



The maximal regularity operator on tent spaces

Pascal Auscher, Sylvie Monniaux, Pierre Portal

► To cite this version:

Pascal Auscher, Sylvie Monniaux, Pierre Portal. The maximal regularity operator on tent spaces. 2010. hal-00531814v1

HAL Id: hal-00531814

<https://hal.science/hal-00531814v1>

Preprint submitted on 3 Nov 2010 (v1), last revised 9 Dec 2010 (v3)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The maximal regularity operator on tent spaces

Pascal Auscher, Sylvie Monniaux, Pierre Portal

En l'honneur des 60 ans de Michel Pierre

Abstract

Recently, Auscher and Axelsson gave a new approach to non-smooth boundary value problems with L^2 data, that relies on some appropriate weighted maximal regularity estimates. As part of the development of the corresponding L^p theory, we prove here the relevant weighted maximal estimates in tent spaces $T^{p,2}$ for p in a certain open range. We also study the case $p = \infty$.

1 Introduction

Let $-L$ be a densely defined closed linear operator acting on $L^2(\mathbb{R}^n)$ and generating a bounded analytic semigroup $(e^{-tL})_{t \geq 0}$. We consider the maximal regularity operator defined by

$$\mathcal{M}_L f(t, x) = \int_0^t L e^{-(t-s)L} f(s, \cdot)(x) ds,$$

for functions $f \in C_c(\mathbb{R}_+ \times \mathbb{R}^n)$. The boundedness of this operator on $L^2(\mathbb{R}_+ \times \mathbb{R}^n)$ was established by de Simon in [15]. The $L^p(\mathbb{R}_+ \times \mathbb{R}^n)$ case, for $1 < p < \infty$, turned out, however, to be much more difficult. In [10], Kalton and Lancien proved that \mathcal{M}_L could fail to be bounded on L^p as soon as $p \neq 2$. The necessary and sufficient assumption for L^p boundedness was then found by Weis [16] to be a vector-valued strengthening of analyticity, called R-analyticity. As many differential operators L turn out to generate R-analytic semigroups, the L^p boundedness of \mathcal{M}_L has subsequently been successfully used in a variety of PDE situations (see [13] for a survey).

Recently, maximal regularity was used as an important tool in [2], where a new approach to boundary value problems with L^2 data for divergence form elliptic systems on Lipschitz domains, is developed. More precisely, in [2], the authors establish and use the boundedness of \mathcal{M}_L on weighted spaces $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dt dx)$, for certain values of $\beta \in \mathbb{R}$. The use of these spaces is common in the study of boundary value problems, where they are seen as variants of the tent space $T^{2,2}$, introduced by Coifman, Meyer and Stein in [6]. For $p \neq 2$, the corresponding spaces are weighted versions of the tent spaces $T^{p,2}$, which are defined, for parameters $\beta \in \mathbb{R}$ and $m \in \mathbb{N}$, as the completion of $C_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$\|g\|_{T^{p,2,m}(t^\beta dt dy)} = \left(\int_{\mathbb{R}^n} \left(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x, t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}} |g(t, y)|^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}},$$

the classical case corresponding to $\beta = -1$, $m = 1$, and being denoted simply by $T^{p,2}$. The parameter m is used to allow various homogeneities, and thus to make these spaces relevant in the study of differential operators L of order m . To develop an analogue of [2] for L^p data, we need boundedness results for the maximal operator \mathcal{M}_L on these tent spaces. This is the purpose of this note. It will be used in the study of such boundary value problems, as well as in the context of well-posedness of non-autonomous Cauchy problems, in subsequent papers. The proof is based on recent results and methods developed in [9], building on ideas from [5] and [8]. In Section 2 we recall the relevant definitions and result from [9]. In Section 3 we then state and prove the adequate boundedness results.

2 Tools

When dealing with tent spaces, the key estimate needed is a change of aperture formula, i.e., a comparison between the $T^{p,2}$ norm and the norm

$$\|g\|_{T_\alpha^{p,2}} := \left(\int_{\mathbb{R}^n} \left(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x,\alpha t)}(y)}{t^n} |g(t,y)|^2 \frac{dy dt}{t} \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}},$$

for some parameter $\alpha > 0$. Such a result was first established in [6], building on similar estimates in [7], and analogues have since been developed in various contexts. Here we use the following version given in [9, Theorem 4.3].

Theorem 2.1. *Let $1 < p < \infty$ and $\alpha \geq 1$. There exists a constant $C > 0$ such that, for all $f \in T^{p,2}$,*

$$\|f\|_{T^{p,2}} \leq \|f\|_{T_\alpha^{p,2}} \leq C(1 + \log \alpha) \alpha^{n/\tau} \|f\|_{T^{p,2}},$$

where $\tau = \min(p, 2)$.

Theorem 2.1 is actually a special case of the Banach space valued result obtained in [9] corresponding to the scalar valued situation. Note, however, that it improves the power of α appearing in the inequality from the n given in [6] to $\frac{n}{\tau}$. This is crucial in what follows, and has been shown to be optimal in [9].

Applying this to $(t, y) \mapsto t^{\frac{m(\beta+1)}{2}} f(t^m, y)$ instead of f , we also have the weighted result, where

$$\|g\|_{T_\alpha^{p,2,m}(t^\beta dt dy)} = \left(\int_{\mathbb{R}^n} \left(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x,\alpha t^{\frac{1}{m}})}(y)}{t^{\frac{n}{m}}} |g(t,y)|^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}}.$$

Corollary 2.2. *Let $1 < p < \infty$, $m \in \mathbb{N}$, $\alpha \geq 1$, and $\beta \in \mathbb{R}$. There exists a constant $C > 0$ such that, for all $f \in T^{p,2,m}(t^\beta dt dy)$,*

$$\|f\|_{T^{p,2,m}(t^\beta dt dy)} \leq \|f\|_{T_\alpha^{p,2,m}(t^\beta dt dy)} \leq C(1 + \log \alpha) \alpha^{n/\tau} \|f\|_{T^{p,2,m}(t^\beta dt dy)},$$

where $\tau = \min(p, 2)$.

To take advantage of this result, one needs to deal with families of operators, that behave nicely with respect to tent norms. As pointed out in [9], this does not mean considering R-bounded families (which means R-analytic semigroups when one considers $(tLe^{-tL})_{t \geq 0}$) as in the $L^p(\mathbb{R}_+ \times \mathbb{R}^n)$ case, but tent bounded ones, i.e. families of operators with the following L^2 off-diagonal decay, also known as Gaffney-Davies estimates.

Definition 2.3. A family of bounded linear operators $(T_t)_{t \geq 0} \subset B(L^2(\mathbb{R}^n))$ is said to satisfy off-diagonal estimates of order M , with homogeneity m , if, for all Borel sets $E, F \subset \mathbb{R}^n$, all $t > 0$, and all $f \in L^2(\mathbb{R}^n)$:

$$\|1_E T_t 1_F f\|_2 \lesssim \left(1 + \frac{\text{dist}(E, F)^m}{t} \right)^{-M} \|1_F f\|_2.$$

In what follows $\|\cdot\|_2$ denotes the norm in $L^2(\mathbb{R}^n)$.

As proven, for instance, in [4], many differential operators of order m , such as (for $m = 2$) divergence form elliptic operators with bounded measurable complex coefficients, are such that $(tLe^{-tL})_{t \geq 0}$ satisfies off-diagonal estimates of any order, with homogeneity m . This condition can, in fact, be seen as a replacement for the classical gaussian kernel estimates satisfied in the case of more regular coefficients.

3 Results

Theorem 3.1. *Let $m \in \mathbb{N}$, $\beta \in (-\infty, 1)$, $p \in (\frac{2n}{n+m(1-\beta)}, \infty) \cap (1, \infty)$, and $\tau = \min(p, 2)$. If $(tLe^{-tL})_{t \geq 0}$ satisfies off-diagonal estimates of order $M > \frac{n}{m\tau}$, with homogeneity m , then \mathcal{M}_L extends to a bounded operator on $T^{p,2,m}(t^\beta dt dy)$.*

Proof. The proof is very much inspired by similar estimates in [5] and [9]. Let $f \in \mathcal{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$. Given $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$, and $j \in \mathbb{Z}_+$, we consider

$$C_j(x, t) = \begin{cases} B(x, t) & \text{if } j = 0, \\ B(x, 2^j t) \setminus B(x, 2^{j-1} t) & \text{otherwise.} \end{cases}$$

We write $\|\mathcal{M}_L f\|_{T^{p,2}} \leq \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} I_{k,j} + \sum_{j=0}^{\infty} J_j$ where

$$I_{k,j} = \left(\int_{\mathbb{R}^n} \left(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x, t \frac{1}{m})}(y)}{t^{\frac{n}{m}}} \left| \int_{2^{-k-1}t}^{2^{-k}t} Le^{-(t-s)L} (1_{C_j(x, 4t \frac{1}{m})} f(s, \cdot))(y) ds \right|^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}},$$

$$J_j = \left(\int_{\mathbb{R}^n} \left(\int_0^\infty \int_{\mathbb{R}^n} \frac{1_{B(x, t \frac{1}{m})}(y)}{t^{\frac{n}{m}}} \left| \int_{\frac{t}{2}}^t Le^{-(t-s)L} (1_{C_j(x, 4s \frac{1}{m})} f(s, \cdot))(y) ds \right|^2 t^\beta dy dt \right)^{\frac{p}{2}} dx \right)^{\frac{1}{p}}.$$

Fixing $j \geq 0$, $k \geq 1$ we first estimate $I_{k,j}$ as follows. For fixed $x \in \mathbb{R}^n$,

$$\begin{aligned} & \int_0^\infty \int_{B(x, t \frac{1}{m})} \left| \int_{2^{-k-1}t}^{2^{-k}t} Le^{-(t-s)L} (1_{C_j(x, 4t \frac{1}{m})} f(s, \cdot))(y) ds \right|^2 t^{\beta - \frac{n}{m}} dy dt \\ & \leq \int_0^\infty \int_{B(x, t \frac{1}{m})} \left(\int_{2^{-k-1}t}^{2^{-k}t} \left| (t-s) Le^{-(t-s)L} (1_{C_j(x, 4t \frac{1}{m})} f(s, \cdot))(y) \right| \frac{ds}{t-s} \right)^2 t^{\beta - \frac{n}{m}} dy dt \\ & \lesssim \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} t \left(\int_{B(x, t \frac{1}{m})} \left| (t-s) Le^{-(t-s)L} (1_{C_j(x, 4t \frac{1}{m})} f(s, \cdot))(y) \right|^2 dy \right) t^{\beta - \frac{n}{m} - 2} ds dt \\ & \lesssim \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} \left(1 + \frac{2^j m t}{t-s} \right)^{-2M} \|1_{B(x, 2^j + 2t \frac{1}{m})} f(s, \cdot)\|_2^2 t^{\beta - \frac{n}{m} - 1} ds dt \\ & \lesssim 2^{-k} 2^{-2jmM} \int_0^\infty \left(\int_{2^k s}^{2^{k+1}s} t^{\beta - \frac{n}{m} - 1} dt \right) \|1_{B(x, 2^j + \frac{k}{m} + 3s \frac{1}{m})} f(s, \cdot)\|_2^2 ds \\ & \lesssim 2^{-k(\frac{n}{m} + 1 - \beta)} 2^{-2jmM} \int_0^\infty \|1_{B(x, 2^j + \frac{k}{m} + 3s \frac{1}{m})} f(s, \cdot)\|_2^2 s^{\beta - \frac{n}{m}} ds. \end{aligned}$$

In the second inequality, we use Cauchy-Schwarz inequality for the integral with respect to t , the fact that $t-s \sim t$ for $s \in \cup_{k \geq 1} [2^{-k-1}t, 2^{-k}t] \subset [0, \frac{t}{2}]$ and Fubini's theorem to exchange the integral in t and the integral in y . The next inequality follows from the off-diagonal estimate verified by $(t-s)Le^{-(t-s)L}$ and again the fact that $t-s \sim t$. By Corollary 2.2 this gives

$$I_{k,j} \lesssim (j+k) 2^{-k(\frac{1}{2}(\frac{n}{m} + 1 - \beta) - \frac{n}{m\tau})} 2^{-j(mM - \frac{n}{\tau})} \|f\|_{T^{p,2,m}(t^\beta dt dy)},$$

where $\tau = \min(p, 2)$. It follows that $\sum_{k=1}^{\infty} \sum_{j=0}^{\infty} I_{k,j} \lesssim \|f\|_{T^{p,2,m}(t^\beta dt dy)}$ since $M > \frac{n}{m\tau}$ and $\frac{n}{m} + 1 - \beta > \frac{2n}{m\tau}$ (Note that for $p \geq 2$, this requires $\beta < 1$).

We now turn to J_0 and remark that $J_0 \leq (\int_{\mathbb{R}^n} J_0(x)^{\frac{p}{2}} dx)^{\frac{1}{p}}$, where

$$J_0(x) = \int_0^\infty \int_{\mathbb{R}^n} \left| \int_{\frac{t}{2}}^t L e^{-(t-s)L} (g(s, \cdot))(y) ds \right|^2 t^{\beta - \frac{n}{m}} dy dt$$

with $g(s, y) = 1_{B(x, 4s^{\frac{1}{m}})}(y) f(s, y)$. The inside integral can be rewritten as

$$\mathcal{M}_L g(t, \cdot) - e^{-\frac{t}{2}L} \mathcal{M}_L g\left(\frac{t}{2}, \cdot\right).$$

As \mathcal{M}_L is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^{\beta - \frac{n}{m}} dy dt)$ by [3, Theorem 3.1] and $(e^{-tL})_{t \geq 0}$ is uniformly bounded on $L^2(\mathbb{R}^n)$, we get

$$J_0(x) \lesssim \int_0^\infty \|1_{B(x, 4s^{\frac{1}{m}})} f(s, \cdot)\|_2^2 s^{\beta - \frac{n}{m}} ds.$$

We finally turn to J_j , for $j \geq 1$. For fixed $x \in \mathbb{R}^n$,

$$\begin{aligned} & \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x, t^{\frac{1}{m}})}(y) \left| \int_{\frac{t}{2}}^t L e^{-(t-s)L} (1_{C_j(x, 4s^{\frac{1}{m}})} f(s, \cdot))(y) ds \right|^2 t^{\beta - \frac{n}{m}} dy dt \\ & \leq \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x, t^{\frac{1}{m}})}(y) \left(\int_{\frac{t}{2}}^t |(t-s) L e^{-(t-s)L} (1_{C_j(x, 4s^{\frac{1}{m}})} f(s, \cdot))(y)| \frac{ds}{t-s} \right)^2 t^{\beta - \frac{n}{m}} dy dt \\ & \lesssim \int_0^\infty \int_{\mathbb{R}^n} 1_{B(x, t^{\frac{1}{m}})}(y) \int_{\frac{t}{2}}^t |(t-s) L e^{-(t-s)L} (1_{C_j(x, 4s^{\frac{1}{m}})} f(s, \cdot))(y)|^2 \frac{ds}{(t-s)^2} t^{\beta - \frac{n}{m} + 1} dy dt \\ & \lesssim \int_0^\infty \int_{\frac{t}{2}}^t (t-s)^{-2} \left(1 + \frac{2^{jm}t}{t-s}\right)^{-2M} \|1_{B(x, 2^{j+2}s^{\frac{1}{m}})} f(s, \cdot)\|_2^2 s^{\beta - \frac{n}{m} + 1} ds dt \\ & \lesssim 2^{-jm(2M-2)} \int_0^\infty \left(\int_s^{2s} s(t-s)^{-2} \left(1 + \frac{2^{jm}t}{t-s}\right)^{-2} dt \right) \|1_{B(x, 2^{j+2}s^{\frac{1}{m}})} f(s, \cdot)\|_2^2 s^{\beta - \frac{n}{m}} ds \\ & \lesssim 2^{-2jmM} \int_0^\infty \|1_{B(x, 2^{j+2}s^{\frac{1}{m}})} f(s, \cdot)\|_2^2 s^{\beta - \frac{n}{m}} ds, \end{aligned}$$

where we have used Cauchy-Schwarz inequality in the second inequality, the off-diagonal estimates and the fact that $s \leq t$ in the third, Fubini's theorem and the fact that $s \geq \frac{t}{2}$ in the fourth, and the change of variable $\sigma = \frac{t}{t-s}$ in the last. An application of Corollary 2.2, then gives

$$J_j \lesssim 2^{-jmM} j 2^{j\frac{n}{\tau}} \|f\|_{T^{p,2,m}(t^\beta dt dy)} = j 2^{-j(mM - \frac{n}{\tau})} \|f\|_{T^{p,2,m}(t^\beta dt dy)},$$

and the proof is concluded by summing the estimates. \square

An end-point result holds for $p = \infty$. In this context the appropriate tent space consists of functions such that $|g(t, x)|^2 \frac{dx dt}{t}$ is a Carleson measure, and is defined as the completion of the space $\mathcal{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$\|g\|_{T^{\infty,2}}^2 = \sup_{(x,r) \in \mathbb{R}^n \times \mathbb{R}_+} r^{-n} \int_{B(x,r)} \int_0^r |g(t, x)|^2 \frac{dx dt}{t}.$$

We also consider the weighted version defined by

$$\|g\|_{T^{\infty,2,m}(t^\beta dtdy)}^2 := \sup_{(x,r) \in \mathbb{R}^n \times \mathbb{R}_+} r^{-\frac{n}{m}} \int_{B(x,r^{\frac{1}{m}})} \int_0^r |g(t,x)|^2 t^\beta dx dt.$$

Theorem 3.2. *Let $m \in \mathbb{N}$, and $\beta \in (-\infty, 1)$. If $(tLe^{-tL})_{t \geq 0}$ satisfies off-diagonal estimates of order $M > \frac{n}{2m}$, with homogeneity m , then \mathcal{M}_L extends to a bounded operator on $T^{\infty,2,m}(t^\beta dtdy)$.*

Proof. Pick a ball $B(z, r^{\frac{1}{m}})$. Let

$$I^2 = \int_{B(z,r^{\frac{1}{m}})} \int_0^r |(\mathcal{M}_L f)(t,x)|^2 t^\beta dx dt.$$

We want to show that $I^2 \lesssim r^{\frac{n}{m}} \|f\|_{T^{\infty,2}(t^\beta dtdy)}^2$. We set

$$I_j^2 = \int_{B(x,r^{\frac{1}{m}})} \int_0^r |(\mathcal{M}_L f_j)(t,x)|^2 t^\beta dx dt$$

where $f_j(s,x) = f(s,x) 1_{C_j(z,4r^{\frac{1}{m}})}(x) 1_{(0,r)}(s)$ for $j \geq 0$. Thus by Minkowsky inequality, $I \leq \sum I_j$. For I_0 we use again [3, Theorem 1.3] which implies that \mathcal{M}_L is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n, t^\beta dx dt)$. Thus

$$I_0^2 \lesssim \int_{B(z,4r^{\frac{1}{m}})} \int_0^r |f(t,x)|^2 t^\beta dx dt \lesssim r^{\frac{n}{m}} \|f\|_{T^{\infty,2,m}(t^\beta dtdy)}^2.$$

Next, for $j \neq 0$, we proceed as in the proof of Theorem 3.1 to obtain

$$\begin{aligned} I_j^2 &\lesssim \sum_{k=1}^{\infty} \int_0^r \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k}t \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 t^{\beta-2} ds dt \\ &\quad + \int_0^r \int_{\frac{t}{2}}^t t(t-s)^{-2} \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 t^\beta ds dt. \end{aligned}$$

Exchanging the order of integration, and using the fact that $t \sim t-s$ in the first part and that $t \sim s$ in the second, we have the following.

$$\begin{aligned} I_j^2 &\lesssim \sum_{k=1}^{\infty} 2^{-k} 2^{-2jmM} r^{-2M} \int_0^{2^{-k}r} \int_{2^k s}^{2^{k+1}s} t^{\beta+2M-1} \|f_j(s, \cdot)\|_{L^2}^2 dt ds \\ &\quad + \int_0^r \int_s^{2s} r(t-s)^{-2} \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 s^\beta dt ds \\ &\lesssim \sum_{k=1}^{\infty} 2^{-k} 2^{-2jmM} \int_0^{2^{-k}r} (2^k s)^\beta \|f_j(s, \cdot)\|_{L^2}^2 ds + \int_0^r \int_1^\infty (1 + 2^{jm}\sigma)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 s^\beta d\sigma ds \\ &\lesssim 2^{-2jmM} \int_0^r \|f_j(s, \cdot)\|_{L^2}^2 s^\beta ds, \end{aligned}$$

where we used $\beta < 1$. We thus have

$$I_j^2 \lesssim 2^{-2jmM} (2^j r^{\frac{1}{m}})^n \|f\|_{T^{\infty,2,m}(t^\beta dt dy)}^2,$$

and the condition $M > \frac{n}{2m}$ allows us to sum these estimates. \square

Remark 3.3. Assuming off-diagonal estimates, instead of kernel estimates, allows to deal with differential operators L with rough coefficients. The harmonic analytic objects associated with L then fall outside the Calderón-Zygmund class, and it is common (see for instance [1]) for their boundedness range to be a proper subset of $(1, \infty)$. Here, our range $(\frac{2n}{n+m(1-\beta)}, \infty]$ includes $[2, \infty]$ as $\beta < 1$, which is consistent with [2]. In the case of classical tent spaces, i.e., $m = 1$ and $\beta = -1$, it is the range $(2_*, \infty]$, where 2_* denotes the Sobolev exponent $\frac{2n}{n+2}$. We do not know, however, if this range is optimal.

Remark 3.4. Theorem 3.2 is a maximal regularity result for parabolic Carleson measures norms. This is quite natural from the point of view of non-linear parabolic PDE (where maximal regularity is often used), and such norm have, actually, already been used in the context of Navier-Stokes equations in [11], and, subsequently, for some geometric non-linear PDE in [12]. Theorem 3.1 is also reminiscent of Krylov's Littlewood-Paley estimates, and of their recent far-reaching generalization in [14]. In fact, the methods and results from [9], on which this paper relies, use the same circle of ideas (R-boundedness, Kalton-Weis γ multiplier theorem...) as [14]. The combination of these ideas into a "conical square function" approach to stochastic maximal regularity will be the subject of another forthcoming paper.

References

- [1] P. Auscher, On necessary and sufficient conditions for L^p estimates of Riesz transforms associated to elliptic operators on \mathbb{R}^n and related estimates. *Mem. Amer. Math. Soc.* 871 (2007).
- [2] P. Auscher, A. Axelsson, Weighted maximal regularity estimates and solvability of elliptic systems I. To appear in *Inventiones Math.*, arXiv:0911.4344.
- [3] P. Auscher, A. Axelsson, Remarks on maximal regularity estimates To appear in *Parabolic Problems : Herbert Amann Festschrift, Birkhäuser*. arXiv:0912.4482.
- [4] P. Auscher, S. Hofmann, M. Lacey, A. McIntosh, P. Tchamitchian, The solution of the Kato square root problem for second order elliptic operators on \mathbb{R}^n . *Ann. of Math.* 156(2) (2002) 633–654.
- [5] P. Auscher, A. McIntosh, E. Russ, Hardy spaces of differential forms and Riesz transforms on Riemannian manifolds. *J. Geom. Anal.* 18 (2008) 192–248.
- [6] R. Coifman, Y. Meyer, E.M. Stein, Some new function spaces and their applications to harmonic analysis. *J. Funct. Anal.* 62 (1985) 304–335.
- [7] C. Fefferman; E.M. Stein, H^p spaces of several variables. *Acta Math.* 129 (1972) 137–193.
- [8] T. Hytönen, A. McIntosh, P. Portal, Kato's square root problem in Banach spaces. *J. Funct. Anal.* 254 (2008) no. 3, 675–726.
- [9] T. Hytönen, J. van Neerven, P. Portal, Conical square function estimates in UMD Banach spaces and applications to H^∞ -functional calculi. *J. Analyse Math.* 106 (2008) 317–351.
- [10] N. Kalton, G. Lancien, A solution to the problem of L_p maximal-regularity. *Math. Z.* 235 (2000), 559–568.
- [11] H. Koch, D. Tataru, Well-posedness for the Navier-Stokes equations. *Adv. Math.* 157 (2001), 22–35.

- [12] H. Koch, T. Lamm, Geometric flows with rough initial data. preprint, arXiv:0902.1488v1.
- [13] P.C. Kunstmann, L. Weis, Maximal L^p regularity for parabolic problems, Fourier multiplier theorems and H^∞ -functional calculus, in *Functional Analytic Methods for Evolution Equations* (Editors: M. Iannelli, R. Nagel, S. Piazzera). *Lect. Notes in Math.* 1855, Springer-Verlag (2004).
- [14] J. van Neerven, M. Veraar, L. Weis, Stochastic maximal L^p regularity. submitted, ArXiv:1004.1309v2.
- [15] L. de Simon. Un'applicazione della theoria degli integrali singolari allo studio delle equazioni differenziali lineare astratte del primo ordine. *Rend. Sem. Mat., Univ. Padova* (1964) 205-223.
- [16] L. Weis, Operator-valued Fourier multiplier theorems and maximal L_p -regularity. *Math. Ann.* 319 (2001), 735–758.

PASCAL AUSCHER

Univ. Paris-Sud, laboratoire de Mathématiques, UMR 8628, F-91405 ORSAY; CNRS, F-91405 ORSAY.

`pascal.auscher@math.u-psud.fr`

SYLVIE MONNIAUX

LATP-UMR 6632, FST Saint-Jérôme - Case Cour A, Univ. Paul Cézanne, F-13397 MARSEILLE Cédex 20.

`sylvie.monniaux@univ-cezanne.fr`

PIERRE PORTAL

Permanent Address:

Université Lille 1, Laboratoire Paul Painlevé, F-59655 VILLENEUVE D'ASCQ.

Current Address:

Australian National University, Mathematical Sciences Institute, John Dedman Building, Acton ACT 0200, Australia.

`pierre.portal@math.univ-lille1.fr`