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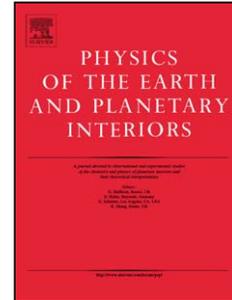
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A statistical analysis of polarity reversals of the geomagnetic field

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Abstract

We investigate the temporal distribution of polarity reversals of the geomagnetic field. In spite of the common assumption that the reversal sequence can be modeled as a realization of a renewal Poisson process with a variable rate, we show that the polarity reversals strongly depart from a local Poisson statistics. The origin of this failure can be attributed to temporal clustering, thus suggesting the presence of long-range correlations in the underlying dynamo process. In this framework we compare our results with the behavior of different models that describe the time evolution of the reversals.

Key words: Geomagnetism, Paleomagnetic measurements, Dynamo effect

PACS: 91.25.Mf, 91.25.-r, 02.50.-r, 91.25.Cw

1 Introduction

Local paleomagnetic measurements of the geomagnetic field (Merrill et al., 1996; Hollerbach, 2003; Cande and Kent, 1995) are currently used to extract informations about the geomagnetic dipole, thus providing information regarding the geodynamo process. Unlike the solar magnetic field, where the polarity reversals are strictly periodic, with a main period of 22 years, geomagnetic

measurements reveal a sequence of sudden and occasional global polarity reversals in the last 160 million years. The typical duration of reversals is a few thousand years, that is much shorter than the typical time interval between successive reversals, which may range from 10^4 up to 10^7 years (Merrill et al., 1996; Cande and Kent, 1995; Valet and Meynadier, 1993). Despite the considerable work performed both on data analysis and on theoretical modeling, the main fundamental questions concerning the relation between the Earth's magnetic dynamo process and polarity reversals remain still unanswered (Merrill et al., 1996; Roberts and Glatzmaier, 2000; Moffatt, 1978; Stefani and Gerbeth, 2005). The nature of the triggers (external or internal to Earth) and the physical mechanisms giving rise to the reversals, the reason for the long time variations in the average reversal rate (cf. e.g. Hollerbach (2003); Yamazaki and Oda (2002)), are all open problems.

The sequence of geomagnetic reversals displays a behavior which seems to be the result of a chaotic (or stochastic) process. An example of reversal dataset, namely the well known CK95 database (Cande and Kent, 1995), is shown in Fig. 1. In principle, the geodynamo should be described by 3D global Magnetohydrodynamics (MHD) that self-consistently solve for the fluid flow, thermodynamics and magnetic fields with all nonlinear feedbacks (for a review see Roberts and Glatzmaier (2000) and references therein, and the results of the recent 15.2 TFlops simulation of geodynamo on the Earth Simulator (Kageyama et al., 2004; Kageyama and Yoshida, 2005)). However, even the most advanced numerical codes developed so far do not have a high enough resolution to be confident that the critical dynamics is being captured by the simulation, and moreover they are able to simulate only short series of spontaneous reversals. Non regular reversals can be observed also in several types of simplified models, such as purely deterministic toy models mimicking the dynamics of the dynamo effect with just few modes (Rikitake, 1958; Crossley et al., 1986; Turcotte, 1992), models of noise-induced switchings between two metastable states (Schmitt et al., 2001; Hoyng et al., 2002; Hoyng and Duijstermaat, 2004), or mean-field dynamo models with a noise-perturbed α profile (Giesecke et al., 2005; Stefani and Gerbeth, 2005).

A statistical approach is extremely useful for the characterization of a complex process such as the time evolution of the geomagnetic field. Notwithstanding the paucity of datasets, it was commonly assumed that the reversal sequence is produced by an underlying Poisson process. This assumption relies on the fact that the distribution of persistence times, defined as the time intervals between two consecutive reversals $\Delta t = t_{i+1} - t_i$ seems to follow an exponential (Merrill et al., 1996; Hoyng et al., 2002; McFadden and Merrill, 1997; Constable, 2000), namely $P(\Delta t) = \lambda \exp(-\lambda \Delta t)$, where λ represents the reversal occurrence rate. However, also other distributions are invoked in the literature, see e. g. Jonkers (2003), where the author claims that the frequency distribution of time intervals between Cenozoic geomagnetic reversals approximates a power law

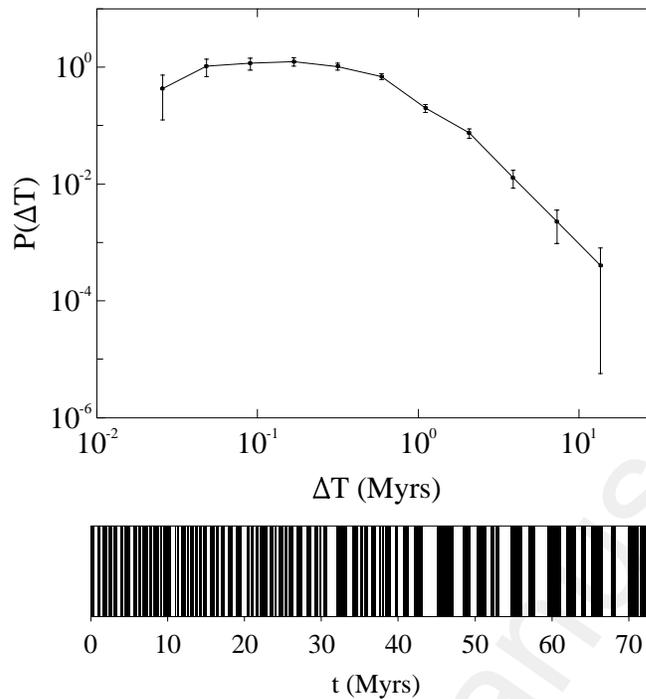


Fig. 1. Bottom: Polarity of the earth’s magnetic field (from today) as in the CK95 record (partial). The black bars are the normal (present) polarity. Top: the probability density function $P(\Delta t)$ of persistence times Δt for CK95 database (statistical errors are shown as vertical bars).

for large Δt . Due to the small number of events in the datasets, the probability distribution function $P(\Delta t)$, shown in Fig. 1 for the CK95 dataset, does not clearly show, at a first look, a clear behaviour.

Another interesting feature which has been recently brought to the attention of the scientific community by Constable (2000) is the time dependence of the occurrence rate of events $\lambda = \lambda(t)$. The author showed that a Poisson model with a monotonic varying rate, either increasing or decreasing, fails in describing the reversal process. Nevertheless, reversals could perhaps be modeled as a renewal Poisson process with a rate that must change sign at some interval before 158 My (Constable, 2000). In any case, modeling the variations of $\lambda(t)$ over the entire time interval is a very delicate issue (cf. also McFadden and Merrill (1984); Gallet and Hulot (1997); McFadden and Merrill (1997)). Moreover, when the occurrence rate λ depends on time, it is difficult to determine the Poisson character of events and to give a clear physical interpretation to the persistence time distribution (Feller, 1968).

Here, starting from the above empirical evidences, and using a simple statistical test on some databases, we investigate whether a conjecture based on

the occurrence of a Poisson process for reversals is correct or not. We show in Section 2 that this is indeed not the case, and that geomagnetic reversals are clustered in time, a result which can be attributed to the presence of memory in the process generating polarity reversals. In Section 3 the results obtained from data analysis are compared with those arising from some dynamical models of the geodynamo. Conclusions are given in Section 4.

It is also worth mentioning that the problem studied in this paper is of broader interest, as abrupt flow reversals have been observed also in the large-scale circulation during turbulent Rayleigh-Benard convection (Benzi, 2005; Araujo et al., 2005; Tsuji et al., 2005; Brown et al., 2005), or in the wind direction in atmosphere (van Doorn et al., 2000). In all these cases, it is assumed that reversals are Poisson events.

2 Local Poisson hypothesis

Following the empirical evidence that the rate of reversals is not constant, we can test, as a zero-th order hypothesis, whether the reversal sequence is consistent with a *Local Poisson Process*. More precisely, we will test the hypothesis (hypothesis H_0) that the geomagnetic reversals are originated by a time-varying Poisson process. Since the reversals rate $\lambda(t)$ is not known, the test must be independent on the rate λ . This can be done through a method introduced some years ago in cosmology (Bi et al., 1989) and used more recently in the context of solar flares (Boffetta et al., 1999; Lepreti et al., 2001) and geomagnetic activity (Lepreti et al., 2004). We consider the time sequence of reversals as a point-like process and suppose that each reversal occurs at a discrete time t_i . We then introduce in the following way a quantity, which we denote by h , that is nothing but the suitably normalized local time interval between reversals.

We define δt_i as

$$\delta t_i = \min\{t_{i+1} - t_i; t_i - t_{i-1}\}, \quad (1)$$

and τ_i by

$$\tau_i = \begin{cases} t_{i-1} - t_{i-2} & \text{if } \delta t_i = t_i - t_{i-1} \\ t_{i+2} - t_{i+1} & \text{if } \delta t_i = t_{i+1} - t_i \end{cases} \quad (2)$$

δt_i and τ_i are then the two persistence times following or preceding a given reversal at t_i . If the local Poisson hypothesis H_0 holds, both δt_i and τ_i are

independently distributed according to an exponential probability density: $p(\delta t) = 2\lambda_i \exp(-2\lambda_i \delta t)$ and $p(\tau) = \lambda_i \exp(-\lambda_i \tau)$ with local rate λ_i . The distribution of the variable h defined by

$$h(\delta t_i, \tau_i) = \frac{2\delta t_i}{2\delta t_i + \tau_i} \quad (3)$$

does not depend on λ_i .

We define now the surviving function of the probability density

$$P(h \geq H) = \int_H^\infty P(h) dh = \int_0^\infty dx 2\lambda e^{-2\lambda x} \int_0^{g(x,H)} dy \lambda e^{-\lambda y} \quad (4)$$

where $P(h)$ is the probability density function of h and

$$g(x, H) = 2x \left[\frac{1}{H} - 1 \right].$$

It can be easily shown that, under the hypothesis H_0 ,

$$P(h \geq H) = 1 - H,$$

that is, h is a stochastic variable uniformly distributed in $h \in [0; 1]$. In a process where τ_i 's are systematically smaller than $2\delta t_i$'s, clusters are present and the average value of h is greater than $1/2$. On the contrary, when the process is characterized by voids, the average value of h is less than $1/2$. From time series, it is easy to calculate the surviving function $P(h \geq H)$ and the probability density function $P(h)$.

The test described above was applied to four different datasets of geomagnetic polarity reversals, namely to the already mentioned CK95, including 185 reversals (Cande and Kent, 1995), to H68 with 167 reversals (Heirtzler et al., 1968), HA97, 127 reversals (Huestis and Acton, 1997) and KG86, 283 events (Kent and Gradstein, 1986). The probability density function $P(h)$ is reported in Fig. 2 for all the considered datasets. A significant deviation from the uniform distribution can clearly be observed in all the datasets. Moreover, the different datasets have the same surviving function, showing the robustness of this result. This departure of polarity reversals from local Poisson statistics is due the presence of clusters. As a consequence, a clear deviation of the cumulative probability $P(h \geq H)$ from a linear law, expected under H_0 , is clearly seen in Fig. 2. A Kolmogorov-Smirnov (KS) test applied to the cumulative distributions confirms that the assumed hypothesis H_0 is not reliable and must

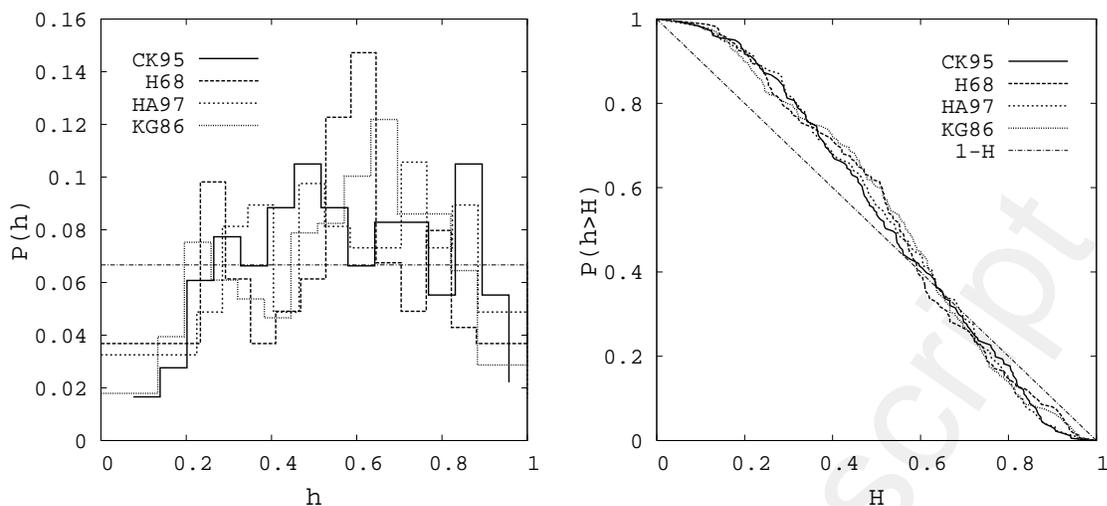


Fig. 2. Probability densities $P(h)$ of the stochastic variable h and corresponding surviving functions $P(h \geq H)$ for all the empirical datasets. The theoretical probability expected under a Poisson statistics is also shown.

be rejected (the significance level of the KS test being smaller than 0.5% for all datasets). In order to better emphasize the significance of the deviation of the data from Poisson statistics, in Fig. 3 we compare the results of the test performed on the CK95 dataset, with those arising from a dataset obtained by generating random intervals (from a Poisson deviates) with a variable rate. This dataset represents an example of renewal Poisson process, as discussed in Constable (2000). The events variable rate $\lambda(t)$ is chosen to approximately represent the empirical result of Constable (2000). In order to get this behaviour, we adopted a piecewise linear fit of the Constable measure, capturing the decreasing part, the flat part, and the reversed, increasing rate period. We believe that the details of the curve presented in Constable (2000) do not affect the statistical properties of the persistence times, which are in any case, by definition, locally Poisson. We considered a small number of events (250), of the order of the real dataset. Thus, this plot shows that the deviation of the CK95 dataset from local Poisson statistics is strongly significant, and the renewal Poisson process is unable to capture the presence of correlations in the data. This is also confirmed by a KS test, which does not allow to reject the H_0 hypothesis (the significance level of the KS test being 63% for the synthetic data).

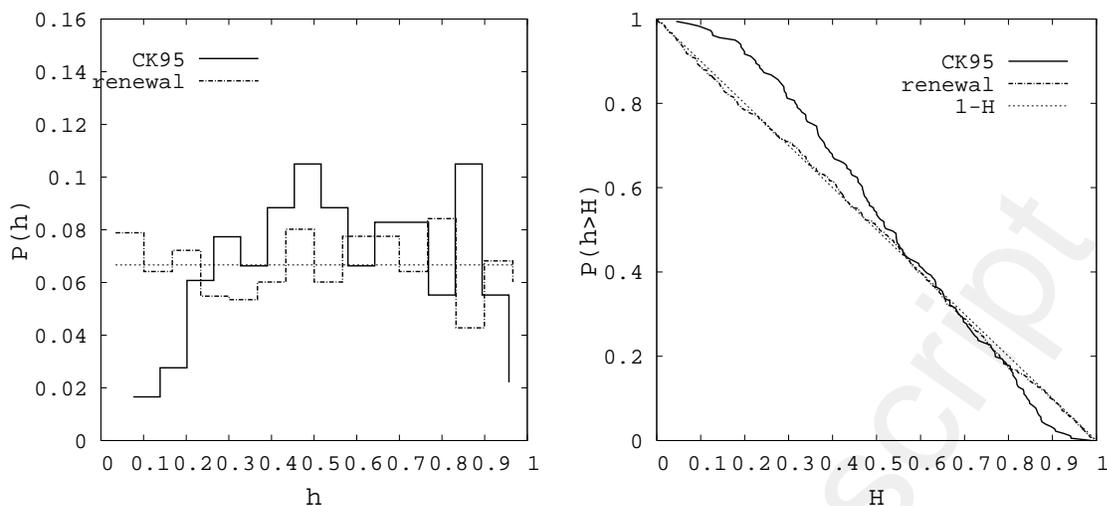


Fig. 3. Probability densities $P(h)$ of the stochastic variable h and corresponding surviving functions $P(h \geq H)$ for the CK95 dataset and for a renewal Poisson process, as described in the text. The theoretical probability expected under a Poisson statistics is also shown.

3 Modeling geodynamo

In the previous section we showed that clustering is present in the data (cf. Sec. 2). This indicates that the process underlying the polarity reversals is characterized by memory effects, due to the presence of long-range correlations. The fact that the dynamics of the fluid earth core is affected by its history, with generation of correlations in the reversal sequence, is not surprising from a physical point of view. Indeed, a similar behaviour has been recently observed for the solar dynamo (Dikpati, 2005). However, it should be pointed out that many different stochastic processes can reproduce the departure from a Poisson statistics. With the next generations of computers, it will be hopefully possible that 3D global MHD simulations of geodynamo will produce datasets with enough reversals to allow us to investigate in detail the occurrence of long-range correlations. In this framework, statistical analysis on real data, among other, plays a key role in discriminating among different stochastic processes that can reproduce the departure from Poisson statistics, thus increasing our knowledge of the geodynamo process. A further understanding of the phenomenon can be provided by dynamical models which describe, with only few physical ingredients, some gross features of the enormous complexity of the geodynamo process. In this perspective, it is useful to compare the statistics of reversals observed in toy models with statistics obtained on real datasets.

3.1 The “Rikitake dynamo”

As a first example we investigate the statistical properties of the sequence of random reversals generated by the two-disk chaotic geodynamo model, also known as “Rikitake model” (Rikitake, 1958). The model is described by the following ordinary differential equations

$$\begin{aligned}\frac{dx}{dt} &= -x + yz, \\ \frac{dy}{dt} &= -y + x \left(z - \frac{15}{4} \right), \\ \frac{dz}{dt} &= 1 - xy.\end{aligned}\tag{5}$$

The quantities $x(t)$, $y(t)$ and $z(t)$ represent respectively the two currents and the dynamo disc angular velocity of the Rikitake dynamo. A sample of the current $x(t)$, which sign is reversed a number of times, is shown in Fig. 4. The probability density $P(h)$ of the stochastic variable h , obtained from 489 reversals of the current $x(t)$, is reported in Fig. 4, together with the surviving function of the distribution $P(h \geq H)$. The results for the CK95 dataset are also reported for comparison. It can be seen that a departure from Poisson statistics is present, which can be related to the correlations produced by the chaotic dynamics of the model. However, from the different shape of the distributions (namely the Rikitake dynamo and CK95 dataset) we can infer that the clustering effects in the Rikitake model seem to be affected by the presence of sudden jumps in the surviving functions of the distribution of h . In other words, the reversal time series $x(t)$ seems to present recurrent pairs of persistence times of equal duration ratios. This effect can be considered a limit of the Rikitake model, which on the other hand is able to capture the gross feature of the real data, as far as deviation from Poissonianity is concerned.

3.2 A bistable geodynamo model

The second dynamical model we investigate (Hoyng and Duistermaat, 2004) is based on stochastic excitation of the axisymmetric component of the magnetic field. In this model, the ordinary differential equations which describe the time evolution of the amplitude $x(t)$ and complex amplitude $y(t) + iz(t)$ of the magnetic field fundamental mode are the following

$$\frac{dx}{dt} = (1 - x^2)x + V_{11}x + V_{12}y + V_{13}z,$$

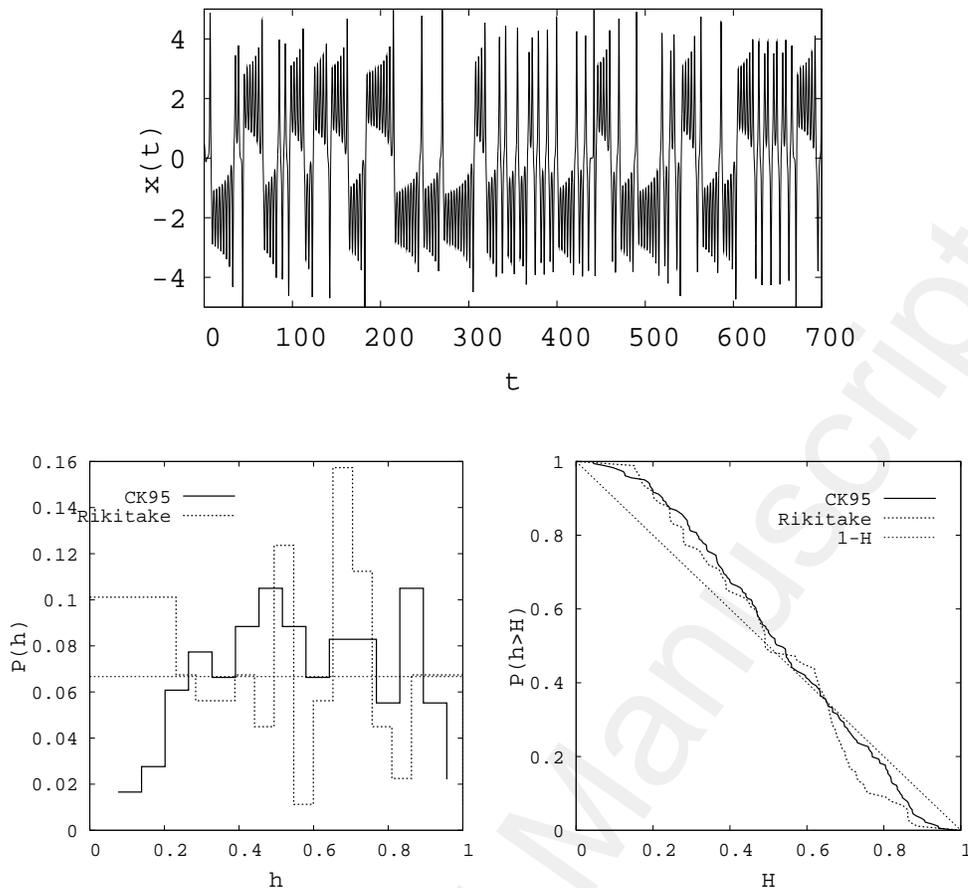


Fig. 4. Top panel: the time evolution of the variable $x(t)$ (in arbitrary units) as obtained from the Rikitake model. Few reversals of the current are clearly visible. Time is also given in arbitrary units. Bottom panels: probability densities $P(h)$ of the stochastic variable h , and corresponding surviving functions $P(h \geq H)$, for the Rikitake model and for CK95 dataset. The theoretical probability expected for a Poisson statistics is also shown.

$$\begin{aligned} \frac{dy}{dt} &= -ay - cz + V_{21}x + V_{22}y + V_{23}z, \\ \frac{dz}{dt} &= cy - az + V_{31}x + V_{32}y + V_{33}z. \end{aligned} \quad (6)$$

The $V_{ij}(t)$ are 9 independent random functions of time, with zero mean and equal r.m.s. magnitude, that are renewed after a time τ_c . We use the standard parameters $a = 2$, $c = 5$, $\tau_c = 0.01$, and $D = \langle V_{ij}^2 \rangle \tau_c$, where $D = 0.4$ (Hoyng and Duistermaat, 2004). The probability density $P(h)$ of the stochastic variable h , calculated by using the times of the random reversals of the amplitude $x(t)$, and its surviving function $P(h \geq H)$ are shown in Fig. 5. In this case we considered 409 reversals. As in previous case, a departure from a Poisson distribution is observed, due to the correlations introduced through τ_c . However,

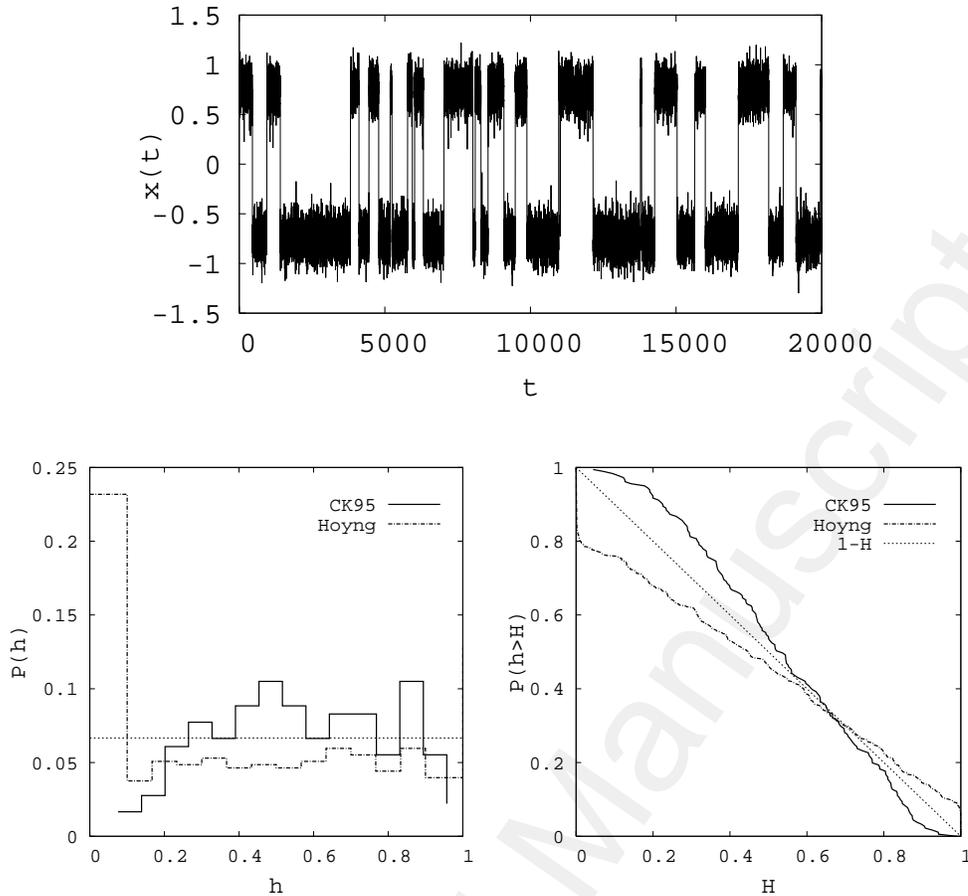


Fig. 5. Top panel: The time evolution of the variable $x(t)$ as obtained from the bistable model (in arbitrary units). The reversals of the magnetic amplitude are clearly visible. Time is also in arbitrary units. Bottom panels: probability densities $P(h)$ of the stochastic variable h and corresponding surviving functions $P(h \geq H)$, for the Hoyng model and for CK95 dataset. The theoretical probability expected for a Poisson statistics is also shown.

this departure is different from that observed for the data (CK95) and for the Rikitake dynamo. In this case, in fact, the departure is mainly due to an excess of very low values of h , as easily seen from $P(h)$, corresponding to pairs of one short time followed by a very long one (and vice-versa). The remaining part of the distribution is rather flat, as would be for a Poisson process. The time behaviour of reversals in this model is then hardly compatible with the clustering process evidenced through our statistical data analysis.

3.3 A shell model for MHD turbulence

It is well known that abrupt flow reversals, similar to the field reversals observed in the geodynamo records, occur in some turbulent systems, as for example the Rayleigh-Benard convection (Benzi, 2005; Araujo et al., 2005; Tsuji et al., 2005; Brown et al., 2005), or the atmosphere wind direction (van Doorn et al., 2000). The problem of reproducing the large-scale reversals in such situations have been addressed by Benzi (2005), which proposed the existence of a one-dimensional unstable manifold arising by a large-scale pitchfork bifurcation. The instability is driven by usual turbulence, but, rather than the whole Navier-Stokes equation, Benzi (2005) considered the GOY hydrodynamical shell model to describe the typical features of the turbulent energy cascade. The shell model approach to turbulence is indeed viewed as a consistent and relevant tentative to describe the energy cascade of turbulence (Bohr et al., 1998). These models mimic the gross features of the time evolution of spectral Navier-Stokes or Magnetohydrodynamic (MHD) equations. The 3D hydrodynamic GOY shell model has been introduced some time ago by Gledzer (1973) and by Ohkitani & Yamada (1989) (for a recent review see Biferale (2003)). The MHD shell model, which coincides with the GOY model when the magnetic variables are set to zero, has been introduced independently by Frick and Sokoloff (1998) and Giuliani & Carbone (1998) (see also Giuliani (1999)). A typical shell model can be built up through the following steps. The wave-vector space is divided into a finite number N of discrete shells whose radii grow according to a power $k_n = 2^n k_0$ (k_0 is the fundamental wave vector and $n = 1, 2, \dots, N$). Each shell is assigned complex scalar variables $u_n(t)$ and $b_n(t)$, describing the chaotic dynamics of velocity and magnetic Fourier modes in the shell of wave vectors between k_n and k_{n+1} . A dynamical model is then written in a way to describe the nonlinear evolution through quadratic nonlinearities among modes of different neighbouring and next neighbouring shells, by allowing the same couplings and the invariance of the same quadratic quantities as in the original magnetohydrodynamic equations

$$\frac{du_n}{dt} = -\nu k_n^2 u_n + ik_n [(u_{n+1}^* u_{n+2} - b_{n+1}^* b_{n+2}) - \frac{1}{4}(u_{n-1}^* u_{n+1} - b_{n-1}^* b_{n+1}) - \frac{1}{8}(u_{n-2} u_{n-1} - b_{n-2} b_{n-1})] + f_n \quad (7)$$

$$\frac{db_n}{dt} = -\eta k_n^2 b_n + ik_n \frac{1}{6} [(u_{n+1}^* b_{n+2} - b_{n+1}^* u_{n+2}) + (u_{n-1}^* b_{n+1} - b_{n-1} u_{n+1}) + (u_{n-2} b_{n-1}^* - b_{n-2} u_{n-1})] \quad (8)$$

Here the first terms in the right-hand-sides represent dissipative effects, η and ν being respectively the resistivity and the viscosity of the MHD flow, while f_n is an external forcing term. This forcing, that drives the fluid flow to become

unstable at large-scales, is reproduced by a Langevin stochastic equation

$$df_n/dt = -f_n/\tau + \gamma \quad (9)$$

where τ is a correlation time, assumed to be $\tau = 1$ and γ is a gaussian stochastic process δ -correlated in time $\langle \gamma(t)\gamma(t') \rangle = \delta(t - t')$.

The shell model is able to reproduce the turbulent dynamo effect, that is a seed of magnetic fluctuations can grow in time (Frick and Sokoloff, 1998; Giuliani & Carbone, 1998). However the effects of the turbulent small scales fluctuations on the largest scale, which are of great importance for the description of the large-scale dynamo phenomenology (Moffatt, 1978) is lost in the MHD shell model (Giuliani & Carbone, 1998) because of the assumption of locality of interactions that is one of the main characteristics of the nonlinear energy cascade in turbulence. Then, in the same spirit as in the model introduced by Benzi (2005), the evolution equation of the first shell $n = 1$ for the magnetic variable b_1 , is now modified by introducing a cubic interaction that reproduces the large-scale instability related to the ambient field B_0

$$\frac{db_1}{dt} = -\eta k_n^2 b_1 + ik_1 \frac{1}{6} (u_2^* b_3 - b_2^* u_3) + \mu b_1 \left(1 - \frac{b_1^2}{B_0^2} \right) \quad (10)$$

(μ is a free parameter representing the pitchfork amplitude). In our model the last term in Eq.(10) can then be viewed as a tentative to reproduce, in a simple way, the effects of the turbulent small scale fluctuations on the largest scale.

The simulation is run considering $N = 19$ shells, $B_0 = 1$, $\mu = 1$ and $\nu = \eta = 10^{-6}$, all in normalized units. From the time series resulting from the run, we identify more than

It should be noted that recent results have been obtained considering a shell model citepalexakis including also non-local interactions citeppoulain. This interesting approach is owever not considered for the moment.

3.4 α^2 dynamo

The last model we consider is a mean-field dynamo model with a spherically symmetric helical turbulence parameter α , assumed to have at least one sign change of $\alpha(r)$ along the radius. This model had been shown to provide asymmetric polarity reversals, a bimodal field distribution, and a positive correlation of field strength and persistence time (Stefani and Gerbeth, 2005). All these features were attributed to the magnetic field dynamics in the vicinity of

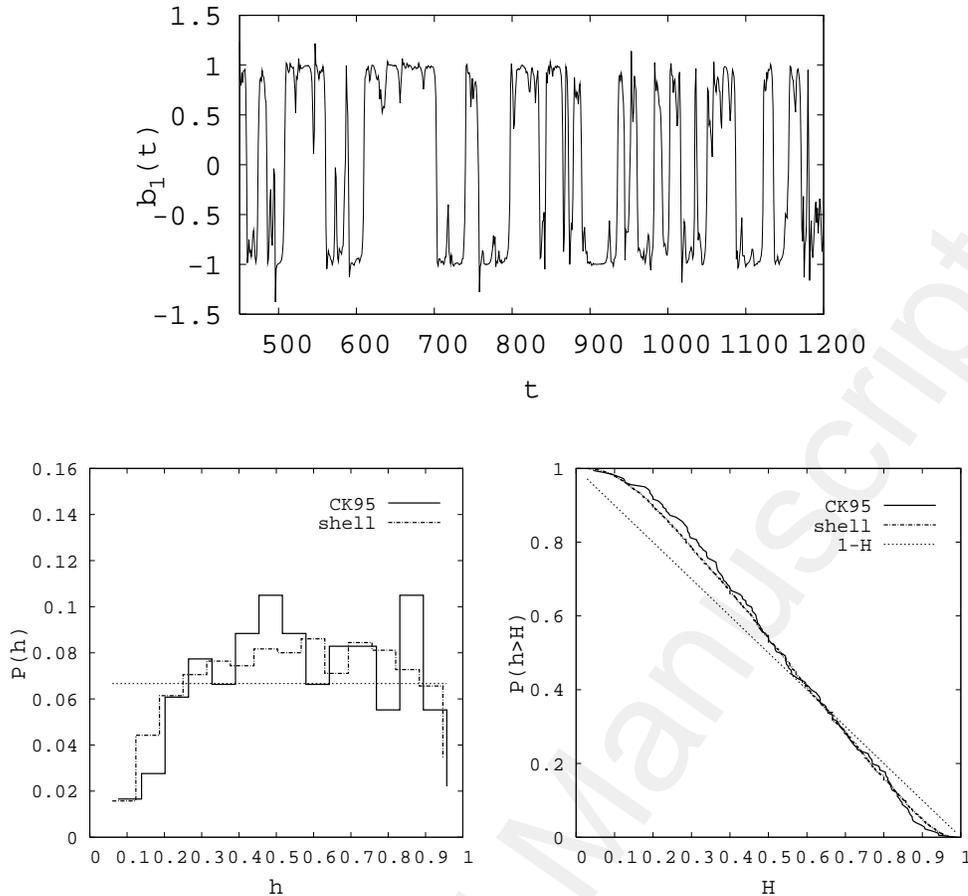


Fig. 6. Top panel: the time evolution of the large scale field $b_1(t)$ as obtained from the shell model (in arbitrary units). The reversals of the component are clearly visible. Time is also in arbitrary units. Bottom panels: probability densities $P(h)$ of the stochastic variable h and the corresponding surviving functions $P(h \geq H)$, for the shell model and for CK95 dataset. The theoretical probability expected for a Poisson statistics is also shown.

an *exceptional point* of the spectrum of the non-selfadjoint dynamo operator where two real eigenvalues coalesce and continue as a complex conjugated pair of eigenvalues. Such an exceptional point is usually associated with a nearby local maximum of the growth rate dependence on the magnetic Reynolds number. Actually, it is the negative slope of this curve between the local maximum and the exceptional point that makes the system unstable and drives it to the exceptional point and beyond into the oscillatory branch where the sign change happens. The apparent weakness of this reversal model, namely the necessity to fine-tune the radial profile of α in order to adjust the operator spectrum in an appropriate way, was overcome in a follow-up paper (Stefani et al., 2006a). For highly supercritical dynamos the exceptional point and the associated local growth rate maximum were demonstrated to tend towards the zero growth rate line where the indicated reversal scenario can be actual-

ized. Reversals from such a highly supercritical dynamo model were compared in Stefani et al. (2006b) with paleomagnetic measurements, exhibiting a nice correspondence of the time-scale and the asymmetry.

Here, we use one of the time-series of (Stefani et al., 2006b), resulting from the stochastic partial differential equation system for the expansion coefficients of the poloidal and toroidal axial field components $s(r, \tau)$ and $t(r, \tau)$:

$$\frac{\partial s_l}{\partial \tau} = \frac{1}{r} \frac{d^2}{dr^2} (r s_l) - \frac{l(l+1)}{r^2} s_l + \alpha(r, \tau) t_l, \quad (11)$$

$$\frac{\partial t_l}{\partial \tau} = \frac{1}{r} \frac{d}{dr} \left[\frac{d}{dr} (r t_l) - \alpha(r, \tau) \frac{d}{dr} (r s_l) \right] - \frac{l(l+1)}{r^2} [t_l - \alpha(r, \tau) s_l], \quad (12)$$

with boundary conditions $\partial s_l / \partial r|_{r=1} + (l+1) s_l(1) = t_l(1) = 0$. The helical turbulence parameter $\alpha(r, \tau)$ is assumed to be quenched according to

$$\alpha(r, \tau) = C \frac{\alpha_{kin}(r)}{1 + E_{mag}(r, \tau)/E_{mag}^0} + \Xi(r, \tau), \quad (13)$$

by the magnetic field energy E_{mag} averaged over the angles:

$$E_{mag}(r, \tau) = \frac{2s^2(r, \tau)}{r^2} + \frac{1}{r^2} \left(\frac{\partial(rs(r, \tau))}{\partial r} \right)^2 + t^2(r, \tau). \quad (14)$$

The stochastic term $\Xi(r, \tau)$ is Taylor expanded,

$$\Xi(r, t) = \xi_1(\tau) + \xi_2(\tau) r^2 + \xi_3(\tau) r^3 + \xi_4(\tau) r^4, \quad (15)$$

with the noise correlation given by $\langle \xi_i(\tau) \xi_j(\tau + \tau_1) \rangle = D^2 (1 - |\tau_1|/\tau_{corr}) \Theta(1 - |\tau_1|/\tau_{corr}) \delta_{ij}$.

In the following, we will focus on a time series resulting from the particular choice

$$\alpha_{kin} = 1.916 \times C \times (1 - 6r^2 + 5r^4), \quad (16)$$

with dynamo strength $C = 100$ and a noise level $D = 7$. The duration of the run is 10^5 diffusion times which corresponds approximately to 20 Billion years for the Earth if we assume a diffusion time of 200 kyr. Within this time we observed 56613 reversals. Note that the reversal rate is very sensitive to the choice of C and D . For example, when replacing $D = 7$ by $D = 6$, the mean reversal rate drops by a factor two. More detailed discussion

on statistical properties of reversals is left for future works. The probability density $P(h)$ and the surviving function $P(h \geq H)$ of the stochastic variable h , calculated by using the times of reversal obtained from the model, are shown in Fig. 7. The result of the test shows significant deviation from a Poisson process. The deviation is even enhanced with respect to the CK95 data, but the main characteristics are well reproduced, as for example the strong evidence of clustering. This shows that the α^2 model is able to capture the statistical properties of the geodynamo reversals. Anyway, the parameters dependence of the results still needs to be investigated, so that a more quantitative agreement between real data and this model could be expected. This study is left for a future work. It might be noted here that the results of the test are very robust with respect to the sample length. Indeed, when reducing the dataset to 160 million years (comparable with the CK95 data), only 442 reversals are observed, and the test gives the same results as for the complete run (not shown).

4 Conclusion

In this paper the statistical properties of persistence times between geomagnetic reversals have been investigated. We performed a statistical test which showed that geomagnetic reversals are produced by an underlying process that is far from being locally Poissonian, as conjectured by (Constable, 2000). Thus, the sequence of geomagnetic reversals is characterized by time correlations. As spontaneous reversals of the geodynamo field have been observed in high resolution numerical simulations (Kageyama et al., 2004; Kageyama and Yoshida, 2005), the main results contained in this paper seem to indicate that such reversals could be related to the non-linear nature of the turbulent dynamo. In order to confirm this conjecture, we performed the statistical test mentioned above on four different dynamical models of the geodynamo process. First, two “toy” models were considered, namely the Rikitake dynamo and the bistable model by Hoyng and Duistermaat (2004). Notwithstanding the high simplified character of these toy models, our analysis has shown that the departure from Poisson statistics found in the data can be related with the long range correlations introduced by the chaotic dynamic of the system (Rikitake case), and is not fully compatible with the phenomenology of stochastic resonance (bistable model). Furthermore, two more complex dynamical systems have been considered, namely a modified shell model reproducing the non-linear features of the MHD equations, and a turbulent α^2 dynamo model. The results obtained with those two models are in agreement with the data. In particular, time correlations due to the non-linear character of the models are observed as clustering of the reversals, which reproduces the behaviour observed in the geomagnetic reversals. The clustering property, which can be

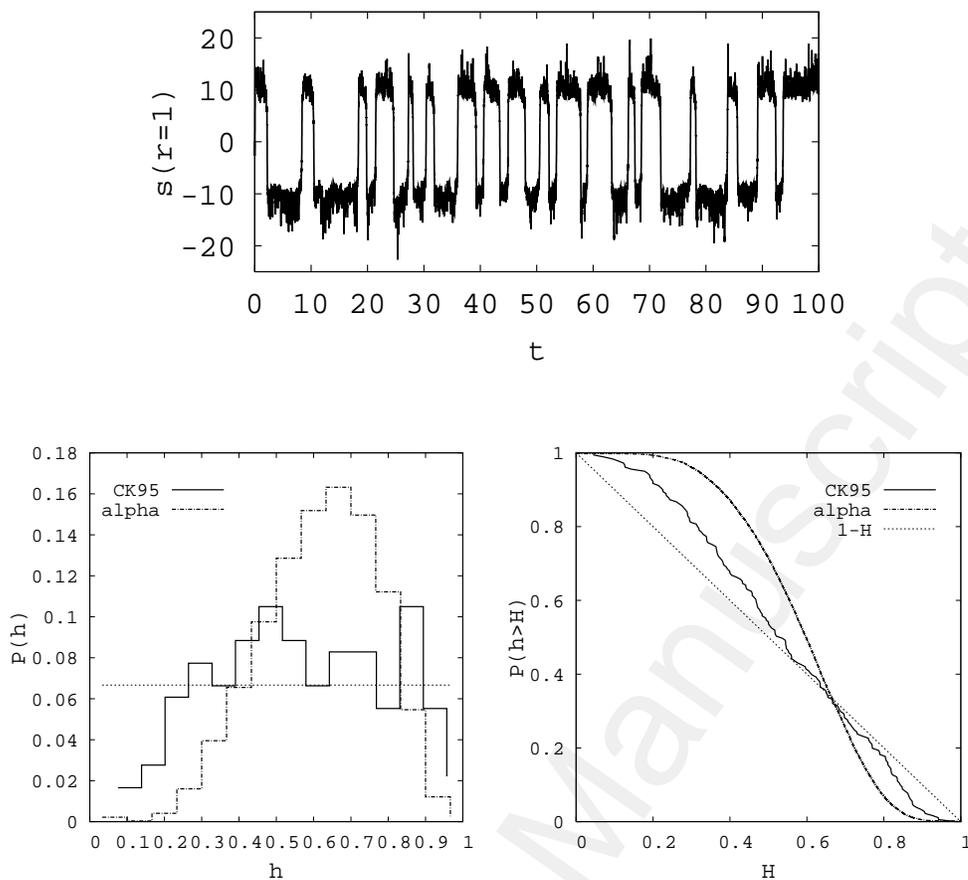


Fig. 7. Top panel: the time evolution of the poloidal field $s(r = 1)$ as obtained from the α^2 model, for $C = 100$ and $D = 6$. The reversals of the component are clearly visible. Time is in diffusion time units. Bottom panels: probability densities $P(h)$ of the stochastic variable h and their surviving functions $P(h \geq H)$, for the α^2 model with $C=100$ and $C=7$, and for CK95 dataset. The theoretical probability expected for a Poisson statistics is also shown.

visualized as a “devil’s staircase” of very long and very short time intervals alternating in a random way (De michelis and Consolini, 2003), may be a signature of “punctuated equilibrium” (Gould and Eldredge, 1993) which, in turn, is known to be a typical feature of metastable systems. In the α^2 model, this metastability results from the existence of a stable and an unstable fix-point which are the zeros of the growth rate curve situated to the left and to the right of its local maximum, close to the exceptional point. This way one might speculate that the clustering property traces back to metastability and ultimately to the spectral behaviour of the dynamo operator. However, the complex nature of the problem still need to be understood.

In this view, our results can be interpreted as a strong evidence for the presence of correlations between reversal events in the geomagnetic field. Such

correlations are only observed in models which include non-linear dynamics, and can be associated with the presence of some degree of memory in the underlying geodynamo process (Valet et al., 2005; Stefani and Gerbeth, 2005) which gives rise to clustering of reversals.

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