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Regional Perturbations in a Global Background Model of Glacial Isostasy

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Abstract

In Schotman and Vermeersen (2005, Earth Planet. Sci. Lett. 236), the effect of crustal and asthenospheric low-viscosity zones (LVZs) on geoid heights was shown, as predicted by models of glacial-isostatic adjustment (GIA). The governing equations were solved analytically in the spectral domain, which makes the method used accurate and fast. However, it does not allow for (large) lateral variations in earth stratification. As the properties of shallow LVZs can be expected to vary laterally, we have developed a finite-element model based on ABAQUS. Global (spherical-3D) finite-element models are currently not capable of providing high-resolution predictions, which we expect due to the shallowness of the LVZs. We therefore use a regional (flat-3D) model and compute geoid heights from the predicted displacements at density boundaries by solving Laplace's equation in the Fourier-transformed domain. The finite-element model is not self-gravitating, but we compare the results

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with a self-gravitating spectral model under the assumption that the lack of selfgravitation is partly compensated by the lack of sphericity, and that long-wavelength differences largely cancel out when using perturbations, which are the difference between a model with and without an LVZ. We show that geoid height perturbations due to an LVZ can be computed accurately, though the accuracy deteriorates somewhat with the depth of the LVZ. Moreover, we show that horizontal rates of displacement, though not accurate for total displacements, are accurate for perturbations in the near field. We show the effect of lateral variations in the properties of the LVZ and in lithospheric thickness, and compute geoid height perturbations for Northern Europe based on a simple laterally heterogeneous model. The model is forced with a realistic ice-load history and a eustatic ocean-load history. The errors introduced by using a eustatic ocean-load history instead of realistic oceans are generally smaller than 10%, but might be critical for perturbations due to crustal LVZs.

Key words: glacial isostasy, satellite gravity, GPS measurements, finite-element modeling, rheology of the crust and upper mantle, halfspace modeling
PACS: 91.10.Fc, 91.10.Kg, 91.10.Op, 91.10.Vr, 91.32.De, 91.32.Gh

1 1 Introduction

The process of glacial-isostatic adjustment (GIA), in which the solid earth is
deformed by load changes due to variations in the volume of continental ice
sheets, is commonly associated with long-wavelength phenomena (spherical
harmonic degree < 20) as land-uplift in Scandinavia ("postglacial rebound",
e.g. Milne et al., 2001), part of the geoid low in Hudson Bay-area (e.g. Tamisiea
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et al., 2007), and variations in the position and speed of the earth's axis of ro-7 tation (polar wander and changes in length-of-day, e.g. Mitrovica et al., 2005). 8 These phenomena are mainly influenced by the thickness of the lithosphere and the viscosity of the mantle, and can to a large extent be explained using 10 spherical, radially stratified (*laterally homogeneous* or 1D) viscoelastic earth 11 models, which often use a spherical harmonic expansion to solve the governing 12 equations in the spectral (SP) domain. In the case of lateral variations, the 13 transformed differential equations in SP methods can no longer be solved sep-14 arately for each degree due to mode coupling (Wu, 2002), which complicates 15 the use of the SP method, especially for large viscosity gradients. Recently, 16 laterally heterogeneous (3D) spherical earth models based on the finite-element 17 (FE) method have been developed, showing the effect of lateral variations in 18 lithospheric thickness and mantle viscosity (Wu and van der Wal, 2003; Zhong 19 et al., 2003; Wu et al., 2005; Latychev et al., 2005). 20

The upcoming GOCE satellite gravity mission, planned for launch by ESA on 21 May 15 2008, is predicted to measure the static gravity field with centimeter 22 accuracy at a resolution of 100 km or less (Visser et al., 2002). This makes 23 the detection of *high-resolution* signals as expected from shallow earth struc-24 tures possible. In GIA, high-harmonic signatures (spherical harmonic degree 25 > 20) can be induced by shallow layers (depth < 200 km) with low viscosity 26 (~ $10^{18} - 10^{20}$ Pas, Schotman and Vermeersen, 2005). Crustal low-viscosity 27 zones (CLVZs) are layers in the lower crust which have viscosities that are sig-28 nificantly smaller than the high-viscosity, for GIA studies effectively elastic, 29 upper crust and lithospheric mantle. As the presence and properties of CLVZs 30 depend strongly on thermal regime (Meissner and Kusznir, 1987; Ranalli and 31 Murphy, 1987; Watts and Burov, 2003), these can in principle not be modeled 32

using 1D-stratified earth models. In Northern Europe for example, CLVZs
are not expected in the cold and thick crust of the Baltic Shield, but might
be present in surrounding younger continental shelves (Schotman and Vermeersen, 2005). Asthenospheric LVZs (ALVZs) can probably be found below
the oceanic lithosphere and perhaps also below the continental shelves, but
not below the Baltic Shield (Schotman and Vermeersen, 2005; Steffen and
Kaufmann, 2005).

As these shallow low-viscosity layers induce high-resolution signals, the use 40 of a 3D-spherical FE model is considered to be not feasible yet. We therefore 41 test if predictions from a 3D-flat FE model, based on the commercial package 42 ABAQUS (Wu, 2004), are accurate enough for computing gooid height per-43 turbations. Perturbations are differences between predictions from a perturbed 44 background model and predictions from the background model itself. Here we 45 assume that we know the background model, consisting of a laterally homo-46 geneous earth stratification and ice-load history, from previous GIA studies. 47 As we concentrate on Northern Europe, this background model will be mainly 48 based on Fennoscandian studies (Lambeck et al., 1998; Milne et al., 2001). The 49 presence and properties of LVZs are, via thermal regime, closely related to the 50 thickness of lithosphere. We are therefore also interested in constraining both 51 properties of LVZs and the thickness of the lithosphere simultaneously. As an 52 additional constraint, apart from high-resolution satellite-gravity data, we will 53 consider the use of 3D-velocities as for example measured by the BIFROST 54 network (Milne et al., 2001), which has an accuracy of 0.8-1.3 mm/yr in the 55 vertical direction and 0.2-0.4 mm/yr in the horizontal direction (Johansson et 56 al., 2002). 57

58 A change in the volume of continental ice sheets leads to a global change in

relative sea level (RSL), which is defined as the difference between a change in 59 the ocean bottom topography and in the equipotential surface that coincides 60 with mean sea level, the geoid. This makes GIA essentially a global process. 6 In this global process, mass is conserved and redistributed gravitationally self-62 consistently, i.e. the mass redistribution on the surface and in the interior of 63 the earth affect the gravitational potential, which in turn affects the mass 64 redistribution. This is called *self-gravitation* and leads for example to a higher 65 than *eustatic* (= uniformly distributed ice-mass equivalent) RSL near an ice 66 sheet, because the mass of the ice sheet attracts the nearby ocean. Another 67 effect is the long-wavelength effect in the solid earth: If we place an ice mass on 68 the North Pole, not only the top of the earth will depress, but also the opposite 69 side. This can be understood by considering the northward movement of the 70 center of mass due to accumulation of mass at the North Pole, the subsequent 71 flattening of the North Pole due to outward movement of the solid earth and 72 the resulting change in gravitational potential. Both effects are illustrated in 73 Figure 1 for an elliptical ice load, with a height at the center of 2500 m and 74 a radius of 8° , which is on the North Pole for 10 kyrs. 75

In Section 6 we will test if we can use a eustatic ocean-load history to force 76 the flat, non-self-gravitating FE model. The reason we do not want to include 77 self-gravitation in the flat model is that we then have to iterate several times 78 over a glacial cycle (see Section 2, and Wu (2004) for a spherical model), 79 which is very demanding in terms of CPU time. We assume that we can ne-80 glect self-gravitation in the solid earth, because in a flat model the lack of 81 self-gravitation is partly compensated by the lack of sphericity (Amelung and 82 Wolf, 1994) and because we assume that some long-wavelength phenomena 83 largely cancel out when using perturbations. Using an axisymmetric Heavi-84

side load, we test in Section 4 the accuracy of predictions from the FE model 85 by comparing differences with a spherical, self-gravitating SP model with the 86 expected accuracies of the GOCE mission and the realized accuracy of the 8 BIFROST network. Using the RSES ice-load history (Lambeck et al., 1998) 88 and a simple model of Northern Europe, with CLVZs in relatively young con-89 tinental crust and ALVZs below young continental and oceanic areas, we try 90 to deduce in Section 5 if it is important to consider lateral heterogeneities in 91 LVZs. We start with a short description of the theory, especially how we com-92 pute geoid heights from the output of the flat FE model (Section 2), followed 93 by a description of the FE and SP model used in this study (Section 3). 94

95 2 Theory

The linearized, *elastic* equation of motion for geophysical problems in which
inertia can be neglected is (e.g. Wu, 2004; Sabadini and Vermeersen, 2004,
p. 5):

99
$$\vec{\nabla} \cdot \vec{\sigma}_{\delta} - \vec{\nabla} (\vec{u} \cdot \rho_0 g_0 \vec{e}_r) - \rho_{\delta} g_0 \vec{e}_r - \rho_0 \vec{\nabla} \phi_{\delta} = 0 \tag{1}$$

with $\vec{\sigma}$ the stress tensor, \vec{u} the displacement vector, ρ the density, g the gravity acceleration, ϕ the gravity potential, and where the subscripts ('0', ' δ ') denote the *initial* and *incremental* state (Wolf, 1998), respectively. For an *incompressible* material (Poisson's ratio $\nu = 0.5$), $\rho_{\delta} = 0$ and the third term (*internal buoyancy* due to material compressibility) vanishes. In this case, the gravity potential ϕ_{δ} can be found from Laplace's equation:

106
$$\vec{\nabla}\phi_{\delta} = 0$$
 (2)

The viscoelastic problem, mostly using Maxwell rheology, is generally solved 107 using normal-mode techniques which involves a Laplace transformation of 108 Eq. (1) and expanding the system of equations (1) and (2) in spherical har-109 monics (e.g. Wu and Peltier, 1982; Vermeersen and Sabadini, 1997). We will 110 call these therefore *spectral* (SP) methods. The system of equations is solved 111 from the core-mantle boundary upwards using a propagator matrix technique 112 (Sabadini and Vermeersen, 2004, p. 18). For a surface load, boundary con-113 ditions at the surface of the earth are that the normal stress is equal to the 114 load-induced pressure and that the shear stress is zero (Wu, 2004; Sabadini and 115 Vermeersen, 2004, p. 31). The boundary conditions for the gravity potential 116 are treated further on in Eq. (7). 117

The finite-element (FE) method solves the stiffness equation, which is by the principle of virtual work equivalent to the equation of motion (Wu, 2004):

$$\vec{\nabla} \cdot \vec{\sigma}_{\delta} = 0 \tag{3}$$

It thus neglects the *advection of pre-stress* (second term in Eq. (1)), the effect of internal buoyancy (third term) and the effect of *self-gravitation* (fourth term). In Section 3 we will describe how we will handle these terms when using the FE method.

To compute geoid heights with the flat FE model, we solve Eq. (2) by separation of variables. This leads for planar boundaries to a 2D Fourier transform of the gravity potential ϕ_{δ} on the horizontal (x- and y-) coordinates:

¹²⁸
$$\frac{\partial^2}{\partial z^2} \Phi(k_x, k_y, z) = (k_x^2 + k_y^2) \Phi(k_x, k_y, z)$$
(4)

with $k_x = 2\pi/\lambda_x$, $k_y = 2\pi/\lambda_y$, and λ_x , λ_y the wavelength in the x- and ydirection, respectively. The general solution to this differential equation is

131
$$\Phi(k_x, k_y, z) = A(k_x, k_y)e^{k_z z} + B(k_x, k_y)e^{-k_z z}$$
(5)

132 with $k_z = \sqrt{k_x^2 + k_y^2}$.

Suppose we have a finite earth layer with the surface at z = 0 and the bottom at $z = z_1$, overlying an infinite substratum. In the substratum, $B_2(k_x, k_y)$ should be zero to ensure that the potential is zero for $z \to -\infty$. At the boundary with the finite layer ($z = z_1$), we have the following boundary conditions (Wu, 2004; Cathles, 1975, p. 19):

¹³⁸
$$\Phi_1(z_1) = \Phi_2(z_1), \ \partial_z \Phi_1(z_1) - \partial_z \Phi_2(z_1) = 4\pi G(\rho_2 - \rho_1)U_1$$
 (6)

with Φ_1 , Φ_2 the potential in the finite layer, respectively the substratum, ρ_1 , ρ_2 the respective densities, and U_1 the vertical displacement at the boundary. From this we find that $B_1(k_x, k_y) = -(2\pi G/k_z)(\rho_2 - \rho_1)U_1e^{k_z z_1}$ in the finite layer.

At the surface of the earth (z = 0) the transformed boundary conditions are (Wu, 2004; Cathles, 1975, p. 19):

$$\Phi_0(0) = \Phi_1(0), \partial_z \Phi_0(0) - \partial_z \Phi_1(0) = 4\pi G(\Gamma + \rho_1 U_0)$$
(7)

with Φ_0 the potential above the surface and Φ_1 below the surface, and Γ the transformed surface density layer (i.e. water or ice).

As the solution should be finite outside the earth $(A_0(k_x, k_y) = 0)$, we find from Eq. (7) that $A_1(k_x, k_y) = -(2\pi G/k_z)(\Gamma + \rho_1)U_0$ in the finite layer, so

150 that the total solution becomes:

151
$$\Phi(k_x, k_y, z) = -\frac{2\pi G}{k_z} \left((\Gamma + \rho_1 U_0) e^{k_z z} + (\rho_2 - \rho_1) U_1 e^{k_z (z_1 - z)} \right)$$
(8)

¹⁵² or, at the surface of the earth:

153
$$\Phi(k_x, k_y, 0) = -\frac{2\pi G}{k_z} \left(\Gamma + \rho_1 U_0 + (\rho_2 - \rho_1) U_1 e^{k_z z_1} \right)$$
(9)

154 For a multilayer model with N layers, this can be generalized to:

155
$$\Phi(k_x, k_y, 0) = -\frac{2\pi G}{k_z} \left(\Gamma + \sum_{n=0}^N (\rho_{n+1} - \rho_n) U_n e^{k_z z_n} \right)$$
(10)

We will refer to the individual parts in the summation as displacement poten-*tials*.

To make the FE earth model self-gravitating, the displacement potential has 158 to be computed at every density contrast and applied as an extra load on the 159 corresponding boundaries, after which the FE model has to be updated, and 160 so on (see Wu, 2004, for a spherical FE model). This is very time-consuming, 161 and is shown below not to be necessary for a flat model of Northern Europe, 162 because of the relatively small size of the former Fennoscandian ice sheet and 163 the relatively small density contrasts at shallow depths (compare Table 1), 164 and because the effect of self-gravitation is partly compensated by the lack 165 of sphericity in the flat FE model (Amelung and Wolf, 1994). As the neglect 166 of self-gravitation increases the rate of displacement and the lack of spheric-167 ity seems to increase the displacement (see Figure 7 of Amelung and Wolf, 168 1994), the two effects seem to partly compensate each other under the load 160 only upon unloading the earth (positive rate of displacement, negative dis-170 placement) and enhance each other upon loading (negative displacement rate 171

¹⁷² and displacement).

173 **3** Model Description

The 3D-flat FE model used in this study is based on the commercially available 174 package ABAQUS. The area of loading is 5,840x5,840 km with a resolution of 80 175 km (73x73 elements), while the total surface area of the model is 10,000x10,000 176 km (97x97 elements) to minimize edge effects. The model consists of 14 finite 177 elements in the vertical direction, of which 11 are used to model the crust 178 and upper mantle (to 670 km depth) and 3 to simulate the lower mantle (to 179 10,000 km depth). This gives a total of 131,726 elements, all 8-node linear 180 bricks. We fix the bottom of the model and restrict the movement of the 181 edges to the vertical direction. Computations which include LVZs are very 182 time-consuming: A model with a LVZ of 10^{19} Pas takes about two weeks of 183 CPU-time on a single core 2.8 GHz processor, which gives a computation time 184 of more than four days on a parallel machine with two dual core 2.8 GHz INTEL 185 XEON processors. For comparison, the SP model we use takes less than half an 186 hour on a single core processor, independent of the viscosity stratification. As 187 we are mainly interested in perturbations due to shallow LVZs, which might 188 make the exact modeling of the lower mantle of minor importance, we test if 189 we can approximate the lower mantle with *infinite* elements or with *dashpots*. 190 The latter model is based on ideas of Hetzel and Hampel (2005) and Hampel 191 and Hetzel (2006) for earthquake triggering by GIA. 192

To test the accuracy of the FE model, we compare predictions with results from an analytical, spectral model, based on normal-mode analysis (see Section 2 and Wu and Peltier, 1982; Vermeersen and Sabadini, 1997). This model

provides predictions on an *incompressible* ($\rho_1 = 0$ in Eq. (1)), Maxwell vis-196 coelastic, spherical, self-gravitating earth and is benchmarked with other GIA 197 models (see Spada and Boschi, 2006, for comparison with an independent 198 method). To compare the output of the SP model with the output of the 199 flat FE model (Ivins and James, 1999), we map the former to a Cartesian 200 grid using the Lambert equal area projection. We take the FE model also 201 to be incompressible, and moreover, to be not self-gravitating, as explained 202 in Section 2. We simulate the advection of pre-stress by attaching Winkler 203 foundations to each density boundary (Ricchio and Cozzarelli, 1980; Williams 204 and Richardson, 1991; Wu, 2004). The Winkler foundations acts like springs 205 with a spring constant equal to the product of the gravity acceleration at 206 and the density contrast across the boundary. Upon loading, the foundations 207 act instantaneously, i.e. in the elastic limit of a viscoelastic material. In a 208 purely viscous or inviscid material, the foundations only work after a certain 209 amount of time, acting as buoyancy forces, as there can be no instantaneous 210 displacement in a viscous material (see the footnote in Cathles, 1975, p. 14). 211

The radial earth stratification used is given in Table 1, where the background 212 model consists of a 30 km fully elastic crust, a lithospheric thickness LT =213 80 km, and an upper¹ and lower mantle viscosity $\eta_{\rm UM} = \eta_{\rm LM} = 10^{21}$ Pas. 214 The perturbed model can have a lower crust or asthenosphere with a low 215 viscosity of 10¹⁹ Pas (CLVZ respectively ALVZ), either laterally homogeneous 216 or heterogeneous. Moreover, the thickness of the lithosphere can increase to 217 140 km for a cratonic area, and the crustal thickness can vary from 10 km in 218 oceanic areas to 50 km in cratonic areas. Values for the density ρ and rigidity 219

¹ this includes the asthenosphere, the low-velocity zone, the upper mantle and the transition zone as defined in Table 1.

 μ are volume-averaged from PREM (Dziewonski and Anderson, 1981).

221 4 Test Results

To test the validity of using a flat FE model for low-viscosity layers in GIA 222 studies, the results are compared with the output of a spherical SP model for 223 a Heaviside loading (Wolf, 1985; Ivins and James, 1999). The Heaviside load 224 is put on the North Pole at t = 0 s and stays there infinitely long. In most 225 cases, we evaluate predictions after 10 kyrs. The load consists of an elliptical 226 ice load, with a maximum height (at the center) of 2500 m, and a radius of 227 8° (~ 890 km, comparable with the dimensions of the former Fennoscandian 228 ice sheet), complemented with a eustatic ocean load of about -7.5 m. We will 229 first consider *total* predictions from the background model, after which we will 230 concentrate on *perturbations* due to CLVZs and ALVZs. We model a CLVZ 231 from a depth of 20 to 30 km (10 km thickness) and a viscosity of 10^{19} Pas. 232 The perturbations are the difference between the predictions of a model with a 233 CLVZ and the background model, which has a fully elastic crust. An ALVZ is 234 modeled at the bottom of the lithosphere (80 km) to 140 km, with a viscosity 235 of 10^{19} Pas. Perturbations are now due to the difference in viscosity with the 236 background model, which has a viscosity of 10^{21} Pas in this depth range, see 23 Table 1. 238

In Figure 2a we show a comparison of the vertical displacement at t = 0 s as predicted by the background model for the SP and FE method. Also the difference between the two methods is plotted, in *decimeters*. The deformation is determined completely by the density and rigidity of the earth model and shows a relatively large difference (~ 1 m) below the center of loading.

This difference can probably be attributed to the lack of self-gravitation in 244 our implementation of the FE method, which leads to a larger gravitational 245 downpull (Wu and Ni, 1996). The effect of incompressibility (Poisson's ratio 246 $\nu = 0.5$) is large in the elastic limit. If we take the earth to be a Poisson 247 solid (Haskell, 1953; Dahlen and Tromp, 1998, p. 4, 297), which is an approx-248 imation often used to simplify seismological problems and in which the Lamé 249 parameter $\lambda = \mu$ with the result that $\nu = 0.25$, the vertical displacement will 250 be about 50% larger. Note that we did only test *material* compressibility and 251 that we did not include *internal buoyancy* (third term in Eq. (1), see for a 252 short discussion Wu, 2004). The difference between the FE and SP model for 253 the subsequent (t > 0 s) viscoelastic deformation is smaller, see Figure 2b, 254 where the total vertical displacement after 10 kyrs minus the elastic displace-255 ment is plotted. Besides a long-wavelength difference, there are some smaller 256 differences in the steep part of the curve, which might be due to a lack of 257 bending in the FE model. This suggests the use of a higher resolution grid 258 or quadratic elements, but, because the induced errors are very small, only 259 linear elements are used in this study. The effect of material compressibility 260 is small for the viscoelastic deformation, decreasing the vertical displacement 261 with only 5% under the center of the load. 262

If now *infinite elements* are used in the lower mantle, instead of 3 layers of elements for the lower mantle as above (from 670 km to 10,000 km, *deep* model), we see from Figure **2c** that the response is reasonably good in the elastic limit, only showing deviation in the center. The viscoelastic response however is not large enough, see Figure **2d**. In contrast, the model with *dashpots* for the lower mantle does not show enough response in the elastic limit. This because there is no response of the upper mantle-lower mantle boundary at

670 km, as a dashpot, which represents a purely viscous material, only acts
after a certain amount of time (see the discussion about Winkler foundations
in Section 3). The model shows however a reasonable viscoelastic response.
For this work it is especially important if *perturbations* generated with these
models are accurate enough, which we will consider further on.

Continuing with the deep model, we see from Figure **3a** that total geoid heights 275 can be computed very accurately with the FE method and the use of displace-276 ment potentials as described in Section 2. Note that the differences are again 277 given in *decimeters* and that the direct effect of the load on the geoid heights 278 (the Γ -term in Eq. (10)) is not included. The comparison for vertical velocities 279 (or *uplift rates*, Figure **3b**) is also excellent, and from tests (not shown) we 280 have found that for both good heights and uplift rates the effect of material 281 compressibility is negligibly small. For horizontal velocities (Figure 4a), the 282 quality of the FE predictions deteriorates. This is probably at least partly due 283 to the horizontal boundary conditions in the FE model and partly due to the 284 lack of sphericity. However, the FE model shows the behavior that is generally 285 expected (Wu, 2005; Mitrovica et al., 1994, Figure 1) and found (Milne et al., 286 2001) for loading of the earth in GIA studies, i.e. an inward velocity near the 287 center of the load and an outward velocity just outside the loading area. This 288 corresponds with a pattern in which the peaks in the horizontal velocities cor-280 respond to the steepest parts in the uplift rates and small horizontal velocities 290 correspond to the flat parts in the uplift rates, i.e. the pattern of horizontal 291 velocities can for a large extent be explained by the (inward) gradient of the 292 uplift rates, as also plotted Figure 4a. A strong change in vertical motion thus 293 induces a large horizontal velocity in the direction of the largest stress, which 294 is under the load and at the forebulge, due to buoyancy forces on upper mantle 295

²⁹⁶ material.

For a compressible material, the FE model is in much closer agreement with the 297 (incompressible) SP model, which seems to indicate that the lack of spheric-298 ity is partly compensated by material compressibility. However, for an earth 299 model which has an order of magnitude increase in viscosity from the upper 300 to the lower mantle ($\eta_{\rm UM} = 0.5 \cdot 10^{21}$ Pas, $\eta_{\rm LM} = 5.0 \cdot 10^{21}$ Pas), the effect 301 of compressibility is smaller, see Figure 4b. Note that now also the SP model 302 predicts inward motion under the load, but that the differences with the FE 303 model are still large. We have tested if also part of the differences might be due 304 to the neglect of degree 1 in the SP model, which changes the reference frame 305 from the center of mass of the *initial* earth to the center of mass of the *incre*-306 mental earth (Cathles, 1975, p. 101). However, including degree 1 (following 307 the implementation of Greff-Lefftz and Legros, 1997) does only significantly 308 affect the far field (> 2,500 km) predictions, preventing the outward motion 309 to change to an inward motion from about 7,500 to 20,000 km. We will show 310 below that differences in horizontal velocity *perturbations* are much smaller, 311 suggesting that it is preferable to compute perturbations on a spherical back-312 ground model. Finally, it might be of interest to mention that in the elastic 313 limit (as in Figure 2a), the model predicts inward motion only for a *com*-314 pressible material (O'Keefe and Wu, 2002). For an incompressible material, 315 only outward motion is predicted due to the horizontal elongation of elements 316 under the load. 317

Differences in vertical displacement perturbations due to a CLVZ do not show the long-wavelength behavior, because it is largely differenced out, see Figure **5a**. The short-wavelength difference is still present, though the results of the SP method look suspicious near the edge of the load, which might indicate

an inaccuracy due to Gibbs' phenomenon. The shape of the displacement per-322 turbations can be attributed to *channel flow*, in which, opposite to *deep flow* 323 where relaxation times are inversely proportional to the wavelength (Cathles, 324 1975, p. 43), relaxation times are proportional to the square of the wavelength 325 (Cathles, 1975, p. 49), see the relaxation spectrum of the MC-mode in Schot-326 man and Vermeersen (2005) and the results in Klemann et al. (2007). This 327 leads to a lack of outward material flow below the center of the load and a 328 relatively large deformation at the edge of the load (see Figure IV-21 in Cath-329 les, 1975, p. 158) and in some cases even to a tilting of the upper crust under 330 the center of the load (e.g. in the presence of a weaker upper mantle, compare 331 the geoid height perturbations in Figure 7a). The comparison of Figure 5a 332 with Figure IV-21 in Cathles (1975) needs some comment, as there *total* flow 333 of a channel is shown, whereas we consider *perturbations* due to a channel in 334 a background model that also consists of a lithosphere and mantle. We ar-335 gue however that the comparison is valid, as a large part of the response of 336 the lithosphere and mantle is differenced out, and because the sensitivity to 337 the background model is not changing the *pattern* of the perturbations, see 338 Figure 7a. 339

The computed geoid height perturbations agree very well, smoothing the 340 short-wavelength differences visible for the vertical displacement perturba-341 tions, see Figure 5c. Amplitudes are small, but significantly larger than the 342 expected accuracy of the GOCE mission, which is smaller than 1 cm for wave-343 lengths longer than a few 100 km. The maximum difference with the SP model 344 is a few centimeters, which is comparable to the accuracy of GOCE, especially 345 if we take into account the suspicious behavior of the SP method near the edge 346 of the ice load (see Figure 5a). From Figure 5b we see that for an ALVZ, the 347

quality of the predictions of the FE model seems to deteriorate. We found that 348 at least 4 element-layers are needed to model the 60 km thick ALVZ and that 349 the improvement using 6 layers is only marginal. The remaining discrepancy 350 can probably be attributed to the lack of self-gravitation in the flat model, as 351 the difference for the 4 element-layer shows a trend from negative under the 352 load towards zero in the far field, as was also found for the elastic limit of the 353 total displacements (Figure 2a). The short wavelength difference is probably 354 due to the use of linear elements, as explained in discussing the differences 355 between the results of the FE and SP method in Figure 2b. The same con-356 clusions seem to hold for ALVZ-induced geoid height perturbations as shown 357 in Figure 5d. 358

As discussed before, it would be useful if we could replace the layering of the 350 lower mantle with either infinite elements or dashpots in computing perturba-360 tions. In Figure **6a** we compare the perturbations in vertical displacement due 363 to a CLVZ computed with a deep model (lower mantle to 10,000 km), a model 362 with infinite elements for the lower mantle, and a model with dashpots for the 363 lower mantle. Differences are small, though in general larger than the differ-364 ences between the deep model and the SP model. However, for geoid height 365 perturbations (Figure 6b), differences are larger than a few centimeters and 366 it seems necessary for our purposes to use the deep model. The differences 367 can probably be explained by the contribution of the displacement potential 368 (see Eq. (10) and the discussion there) of the upper mantle-lower mantle (670 369 km) boundary to the geoid height perturbations, of which the movement is 370 not modeled very accurately for especially the model with infinite elements 371 (see Figure 2d). The contribution of this boundary is small compared to the 372 surface contribution, but not much smaller in magnitude than the contribu-373

tion of the much shallower (30 km) crust-lithosphere boundary. This is due 374 to the larger density contrast at 670 km (1020 vs. 680 kg/m³ at 30 km, see 375 Table 1) and due to the longer wavelength induced at 670 km (about five times 376 larger than at 30 km). For an ALVZ, the displacement potential of the upper 377 mantle-lower mantle boundary has, due to the large effect of an ALVZ on the 378 movement of this boundary (amplitude of ~ 10 m to ~ 2 m for a CLVZ), a 379 significant contribution to the geoid height perturbations, being only smaller 380 than the surface contribution. Predictions using a model with the lower man-383 the replaced by dashpots or infinite elements are thus worse than for a CLVZ, 382 and are therefore not shown. 383

The perturbations are sensitive to the background model, see Figure 7a, where 384 we have plotted the CLVZ-induced perturbations for our reference background 385 model (LT = 80 km, $\eta_{\rm UM} = \eta_{\rm LM} = 1.0 \cdot 10^{21}$ Pas, '80/ 1.0/ 1.0'), and mod-386 els with changes in $\eta_{\rm LM}$, $\eta_{\rm UM}$ and LT. The effect of a weaker upper mantle 387 (compare '80/ 0.5/ 5.0' and '80/ 1.0/ 5.0') is largest, because of additional 388 outward material flow in the upper mantle and larger bulges just outside of 380 the load, i.e. the depression under the load is deeper and steeper. This in-390 creases the stress in the CLVZ just inside and outside the loading area (in the 393 steep part of the depression and the bulge), enhancing mass flow through the 392 CLVZ from the edges outwards, and tilting of the upper crust in the center. 393 The background model does not change the pattern of channel flow, which 394 is not the case for ALVZ-induced perturbations, see Figure 7b. Where the 395 perturbations for our reference background model are determined by channel 396 flow (small perturbations under the center of the load), the perturbations for 397 a weaker upper mantle ($\eta_{\rm UM} = 0.5 \cdot 10^{21}$ Pas) show deep flow behavior. This 398 effect is, somewhat less pronounced, also present for only a stiffer lower mantle 399

400 $(\eta_{\rm LM} = 5.0 \cdot 10^{21} \text{ Pas}).$

Perturbations in horizontal velocities show (see Figure 8), in contrast with 401 total horizontal velocities (Figure 4a), the same behavior for the FE and SP 402 method. Both for a CLVZ and an ALVZ, the quality deteriorates towards the 403 far field, at least partly due to the boundary conditions in the FE model, 404 which are fixed in the horizontal direction at 10,000 km. If we compare this 405 with the SP model, in which, due to symmetry considerations, the horizontal 406 direction is fixed at the south pole ($\sim 20,000$ km), we argue that part of the 407 difference can be explained by the larger horizontal elongation or stretching 408 of the crust/lithosphere for a shorter distance to a fixed point, i.e. for the FE 409 model. However, due to the large differences between the FE and SP model for 410 total horizontal velocities (Figure 4a), other causes such as the lack of spheric-411 ity cannot be ruled out. The patterns can again for a large part be explained 412 by the inward gradient of the uplift rates (not shown). The noisy character 413 of the perturbations, especially for a CLVZ, are due to the double differenc-414 ing and the relatively low precision ('single', default) used for the output of 415 ABAQUS, which can be changed to 'double' in the execution procedure. Note 416 that the differences with the SP model are small compared to the accuracy 417 of the BIFROST network ($\sim 0.3 \text{ mm/yr}$), however, so are the perturbations. 418 This means that LVZs can probably not be detected by BIFROST, especially 419 because high-resolution spatial information is not provided. 420

In Figure **9a** perturbations are shown due to a laterally heterogeneous CLVZ, that extends from just under the load ('A', 670 km) outward. This means that the laterally heterogeneous model is equal to the background model from the center of the load to 'A', and equal to the model with a laterally homogeneous CLVZ from 'A' outwards. Because under the load the induced stresses

are largest, the response is comparable to the homogeneous case from 670 km 426 outwards. Inwards, there seems to be some tilting of the crust relative to the 427 displacement of the crust in the background model, due to the indentation 428 of the crust from 'A' outwards. For a CLVZ from 1110 km ('B') outwards, 420 the effect of the lateral heterogeneity is much more pronounced, and leads 430 to a significant relative tilting of the crust. For an ALVZ the effect of lateral 431 heterogeneities is larger, substantially weakening the amplitudes of the homo-432 geneous case, but mainly showing the same effect as for a CLVZ and therefore 433 not shown. As shallow viscosity and lithospheric thickness LT are via thermal 434 regime closely related, we show in Figure **9b** the effect of the inclusion of a 435 larger LT ('craton') under the load only, and the inclusion of both a larger 436 LT under the load and a laterally heterogeneous CLVZ. The effect of a cra-437 ton under the load is to strengthen the lithosphere, thus de-amplifying the 438 vertical displacement in the center, and thus inducing a positive geoid height 439 perturbation. If the craton extends to outside the load ('craton < B (1110 440 km)'), there is also a negative perturbation outside the load, because, due to 441 the larger lithospheric strength, the bulge area is broader and smoother. If we 442 now compare the perturbations due to variations in LT only with perturba-443 tions due to both a laterally heterogeneous LT and CLVZ, we see that only 444 for the case that part of the ice load is in an area of small LT and a CLVZ 445 (craton < A, CLVZ > A'), differences are as large as 10 cm. Due to the differ-446 ent scales of deformation for variations in lithospheric thickness (wavelengths 447 mainly larger than 1000 km) and for a CLVZ (mainly smaller than 1000 km), 448 the two effects might be separated using medium- to high-resolution gravity 440 field information as provided by GOCE. 450

451 **5** Example for Northern Europe

We will now investigate the effect of lateral heterogeneities in LVZs and LTusing a simple regional model for Northern Europe, in which we assume 3 different types of areas, see also Figure **10a-c**:

- I. Continental areas with a 50 km thick elastic crust in a thick elastic lithosphere (LT = 140 km), on top of an upper mantle with a viscosity of 10^{21} Pas, which represent old, cratonic areas that are mainly Archean and Early Proterozoic (> 1.5 Gyr, Pérez-Gussinyé and Watts, 2005). This type corresponds well with estimates of thermal thickness larger than 140 km (Artemieva and Mooney, 2001);
- II. Continental areas with 20 km elastic upper crust and a 10 km lower 461 crust that is either elastic or has low viscosity (10^{19} Pas, CLVZ). The 462 total lithospheric thickness is thinner now (LT = 80 km) and overlies 463 a 60 km thick as then on that has either the viscosity of the upper 464 mantle (10^{21} Pas) or a low viscosity $(10^{19} \text{ Pas}, \text{ALVZ})$. These areas are 465 younger (< 1.5 Gyr) and have a thermal thickness smaller than 140 km; 466 III. Oceanic areas with a 10 km crust in a lithosphere of 80 km, overlying a 60 467 km thick as then opphere with a viscosity of 10^{21} Pas or 10^{19} Pas (ALVZ). 468

The different areas are derived from CRUST2.0 (http://mahi.ucsd.edu/Gabi/ rem.dir/crust/crust2.html, accessed 3 May 2007), in which an area with a crustal thickness between 15 and 35 km is considered as Type II. Areas with a crustal thickness thinner than 15 km are then Type III and thicker than 35 km Type I. The different areas are indicated in Figures **10d**. Note that this is a first approximation, which means that some areas in Type II are

oceanic and that some areas in Type I are not cratonic. We use the ice-load 475 history as derived by Lambeck (RSES, Lambeck et al., 1998), which is given 476 from 30 kyrs BP to present for Fennoscandia, the Barents Sea area and the 477 British Isles. We complement this with the eustatic ocean-load history of the 478 global RSES model, which is about -130 m from 30 kyrs BP to the last glacial 479 maximum (LGM, 21 kyrs BP), and include a linear glaciation phase of 90 480 kyrs to have a glacial cycle of 120 kyrs. We have plotted the ice heights at 481 LGM in Figure **10e**. 482

In Figure **11a** we show total (i.e. background-induced) 3D-velocity predictions 483 for Northern Europe computed with the FE model. Contour lines show the 484 difference in uplift rates with the SP model, the black arrows are horizontal 485 velocities computed with the FE model and the white arrows are computed 486 with the SP model. As expected from Figure 3b, the uplift rates compare 487 well, with only some small areas where the difference is larger than 0.2 mm/yr488 (which is much smaller than the accuracy of about 1 mm/yr as reported for 480 the BIFROST network, Johansson et al., 2002). Note that the predicted uplift 490 rate is sensitive to the earth stratification, which explains differences with 491 e.g. Milne et al. (2001), who use the same ice-load history. The horizontal 492 rates compare less well, though not as bad as expected from Figure 4a. In 493 Figure 11b we show the total geoid height predictions. Contour lines are 494 again differences with the SP model, which are generally smaller than 2 cm, 495 which is comparable to the accuracy of the GOCE mission. Note that we have 496 subtracted the mean from both fields, as we have in our FE computations no 497 self-gravitating sea level that can serve as a reference for the geoid height. 498

⁴⁹⁹ Also for LVZ-induced perturbations in 3D-velocities we see some differences ⁵⁰⁰ between FE and SP (Figure **12**), though the general pattern agrees well.

Note that perturbations in horizontal velocities are up to 0.5 mm/yr, which is 501 slightly larger than the accuracy of the BIFROST network of about 0.3 mm/yr. 502 However, due to the point-like character of GPS measurements, which makes 503 the extraction of relatively high-resolution spatial signatures difficult, and the 504 fact that part of the perturbative signal is generated by assuming LVZs be-505 low Scandinavia, we expect that BIFROST cannot add information on LVZs. 506 This is confirmed by Milne et al. (2001), who indicate that the radial resolving 507 power of their inversion is ~ 200 km in the shallow earth. 508

In Figure 13a we show geoid height perturbations due to a laterally homo-509 geneous CLVZ, in which contour lines indicate again differences with the SP 510 model. The pattern can be explained by lower crustal material flow through 511 the low-viscosity channel from under the ice load outwards during glaciation, 512 and the long relaxation times of crustal channel flow (Schotman and Ver-513 meersen, 2005). Due to the long glaciation period (~ 100 kyrs), the earth is 514 close to isostatic equilibrium at LGM and during the much shorter deglacia-515 tion period (~ 20 kyrs) only part of the crustal material has returned from the 516 perturbative bulges. For a homogenous ALVZ the differences are also small, 517 see Figure 13b. The pattern is broader, and moreover of opposite sign. This 518 is due to the fact that an ALVZ accelerates the deformation process, as de-519 scribed in Schotman and Vermeersen (2005). To highlight the effect of lateral 520 heterogeneities in the LVZ, we show in Figure 13c, d the difference between 521 a model which includes a laterally heterogeneous LVZ and a model which in-522 cludes a laterally homogeneous LVZ (as shown in **a**, **b**). To focus on the LVZ 523 only, we have not yet included variations in the lithospheric thickness. For a 524 CLVZ the differences are relatively small, though still with amplitudes up to 525 20 cm. Of course, the differences are large in the Baltic Shield, but also in 526

⁵²⁷ southern Scandinavia, at the edge of the cratonic area, and off the Norwegian
⁵²⁸ coast, where the geoid high predicted by the model with a homogeneous CLVZ
⁵²⁹ is for the model with a laterally heterogeneous CLVZ significantly smaller. For
⁵³⁰ an ALVZ, the effect of lateral heterogeneities is somewhat larger, showing a
⁵³¹ clear difference under the main domes of the RSES ice-load history, but also
⁵³² further outwards, especially to the east, due to the larger depth of the ALVZ
⁵³³ compared to the CLVZ.

Before we look at the simultaneous effect of lateral variations in LT and LVZs, 534 we show the effect of variations in LT in the absence of an LVZ. The geoid 535 height perturbations in Figure 14a are thus only due to the thicker lithosphere 536 (LT = 140 km) of the Baltic Shield. The effect is largest over the Baltic Sea, 537 as there the ice load is largest, see Figure 10e. In the perturbations due to 538 lateral heterogeneities in both LT and a CLVZ (Figure 14b), we can still see 539 the effect of the CLVZ around the formerly glaciated area (geoid highs of 20-540 30 cm, see Figure 13a) and the effect of the large LT below the largest ice 541 load (geoid high of 50 cm, compare with Figure 14a). Due to the different 542 spatial signatures of both effects, it seems to be possible to separate the two 543 and extract information on both LT and crustal low-viscosity. For an ALVZ 544 (Figure 14c) the separation of the two effects seems to be difficult, as the effect 545 of variations in LT (Figure 14a) is very similar to ALVZ-induced perturbations 546 (Figure 13b), especially along the Norwegian coast and under the major ice 547 domes. 548

549 6 Realistic Ocean-Load History

For loading the FE model for Northern Europe in Section 5, we used the 550 eustatic ocean-load output of the SP model. Here we show which error this 551 introduces in geoid height predictions compared with predictions of a self-552 gravitating model that includes realistic oceans ('RO', which includes the effect 553 of self-gravitation in the oceans, but also the effect of time-dependent coast-554 lines and meltwater influx, see Section 1). For total good heights predicted 555 with the RSES ice-load history, the errors introduced by using the eustatic 556 load output of the SP model is shown in Figure 15a. The maximum error 557 is about 20 cm, which is small compared to the total amplitudes of several 558 meters (Figure 11b). Moreover, the relatively large error over the Northern 559 Atlantic (geoid height predictions in this area are about zero) can be removed 560 if we use the realistic ocean-load output of the SP model. However, the er-561 rors in areas that were once glaciated and are now ocean covered increase to 562 30 cm (Barents Sea, North Sea) and 50 cm (Gulf of Bothnia). For CLVZ-563 induced perturbations, the error is generally smaller than 5 cm (Figure 15b), 564 compared to CLVZ-induced geoid height perturbations of tens of centimeters 565 (Figure 13a). Note the clear effect of time-dependent coastlines and meltwater 566 influx in the Gulf of Bothnia, which is also present if we use the realis-567 tic ocean-load output of the SP model. Overall errors are somewhat smaller 568 then, but probably not negligibly small. For an ALVZ, the errors are up to 569 10 cm (Figure 15c), but ALVZ-induced geoid height perturbations have am-570 plitudes that are about 5 times as large, compare Figure 13b, so these errors 571 can probably be neglected. 572

573 7 Conclusion

We have shown that a 3D-flat, non-self-gravitating finite-element (FE) model 574 of glacial-isostatic adjustment (GIA) can deliver accurate predictions of ra-575 dial displacements, uplift rates and geoid heights when compared to conven-576 tional spherical, self-gravitating spectral (SP) models. Perturbations in these 577 predictions due to shallow low-viscosity zones (LVZs) can also be computed 578 accurately, though the accuracy seems, due to the neglect of self-graviation, to 579 decrease somewhat with increasing depth, and moreover, the sensitivity to the 580 background model increases considerably. Predictions of horizontal velocities 581 are less accurate, showing considerable differences with the predictions of an 582 SP model, probably due to the lack of sphericity of the model. We have to 583 keep in mind however that horizontal velocities are very sensitive to model 584 parameters in general. We have shown that material compressibility can have 585 a significant effect on horizontal velocities. The effect on uplift rates and good 586 heights is found to be negligibly small. 587

For a simple model of Northern Europe, horizontal and vertical velocity per-588 turbations due to shallow LVZs are only slightly above the current BIFROST 589 performance and due to the relatively low spatial resolution of BIFROST, we 590 do not expect BIFROST to add information on LVZs. We have found that lat-591 eral heterogeneities do not necessarily have a large influence on CLVZ-induced 592 perturbations, though amplitudes are still up to 20 cm in geoid height, which 593 is an order of magnitude larger than the expected GOCE performance. In real-594 ity, however, we also have to consider unmodeled thermally- and chemically-595 induced shallow mass inhomogeneities, that are expected to be effective in 596 masking the gravity signal due to CLVZs. We find, for example, from the 597

GRACE satellite gravity mission (solution GGM02S, Tapley et al., 2005), 598 only retaining the spherical harmonic coefficients from degree 40 to 90, where 599 CLVZs have their largest signal, geoid heights of a few meters. Effective spatio-600 spectral filtering tools have to be developed to extract the GIA-related signal 601 (see Schotman and Vermeersen, 2005, for a discussion), in which we expect 602 the specific signatures of CLVZs to be useful. For example, because of the 603 different signature of perturbations due to a CLVZ and due to variations in 604 lithospheric thickness, it seems possible to solve for both lithospheric thickness 605 and CLVZs at the same time. 606

Forcing the FE model with a eustatic ocean load gives in general acceptable 607 results, which can be improved somewhat using the realistic load output of 608 the self-gravitating SP model. Errors in CLVZ-induced geoid height pertur-600 bations are about 5 cm, which is small but not negligible compared to the 610 amplitudes of the perturbations (tens of centimers). For an ALVZ, the errors 611 are somewhat larger, but small compared to the perturbations. Based on the 612 above we have given in Table 2 overview of error estimates using the flat, not 613 self-gravitating FE model and the spherical, self-gravitating SP model ('FE vs. 614 SP'), for geoid heights and geoid height perturbations ('GEO') and horizontal 615 velocities and velocity perturbations ('VEL'). We have also given the error 616 estimates of using a eustatic instead of realistic ocean load ('EU vs. RO'). If 617 we accept errors equal to or smaller than 10%, we have to be critical when 618 computing (perturbations in) horizontal velocities and when using a eustatic 619 ocean load for computing CLVZ-induced geoid height perturbations. For this 620 error bound, we cannot use infinite or dashpot elements to replace the deep 621 layering of the lower mantle (to 10,000 km), as the errors are larger than 10%622 for LVZ-induced perturbations. 623

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Fig. 1. Near- (a) and far- (b) field effect of an elliptical iceload (8° radius) on the



Fig. 2. Elastic (t = 0 s, 'E', left) and viscoelastic (t > 0 s, total response minusresponse at t = 0 s, 'VE', right) response for the SP and FE model (**a**, **b**) and a *deep* model (lower mantle to 10,000 km), and *shallow* models using *infinite* elements for the lower mantle, or using *dashpots* for the lower mantle (**c**, **d**)



Fig. 3. Comparison of geoid heights (a) and uplift rates (b) as predicted by the SP and FE method



Fig. 4. Horizontal velocities (positive is outward motion) as predicted by the SP and FE method for a univiscous (Table 1) earth model (a) and an earth model with an order of magnitude increase in viscosity from the upper to lower mantle (b)



CRI =1 D P1 CCE РΤ



Fig. 5. CLVZ- (left) and ALVZ- (right) induced vertical displacement (a, b) and geoid height (c, d) perturbations due to a laterally homogeneous LVZ

A Cos

(b) ALVZ, vertical displacement



Fig. 6. CLVZ-induced vertical displacement (a) and geoid height (b) perturbations for a *deep* model (lower mantle to 10,000 km), a model using *infinite* elements for the lower mantle, and a model using *dashpots* for the lower mantle



Fig. 7. Effect of the background earth model on CLVZ- (a) and ALVZ- (b) induced perturbations (the legend gives: LT [km]/ $\eta_{\rm UM}$ [10²¹ Pas]/ $\eta_{\rm LM}$ [10²¹ Pas])





Fig. 8. CLVZ- (a) and ALVZ- (b) induced horizontal velocity perturbations (positive direction is outward motion)



Fig. 9. Geoid height perturbations due to a laterally heterogeneous CLVZ (a) and due to a laterally heterogeneous LT and CLVZ (b)

P-Cox



Fig. 10. Earth stratification for the 3 areas: type I (a), type II (b) and type III (c), corresponding geographical areas based on CRUST2.0 (d), and RSES ice heights at LGM (e)



Fig. 11. Total (no LVZ) 3D-velocities (a) and good heights (b) at present, computed with FE (white arrows are computed with SP, contour lines are difference with SP)



Fig. 12. CLVZ- (a) and ALVZ- (b) induced 3D-velocity perturbations (white arrows are computed with SP, contour lines are difference with SP)



(a) CLVZ, lat. hom.

(b) ALVZ, lat. hom.

Fig. 13. Geoid height perturbations for a laterally homogeneous CLVZ (a) and ALVZ (b), and differences between a laterally heterogeneous and homogeneous CLVZ (c) and ALVZ (d)



Fig. 14. Geoid height perturbations due to a laterally heterogeneous LT in the absence of an LVZ (a), and combined effect of a laterally heterogeneous LT and CLVZ (b) or ALVZ (c)



Fig. 15. Errors introduced by using a eustatic ocean load in geoid heights for the background model (a), and in geoid height perturbations for a CLVZ (b) and ALVZ

Table 1

Radial earth stratification in which the lower crust and the asthenosphere can have an LVZ of 10^{19} Pas

Layer	Depth	Density ρ	Rigidity ν	Viscosity η
	[km]	[kg/m3]	[GPa]	[Pas]
upper crust	0	2700	27	elastic
lower crust	20	:	\sim	elastic or 10^{19}
lithosphere	30	3380	68	elastic
asthenosphere	80		:	$10^{21} \text{ or } \mathbf{10^{19}}$
low-velocity zone	140	2	:	10^{21}
upper mantle	220	3480	77	:
transition zone	400	3870	108	:
lower mantle	670	4890	221	10^{21}
core	2891	10925	0	0

Table 2

Error estimates for the use of a regional model in a global background model. GEO are geoid heights and VEL are horizontal velocities, L indicates that the error has a long-wavelength (> \sim 1000 km) signature and S a short-wavelength (< \sim 1000 km).

Source	Quantity	total	CLVZ pert.	ALVZ pert.
FE vs. SP	GEO	1% (L)	5% (S ^a)	10% (S ^a) + 10% (L)
	VEL	50% (L)	$5\% (L^{b})$	$25\%~({\rm L~^b})$
EU vs. RO	GEO	1% (L)	25% (S ^c)	10% (L)

^a can probably be removed using smaller gridsize or quadratic elements (Sec-

tion 4)

^b can probably be reduced by increasing the horizontal extent of the FE model

(Section 4)

^c can be somewhat reduced using the RO instead of the EU load output of the CD_{const} = 1.1 (C_{const} : C_{const})

SP model (Section 6)