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# Unit Root Testing Based on BLUS Residuals

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## Abstract

This paper introduces an efficient version of the Dickey-Fuller unit root test, which is based on BLUS residuals. Simulated critical values are provided, along with power simulation and an empirical example.

**Keywords:** BLUS Residuals; Dickey-Fuller BLUS unit root test.

**JEL classification:** C12, C15, C22.

## 1 Introduction

There are many more powerful tests than the original Dickey and Fuller (1979) unit root test (DF). Although efficient versions of the DF test have been proposed, see Elliott *et al.* (1996) (ERS, hereafter), they may appear arbitrary or unjustified since they depend on arbitrary generalized least squares (GLS) initial estimation. Also, DF-type unit root tests, that are based on recursive LS, have been criticized for lack of efficiency in general, see Sul *et al.* (2005). Further, it appears that no one has employed BLUS residuals in unit root testing, although these residuals have been recently recommended as efficient by Magnus and Sinha (2005). It is of interest to employ BLUS residuals from first-step de-meaning or de-trending into second-step unit root testing. The resulting test, to be called DF-BLUS, is expected to have good power properties, resulting from the efficiency of BLUS residuals. This paper introduces the new, two-step DF-BLUS unit root test, providing simulated critical values, power experiment, and an application to the data of Nelson and Plosser (1982). The paper is organized as follows: Section 2 discusses BLUS residuals calculation and implemented unit root testing, while Section 3 provides Monte Carlo design and results, their discussion, and an empirical example. Finally, Section 4 concludes.

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## 2 BLUS Residuals and Unit Root Testing

Let  $y$  ( $T \times 1$ ) be generated via

$$y = W\delta + u, \quad (1)$$

where  $W$  ( $T \times k$ ) ( $T > k$ ) is a matrix of deterministics,<sup>1</sup>  $u$  ( $T \times 1$ ) is the error, and  $\delta$  ( $k \times 1$ ) the parameter vector. The error vector  $u$  need not be white; it is assumed to be autoregressive (AR) with one possible unit root. In fact, if the assumed AR order is  $p$ , then  $T > k + p$  is required for the DF-BLUS test to be calculable. The computationally most economical approach for calculating BLUS residuals is given in Theil (1968). Conventionally,  $k$  BLUS residuals are not identified; they are chosen to be the first  $k$  ones here. Let  $\hat{u}$  be the LS residual vector from Eq. (1) and partition  $\hat{u} = [\hat{u}'_0, \hat{u}'_1]'$ , with  $\hat{u}_0$  ( $k \times 1$ ) and  $\hat{u}_1$  ( $(T - k) \times 1$ ), and  $W = [W'_0 : W'_1]'$ , with  $W_0$  ( $k \times k$ ) and  $W_1$  ( $(T - k) \times k$ ). Then,  $\tilde{u}_1$  ( $(T - k) \times 1$ ), the BLUS residual vector, is given by

$$\tilde{u}_1 = \hat{u}_1 - W_1 W_0^{-1} \left[ \sum_1^k \left( \frac{d_i}{1 + d_i} \right) q_i q_i' \right] \hat{u}_0, \quad (2)$$

where  $d_i^2$  and  $q_i$  are the eigenvalues and eigenvectors of  $W_0(W'W)^{-1}W'_0$ , respectively.<sup>2</sup> Let  $\tilde{u}_{1t}$  be the  $t$ -th element of  $\tilde{u}_1$ , the proposed DF-BLUS unit root test is based on the autoregression

$$\Delta \tilde{u}_{1t} = \phi \tilde{u}_{1t-1} + \sum_{j=1}^l \gamma_j \Delta \tilde{u}_{1t-j} + \varepsilon_t \quad t = p + 1, \dots, T - k, \quad (3)$$

with  $l = p - 1 \geq 0$ ,  $\gamma_j = -\sum_{k=j+1}^p \rho_k$ , and  $\phi = \rho - 1$  with  $\rho = \sum_{j=1}^p \rho_j$ , assuming  $u_t$  (the  $t$ -th element of  $u$ ) follows  $u_t = \sum_{j=1}^p \rho_j u_{t-j} + \varepsilon_t$ .<sup>3</sup> Unit root testing, in this context, requires testing  $H_0 : \phi = 0$ .<sup>4</sup> Note that the BLUS residual  $\tilde{u}_1$  vector is invariant with respect to  $\delta$ . To prove this let  $\hat{\delta}$  be the OLS estimator. Thus,

$$W\delta + u = W\hat{\delta} + \hat{u} \implies W(\delta - \hat{\delta}) + u = \hat{u}, \quad (4)$$

so that tests that depend on  $\hat{u}$  are invariant with respect to  $\delta$ , they depend on  $W$  and the sampling error  $(\delta - \hat{\delta})$ . It is well known that  $\hat{\delta} - \delta = (W'W)^{-1}W'u$ . The choice of  $\delta$  does not matter in Monte Carlo simulations of both size and

<sup>1</sup>Stochastic variables can be used in testing for no cointegration. However, this paper focuses on unit root testing. Matrix  $W$  contains no lags of  $y$  for identifiability. This DGP guarantees that the  $\delta$  parameters do not change their meaning under the null and alternative hypotheses.

<sup>2</sup>The original and computationally far more expensive approach of Theil (1965) is further clarified in Chow (1976).

<sup>3</sup>If  $l = 0$  ( $p = 1$ ), no sum enters in Eq. (3).

<sup>4</sup>Note that the original DF test can be derived from an autoregression, like Eq. (3), on  $\hat{u}_t$  (element of the LS residual vector,  $\hat{u}$ ), augmented with the columns of  $W$  as regressors.

power. For the BLUS residuals, partition  $u$  so that  $u = [u'_0, u'_1]'$ , in line with the partition of  $\hat{u}$ . From Eq. (2), it follows:

$$W_0(\delta - \hat{\delta}) + u_0 = \hat{u}_0, \quad (5)$$

$$W_1(\delta - \hat{\delta}) + u_1 = \hat{u}_1. \quad (6)$$

From the definition of BLUS residuals, it follows (using Eq. (5) and Eq. (6)) that:

$$\begin{aligned} \tilde{u}_1 &= \hat{u}_1 - W_1 W_0^{-1} \left[ \sum_1^k \left( \frac{d_i}{1 + d_i} \right) q_i q_i' \right] \hat{u}_0 = \\ & [W_1(\delta - \hat{\delta}) + u_1] - \{ W_1 W_0^{-1} \left[ \sum_1^k \left( \frac{d_i}{1 + d_i} \right) q_i q_i' \right] [W_0(\delta - \hat{\delta}) + u_0] \}. \end{aligned} \quad (7)$$

Thus, BLUS residuals only depend on the partitions of  $W$ ,  $W_0$  and  $W_1$ , and the sampling error  $(\delta - \hat{\delta})$  and not  $\delta$  alone. This invariance is irrespective of the structure of  $u$ , that is neither power nor the critical values are affected by the choice of  $\delta$ . Similar invariant methods have been employed in the derivation of the LM and DF-GLS unit root tests.

### 3 Monte Carlo and Empirical Results

{ ----- Table 1, about here ----- }

Simulations are based on the AR(1) generating mechanism

$$y_t = u_t = \rho u_{t-1} + \varepsilon_t \quad \varepsilon_t \sim iidN(0, 1) \quad t = 2, \dots, T \quad (8)$$

with  $u_1 = 0$  for  $|\rho| = 1$  and  $u_1 = \varepsilon_1 / \sqrt{1 - \rho^2}$  with  $\varepsilon_1 \sim iidN(0, 1)$  for  $|\rho| < 1$  (stationarity), in line with Schmidt and Lee (1991), Schmidt and Phillips (1992), ERS, and Elliott (1999). All unit root tests of this paper are invariant with respect to  $u_1$  under the null, hence there is no loss of generality by setting  $u_1 = 0$ . Without loss of generality,  $\delta = 0$  is set, when simulating critical values and power. This is standard practice in unit root testing literature since the tests depend on sampling errors of the form  $(\bar{\delta} - \delta)$  (where  $\bar{\delta}$  is an estimator) only. What matters is, in effect, the context of  $W$ . Table 1 presents simulated critical values for coefficient and  $t$ -ratio DF-BLUS unit root tests, based on half-a-million replications.<sup>5</sup> Only the cases with intercept only or linear trend are of interest in this paper, with typical element of  $W$ ,  $w'_t = \{1\}$  or  $w'_t = \{1, t\}$ , respectively. Note that sample sizes  $T \in \{25, 50, 100, 200, 400\}$  are employed. A small power experiment of the  $t$ -ratio DF-BLUS unit root test is undertaken to evaluate its finite sample (size-adjusted) power properties. Alternatives  $\rho \in \{.95, .9, .8, .7\}$

<sup>5</sup>One may observe that the DF-BLUS critical values stand between the DF-GLS and the DF critical values in absolute terms. Also, the DF-BLUS critical values are marginally, absolutely higher than the corresponding LM critical values.

are considered with  $T \in \{25, 50, 100, 200\}$  and half-a-million replications are also employed. Table 2 presents these power results. For comparison, one may consult Schmidt and Lee (1991), Schmidt and Phillips (1992), Elliott *et al.* (1996), and Elliott (1999). However, reported power in these studies is grossly inaccurate (due to the low number of replications) and not actually comparable to power reported in Table 2. Accurate, comparable power figures are provided in Vougas (2007 a (especially), b) who examines power of a number of unit root tests, in a model with linear trend, including the DF-GLS test, the weighted symmetric test of Pantula *et al.* (1994), as well as various likelihood ratio (LR) unit root tests. It is apparent that power differences of unit root tests (a) are non-uniform and (b) (relatively) small. The proposed DF-BLUS test has better power than all examined unit root tests (except the modified DF-GLS tests of Vougas (2007, b)) for small samples,  $T \in \{25, 50\}$ . However, this is not the case for larger samples,  $T \in \{100, 200\}$ , although the DF-BLUS test still remains among the four most powerful unit root tests. It is the efficiency of the first-step BLUS residuals (see Magnus and Sinha (2005)) that gives good power to the DF-BLUS test. It makes it preferable to many unit root tests (including the DF and Lagrange multiplier (LM) tests) and a very good competitor to optimal unit root tests.<sup>6</sup>

{ ----- Table 2, about here ----- }

Finally, the  $t$  version of the new test is applied to the data of Nelson and Plosser (1982) (NP) for comparison. Table 3 presents these results. Note that linear trend is employed in the first-step de-trending and unit root test calculations are done with  $l = 1$  for convenience. The tests indicated  $t_1$  are calculated with the usual standard errors, while the tests indicated  $t_2$  are calculated with White's (1980) heteroskedasticity-consistent standard errors. The latter standard errors maintain size robustness of unit root tests in the presence of heteroskedasticity, see Kim and Schmidt (1993). Using  $t_1$ , industrial production, employment, and unemployment appear trend stationary at 5% level,<sup>7</sup> while real GNP, real per capita GNP, and money stock appear trend stationary at 10% level. On the basis of  $t_2$ , similar trend stationarity conclusions hold, with the number of time series, that appear stationary at 5% level, increased. This is additional *prima facie* evidence in favor of the usefulness of the DF-BLUS unit root test, which is to be an additional tool towards accurate inference about the presence of a unit root in a time series.

{ ----- Table 3, about here ----- }

## 4 Conclusions

Inspired by the recent paper of Magnus and Sinha (2005), this paper introduces unit root testing applied to first-step best linear unbiased with scalar covariance

<sup>6</sup>Although asymptotic considerations are far beyond the scope of this paper, one observes test consistency since power goes to 1 as the sample increases for fixed  $\rho$ .

<sup>7</sup>Note that using the DF test, NP only find unemployment trend stationary at 5% level.

(BLUS), also called Theil's, residuals. The power of the DF-BLUS unit root test compares favorably to that of the most powerful unit root tests in the literature. Thus, BLUS residuals are helpful in efficient unit root testing.

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## Appendix A: Tables

Table 1. Critical Values for the new DF-BLUS tests										
Probability of a Lower Value										
	<i>Intercept</i>					<i>Linear Trend</i>				
<i>n</i>	1%	2.5%	5%	10%	20%	1%	2.5%	5%	10%	20%
Coefficient Tests										
25	-16.11	-13.55	-11.48	-9.30	-6.99	-20.97	-18.39	-16.29	-13.99	-11.42
50	-17.95	-14.89	-12.53	-10.1	-7.52	-24.20	-20.97	-18.34	-15.55	-12.56
100	-19.09	-15.78	-13.22	-10.6	-7.85	-26.18	-22.55	-19.66	-16.60	-13.29
200	-19.77	-16.25	-13.56	-10.8	-8.05	-27.51	-23.51	-20.44	-17.20	-13.76
400	-20.12	-16.51	-13.78	-11.0	-8.16	-28.38	-24.18	-20.93	-17.59	-14.07
<i>t</i> -ratios										
25	-3.26	-2.87	-2.58	-2.27	-1.92	-3.90	-3.49	-3.17	-2.83	-2.46
50	-3.24	-2.91	-2.65	-2.36	-2.02	-3.82	-3.48	-3.20	-2.89	-2.54
100	-3.28	-2.98	-2.72	-2.43	-2.09	-3.80	-3.49	-3.23	-2.94	-2.61
200	-3.32	-3.02	-2.76	-2.47	-2.13	-3.82	-3.52	-3.27	-2.99	-2.66
400	-3.36	-3.05	-2.79	-2.50	-2.16	-3.85	-3.55	-3.31	-3.03	-2.70

**Table 2.** Power of DF-BLUS ( $t$ ).

	.0950	.9000	.8000	.7000
	Intercept Only			
25	.0676	.0952	.1845	.3278
50	.0909	.1717	.4586	.7844
100	.1549	.4123	.9289	.9990
200	.3823	.9084	1.000	1.000
	Linear Trend			
25	.0536	.0629	.0987	.1623
50	.0632	.0988	.2533	.5215
100	.0971	.2416	.7527	.9836
200	.2267	.7165	.9995	1.000

**Table 3.** DF-BLUS applied to NP's data.

	$T$	$t_1$	$t_2$
Real GNP	62	-2.96	-3.41*
Nominal GNP	62	-2.34	-1.95
Real per Capita GNP	62	-3.00	-3.39*
Industrial Production	111	-3.29*	-3.38*
Employment	81	-3.36*	-3.19
Unemployment	81	-4.06*	-4.13*
GNP Deflator	82	-2.21	-1.47
Consumer Prices	111	-2.26	-2.18
Wages	71	-2.48	-2.15
Real Wages	71	-2.56	-2.56
Money Stock	82	-3.10	-2.80
Velocity	102	-1.61	-1.70
Interest Rate	71	0.79	0.71
Common Stock Prices	100	-2.47	-2.68

A (\*) indicates significance at 5% level.