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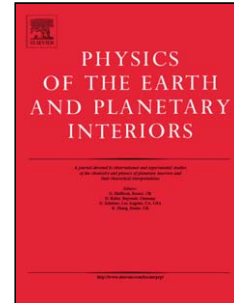
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Regional Perturbations in a Global Background Model of Glacial Isostasy

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Abstract

In Schotman and Vermeersen (2005, *Earth Planet. Sci. Lett.* 236), the effect of crustal and asthenospheric low-viscosity zones (LVZs) on geoid heights was shown, as predicted by models of glacial-isostatic adjustment (GIA). The governing equations were solved analytically in the spectral domain, which makes the method used accurate and fast. However, it does not allow for (large) lateral variations in earth stratification. As the properties of shallow LVZs can be expected to vary laterally, we have developed a finite-element model based on ABAQUS. Global (spherical-3D) finite-element models are currently not capable of providing high-resolution predictions, which we expect due to the shallowness of the LVZs. We therefore use a regional (flat-3D) model and compute geoid heights from the predicted displacements at density boundaries by solving Laplace's equation in the Fourier-transformed domain. The finite-element model is not self-gravitating, but we compare the results

with a self-gravitating spectral model under the assumption that the lack of self-gravitation is partly compensated by the lack of sphericity, and that long-wavelength differences largely cancel out when using perturbations, which are the difference between a model with and without an LVZ. We show that geoid height perturbations due to an LVZ can be computed accurately, though the accuracy deteriorates somewhat with the depth of the LVZ. Moreover, we show that horizontal rates of displacement, though not accurate for total displacements, are accurate for perturbations in the near field. We show the effect of lateral variations in the properties of the LVZ and in lithospheric thickness, and compute geoid height perturbations for Northern Europe based on a simple laterally heterogeneous model. The model is forced with a realistic ice-load history and a eustatic ocean-load history. The errors introduced by using a eustatic ocean-load history instead of realistic oceans are generally smaller than 10%, but might be critical for perturbations due to crustal LVZs.

Key words: glacial isostasy, satellite gravity, GPS measurements, finite-element modeling, rheology of the crust and upper mantle, halfspace modeling

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1 Introduction

2 The process of glacial-isostatic adjustment (GIA), in which the solid earth is
3 deformed by load changes due to variations in the volume of continental ice
4 sheets, is commonly associated with long-wavelength phenomena (spherical
5 harmonic degree < 20) as land-uplift in Scandinavia ("postglacial rebound",
6 e.g. Milne et al., 2001), part of the geoid low in Hudson Bay-area (e.g. Tamisiea

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7 et al., 2007), and variations in the position and speed of the earth's axis of ro-
8 tation (polar wander and changes in length-of-day, e.g. Mitrovica et al., 2005).
9 These phenomena are mainly influenced by the thickness of the lithosphere
10 and the viscosity of the mantle, and can to a large extent be explained using
11 spherical, radially stratified (*laterally homogeneous* or 1D) viscoelastic earth
12 models, which often use a spherical harmonic expansion to solve the governing
13 equations in the spectral (SP) domain. In the case of lateral variations, the
14 transformed differential equations in SP methods can no longer be solved sep-
15 arately for each degree due to mode coupling (Wu, 2002), which complicates
16 the use of the SP method, especially for large viscosity gradients. Recently,
17 *laterally heterogeneous* (3D) spherical earth models based on the finite-element
18 (FE) method have been developed, showing the effect of lateral variations in
19 lithospheric thickness and mantle viscosity (Wu and van der Wal, 2003; Zhong
20 et al., 2003; Wu et al., 2005; Latychev et al., 2005).

21 The upcoming GOCE satellite gravity mission, planned for launch by ESA on
22 May 15 2008, is predicted to measure the static gravity field with centimeter
23 accuracy at a resolution of 100 km or less (Visser et al., 2002). This makes
24 the detection of *high-resolution* signals as expected from shallow earth struc-
25 tures possible. In GIA, high-harmonic signatures (spherical harmonic degree
26 > 20) can be induced by shallow layers (depth < 200 km) with low viscosity
27 ($\sim 10^{18} - 10^{20}$ Pas, Schotman and Vermeersen, 2005). Crustal low-viscosity
28 zones (CLVZs) are layers in the lower crust which have viscosities that are sig-
29 nificantly smaller than the high-viscosity, for GIA studies effectively elastic,
30 upper crust and lithospheric mantle. As the presence and properties of CLVZs
31 depend strongly on thermal regime (Meissner and Kusznir, 1987; Ranalli and
32 Murphy, 1987; Watts and Burov, 2003), these can in principle not be modeled

33 using 1D-stratified earth models. In Northern Europe for example, CLVZs
34 are not expected in the cold and thick crust of the Baltic Shield, but might
35 be present in surrounding younger continental shelves (Schotman and Ver-
36 meersen, 2005). Asthenospheric LVZs (ALVZs) can probably be found below
37 the oceanic lithosphere and perhaps also below the continental shelves, but
38 not below the Baltic Shield (Schotman and Vermeersen, 2005; Steffen and
39 Kaufmann, 2005).

40 As these shallow low-viscosity layers induce high-resolution signals, the use
41 of a 3D-spherical FE model is considered to be not feasible yet. We therefore
42 test if predictions from a 3D-flat FE model, based on the commercial package
43 ABAQUS (Wu, 2004), are accurate enough for computing geoid height *per-*
44 *turbations*. Perturbations are differences between predictions from a *perturbed*
45 background model and predictions from the background model itself. Here we
46 assume that we know the background model, consisting of a laterally homo-
47 geneous *earth stratification* and *ice-load history*, from previous GIA studies.
48 As we concentrate on Northern Europe, this background model will be mainly
49 based on Fennoscandian studies (Lambeck et al., 1998; Milne et al., 2001). The
50 presence and properties of LVZs are, via thermal regime, closely related to the
51 thickness of lithosphere. We are therefore also interested in constraining both
52 properties of LVZs and the thickness of the lithosphere simultaneously. As an
53 additional constraint, apart from high-resolution satellite-gravity data, we will
54 consider the use of 3D-velocities as for example measured by the BIFROST
55 network (Milne et al., 2001), which has an accuracy of 0.8-1.3 mm/yr in the
56 vertical direction and 0.2-0.4 mm/yr in the horizontal direction (Johansson et
57 al., 2002).

58 A change in the volume of continental ice sheets leads to a global change in

59 relative sea level (RSL), which is defined as the difference between a change in
60 the ocean bottom topography and in the equipotential surface that coincides
61 with mean sea level, the geoid. This makes GIA essentially a global process.
62 In this global process, mass is conserved and redistributed gravitationally self-
63 consistently, i.e. the mass redistribution on the surface and in the interior of
64 the earth affect the gravitational potential, which in turn affects the mass
65 redistribution. This is called *self-gravitation* and leads for example to a higher
66 than *eustatic* (= uniformly distributed ice-mass equivalent) RSL near an ice
67 sheet, because the mass of the ice sheet attracts the nearby ocean. Another
68 effect is the long-wavelength effect in the solid earth: If we place an ice mass on
69 the North Pole, not only the top of the earth will depress, but also the opposite
70 side. This can be understood by considering the northward movement of the
71 center of mass due to accumulation of mass at the North Pole, the subsequent
72 flattening of the North Pole due to outward movement of the solid earth and
73 the resulting change in gravitational potential. Both effects are illustrated in
74 Figure 1 for an elliptical ice load, with a height at the center of 2500 m and
75 a radius of 8° , which is on the North Pole for 10 kyrs.

76 In Section 6 we will test if we can use a eustatic ocean-load history to force
77 the flat, non-self-gravitating FE model. The reason we do not want to include
78 self-gravitation in the flat model is that we then have to iterate several times
79 over a glacial cycle (see Section 2, and Wu (2004) for a spherical model),
80 which is very demanding in terms of CPU time. We assume that we can ne-
81 glect self-gravitation in the solid earth, because in a flat model the lack of
82 self-gravitation is partly compensated by the lack of sphericity (Amelung and
83 Wolf, 1994) and because we assume that some long-wavelength phenomena
84 largely cancel out when using perturbations. Using an axisymmetric Heavi-

85 side load, we test in Section 4 the accuracy of predictions from the FE model
 86 by comparing differences with a spherical, self-gravitating SP model with the
 87 expected accuracies of the GOCE mission and the realized accuracy of the
 88 BIFROST network. Using the RSES ice-load history (Lambeck et al., 1998)
 89 and a simple model of Northern Europe, with CLVZs in relatively young con-
 90 tinental crust and ALVZs below young continental and oceanic areas, we try
 91 to deduce in Section 5 if it is important to consider lateral heterogeneities in
 92 LVZs. We start with a short description of the theory, especially how we com-
 93 pute geoid heights from the output of the flat FE model (Section 2), followed
 94 by a description of the FE and SP model used in this study (Section 3).

95 2 Theory

96 The linearized, *elastic* equation of motion for geophysical problems in which
 97 inertia can be neglected is (e.g. Wu, 2004; Sabadini and Vermeersen, 2004,
 98 p. 5):

$$99 \quad \vec{\nabla} \cdot \vec{\sigma}_\delta - \vec{\nabla}(\vec{u} \cdot \rho_0 g_0 \vec{e}_r) - \rho_\delta g_0 \vec{e}_r - \rho_0 \vec{\nabla} \phi_\delta = 0 \quad (1)$$

100 with $\vec{\sigma}$ the stress tensor, \vec{u} the displacement vector, ρ the density, g the grav-
 101 ity acceleration, ϕ the gravity potential, and where the subscripts ('0', ' δ ')
 102 denote the *initial* and *incremental* state (Wolf, 1998), respectively. For an
 103 *incompressible* material (Poisson's ratio $\nu = 0.5$), $\rho_\delta = 0$ and the third term
 104 (*internal buoyancy* due to material compressibility) vanishes. In this case, the
 105 gravity potential ϕ_δ can be found from Laplace's equation:

$$106 \quad \vec{\nabla} \phi_\delta = 0 \quad (2)$$

107 The *viscoelastic* problem, mostly using *Maxwell rheology*, is generally solved
 108 using normal-mode techniques which involves a Laplace transformation of
 109 Eq. (1) and expanding the system of equations (1) and (2) in spherical har-
 110 monics (e.g. Wu and Peltier, 1982; Vermeersen and Sabadini, 1997). We will
 111 call these therefore *spectral* (SP) methods. The system of equations is solved
 112 from the core-mantle boundary upwards using a propagator matrix technique
 113 (Sabadini and Vermeersen, 2004, p. 18). For a surface load, boundary con-
 114 ditions at the surface of the earth are that the normal stress is equal to the
 115 load-induced pressure and that the shear stress is zero (Wu, 2004; Sabadini and
 116 Vermeersen, 2004, p. 31). The boundary conditions for the gravity potential
 117 are treated further on in Eq. (7).

118 The finite-element (FE) method solves the stiffness equation, which is by the
 119 principle of virtual work equivalent to the equation of motion (Wu, 2004):

$$120 \quad \vec{\nabla} \cdot \vec{\sigma}_\delta = 0 \quad (3)$$

121 It thus neglects the *advection of pre-stress* (second term in Eq. (1)), the effect
 122 of internal buoyancy (third term) and the effect of *self-gravitation* (fourth
 123 term). In Section 3 we will describe how we will handle these terms when
 124 using the FE method.

125 To compute *geoid heights* with the flat FE model, we solve Eq. (2) by *separa-*
 126 *tion of variables*. This leads for planar boundaries to a 2D Fourier transform
 127 of the gravity potential ϕ_δ on the horizontal (x- and y-) coordinates:

$$128 \quad \frac{\partial^2}{\partial z^2} \Phi(k_x, k_y, z) = (k_x^2 + k_y^2) \Phi(k_x, k_y, z) \quad (4)$$

129 with $k_x = 2\pi/\lambda_x$, $k_y = 2\pi/\lambda_y$, and λ_x , λ_y the wavelength in the x- and y-
 130 direction, respectively. The general solution to this differential equation is

$$131 \quad \Phi(k_x, k_y, z) = A(k_x, k_y)e^{k_z z} + B(k_x, k_y)e^{-k_z z} \quad (5)$$

132 with $k_z = \sqrt{k_x^2 + k_y^2}$.

133 Suppose we have a finite earth layer with the surface at $z = 0$ and the bottom
 134 at $z = z_1$, overlying an infinite substratum. In the substratum, $B_2(k_x, k_y)$
 135 should be zero to ensure that the potential is zero for $z \rightarrow -\infty$. At the
 136 boundary with the finite layer ($z = z_1$), we have the following boundary
 137 conditions (Wu, 2004; Cathles, 1975, p. 19):

$$138 \quad \Phi_1(z_1) = \Phi_2(z_1), \quad \partial_z \Phi_1(z_1) - \partial_z \Phi_2(z_1) = 4\pi G(\rho_2 - \rho_1)U_1 \quad (6)$$

139 with Φ_1 , Φ_2 the potential in the finite layer, respectively the substratum, ρ_1 ,
 140 ρ_2 the respective densities, and U_1 the vertical displacement at the boundary.
 141 From this we find that $B_1(k_x, k_y) = -(2\pi G/k_z)(\rho_2 - \rho_1)U_1 e^{k_z z_1}$ in the finite
 142 layer.

143 At the surface of the earth ($z = 0$) the transformed boundary conditions are
 144 (Wu, 2004; Cathles, 1975, p. 19):

$$145 \quad \Phi_0(0) = \Phi_1(0), \quad \partial_z \Phi_0(0) - \partial_z \Phi_1(0) = 4\pi G(\Gamma + \rho_1 U_0) \quad (7)$$

146 with Φ_0 the potential above the surface and Φ_1 below the surface, and Γ the
 147 transformed surface density layer (i.e. water or ice).

148 As the solution should be finite outside the earth ($A_0(k_x, k_y) = 0$), we find
 149 from Eq. (7) that $A_1(k_x, k_y) = -(2\pi G/k_z)(\Gamma + \rho_1)U_0$ in the finite layer, so

150 that the total solution becomes:

$$151 \quad \Phi(k_x, k_y, z) = -\frac{2\pi G}{k_z} \left((\Gamma + \rho_1 U_0) e^{k_z z} + (\rho_2 - \rho_1) U_1 e^{k_z(z_1 - z)} \right) \quad (8)$$

152 or, at the surface of the earth:

$$153 \quad \Phi(k_x, k_y, 0) = -\frac{2\pi G}{k_z} \left(\Gamma + \rho_1 U_0 + (\rho_2 - \rho_1) U_1 e^{k_z z_1} \right) \quad (9)$$

154 For a multilayer model with N layers, this can be generalized to:

$$155 \quad \Phi(k_x, k_y, 0) = -\frac{2\pi G}{k_z} \left(\Gamma + \sum_{n=0}^N (\rho_{n+1} - \rho_n) U_n e^{k_z z_n} \right) \quad (10)$$

156 We will refer to the individual parts in the summation as *displacement poten-*
157 *tials*.

158 To make the FE earth model self-gravitating, the displacement potential has
159 to be computed at every density contrast and applied as an extra load on the
160 corresponding boundaries, after which the FE model has to be updated, and
161 so on (see Wu, 2004, for a spherical FE model). This is very time-consuming,
162 and is shown below not to be necessary for a flat model of Northern Europe,
163 because of the relatively small size of the former Fennoscandian ice sheet and
164 the relatively small density contrasts at shallow depths (compare Table 1),
165 and because the effect of self-gravitation is partly compensated by the lack
166 of sphericity in the flat FE model (Amelung and Wolf, 1994). As the neglect
167 of self-gravitation increases the rate of displacement and the lack of spheric-
168 ity seems to increase the displacement (see Figure 7 of Amelung and Wolf,
169 1994), the two effects seem to partly compensate each other under the load
170 only upon unloading the earth (positive rate of displacement, negative dis-
171 placement) and enhance each other upon loading (negative displacement rate

172 and displacement).

173 **3 Model Description**

174 The 3D-flat FE model used in this study is based on the commercially available
175 package ABAQUS. The area of loading is 5,840x5,840 km with a resolution of 80
176 km (73x73 elements), while the total surface area of the model is 10,000x10,000
177 km (97x97 elements) to minimize edge effects. The model consists of 14 finite
178 elements in the vertical direction, of which 11 are used to model the crust
179 and upper mantle (to 670 km depth) and 3 to simulate the lower mantle (to
180 10,000 km depth). This gives a total of 131,726 elements, all 8-node linear
181 bricks. We fix the bottom of the model and restrict the movement of the
182 edges to the vertical direction. Computations which include LVZs are very
183 time-consuming: A model with a LVZ of 10^{19} Pas takes about two weeks of
184 CPU-time on a single core 2.8 GHz processor, which gives a computation time
185 of more than four days on a parallel machine with two dual core 2.8 GHz INTEL
186 XEON processors. For comparison, the SP model we use takes less than half an
187 hour on a single core processor, independent of the viscosity stratification. As
188 we are mainly interested in perturbations due to shallow LVZs, which might
189 make the exact modeling of the lower mantle of minor importance, we test if
190 we can approximate the lower mantle with *infinite* elements or with *dashpots*.
191 The latter model is based on ideas of Hetzel and Hampel (2005) and Hampel
192 and Hetzel (2006) for earthquake triggering by GIA.

193 To test the accuracy of the FE model, we compare predictions with results
194 from an analytical, spectral model, based on normal-mode analysis (see Sec-
195 tion 2 and Wu and Peltier, 1982; Vermeersen and Sabadini, 1997). This model

196 provides predictions on an *incompressible* ($\rho_1 = 0$ in Eq. (1)), Maxwell vis-
 197 coelastic, spherical, self-gravitating earth and is benchmarked with other GIA
 198 models (see Spada and Boschi, 2006, for comparison with an independent
 199 method). To compare the output of the SP model with the output of the
 200 flat FE model (Ivins and James, 1999), we map the former to a Cartesian
 201 grid using the Lambert equal area projection. We take the FE model also
 202 to be incompressible, and moreover, to be not self-gravitating, as explained
 203 in Section 2. We simulate the advection of pre-stress by attaching Winkler
 204 foundations to each density boundary (Ricchio and Cozzarelli, 1980; Williams
 205 and Richardson, 1991; Wu, 2004). The Winkler foundations acts like springs
 206 with a spring constant equal to the product of the gravity acceleration at
 207 and the density contrast across the boundary. Upon loading, the foundations
 208 act instantaneously, i.e. in the elastic limit of a viscoelastic material. In a
 209 purely viscous or inviscid material, the foundations only work after a certain
 210 amount of time, acting as buoyancy forces, as there can be no instantaneous
 211 displacement in a viscous material (see the footnote in Cathles, 1975, p. 14).

212 The radial earth stratification used is given in Table 1, where the background
 213 model consists of a 30 km fully elastic crust, a lithospheric thickness $LT =$
 214 80 km, and an upper¹ and lower mantle viscosity $\eta_{UM} = \eta_{LM} = 10^{21}$ Pas.
 215 The perturbed model can have a lower crust or asthenosphere with a low
 216 viscosity of 10^{19} Pas (CLVZ respectively ALVZ), either laterally homogeneous
 217 or heterogeneous. Moreover, the thickness of the lithosphere can increase to
 218 140 km for a cratonic area, and the crustal thickness can vary from 10 km in
 219 oceanic areas to 50 km in cratonic areas. Values for the density ρ and rigidity

¹ this includes the asthenosphere, the low-velocity zone, the upper mantle and the transition zone as defined in Table 1.

220 μ are volume-averaged from PREM (Dziewonski and Anderson, 1981).

221 4 Test Results

222 To test the validity of using a flat FE model for low-viscosity layers in GIA
223 studies, the results are compared with the output of a spherical SP model for
224 a Heaviside loading (Wolf, 1985; Ivins and James, 1999). The Heaviside load
225 is put on the North Pole at $t = 0$ s and stays there infinitely long. In most
226 cases, we evaluate predictions after 10 kyrs. The load consists of an elliptical
227 ice load, with a maximum height (at the center) of 2500 m, and a radius of
228 8° (~ 890 km, comparable with the dimensions of the former Fennoscandian
229 ice sheet), complemented with a eustatic ocean load of about -7.5 m. We will
230 first consider *total* predictions from the background model, after which we will
231 concentrate on *perturbations* due to CLVZs and ALVZs. We model a CLVZ
232 from a depth of 20 to 30 km (10 km thickness) and a viscosity of 10^{19} Pas.
233 The perturbations are the difference between the predictions of a model with a
234 CLVZ and the background model, which has a fully elastic crust. An ALVZ is
235 modeled at the bottom of the lithosphere (80 km) to 140 km, with a viscosity
236 of 10^{19} Pas. Perturbations are now due to the difference in viscosity with the
237 background model, which has a viscosity of 10^{21} Pas in this depth range, see
238 Table 1.

239 In Figure **2a** we show a comparison of the vertical displacement at $t = 0$ s
240 as predicted by the background model for the SP and FE method. Also the
241 difference between the two methods is plotted, in *decimeters*. The deforma-
242 tion is determined completely by the density and rigidity of the earth model
243 and shows a relatively large difference (~ 1 m) below the center of loading.

244 This difference can probably be attributed to the lack of self-gravitation in
245 our implementation of the FE method, which leads to a larger gravitational
246 downpull (Wu and Ni, 1996). The effect of incompressibility (Poisson's ratio
247 $\nu = 0.5$) is large in the elastic limit. If we take the earth to be a Poisson
248 solid (Haskell, 1953; Dahlen and Tromp, 1998, p. 4, 297), which is an approx-
249 imation often used to simplify seismological problems and in which the Lamé
250 parameter $\lambda = \mu$ with the result that $\nu = 0.25$, the vertical displacement will
251 be about 50% larger. Note that we did only test *material* compressibility and
252 that we did not include *internal buoyancy* (third term in Eq. (1), see for a
253 short discussion Wu, 2004). The difference between the FE and SP model for
254 the subsequent ($t > 0$ s) viscoelastic deformation is smaller, see Figure **2b**,
255 where the total vertical displacement after 10 kyrs minus the elastic displace-
256 ment is plotted. Besides a long-wavelength difference, there are some smaller
257 differences in the steep part of the curve, which might be due to a lack of
258 bending in the FE model. This suggests the use of a higher resolution grid
259 or quadratic elements, but, because the induced errors are very small, only
260 linear elements are used in this study. The effect of material compressibility
261 is small for the viscoelastic deformation, decreasing the vertical displacement
262 with only 5% under the center of the load.

263 If now *infinite elements* are used in the lower mantle, instead of 3 layers of ele-
264 ments for the lower mantle as above (from 670 km to 10,000 km, *deep* model),
265 we see from Figure **2c** that the response is reasonably good in the elastic
266 limit, only showing deviation in the center. The viscoelastic response however
267 is not large enough, see Figure **2d**. In contrast, the model with *dashpots* for
268 the lower mantle does not show enough response in the elastic limit. This
269 because there is no response of the upper mantle-lower mantle boundary at

270 670 km, as a dashpot, which represents a purely viscous material, only acts
271 after a certain amount of time (see the discussion about Winkler foundations
272 in Section 3). The model shows however a reasonable viscoelastic response.
273 For this work it is especially important if *perturbations* generated with these
274 models are accurate enough, which we will consider further on.

275 Continuing with the deep model, we see from Figure **3a** that total geoid heights
276 can be computed very accurately with the FE method and the use of displace-
277 ment potentials as described in Section 2. Note that the differences are again
278 given in *decimeters* and that the direct effect of the load on the geoid heights
279 (the Γ -term in Eq. (10)) is not included. The comparison for vertical velocities
280 (or *uplift rates*, Figure **3b**) is also excellent, and from tests (not shown) we
281 have found that for both geoid heights and uplift rates the effect of material
282 compressibility is negligibly small. For horizontal velocities (Figure **4a**), the
283 quality of the FE predictions deteriorates. This is probably at least partly due
284 to the horizontal boundary conditions in the FE model and partly due to the
285 lack of sphericity. However, the FE model shows the behavior that is generally
286 expected (Wu, 2005; Mitrovica et al., 1994, Figure 1) and found (Milne et al.,
287 2001) for loading of the earth in GIA studies, i.e. an inward velocity near the
288 center of the load and an outward velocity just outside the loading area. This
289 corresponds with a pattern in which the peaks in the horizontal velocities cor-
290 respond to the steepest parts in the uplift rates and small horizontal velocities
291 correspond to the flat parts in the uplift rates, i.e. the pattern of horizontal
292 velocities can for a large extent be explained by the (inward) gradient of the
293 uplift rates, as also plotted Figure **4a**. A strong change in vertical motion thus
294 induces a large horizontal velocity in the direction of the largest stress, which
295 is under the load and at the forebulge, due to buoyancy forces on upper mantle

296 material.

297 For a compressible material, the FE model is in much closer agreement with the
298 (incompressible) SP model, which seems to indicate that the lack of spheric-
299 ity is partly compensated by material compressibility. However, for an earth
300 model which has an order of magnitude increase in viscosity from the upper
301 to the lower mantle ($\eta_{UM} = 0.5 \cdot 10^{21}$ Pas, $\eta_{LM} = 5.0 \cdot 10^{21}$ Pas), the effect
302 of compressibility is smaller, see Figure 4b. Note that now also the SP model
303 predicts inward motion under the load, but that the differences with the FE
304 model are still large. We have tested if also part of the differences might be due
305 to the neglect of degree 1 in the SP model, which changes the reference frame
306 from the center of mass of the *initial* earth to the center of mass of the *incre-*
307 *mental* earth (Cathles, 1975, p. 101). However, including degree 1 (following
308 the implementation of Greff-Lefftz and Legros, 1997) does only significantly
309 affect the far field ($> 2,500$ km) predictions, preventing the outward motion
310 to change to an inward motion from about 7,500 to 20,000 km. We will show
311 below that differences in horizontal velocity *perturbations* are much smaller,
312 suggesting that it is preferable to compute perturbations on a spherical back-
313 ground model. Finally, it might be of interest to mention that in the elastic
314 limit (as in Figure 2a), the model predicts inward motion only for a *com-*
315 *pressible* material (O’Keefe and Wu, 2002). For an incompressible material,
316 only outward motion is predicted due to the horizontal elongation of elements
317 under the load.

318 Differences in vertical displacement perturbations due to a CLVZ do not show
319 the long-wavelength behavior, because it is largely differenced out, see Fig-
320 ure 5a. The short-wavelength difference is still present, though the results of
321 the SP method look suspicious near the edge of the load, which might indicate

322 an inaccuracy due to Gibbs' phenomenon. The shape of the displacement per-
323 turbations can be attributed to *channel flow*, in which, opposite to *deep flow*
324 where relaxation times are inversely proportional to the wavelength (Cathles,
325 1975, p. 43), relaxation times are proportional to the square of the wavelength
326 (Cathles, 1975, p. 49), see the relaxation spectrum of the MC-mode in Schot-
327 man and Vermeersen (2005) and the results in Klemann et al. (2007). This
328 leads to a lack of outward material flow below the center of the load and a
329 relatively large deformation at the edge of the load (see Figure IV-21 in Cath-
330 les, 1975, p. 158) and in some cases even to a tilting of the upper crust under
331 the center of the load (e.g. in the presence of a weaker upper mantle, compare
332 the geoid height perturbations in Figure 7a). The comparison of Figure 5a
333 with Figure IV-21 in Cathles (1975) needs some comment, as there *total* flow
334 of a channel is shown, whereas we consider *perturbations* due to a channel in
335 a background model that also consists of a lithosphere and mantle. We ar-
336 gue however that the comparison is valid, as a large part of the response of
337 the lithosphere and mantle is differenced out, and because the sensitivity to
338 the background model is not changing the *pattern* of the perturbations, see
339 Figure 7a.

340 The computed geoid height perturbations agree very well, smoothing the
341 short-wavelength differences visible for the vertical displacement perturba-
342 tions, see Figure 5c. Amplitudes are small, but significantly larger than the
343 expected accuracy of the GOCE mission, which is smaller than 1 cm for wave-
344 lengths longer than a few 100 km. The maximum difference with the SP model
345 is a few centimeters, which is comparable to the accuracy of GOCE, especially
346 if we take into account the suspicious behavior of the SP method near the edge
347 of the ice load (see Figure 5a). From Figure 5b we see that for an ALVZ, the

348 quality of the predictions of the FE model seems to deteriorate. We found that
349 at least 4 element-layers are needed to model the 60 km thick ALVZ and that
350 the improvement using 6 layers is only marginal. The remaining discrepancy
351 can probably be attributed to the lack of self-gravitation in the flat model, as
352 the difference for the 4 element-layer shows a trend from negative under the
353 load towards zero in the far field, as was also found for the elastic limit of the
354 total displacements (Figure **2a**). The short wavelength difference is probably
355 due to the use of linear elements, as explained in discussing the differences
356 between the results of the FE and SP method in Figure **2b**. The same con-
357 clusions seem to hold for ALVZ-induced geoid height perturbations as shown
358 in Figure **5d**.

359 As discussed before, it would be useful if we could replace the layering of the
360 lower mantle with either infinite elements or dashpots in computing perturba-
361 tions. In Figure **6a** we compare the perturbations in vertical displacement due
362 to a CLVZ computed with a deep model (lower mantle to 10,000 km), a model
363 with infinite elements for the lower mantle, and a model with dashpots for the
364 lower mantle. Differences are small, though in general larger than the differ-
365 ences between the deep model and the SP model. However, for geoid height
366 perturbations (Figure **6b**), differences are larger than a few centimeters and
367 it seems necessary for our purposes to use the deep model. The differences
368 can probably be explained by the contribution of the displacement potential
369 (see Eq. (10) and the discussion there) of the upper mantle-lower mantle (670
370 km) boundary to the geoid height perturbations, of which the movement is
371 not modeled very accurately for especially the model with infinite elements
372 (see Figure **2d**). The contribution of this boundary is small compared to the
373 surface contribution, but not much smaller in magnitude than the contribu-

374 tion of the much shallower (30 km) crust-lithosphere boundary. This is due
 375 to the larger density contrast at 670 km (1020 vs. 680 kg/m³ at 30 km, see
 376 Table 1) and due to the longer wavelength induced at 670 km (about five times
 377 larger than at 30 km). For an ALVZ, the displacement potential of the upper
 378 mantle-lower mantle boundary has, due to the large effect of an ALVZ on the
 379 movement of this boundary (amplitude of ~ 10 m to ~ 2 m for a CLVZ), a
 380 significant contribution to the geoid height perturbations, being only smaller
 381 than the surface contribution. Predictions using a model with the lower man-
 382 tle replaced by dashpots or infinite elements are thus worse than for a CLVZ,
 383 and are therefore not shown.

384 The perturbations are sensitive to the background model, see Figure **7a**, where
 385 we have plotted the CLVZ-induced perturbations for our reference background
 386 model ($LT = 80$ km, $\eta_{UM} = \eta_{LM} = 1.0 \cdot 10^{21}$ Pas, '80/ 1.0/ 1.0'), and mod-
 387 els with changes in η_{LM} , η_{UM} and LT . The effect of a weaker upper mantle
 388 (compare '80/ 0.5/ 5.0' and '80/ 1.0/ 5.0') is largest, because of additional
 389 outward material flow in the upper mantle and larger bulges just outside of
 390 the load, i.e. the depression under the load is deeper and steeper. This in-
 391 creases the stress in the CLVZ just inside and outside the loading area (in the
 392 steep part of the depression and the bulge), enhancing mass flow through the
 393 CLVZ from the edges outwards, and tilting of the upper crust in the center.
 394 The background model does not change the pattern of channel flow, which
 395 is not the case for ALVZ-induced perturbations, see Figure **7b**. Where the
 396 perturbations for our reference background model are determined by channel
 397 flow (small perturbations under the center of the load), the perturbations for
 398 a weaker upper mantle ($\eta_{UM} = 0.5 \cdot 10^{21}$ Pas) show deep flow behavior. This
 399 effect is, somewhat less pronounced, also present for only a stiffer lower mantle

400 ($\eta_{LM} = 5.0 \cdot 10^{21}$ Pas).

401 Perturbations in horizontal velocities show (see Figure 8), in contrast with
402 *total* horizontal velocities (Figure 4a), the same behavior for the FE and SP
403 method. Both for a CLVZ and an ALVZ, the quality deteriorates towards the
404 far field, at least partly due to the boundary conditions in the FE model,
405 which are fixed in the horizontal direction at 10,000 km. If we compare this
406 with the SP model, in which, due to symmetry considerations, the horizontal
407 direction is fixed at the south pole ($\sim 20,000$ km), we argue that part of the
408 difference can be explained by the larger horizontal elongation or stretching
409 of the crust/lithosphere for a shorter distance to a fixed point, i.e. for the FE
410 model. However, due to the large differences between the FE and SP model for
411 total horizontal velocities (Figure 4a), other causes such as the lack of spheric-
412 ity cannot be ruled out. The patterns can again for a large part be explained
413 by the inward gradient of the uplift rates (not shown). The noisy character
414 of the perturbations, especially for a CLVZ, are due to the double differenc-
415 ing and the relatively low precision ('single', default) used for the output of
416 ABAQUS, which can be changed to 'double' in the execution procedure. Note
417 that the differences with the SP model are small compared to the accuracy
418 of the BIFROST network (~ 0.3 mm/yr), however, so are the perturbations.
419 This means that LVZs can probably not be detected by BIFROST, especially
420 because high-resolution spatial information is not provided.

421 In Figure 9a perturbations are shown due to a laterally heterogeneous CLVZ,
422 that extends from just under the load ('A', 670 km) outward. This means that
423 the laterally heterogeneous model is equal to the background model from the
424 center of the load to 'A', and equal to the model with a laterally homoge-
425 neous CLVZ from 'A' outwards. Because under the load the induced stresses

426 are largest, the response is comparable to the homogeneous case from 670 km
427 outwards. Inwards, there seems to be some tilting of the crust relative to the
428 displacement of the crust in the background model, due to the indentation
429 of the crust from 'A' outwards. For a CLVZ from 1110 km ('B') outwards,
430 the effect of the lateral heterogeneity is much more pronounced, and leads
431 to a significant relative tilting of the crust. For an ALVZ the effect of lateral
432 heterogeneities is larger, substantially weakening the amplitudes of the homo-
433 geneous case, but mainly showing the same effect as for a CLVZ and therefore
434 not shown. As shallow viscosity and lithospheric thickness LT are via thermal
435 regime closely related, we show in Figure 9b the effect of the inclusion of a
436 larger LT ('craton') under the load only, and the inclusion of both a larger
437 LT under the load and a laterally heterogeneous CLVZ. The effect of a cra-
438 ton under the load is to strengthen the lithosphere, thus de-amplifying the
439 vertical displacement in the center, and thus inducing a positive geoid height
440 perturbation. If the craton extends to outside the load ('craton < B (1110
441 km)'), there is also a negative perturbation outside the load, because, due to
442 the larger lithospheric strength, the bulge area is broader and smoother. If we
443 now compare the perturbations due to variations in LT only with perturba-
444 tions due to both a laterally heterogeneous LT and CLVZ, we see that only
445 for the case that part of the ice load is in an area of small LT and a CLVZ
446 ('craton < A, CLVZ > A'), differences are as large as 10 cm. Due to the differ-
447 ent scales of deformation for variations in lithospheric thickness (wavelengths
448 mainly larger than 1000 km) and for a CLVZ (mainly smaller than 1000 km),
449 the two effects might be separated using medium- to high-resolution gravity
450 field information as provided by GOCE.

451 5 Example for Northern Europe

452 We will now investigate the effect of lateral heterogeneities in LVZs and LT
453 using a simple regional model for Northern Europe, in which we assume 3
454 different types of areas, see also Figure **10a-c**:

- 455 I. Continental areas with a 50 km thick elastic crust in a thick elastic litho-
456 sphere ($LT = 140$ km), on top of an upper mantle with a viscosity of
457 10^{21} Pas, which represent old, cratonic areas that are mainly Archean
458 and Early Proterozoic (> 1.5 Gyr, Pérez-Gussinyé and Watts, 2005).
459 This type corresponds well with estimates of thermal thickness larger
460 than 140 km (Artemieva and Mooney, 2001);
- 461 II. Continental areas with 20 km elastic upper crust and a 10 km lower
462 crust that is either elastic or has low viscosity (10^{19} Pas, CLVZ). The
463 total lithospheric thickness is thinner now ($LT = 80$ km) and overlies
464 a 60 km thick asthenosphere that has either the viscosity of the upper
465 mantle (10^{21} Pas) or a low viscosity (10^{19} Pas, ALVZ). These areas are
466 younger (< 1.5 Gyr) and have a thermal thickness smaller than 140 km;
- 467 III. Oceanic areas with a 10 km crust in a lithosphere of 80 km, overlying a 60
468 km thick asthenosphere with a viscosity of 10^{21} Pas or 10^{19} Pas (ALVZ).

469 The different areas are derived from CRUST2.0 ([http://mahi.ucsd.edu/Gabi/
470 rem.dir/crust/crust2.html](http://mahi.ucsd.edu/Gabi/rem.dir/crust/crust2.html), accessed 3 May 2007), in which an area with a
471 crustal thickness between 15 and 35 km is considered as Type II. Areas with
472 a crustal thickness thinner than 15 km are then Type III and thicker than
473 35 km Type I. The different areas are indicated in Figures **10d**. Note that
474 this is a first approximation, which means that some areas in Type II are

475 oceanic and that some areas in Type I are not cratonic. We use the ice-load
476 history as derived by Lambeck (RSES, Lambeck et al., 1998), which is given
477 from 30 kyrs BP to present for Fennoscandia, the Barents Sea area and the
478 British Isles. We complement this with the eustatic ocean-load history of the
479 global RSES model, which is about -130 m from 30 kyrs BP to the last glacial
480 maximum (LGM, 21 kyrs BP), and include a linear glaciation phase of 90
481 kyrs to have a glacial cycle of 120 kyrs. We have plotted the ice heights at
482 LGM in Figure 10e.

483 In Figure 11a we show total (i.e. background-induced) 3D-velocity predictions
484 for Northern Europe computed with the FE model. Contour lines show the
485 difference in uplift rates with the SP model, the black arrows are horizontal
486 velocities computed with the FE model and the white arrows are computed
487 with the SP model. As expected from Figure 3b, the uplift rates compare
488 well, with only some small areas where the difference is larger than 0.2 mm/yr
489 (which is much smaller than the accuracy of about 1 mm/yr as reported for
490 the BIFROST network, Johansson et al., 2002). Note that the predicted uplift
491 rate is sensitive to the earth stratification, which explains differences with
492 e.g. Milne et al. (2001), who use the same ice-load history. The horizontal
493 rates compare less well, though not as bad as expected from Figure 4a. In
494 Figure 11b we show the total geoid height predictions. Contour lines are
495 again differences with the SP model, which are generally smaller than 2 cm,
496 which is comparable to the accuracy of the GOCE mission. Note that we have
497 subtracted the mean from both fields, as we have in our FE computations no
498 self-gravitating sea level that can serve as a reference for the geoid height.

499 Also for LVZ-induced perturbations in 3D-velocities we see some differences
500 between FE and SP (Figure 12), though the general pattern agrees well.

501 Note that perturbations in horizontal velocities are up to 0.5 mm/yr, which is
502 slightly larger than the accuracy of the BIFROST network of about 0.3 mm/yr.
503 However, due to the point-like character of GPS measurements, which makes
504 the extraction of relatively high-resolution spatial signatures difficult, and the
505 fact that part of the perturbative signal is generated by assuming LVZs be-
506 low Scandinavia, we expect that BIFROST cannot add information on LVZs.
507 This is confirmed by Milne et al. (2001), who indicate that the radial resolving
508 power of their inversion is ~ 200 km in the shallow earth.

509 In Figure **13a** we show geoid height perturbations due to a laterally homo-
510 geneous CLVZ, in which contour lines indicate again differences with the SP
511 model. The pattern can be explained by lower crustal material flow through
512 the low-viscosity channel from under the ice load outwards during glaciation,
513 and the long relaxation times of crustal channel flow (Schotman and Ver-
514 meersen, 2005). Due to the long glaciation period (~ 100 kyrs), the earth is
515 close to isostatic equilibrium at LGM and during the much shorter deglacia-
516 tion period (~ 20 kyrs) only part of the crustal material has returned from the
517 perturbative bulges. For a homogenous ALVZ the differences are also small,
518 see Figure **13b**. The pattern is broader, and moreover of opposite sign. This
519 is due to the fact that an ALVZ accelerates the deformation process, as de-
520 scribed in Schotman and Vermeersen (2005). To highlight the effect of lateral
521 heterogeneities in the LVZ, we show in Figure **13c, d** the difference between
522 a model which includes a laterally heterogeneous LVZ and a model which in-
523 cludes a laterally homogeneous LVZ (as shown in **a, b**). To focus on the LVZ
524 only, we have not yet included variations in the lithospheric thickness. For a
525 CLVZ the differences are relatively small, though still with amplitudes up to
526 20 cm. Of course, the differences are large in the Baltic Shield, but also in

527 southern Scandinavia, at the edge of the cratonic area, and off the Norwegian
528 coast, where the geoid high predicted by the model with a homogeneous CLVZ
529 is for the model with a laterally heterogeneous CLVZ significantly smaller. For
530 an ALVZ, the effect of lateral heterogeneities is somewhat larger, showing a
531 clear difference under the main domes of the RSES ice-load history, but also
532 further outwards, especially to the east, due to the larger depth of the ALVZ
533 compared to the CLVZ.

534 Before we look at the simultaneous effect of lateral variations in LT and LVZs,
535 we show the effect of variations in LT in the absence of an LVZ. The geoid
536 height perturbations in Figure 14a are thus only due to the thicker lithosphere
537 ($LT = 140$ km) of the Baltic Shield. The effect is largest over the Baltic Sea,
538 as there the ice load is largest, see Figure 10e. In the perturbations due to
539 lateral heterogeneities in both LT and a CLVZ (Figure 14b), we can still see
540 the effect of the CLVZ around the formerly glaciated area (geoid highs of 20-
541 30 cm, see Figure 13a) and the effect of the large LT below the largest ice
542 load (geoid high of 50 cm, compare with Figure 14a). Due to the different
543 spatial signatures of both effects, it seems to be possible to separate the two
544 and extract information on both LT and crustal low-viscosity. For an ALVZ
545 (Figure 14c) the separation of the two effects seems to be difficult, as the effect
546 of variations in LT (Figure 14a) is very similar to ALVZ-induced perturbations
547 (Figure 13b), especially along the Norwegian coast and under the major ice
548 domes.

549 **6 Realistic Ocean-Load History**

550 For loading the FE model for Northern Europe in Section 5, we used the
551 eustatic ocean-load output of the SP model. Here we show which error this
552 introduces in geoid height predictions compared with predictions of a self-
553 gravitating model that includes realistic oceans ('RO', which includes the effect
554 of self-gravitation in the oceans, but also the effect of time-dependent coast-
555 lines and meltwater influx, see Section 1). For total geoid heights predicted
556 with the RSES ice-load history, the errors introduced by using the eustatic
557 load output of the SP model is shown in Figure **15a**. The maximum error
558 is about 20 cm, which is small compared to the total amplitudes of several
559 meters (Figure **11b**). Moreover, the relatively large error over the Northern
560 Atlantic (geoid height predictions in this area are about zero) can be removed
561 if we use the realistic ocean-load output of the SP model. However, the er-
562 rors in areas that were once glaciated and are now ocean covered increase to
563 30 cm (Barents Sea, North Sea) and 50 cm (Gulf of Bothnia). For CLVZ-
564 induced perturbations, the error is generally smaller than 5 cm (Figure **15b**),
565 compared to CLVZ-induced geoid height perturbations of tens of centimeters
566 (Figure **13a**). Note the clear effect of time-dependent coastlines and meltwater
567 influx in the the Gulf of Bothnia, which is also present if we use the realis-
568 tic ocean-load output of the SP model. Overall errors are somewhat smaller
569 then, but probably not negligibly small. For an ALVZ, the errors are up to
570 10 cm (Figure **15c**), but ALVZ-induced geoid height perturbations have am-
571 plitudes that are about 5 times as large, compare Figure **13b**, so these errors
572 can probably be neglected.

573 **7 Conclusion**

574 We have shown that a 3D-flat, non-self-gravitating finite-element (FE) model
575 of glacial-isostatic adjustment (GIA) can deliver accurate predictions of ra-
576 dial displacements, uplift rates and geoid heights when compared to conven-
577 tional spherical, self-gravitating spectral (SP) models. Perturbations in these
578 predictions due to shallow low-viscosity zones (LVZs) can also be computed
579 accurately, though the accuracy seems, due to the neglect of self-gravitation, to
580 decrease somewhat with increasing depth, and moreover, the sensitivity to the
581 background model increases considerably. Predictions of horizontal velocities
582 are less accurate, showing considerable differences with the predictions of an
583 SP model, probably due to the lack of sphericity of the model. We have to
584 keep in mind however that horizontal velocities are very sensitive to model
585 parameters in general. We have shown that material compressibility can have
586 a significant effect on horizontal velocities. The effect on uplift rates and geoid
587 heights is found to be negligibly small.

588 For a simple model of Northern Europe, horizontal and vertical velocity per-
589 turbations due to shallow LVZs are only slightly above the current BIFROST
590 performance and due to the relatively low spatial resolution of BIFROST, we
591 do not expect BIFROST to add information on LVZs. We have found that lat-
592 eral heterogeneities do not necessarily have a large influence on CLVZ-induced
593 perturbations, though amplitudes are still up to 20 cm in geoid height, which
594 is an order of magnitude larger than the expected GOCE performance. In real-
595 ity, however, we also have to consider unmodeled thermally- and chemically-
596 induced shallow mass inhomogeneities, that are expected to be effective in
597 masking the gravity signal due to CLVZs. We find, for example, from the

598 GRACE satellite gravity mission (solution GGM02S, Tapley et al., 2005),
599 only retaining the spherical harmonic coefficients from degree 40 to 90, where
600 CLVZs have their largest signal, geoid heights of a few meters. Effective spatio-
601 spectral filtering tools have to be developed to extract the GIA-related signal
602 (see Schotman and Vermeersen, 2005, for a discussion), in which we expect
603 the specific signatures of CLVZs to be useful. For example, because of the
604 different signature of perturbations due to a CLVZ and due to variations in
605 lithospheric thickness, it seems possible to solve for both lithospheric thickness
606 and CLVZs at the same time.

607 Forcing the FE model with a eustatic ocean load gives in general acceptable
608 results, which can be improved somewhat using the realistic load output of
609 the self-gravitating SP model. Errors in CLVZ-induced geoid height pertur-
610 bations are about 5 cm, which is small but not negligible compared to the
611 amplitudes of the perturbations (tens of centimeters). For an ALVZ, the errors
612 are somewhat larger, but small compared to the perturbations. Based on the
613 above we have given in Table 2 overview of error estimates using the flat, not
614 self-gravitating FE model and the spherical, self-gravitating SP model ('FE vs.
615 SP'), for geoid heights and geoid height perturbations ('GEO') and horizontal
616 velocities and velocity perturbations ('VEL'). We have also given the error
617 estimates of using a eustatic instead of realistic ocean load ('EU vs. RO'). If
618 we accept errors equal to or smaller than 10%, we have to be critical when
619 computing (perturbations in) horizontal velocities and when using a eustatic
620 ocean load for computing CLVZ-induced geoid height perturbations. For this
621 error bound, we cannot use infinite or dashpot elements to replace the deep
622 layering of the lower mantle (to 10,000 km), as the errors are larger than 10%
623 for LVZ-induced perturbations.

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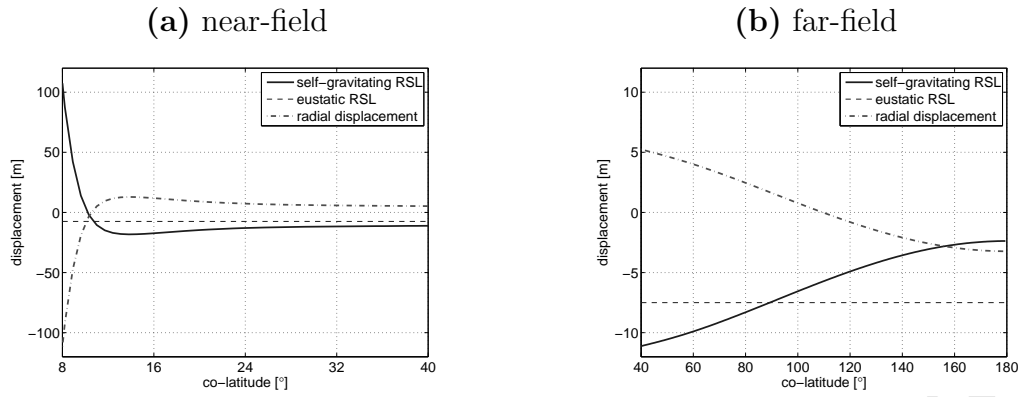


Fig. 1. Near- (a) and far- (b) field effect of an elliptical iceload (8° radius) on the North Pole after 10 kyrs of loading

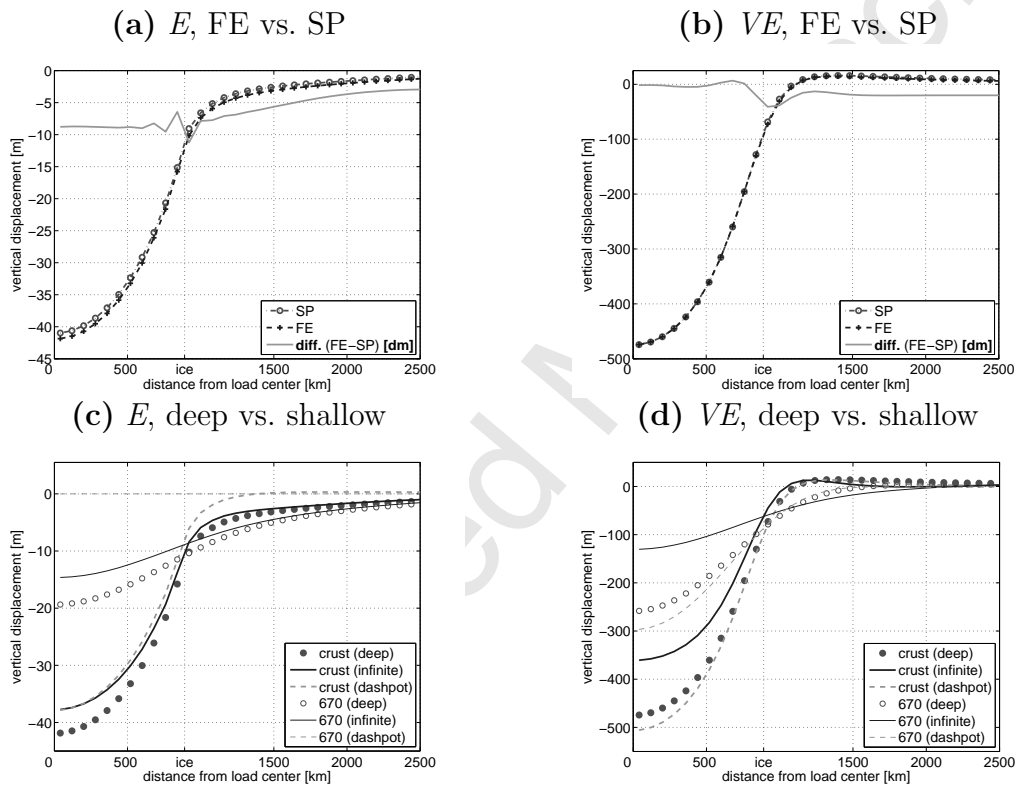


Fig. 2. Elastic ($t = 0$ s, 'E', left) and viscoelastic ($t > 0$ s, total response minus response at $t = 0$ s, 'VE', right) response for the SP and FE model (a, b) and a *deep* model (lower mantle to 10,000 km), and *shallow* models using *infinite* elements for the lower mantle, or using *dashpots* for the lower mantle (c, d)

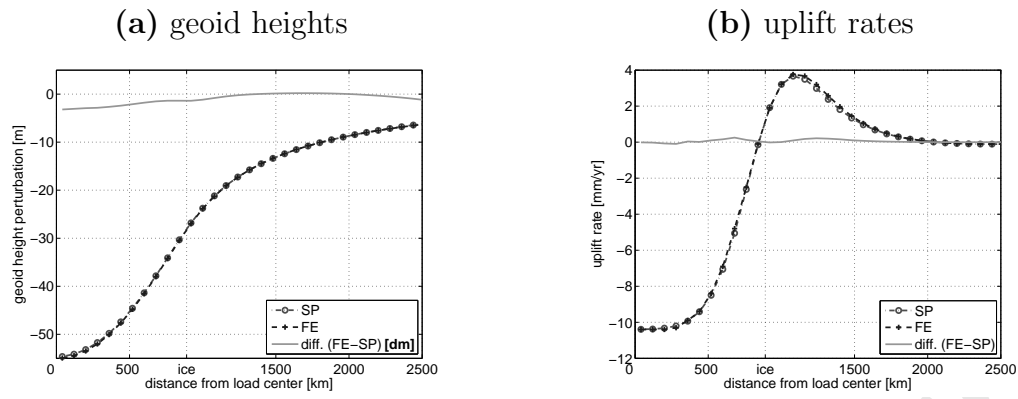


Fig. 3. Comparison of geoid heights (a) and uplift rates (b) as predicted by the SP and FE method

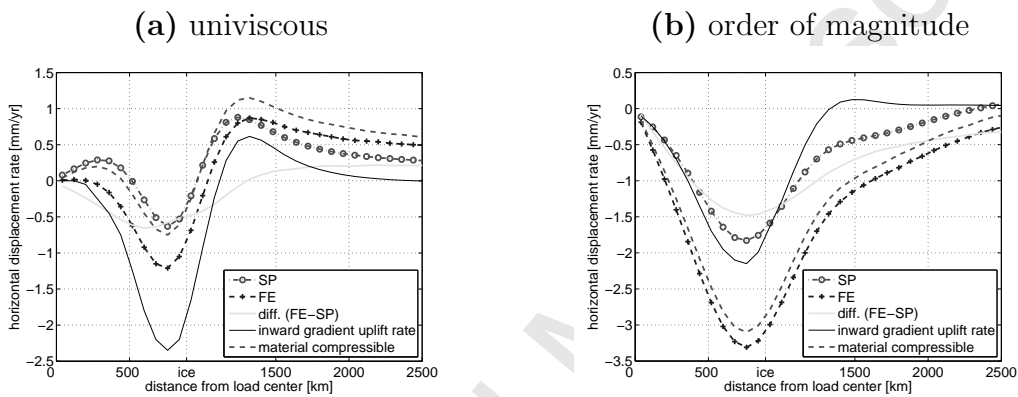
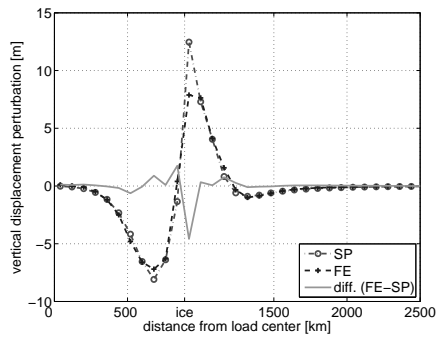


Fig. 4. Horizontal velocities (positive is outward motion) as predicted by the SP and FE method for a univiscous (Table 1) earth model (a) and an earth model with an order of magnitude increase in viscosity from the upper to lower mantle (b)

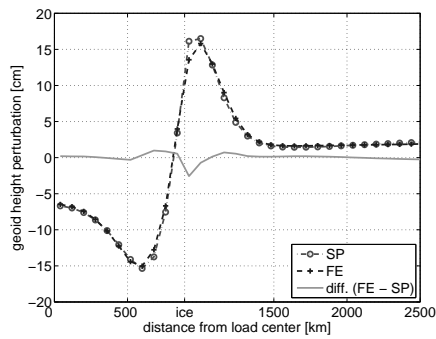
(a) CLVZ, vertical displacement

perturbations



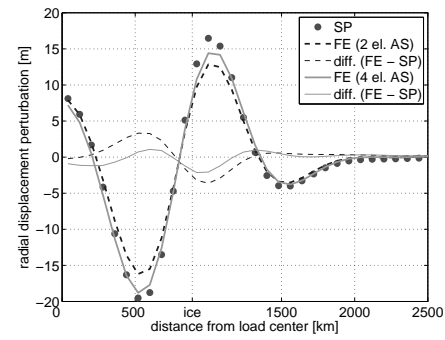
(c) CLVZ, geoid height

perturbations



(b) ALVZ, vertical displacement

perturbations



(d) ALVZ, geoid height

perturbations

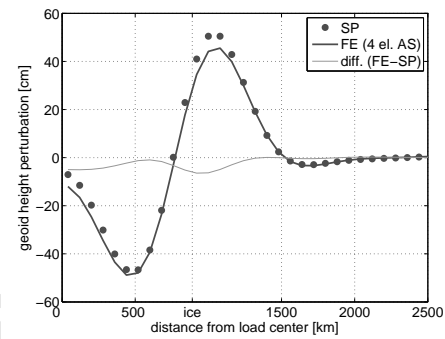
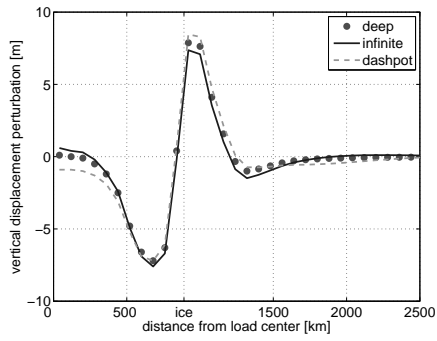


Fig. 5. CLVZ- (left) and ALVZ- (right) induced vertical displacement (a, b) and geoid height (c, d) perturbations due to a laterally homogeneous LVZ

(a) vertical displ. perturbations



(b) geoid height perturbations

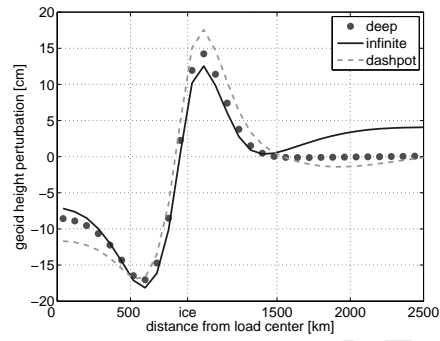
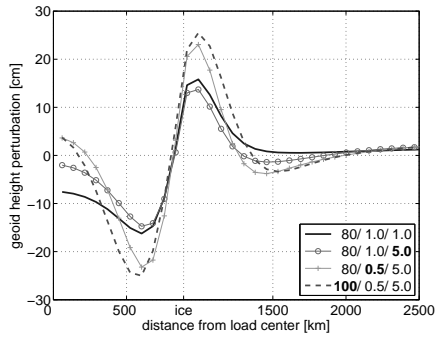


Fig. 6. CLVZ-induced vertical displacement (a) and geoid height (b) perturbations for a *deep* model (lower mantle to 10,000 km), a model using *infinite* elements for the lower mantle, and a model using *dashpots* for the lower mantle

(a) CLVZ



(b) ALVZ

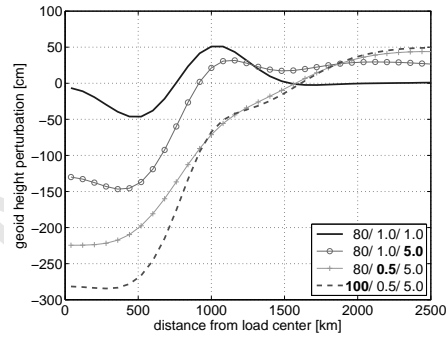


Fig. 7. Effect of the background earth model on CLVZ- (a) and ALVZ- (b) induced perturbations (the legend gives: LT [km]/ η_{UM} [10^{21} Pas]/ η_{LM} [10^{21} Pas])

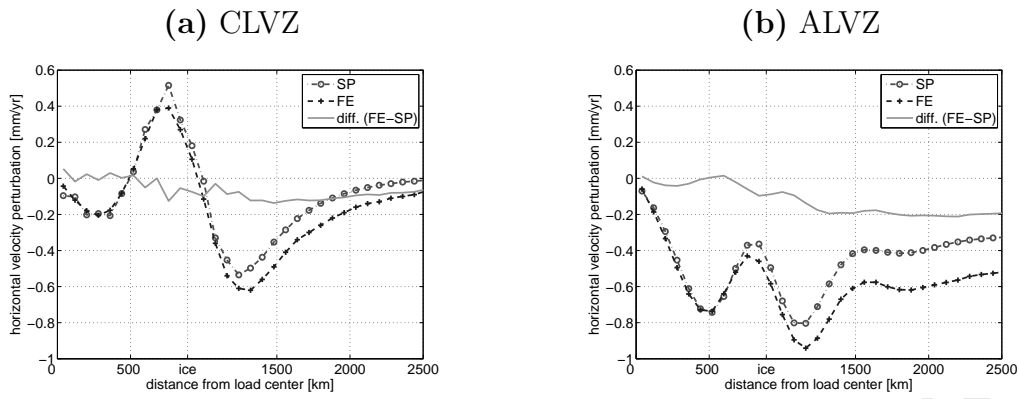


Fig. 8. CLVZ- (a) and ALVZ- (b) induced horizontal velocity perturbations (positive direction is outward motion)

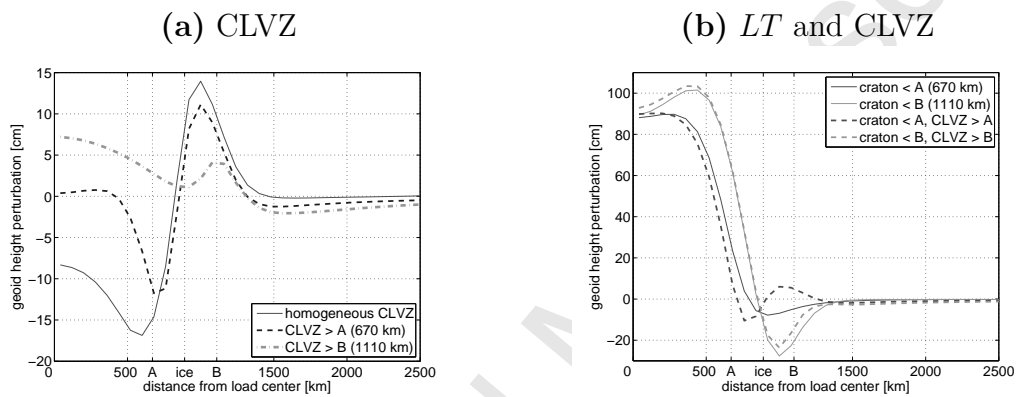


Fig. 9. Geoid height perturbations due to a laterally heterogeneous CLVZ (a) and due to a laterally heterogeneous *LT* and CLVZ (b)

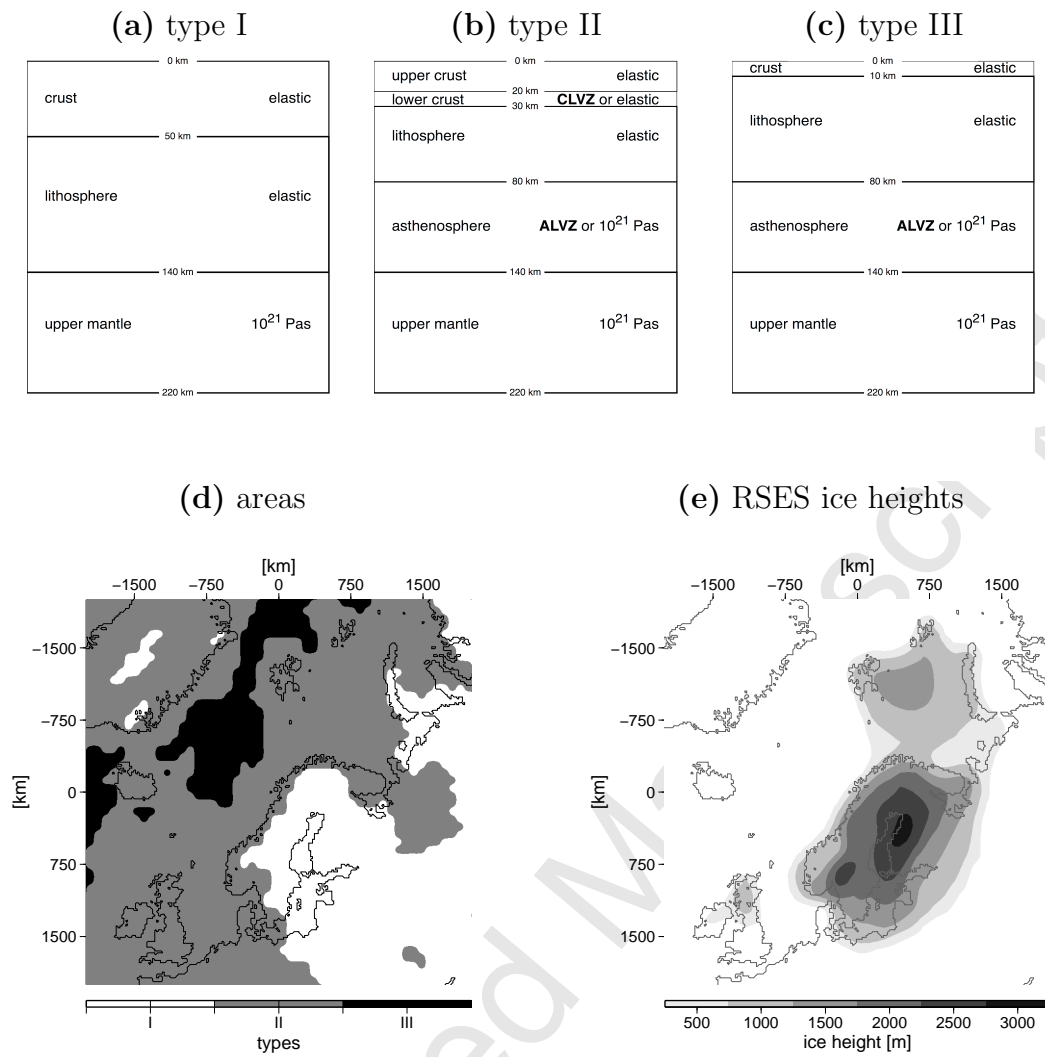


Fig. 10. Earth stratification for the 3 areas: type I (a), type II (b) and type III (c), corresponding geographical areas based on CRUST2.0 (d), and RSES ice heights at LGM (e)

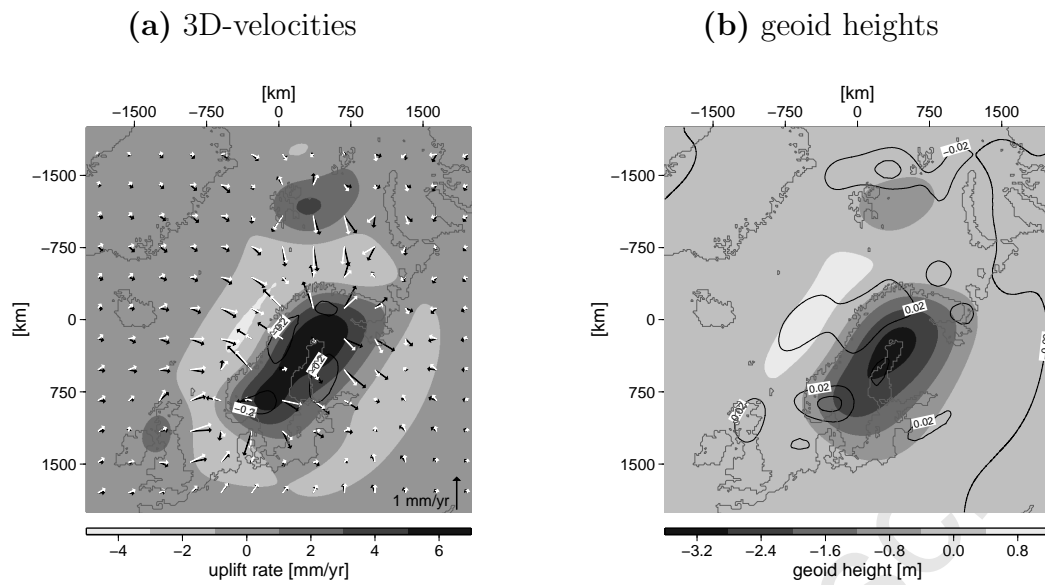


Fig. 11. Total (no LVZ) 3D-velocities (a) and geoid heights (b) at present, computed with FE (white arrows are computed with SP, contour lines are difference with SP)

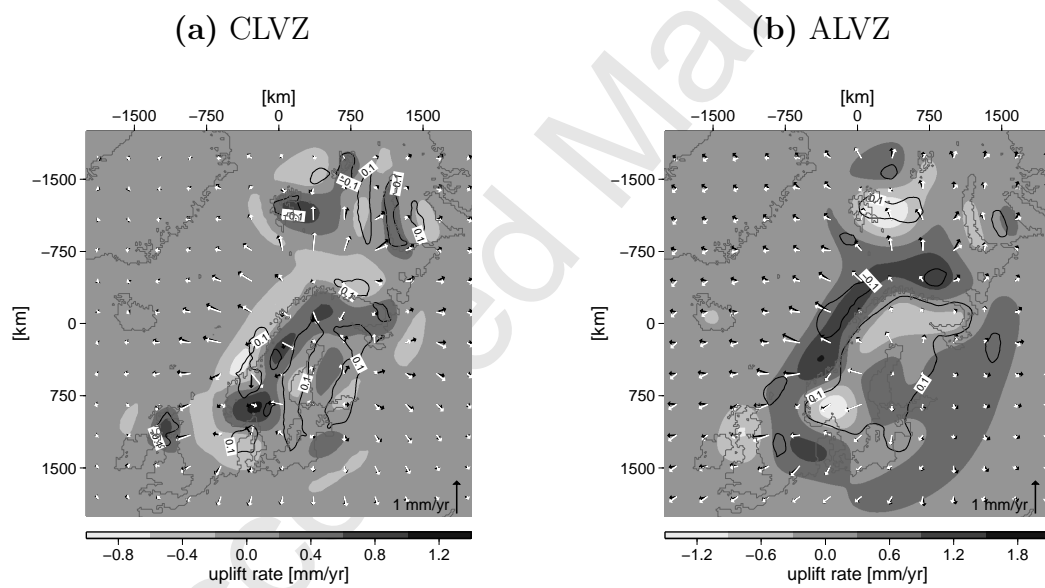


Fig. 12. CLVZ- (a) and ALVZ- (b) induced 3D-velocity perturbations (white arrows are computed with SP, contour lines are difference with SP)

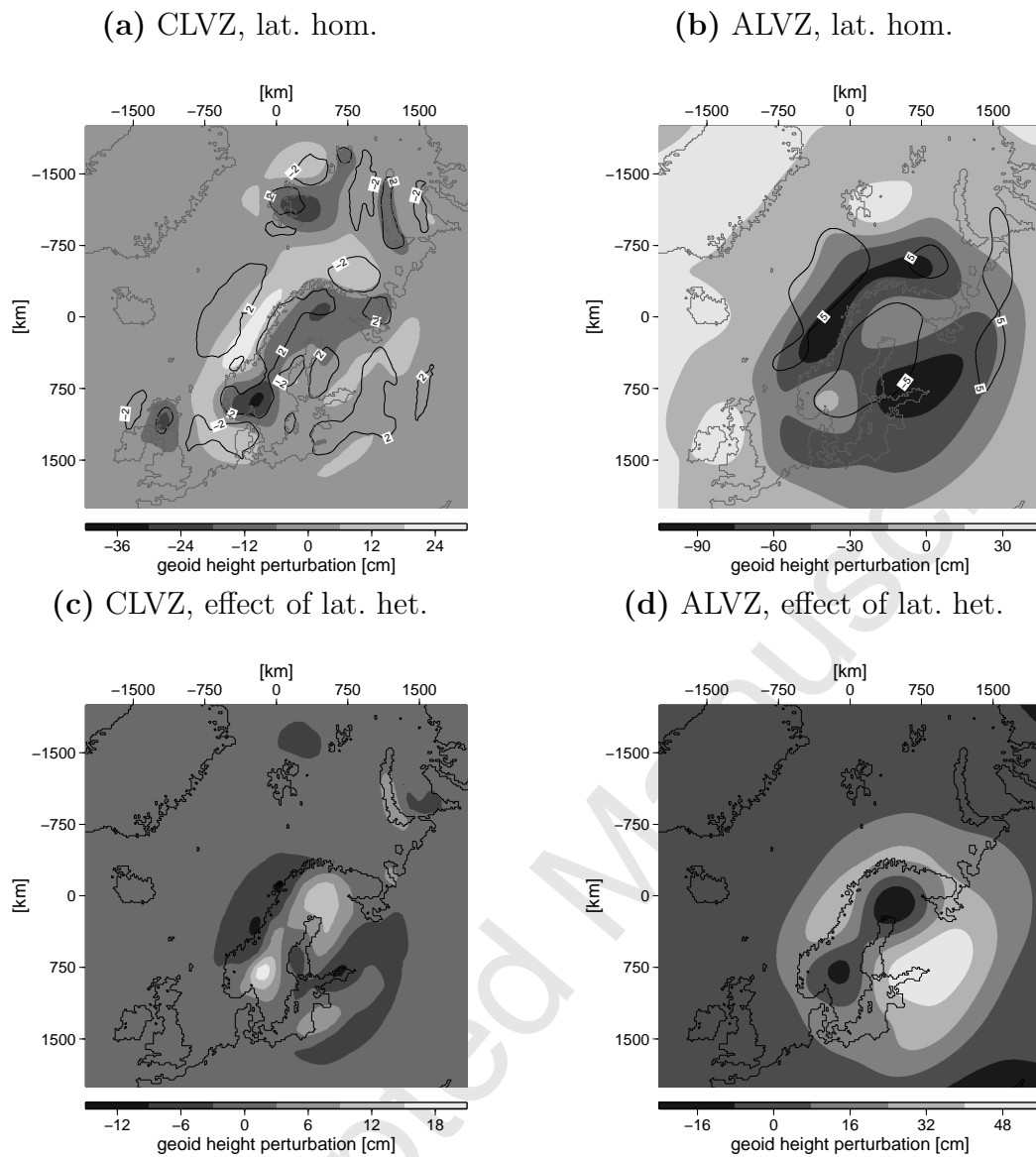


Fig. 13. Geoid height perturbations for a laterally homogeneous CLVZ (a) and ALVZ (b), and differences between a laterally heterogeneous and homogeneous CLVZ (c) and ALVZ (d)

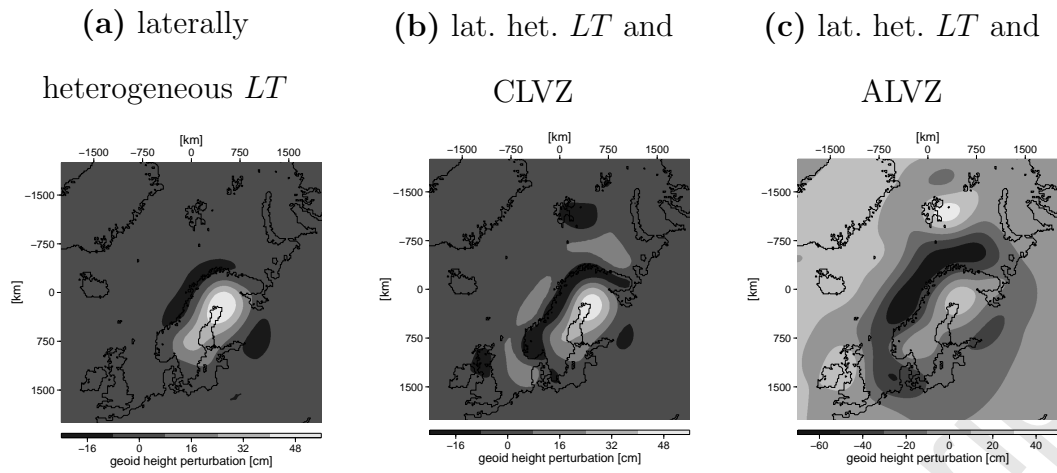


Fig. 14. Geoid height perturbations due to a laterally heterogeneous LT in the absence of an LVZ (a), and combined effect of a laterally heterogeneous LT and CLVZ (b) or ALVZ (c)

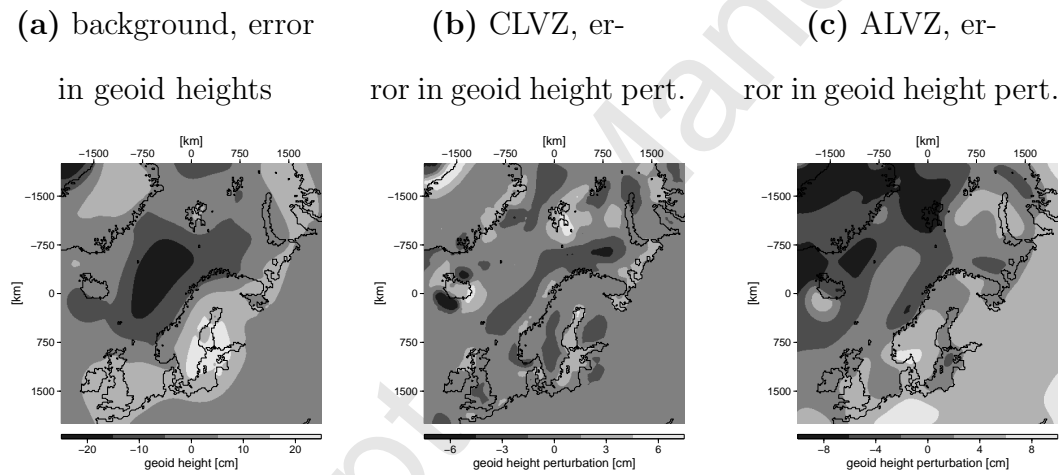


Fig. 15. Errors introduced by using a eustatic ocean load in geoid heights for the background model (a), and in geoid height perturbations for a CLVZ (b) and ALVZ (c)

Table 1

Radial earth stratification in which the lower crust and the asthenosphere can have an LVZ of 10^{19} Pas

Layer	Depth [km]	Density ρ [kg/m ³]	Rigidity ν [GPa]	Viscosity η [Pas]
upper crust	0	2700	27	elastic
lower crust	20	:	:	elastic or 10^{19}
lithosphere	30	3380	68	elastic
asthenosphere	80	:	:	10^{21} or 10^{19}
low-velocity zone	140	:	:	10^{21}
upper mantle	220	3480	77	:
transition zone	400	3870	108	:
lower mantle	670	4890	221	10^{21}
core	2891	10925	0	0

Table 2

Error estimates for the use of a regional model in a global background model. GEO are geoid heights and VEL are horizontal velocities, L indicates that the error has a long-wavelength ($>\sim 1000$ km) signature and S a short-wavelength ($<\sim 1000$ km).

Source	Quantity	total	CLVZ pert.	ALVZ pert.
FE vs. SP	GEO	1% (L)	5% (S ^a)	10% (S ^a) + 10% (L)
	VEL	50% (L)	5% (L ^b)	25% (L ^b)
EU vs. RO	GEO	1% (L)	25% (S ^c)	10% (L)

^a can probably be removed using smaller gridsize or quadratic elements (Section 4)

^b can probably be reduced by increasing the horizontal extent of the FE model (Section 4)

^c can be somewhat reduced using the RO instead of the EU load output of the SP model (Section 6)