MOM	0.673	1.0E-06		-465.255	-467.568	-462.200	0.116	1.570	0.163		-431.516	-434.581	-427.425
	MSE	0.664	1.0E-06	Up. Lim.	-466.999	-470.064	-462.908	0.118	0.464	1.0E-06	Low Lim.	-432.487	-436.296	-427.350
	LPL	0.673	3.2E-05	Up. Lim.	-466.347	-469.413	-462.256	0.119	0.626	3.4E-06	2	-431.204	-435.012	-426.067
cal12	MOM	0.962	0.009		15.087	12.837	18.143	0.055	1.450	0.090		44.980	42.000	49.075
	MSE	0.945	0.010	Up. Lim.	13.480	10.500	17.575	0.060	0.552	0.001	Low Lim.	38.095	34.394	43.238
	LPL	0.961	0.011	Up. Lim.	15.495	12.515	19.590	0.053	1.340	0.064	4.0E+06	43.005	39.305	48.148
cal13	MOM	1.150	0.066		529.989	527.020	533.024	0.016	1.260	0.110		533.566	529.620	537.625
	MSE	0.838	0.020	1	523.966	520.019	528.025	0.027	0.723	0.020	Low Lim.	535.597	530.679	540.686
	LPL	1.130	0.051	Up. Lim.	525.881	521.934	529.940	0.027	1.110	0.054	197	530.123	525.205	535.212
cal14	MOM	1.190	0.037		206.291	204.603	209.372	0.002	1.220	0.045		205.622	203.401	209.759
	MSE	0.794	0.005	1	204.637	202.415	208.774	0.008	0.703	0.003	Low Lim.	209.143	206.401	214.349
	LPL	1.150	0.025	Up. Lim.	203.749	201.527	207.886	0.010	0.945	0.008	40	206.945	204.203	212.152
cal15	MOM	0.951	0.165		2646.788	2641.143	2649.794	0.297	1.290	1.070		2702.098	2694.573	2706.108
	MSE	0.922	0.160	Up. Lim.	2645.082	2637.557	2649.092	0.411	0.462	1.0E-06	12	2782.900	2773.496	2787.915
	LPL	0.925	0.110	Up. Lim.	2634.159	2626.634	2638.169	0.423	0.547	0.002	26	2783.754	2774.350	2788.768
cal16	MOM	0.745	0.003		-365.589	-369.816	-362.574	0.308	1.740	1.150		-194.518	-200.148	-190.492
	MSE	0.732	0.002	Up. Lim.	-370.059	-375.689	-366.033	0.313	0.624	0.240	Low Lim.	-188.049	-195.080	-183.010
	LPL	0.746	0.008	Up. Lim.	-366.548	-372.178	-362.523	0.317	0.489	8.5E-07	6	-184.342	-191.373	-179.303
cal17	MOM	0.994	0.025		-86.333	-87.770	-83.237	0.002	1.010	0.033		-87.325	-89.208	-83.164
	MSE	0.895	0.014	55	-87.350	-89.233	-83.189	0.012	0.745	1.0E-06	7	-87.635	-89.946	-82.391
	LPL	0.976	0.018	Up. Lim.	-87.793	-89.676	-83.632	0.016	0.919	6.2E-06	253	-88.705	-91.016	-83.461
cal18	MOM	0.645	1.0E-06		-377.158	-378.803	-374.074	0.057	0.865	1.0E-06		-368.524	-370.688	-364.383
	MSE	0.639	1.0E-06	Up. Lim.	-378.495	-380.659	-374.354	0.057	0.850	1.0E-06	Up Lim.	-369.634	-372.302	-364.421
	LPL	0.645	1.4E-06	Up. Lim.	-378.249	-380.414	-374.109	0.065	0.856	2.8E-06	711	-369.560	-372.229	-364.347

Table 2 (continued)

Seq.	Model	Without background						With background						
		p1	p2	p3	AICc	BIC		μ	p1	p2	p3	AICc	BIC	
ita01	MOM	0.861	0.038		-202.562	-203.668	-199.444	7.5E-07	0.861	0.038		-203.644	-205.076	-199.444
	MSE	0.745	0.005	1260	-203.462	-204.895	-199.262	0.012	0.654	1.0E-06	60	-204.180	-205.917	-198.877
	LPL	0.829	0.026	6810	-203.817	-205.249	-199.617	0.019	0.730	7.5E-06	194	-204.178	-205.915	-198.875
ita02	MOM	0.908	0.005		-127.406	-128.856	-124.311	0.025	1.210	0.122		-120.859	-122.758	-116.699
	MSE	0.883	0.004	Up. Lim.	-128.684	-130.583	-124.524	0.032	0.559	1.0E-06	3	-115.481	-117.813	-110.239
	LPL	0.890	1.3E-07	Up. Lim.	-128.714	-130.614	-124.554	0.032	0.730	2.3E-06	19	-115.416	-117.748	-110.174
ita03	MOM	0.771	1.0E-06		-243.151	-244.317	-240.037	0.034	0.987	0.007		-236.613	-238.127	-232.420
	MSE	0.759	1.0E-06	Up. Lim.	-244.665	-246.180	-240.473	0.045	0.671	1.0E-06	2	-237.386	-239.228	-232.095
	LPL	0.771	8.1E-07	Up. Lim.	-244.237	-245.752	-240.045	0.047	0.753	4.6E-06	10	-237.106	-238.949	-231.815
ita04	MOM	1.340	4.050		497.048	492.843	500.064	5.8E-06	1.340	4.050		496.038	490.436	500.064
	MSE	0.572	0.048	42	511.769	506.167	515.795	0.046	0.455	1.0E-06	31	526.315	519.320	531.354
	LPL	0.750	0.083	330	491.742	486.141	495.768	0.050	0.551	0.008	71	523.765	516.770	528.804
ita05	MOM	0.960	0.061		-126.564	-128.537	-123.496	1.3E-06	0.960	0.061		-127.610	-130.218	-123.496
	MSE	0.720	0.007	70	-124.837	-127.444	-120.723	0.023	0.515	1.0E-06	15	-115.103	-118.333	-109.931
	LPL	0.899	0.026	3430	-128.744	-131.352	-124.631	0.025	0.623	1.2E-06	48	-112.481	-115.712	-107.310
ita06	MOM	0.974	0.331		-199.462	-201.270	-196.386	0.019	1.110	0.587		-199.809	-202.195	-195.683
	MSE	0.951	0.338	Up. Lim.	-200.571	-202.956	-196.444	0.007	0.998	0.356	2	-201.228	-204.177	-196.037
	LPL	0.974	0.358	Up. Lim.	-199.165	-201.551	-195.039	0.019	1.090	0.508	6.1E+06	-199.329	-202.278	-194.138

499

Table 3

500

Counts of preferred models (including background)

Model	California			Italy			California+Italy		
	AICc	BIC		AICc	BIC		AICc	BIC	
MOM	7	10	1	2	3	0	9	13	1
MSE	2	2	7	1	1	1	3	3	8
LPL	9	6	10	3	2	5	12	8	15
MOM	8	11	6	2	3	0	10	14	6
LPL	10	7	12	4	3	6	14	10	18
MOM	13	13	5	3	3	1	16	16	6
MSE	5	5	13	3	3	5	8	8	18
MSE		7			1			8	
LPL		11			5			16	
LPL	14	13	14	3	3	6	17	16	20
DRL	4	5	4	3	3	0	7	8	4

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Table 4

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Parameters values and goodness-of-fit scores

Seq.	Model	μ	p1	p2	p3	AICc	BIC	
cal01	LPL	0.011	1.130	0.073	9.1E+06	212.352	208.074	217.467
	DRL	1.0E-07		0.025	2.4E+08	205.206	202.616	208.251
cal02	LPL	0.006	1.190	1.150	5.8E+06	-138.479	-141.274	-133.276
	DRL	4.5E-09		0.545	4.4E+09	-144.569	-146.288	-141.489
cal03	LPL	0.014	0.933	0.004	373	-40.007	-42.209	-34.753
	DRL	0.007		0.011	1390	-37.499	-38.872	-34.399
cal04	LPL	0.016	1.010	0.005	94	121.385	118.928	126.615
	DRL	0.008		0.002	1340	116.246	114.725	119.337
cal05	LPL	0.052	0.979	0.036	59	-196.518	-199.450	-191.326
	DRL	0.047		0.029	223	-196.811	-198.610	-193.735
cal06	LPL	0.022	0.992	0.001	190	650.093	645.609	655.198
	DRL	2.0E-06		0.001	1.1E+07	646.748	644.036	649.789
cal07	LPL	0.025	0.799	3.4E-07	104	-81.636	-84.791	-76.459
	DRL	0.004		0.050	3890	-79.000	-80.929	-75.930
cal08	LPL	0.036	1.540	0.139	46	187.865	184.562	193.031
	DRL	0.030		4.5E-08	454	140.317	138.301	143.383
cal09	LPL	0.274	1.030	0.024	773	168.214	160.928	173.249
	DRL	0.290		0.021	173	170.673	166.294	173.687
cal10	LPL	1.2E-05	0.630	3.2E-06	1790	193.924	184.961	198.942
	DRL	0.346		1.760	597	115.124	109.743	118.131
cal11	LPL	0.119	0.626	3.4E-06	2	-431.204	-435.012	-426.067
	DRL	0.114		0.007	61	-434.675	-436.988	-431.621
cal12	LPL	0.053	1.340	0.064	4.0E+06	43.005	39.305	48.148
	DRL	0.043		0.009	346	28.513	26.263	31.569
cal13	LPL	0.027	1.110	0.054	197	530.123	525.205	535.212
	DRL	1.9E-07		0.016	1.6E+08	514.042	511.073	517.077
cal14	LPL	0.010	0.945	0.008	40	206.945	204.203	212.152
	DRL	3.1E-07		0.004	4.1E+07	194.681	192.993	197.762
cal15	LPL	0.423	0.547	0.002	26	2783.754	2774.350	2788.768
	DRL	0.265		0.200	761	2681.258	2675.612	2684.264
cal16	LPL	0.317	0.489	8.5E-07	6	-184.342	-191.373	-179.303
	DRL	0.307		0.077	147	-219.133	-223.360	-216.118
cal17	LPL	0.016	0.919	6.2E-06	253	-88.705	-91.016	-83.461
	DRL	0.003		0.026	4530	-86.282	-87.719	-83.186
cal18	LPL	0.065	0.856	2.8E-06	711	-369.560	-372.229	-364.347
	DRL	0.075		0.013	70	-371.459	-373.104	-368.375

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504

Table 4 (continued)

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Seq.	Model	μ	p1	p2	p3	AICc	BIC	
ita01	LPL	0.019	0.730	7.5E-06	194	-204.178	-205.915	-198.875
	DRL	0.014		0.223	751	-203.952	-205.058	-200.833
ita02	LPL	0.032	0.730	2.3E-06	19	-115.416	-117.748	-110.174
	DRL	0.026		0.019	370	-121.550	-122.999	-118.455
ita03	LPL	0.047	0.753	4.6E-06	10	-237.106	-238.949	-231.815
	DRL	0.043		0.009	121	-236.343	-237.509	-233.229
ita04	LPL	0.050	0.551	0.008	71	523.765	516.770	528.804
	DRL	2.2E-08		0.517	4.4E+09	467.306	463.101	470.322
ita05	LPL	0.025	0.623	1.2E-06	48	-112.481	-115.712	-107.310
	DRL	0.007		0.097	2820	-126.766	-128.739	-123.698
ita06	LPL	0.019	1.090	0.508	6.1E+06	-199.329	-202.278	-194.138
	DRL	0.011		0.365	1760	-199.197	-201.005	-196.121

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Table 5

508

Counts of preferred models using different starting times T_s

509

Model	$T_s=t_1$			$T_s=0.007$			$T_s=0.042$			$T_s=1$		
	AICc	BIC		AICc	BIC		AICc	BIC		AICc	BIC	
MOM	9	13	1	10	13	1	11	14	2	20	21	4
MSE	3	3	8	4	3	8	3	3	4	0	0	2
LPL	12	8	15	10	8	15	10	7	18	4	3	18
LPL	17	16	20	17	16	22	18	16	22	15	12	24
DRL	7	8	4	7	8	2	6	8	2	9	12	0

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Table 6

512

MOM parameter estimates using different starting times T_s

513

<i>Seq.</i>	$T_s=t_1$			$T_s=1$		
	μ	p	c	μ	p	c
cal01	0.012	1.160	0.082	0.003	1.030	0.000
cal02	0.008	1.290	1.780	0.005	1.170	0.100
cal03	0.004	1.000	0.012	0.005	0.988	0.006
cal04	0.010	1.130	0.010	0.015	2.130	9.890
cal05	0.045	1.210	0.108	0.049	1.450	1.140
cal06	0.003	1.060	0.002	0.013	1.340	2.350
cal07	0.002	0.993	0.046	0.009	1.140	1.190
cal08	0.036	1.860	0.229	0.036	1.750	0.000
cal09	0.253	1.070	0.035	0.273	1.220	0.830
cal10	0.000	0.725	0.031	0.000	0.817	2.790
cal11	0.116	1.570	0.163	0.119	4.000	5.120
cal12	0.055	1.450	0.090	0.056	1.500	0.000
cal13	0.016	1.260	0.110	0.020	1.330	0.000
cal14	0.002	1.220	0.045	0.007	1.780	2.000
cal15	0.297	1.290	1.070	0.395	2.180	12.600
cal16	0.308	1.740	1.150	0.315	3.130	9.170
cal17	0.002	1.010	0.033	0.011	1.290	1.450
cal18	0.057	0.865	0.000	0.055	0.906	1.680
ita01	0.000	0.861	0.038	0.004	0.942	0.370
ita02	0.025	1.210	0.122	0.032	2.400	6.090
ita03	0.034	0.987	0.007	0.043	1.330	0.000
ita04	0.000	1.340	4.050	0.032	1.870	18.600
ita05	0.000	0.960	0.061	0.024	2.790	31.000
ita06	0.019	1.110	0.587	0.029	1.210	0.000

514

514

Figure captions

515

516 Figure 1. Number of sequences of Southern California for which each decay model is preferable
517 according to AIC_c (a), BIC (b) and maximum log-likelihood (c) as a function of the duration of the
518 sequence, when the background rate is neglected.

519

520 Figure 2. Number of sequences of Southern California for which each decay model is preferable
521 according to AIC_c (a), BIC (b) and maximum log-likelihood (c) as a function of the duration of the
522 sequence, when the background rate is modeled.

523

524 Figure 3. Number of sequences of Italy for which each decay model is preferable according to AIC_c
525 as a function of the duration of the sequence, when the background rate is neglected (a) or modeled
526 (b).

527

528 Figure 4. Number of sequences of Italy and Souther California for which the LPL (with
529 background) or the DRL models are preferable according to AIC_c (a) or BIC (b) and maximum log-
530 likelihood (c) as a function of the duration of the sequence.

531

532 Figure 5. Observed (symbols) and predicted (lines) aftershock rates as a function of time elapsed
533 after the mainshock when the background rate is modeled for two sequences showing the clear
534 emergence of the exponential decay at relatively long times.

535

536 Figure 6. Differences between observed and predicted cumulative number of aftershocks as a
537 function of time elapsed after the mainshock for the same two sequences of Fig. 5.

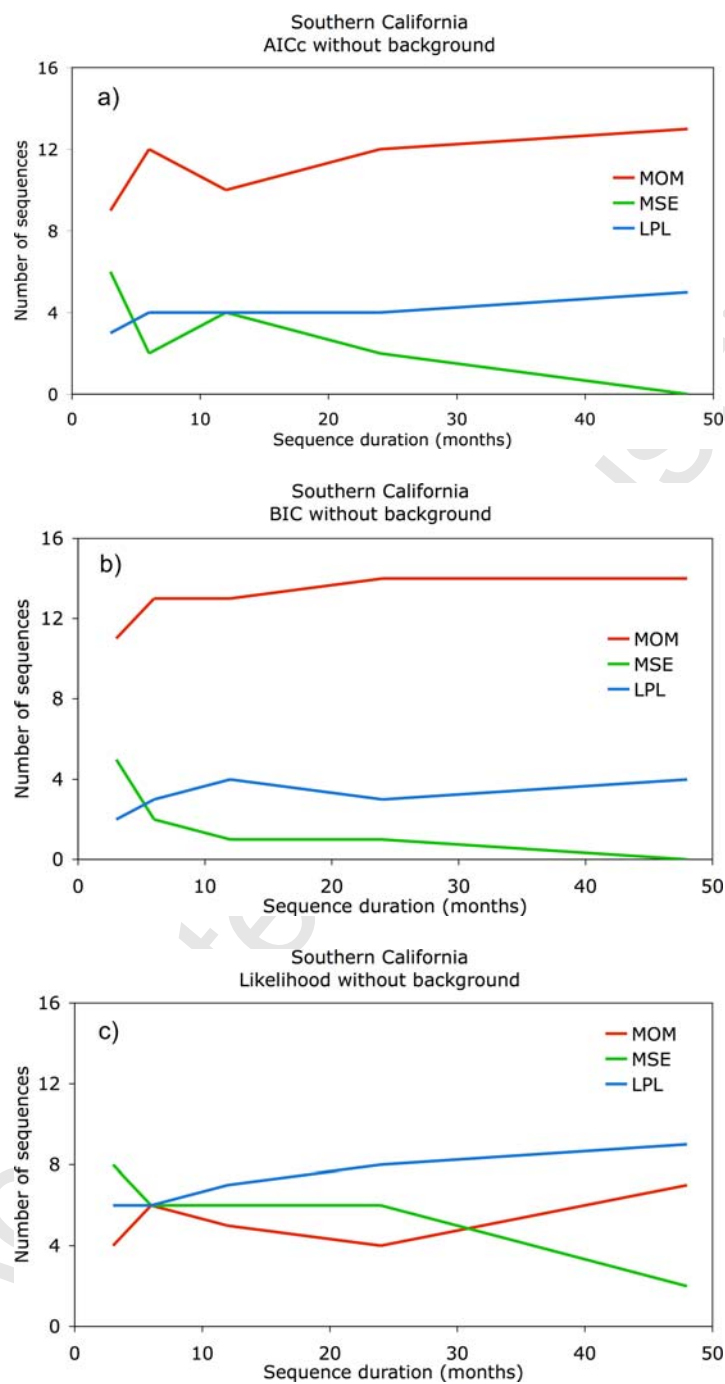
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539 Figure 7. Observed (symbols) and predicted (lines) aftershock rates (a), and differences between
540 observed and predicted cumulative number of aftershocks (b) as a function of time elapsed after the
541 mainshock when the background rate is modeled, for a sequence characterized by an high power
542 law exponent ($p=1.86$ for the MOM and $q=1.54$ for the LPL).
543

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Figure 1



544

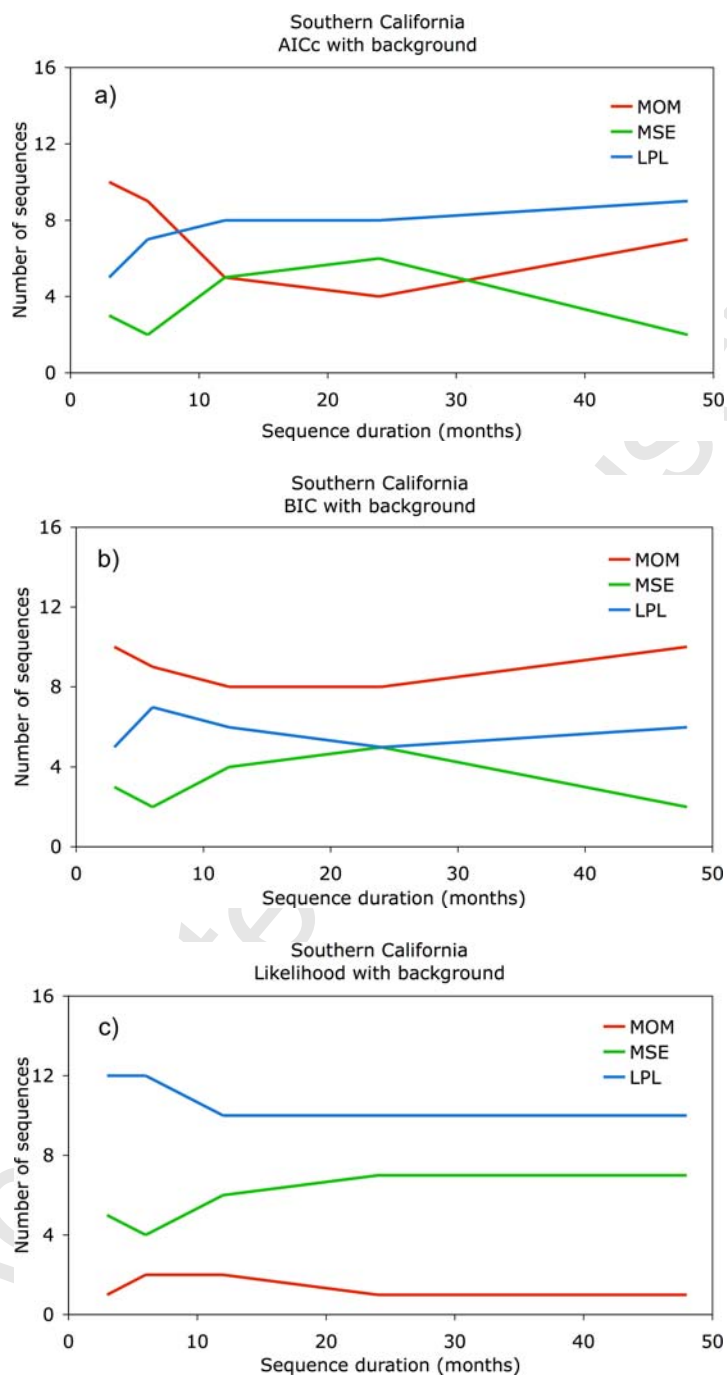
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Figure 2



548

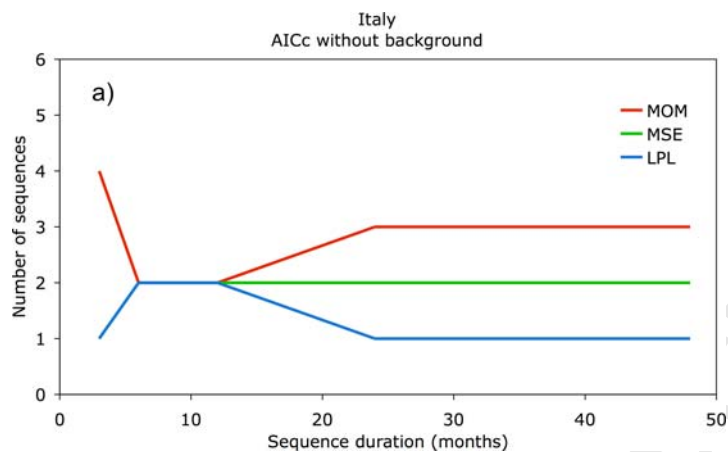
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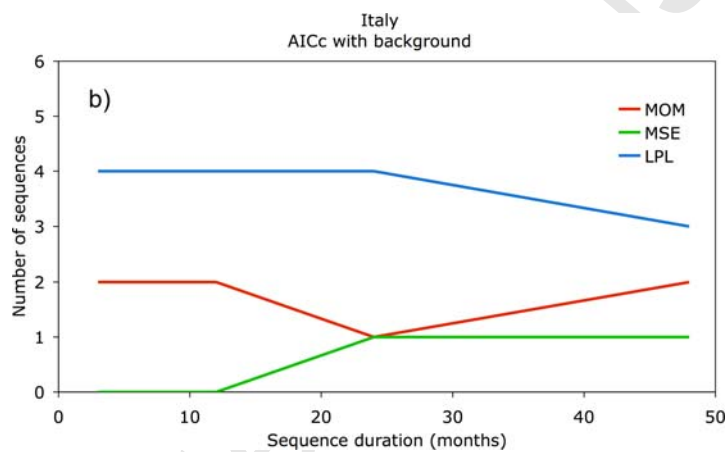
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Figure 3



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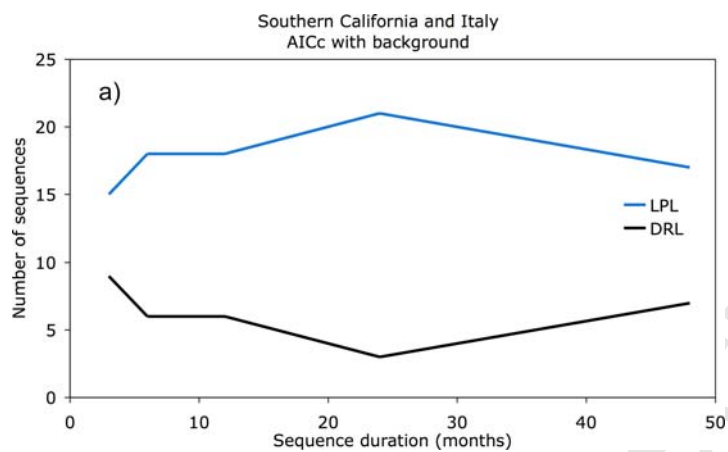


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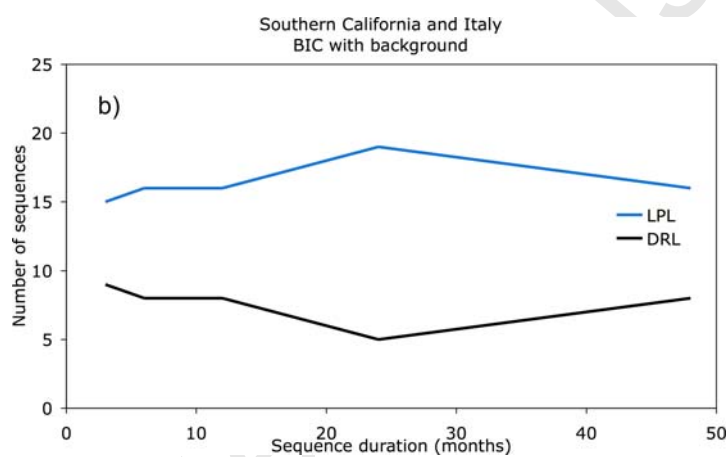
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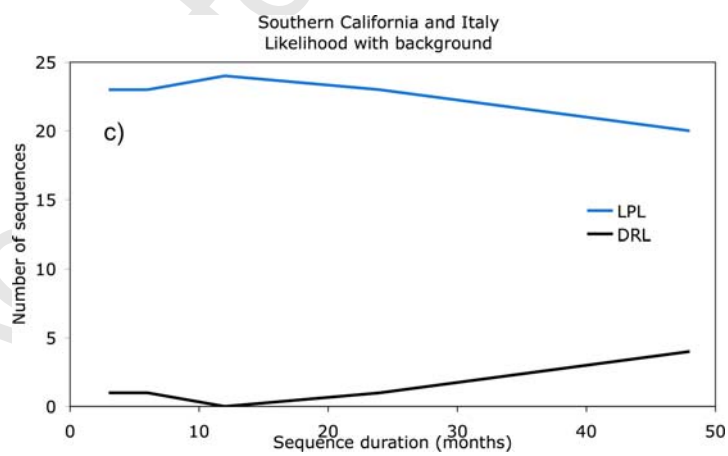
Figure 4



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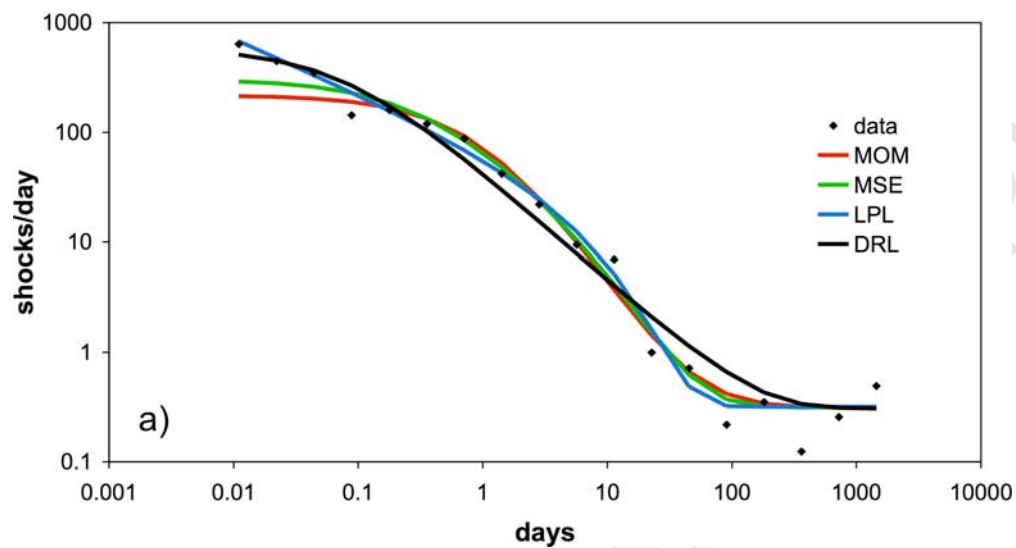


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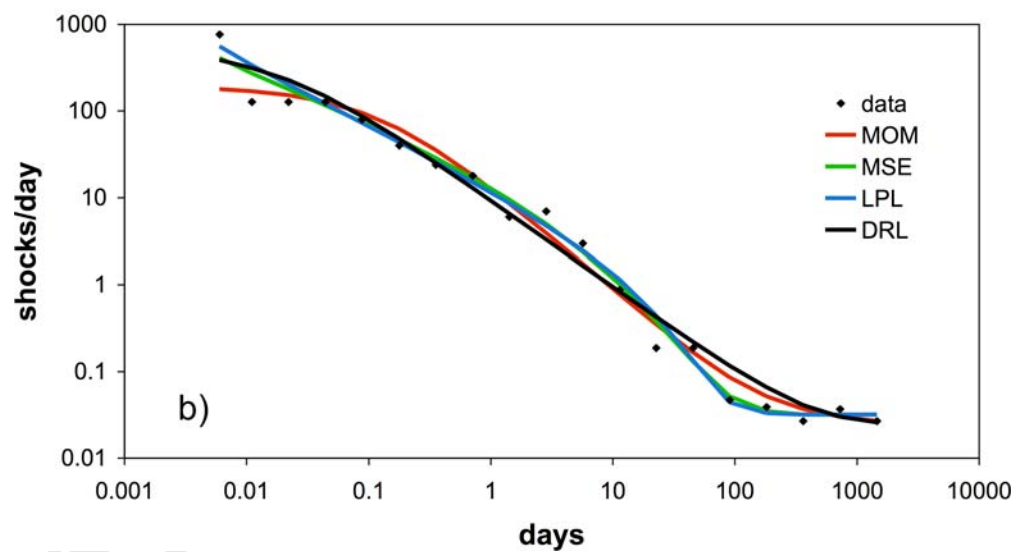
Figure 5

Sequence cal16



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Sequence ita02



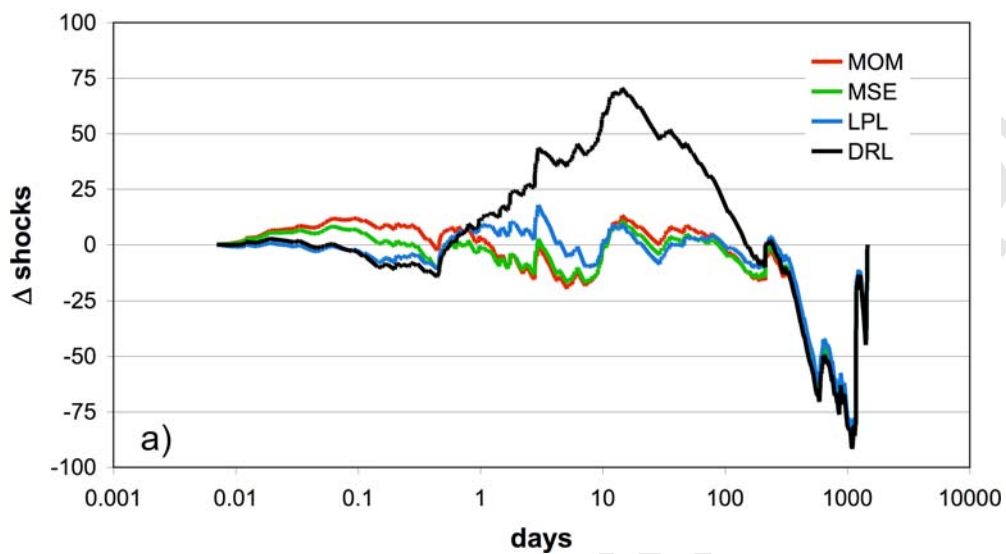
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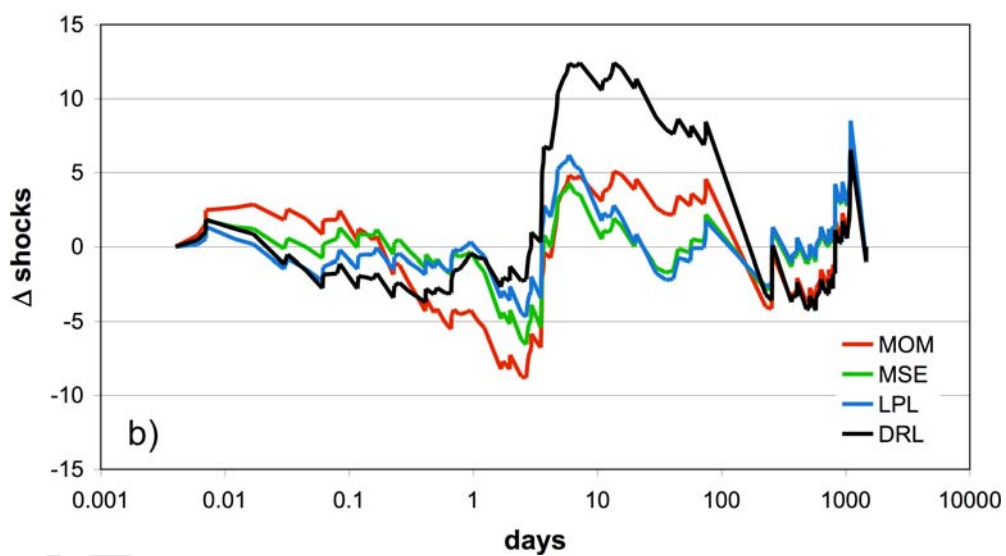
Figure 6

Sequence cal16



562

Sequence ita02

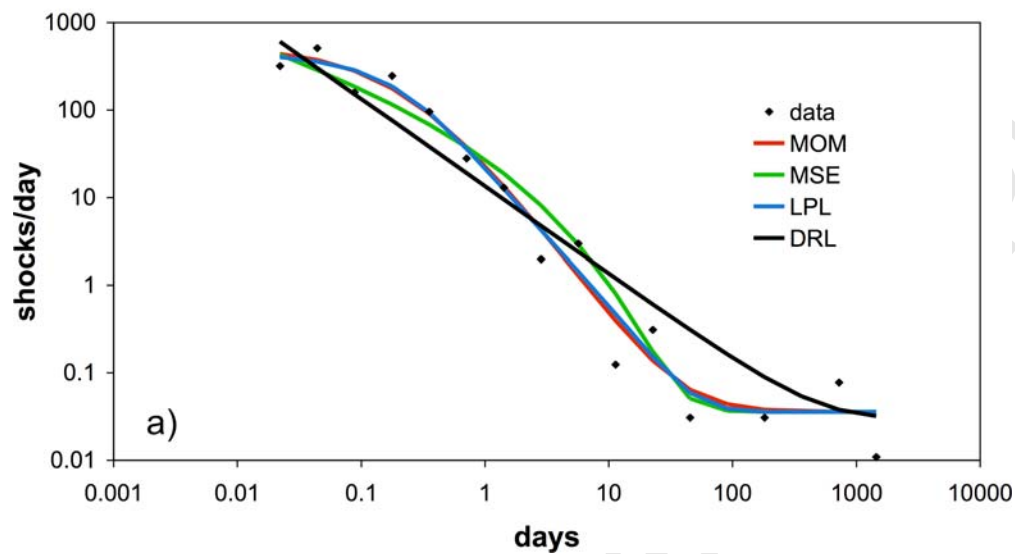


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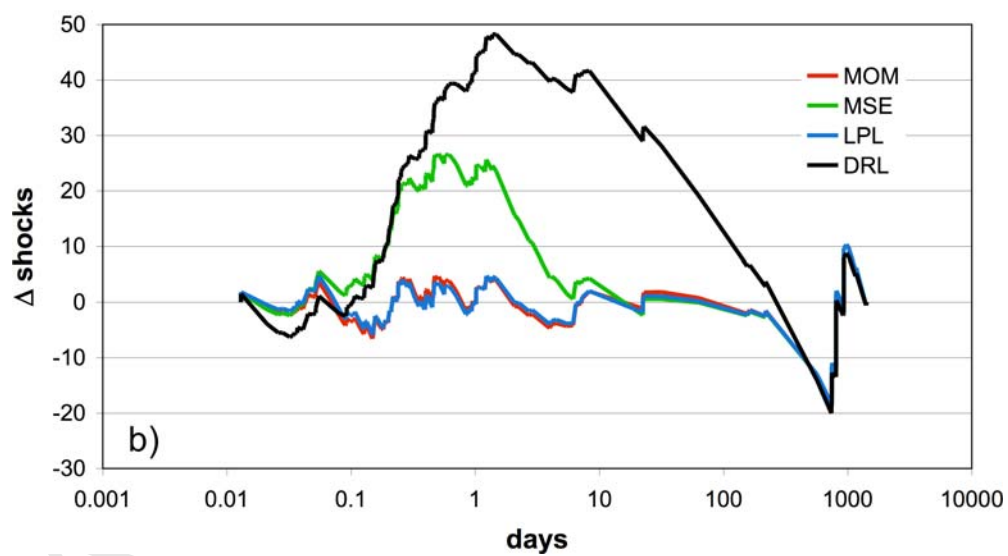
Figure 7

Sequence cal08



565

Sequence cal08



566