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Abstract: In this study a simple hybrid genetic algorithm using a procedure dedicated to the minimum sum coloring problem (MSCP) is proposed. This algorithm combines an original crossover based on the union of independent sets proposed in [1] and local search techniques dedicated to the minimum sum coloring problem for mutation. We call this hybrid method a memetic algorithm for the MSCP.

Keywords: Graph coloring problem, memetic algorithm, local search.

1. Introduction

The minimum sum coloring problem (MSCP) is a problem derived from the graph coloring problem (GCP). It consists in finding a *feasible coloring* for an undirected graph $G = (V, E)$, using natural numbers $1, \dots, k$, such that the sum of colors is minimum. A *feasible coloring* of G corresponds to a partition X of V into k independent subsets called *color classes* : $X_1, \dots, X_k$, where the vertices in X_i are colored with color i. We denote x_i the number of vertices in X_i. The sum of colors associated to X can be written as follows :

$$Sum(X) = 1.x_1 + 2.x_2 + \dots + k.x_k \tag{1}$$

The objective of the MSCP is to compute a feasible coloring of the graph G as described before, such that the sum of colors is minimum. This optimal value is called the *chromatic sum* of G and is denoted $\Sigma(G)$. The smallest number of colors required in an optimal solution for the MSCP is called the *strength* of the graph G, and denoted $s(G)$.

The MSCP was formulated by Kubicka and Schwenk, who have proved its NP-completeness [2]. The MSCP is often related to scheduling problems. A basic example could be the following. Consider a distributed system where a set of jobs have to be executed by several processors and some jobs are in conflict because of sharing the same resource and thus cannot be executed at the same time. The objective of the MSCP is to minimize the average time for jobs to be completed. There are some papers in the literature that deal with theoretical upper or/and lower bounds of the optimal solution for some specific graphs. Nevertheless very few numerical results have been reported for general graphs. In [4] we proposed a greedy algorithm to solve the MSCP and a method based on partial graphs to compute a general lower bound for the MSCP [5]. In this abstract, we are interested to develop a memetic algorithm, that combines evolutionary algorithm with local search techniques.

2. Local search

For any best coloring for the MSCP, the following properties related to the partition of its vertices set [6] hold : let X be any feasible coloring using colors $1, 2, \ldots, k$.

1. for every two colors $i < j$ we have $x_i \geq x_j$.
2. for every two colors $i < j$ and for every $v \in X_j$ there exists $w \in X_i$ adjacent to v.

According to the first property, sort the color classes by decreasing cardinalities improves the sum of colors. So, before evaluating each solution, we sort the color classes.
According to the second property, we define a *declining process* : for each vertex $v \in V$ with color i, we try to move it to a different color class X_j, such that there is no adjacent vertex of v in X_j and $x_j \geq x_i$. During this process, if there is no possible movement then we perform a *jump*. One jump consists in randomly choosing a vertex $v \in V$, and changing its color i to j. The new color j is the smallest available color for v different to i. If there is no such a color in $\{1, 2, \ldots, k\}$ then a new color is added.

Our local search algorithm LS alternatively applies a declining process and a jump Algorithm 1. The stop condition of the LS is a bound on the number of movements.

Algorithm 1: Local search

```
Input : X
Output : Best
begin
    Best := X;
    while NOT(stopping condition) do
        Step1 : declining process(X);
        if No movement in Step 1 then
            jump(X);
        else
            if Sum(X)<Sum(Best) then
                Best :=X;
end
```

3. Memetic algorithm

To solve the MSCP we propose a memetic algorithm MA. The MA is a combination of an evolutionary algorithm and local search LS techniques. This combination has been shown to be effective for Team Orienteering Problem (TOP) in [7] and for GCP in [8]. At start, a small part of the population is created with our greedy algorithm MRLF [4] and the remainder is generated randomly. At each iteration a couple of parents are chosen among the population using the Binary Tournament. The union of independent sets proposed in [1] is used as crossover operator to produce a child chromosome. New chromosomes are evaluated by their sum of colors. They are then inserted into the current population such that the size of the population is maintained. The population is a list of chromosomes sorted with respect to two criteria : the sum of colors and the number of colors. A child chromosome has a probability pm of being mutated, using a LS technique. The stop condition of the MA is a bound on the number of iterations without improvement of the population. At the end of the search the chromosome at the head of the population is reported as the best solution.

4. Conclusion

In this abstract we have proposed local search techniques dedicated to the MSCP. We have presented a memetic algorithm for the sum coloring problem. The proposed algorithm integrates the following original features. First, we used an adaptive crossover operator. Second, we used as local search a method based on properties of the MSCP.

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