Coupling Continuous and Discrete Event System Techniques for Hybrid System Diagnosability Analysis

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Abstract. In this paper we propose a hybrid system modeling framework aimed at analyzing diagnosability. In this framework, the hybrid system is seen as the composition of an underlying discrete event and an underlying continuous systems. Diagnosability of these two underlying systems are fully analyzed and new results are provided for the underlying continuous system (called *the multimode system*). Based on these results, a hybrid language that contains 'natural' discrete events and discrete events capturing the continuous dynamics, is defined. On the basis of this language the diagnosability definition of hybrid systems is provided. With respect to this definition, we prove that the diagnosability of the underlying continuous or the discrete event system is only a sufficient condition. Diagnosability of hybrid systems must be decided by coupling both discrete event and continuous informations. Finally, the necessary and sufficient condition of hybrid diagnosability is given.

1 INTRODUCTION

Diagnosability is the property that guarantees that the system state can be precisely diagnosed after the occurrence of a fault. In a autonomy context, particularly, for autonomous satellites, diagnosability property is required and allows one to perform reconfiguration actions. Diagnosability definition depends mainly on the system modeling, the diagnosis approach and the observation system. Diagnosability was properly defined for Discrete Event Systems (DES) [8] and for Continuous Systems (CS) [9]. But there are few equivalent results for hybrid systems. In [2] diagnosability is studied for Real Time Hybrid Systems (RTHS), the classical DES necessary and sufficient condition of diagnosability from [8] is lightly modified and expressed in terms of reachability. In [4] hybrid diagnosability is studied based on the Hybrid Input/Ouput Automata (HIOA) formalism, the DES necessary and sufficient condition of diagnosability is generalized but requires more restrictive hypotheses.

In this paper, we propose an approach to analyze the diagnosability of hybrid systems based on a hybrid model. The behavior of the hybrid system is seen as the composition of an underlying DES and an underlying CS. The new concept of mode signature is proposed to characterize the diagnosability of the underling CS called *the multimode system*. This concept is used to define a language for the hybrid system. Then, DES diagnosability analysis is extended to hybrid systems based on this language. The diagnosability of hybrid systems is then defined. Based on this definition we show that the diagnosability of the underlying DES or CS is only a sufficient condition. Diagnosability of hybrid systems must be decided by coupling both discrete event and continuous informations. Finally, the necessary and sufficient condition of hybrid diagnosability is given.

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2 Hybrid System Modeling

As mentioned in [5], a hybrid system may be described by a hybrid automaton defined as a tuple $S = (\zeta, Q, \Sigma, T, C, (q_0, \zeta_0))$, where:

- ζ is the set of continuous variables, which includes observable and non observable variables. The set of observable variables is denoted by ζ_{OBS} [3].
- Q is the set of discrete system states. Each state $q_i \in Q$ represents a functional mode of the system. It includes nominal and anticipated fault modes.
- Σ is the set of events. Events correspond to discrete control inputs, spontaneous mode changes and fault events.
  $\Sigma o \subseteq \Sigma$ is the set of observable events. Without loss of generality, we assume that fault events are unobservable (otherwise, these faults are obviously diagnosable).
- T is the transition function, $T : Q \times \Sigma \rightarrow Q$.
- C is the set of system constraints linking continuous variables. It represents the set of differential and algebraic equations modeling the continuous behavior of the system.
- $(\zeta_0, q_0) \in \zeta \times Q$, is the initial condition of the hybrid system.

The discrete part of the hybrid automaton, given by $M = (Q, \Sigma, T, q_0)$, is a discrete automaton that describes the discrete dynamics of the system, i.e. the possible evolutions between functional modes of Q. Modes include nominal and fault modes. An *unknown mode* can be added to model all non anticipated faulty situations.

The continuous behavior of the hybrid system is modeled by an underlying continuous system $\Xi = (\zeta, Q, C, \zeta_0)$ that describes the whole continuous behavior of the system. Notice that transitions between modes are implicit and consequently not constrained in any way. We hence call this system a *multimode system*.

The underlying continuous behavior in each mode q_i is modeled by a set of constraints C_i. A set of constraints linking only observable continuous variables is computed from C_i. This set is denoted C_{obs_i}. Each constraint of C_{obs_i} can be evaluated from observable variables. It must be satisfied when the system evolves in mode q_i.

The hybrid behavior is the result of the contribution of the underlying CS and DES. The diagnosability of the hybrid system is analyzed by considering diagnosability properties of its two underlying systems.

[3] We assume that the set of system observable variables is the same in all system modes. This assumption is generally verified when the set of system's sensors is permanent, and do not depend on the system mode.

3 Diagnosability of the underlying CS

Diagnosing the multimode system consists in determining the current mode of the system. The diagnosability property of the multimode system guarantees that the mode of the system can be determined without ambiguity using continuous observations. In this section, we present the theory to analyze the diagnosability of the underlying continuous system. We introduce the new concepts of *mirror* and *mode signatures*. This study is used later to establish the definition and criteria for the whole hybrid system diagnosability.
To check the consistency of the system model with respect to observations, a set of consistency indicators is linked with every operating mode of the system. To each constraint $C^k_{obs_i}$ from C_{obs_i} a consistency indicator called residual is associated and denoted r_{ik}. The residual is a Boolean indicator. It is zero when the constraint $C^k_{obs_i}$ is satisfied, otherwise it is equal to 1.

3.1 Mirror and Reflexive signatures

The q_k-mirror signature of mode q_j is the vector of residuals of mode q_k evaluated when the system is in mode q_j. We use the term *mirror* because it represents the signature of q_j seen in mode q_k.

Definition 1 *Mirror Signature*
Given the tuple $S_r^{q_k} = [r_{k1}, r_{k2}, ..., r_{kN_{r(q_k)}}]$ of system residuals in mode q_k, the q_k-mirror signature of mode q_j is given by the vector $S_{j/k} = [s_{1_{j/k}}, ..., s_{N_{r(q_k)_{j/k}}}]^T = [S_r^{q_k}(\zeta_{OBS_{q_j}})]^T$, where $\zeta_{OBS_{q_j}}$ denotes the value of observable variables in mode q_j.

The reflexive signature is a particular case of the mirror signature $S_{j/k}$, with $j = k$.

Definition 2 *Reflexive Signature*
The reflexive signature of mode q_j, $S_{j/j} = [S_r^{q_j}(\zeta_{OBS_{q_j}})]^T = [0, 0, ..., 0]^T$, is the vector of residuals of mode q_j, computed with observations when the system is in mode q_j.

3.2 The Mode signature

The new concept of mode signature that characterizes a mode is now introduced.

Definition 3 *Mode Signature*
The signature of a mode q_i is the vector obtained by the concatenation of all the mirror signatures of q_i, $Sig(q_i) = [S^T_{i/1}, S^T_{i/2}, ..., S^T_{i/i}, ..., S^T_{i/m}]^T$, where m is the number of system modes [4].

3.3 From mode signatures to multimode system diagnosability characterization

The concept of mode signature leads us to the characterization of multimode systems diagnosability. Let us notice that in our approach, the faulty behaviors (but the unknown mode) are modeled by fault modes. A given fault corresponds to a set of fault modes in which this fault is present. In this paper, we analyze diagnosability at the level of fault modes, which is somehow more precise than at fault level. Indeed, whereas the signature of a mode is reduced to one single tuple, the signature of a fault is in general a set of tuples.

[4] In our approach, nominal and fault modes have the same status and the signature of a given mode anticipates how it should be seen in terms of the indicator tuples of the different modes of the system (including itself).

By analogy with fault diagnosability of continuous systems, concepts of mode and fault diagnosability of multimode systems are defined as follows:

Definition 4 *Two modes q_i and q_j ($i \neq j$) are diagnosable if $Sig(q_i) \neq Sig(q_j)$.*
The multimode system Ξ is diagnosable if and only if all pairs of modes q_i and q_j, $i \neq j$, are diagnosable.

Definition 5 *The signature of the fault f_i is defined as the set of the signatures of all possible destination modes after the occurrence of the fault event f_i.*
$Sig(f_i) = \{Sig(T(q_k, f_i)), 1 \leq k \leq m\}$

Definition 6 *Two faults f_i and $f_j, i \neq j$ are diagnosable if $Sig(f_i) \cap Sig(f_j) = \emptyset$.*

In our theory, diagnosability of two modes q_i and q_j is interpreted along two complementary ways through introduced definitions of *mutual* and *3-rd* diagnosability:

- **Definition 7** *Mutual Diagnosability Two modes q_i and q_j, $i \neq j$, are not mutually diagnosable if:*
  $S_{i/i} = S_{i/j} = [0, 0, ..., 0]^T_{Nr(q_i)}$ *and* $S_{j/j} = S_{j/i} = [0, 0, ..., 0]^T_{Nr(q_j)}$.
  The mutual diagnosability is equivalent to Mode Discernability *defined in [3].*
- **Definition 8** *3rd-Diagnosability Two modes q_i and q_j are q_k-3rd-diagnosable if they have different signatures with respect to the q_k mode , i.e. they have different q_k-mirror signatures, $k \neq i, j$.*
  Formally, q_i and q_j, $i \neq j$, are q_k-3rd-mirror diagnosable if and only if $S_{i/k} \neq S_{j/k}$.
  Two modes q_i and q_j, $i \neq j$, are 3rd-diagnosable if and only if $\exists k \neq i, j$ such as $S_{i/k} \neq S_{j/k}$.
  The multimode system is 3rd-diagnosable if and only if for all pairs of modes q_i and q_j, $i \neq j$, there exist $k_{i,j} \neq i, j$ such as $S_{i/k_{i,j}} \neq S_{j/k_{i,j}}$.

Then, we have the following result:

Theorem 1 *Two modes q_i and q_j, $i \neq j$ are diagnosable if and only if they are mutually diagnosable or 3rd-diagnosable.*

Proof 1 *Consider two modes q_i and q_j, $i \neq j$.*
Let $Sig(q_i) = [S^T_{i/1}, S^T_{i/2}, ..., S^T_{i/i}, ..., S^T_{i/m}]^T$
and $Sig(q_j) = [S^T_{j/1}, S^T_{j/2}, ..., S^T_{j/j}, ..., S^T_{j/m}]^T$
q_i and q_j are diagnosable if and only if $Sig(q_i) \neq Sig(q_j)$
$\Leftrightarrow \exists k \in [1, m]$ such as $S^T_{i/k} \neq S^T_{j/k}$
$\Leftrightarrow$ q_i and q_j are 3rd (if $k \neq i, j$) or mutually (if $k = i$ or $k = j$) diagnosable. □

Consequently, the multimode system is diagnosable if and only if for every pair of modes (q_i, q_j), $i \neq j$ mutual or/and 3rd-diagnosability holds.

4 Diagnosability of the underlying DES

4.1 DES Diagnosability Reminder

A DES is modeled by a finite state machine $M = (Q, \Sigma, T, q_0)$, where Q is the set of discrete states, Σ is the set of events, $T : Q \times \Sigma \rightarrow Q$ the transition function and q_0 the initial state, as already defined in section 2. The event set Σ is partitioned as $\Sigma = \Sigma_{uo} \cup \Sigma_o$,

where Σ_{uo} (Σ_o) is the unobservable (observable) event set. We consider $\Sigma_F \subseteq \Sigma_{uo}$ as the set of fault events to be diagnosed. In [8] the diagnosis of the DES consists in the deduction of unobservable fault events from the observable traces generated by the system. The event-based point of view introduces temporal aspects in the observations and the diagnosability definition takes the following form:

Definition 9 *A fault f is diagnosable if its occurrence is always followed by a finite observable sequence of events that allows us to diagnose f with certainty [6]. The system is said to be diagnosable if and only if all the anticipated faults are diagnosable.*

We then have the following result from [8]:

Proposition 1 *The DES is diagnosable if and only if* $\forall f \in \Sigma_F$, $\exists n \in \mathbb{N}$ *such as:*
$\forall$ *sequence of events (or trajectory) $s_F t$, such that s_F ends with the occurrence of f, and t it is a continuation of s_F, $||t|| \geq n \Rightarrow$ ($\forall$ trajectory s: $P_{\Sigma_o}(s) = P_{\Sigma_o}(s_F t) \Rightarrow f$ occurs in s), where P_{Σ_o} is the projection operator on the set of observable events.*

4.2 The diagnoser approach

We assume that M has no unobservable cycles (i.e cycles containing unobservable events only). The set of fault events Σ_F is partitioned into disjoint sets corresponding to different fault types F_i, $\Sigma_F = \Sigma_{F_1} \cup \Sigma_{F_2} \cup ... \cup \Sigma_{F_n}$ and $\Sigma_{F_i} \cap \Sigma_{F_j} = \emptyset$, $for\, i \neq j$. The aim of the diagnosis is to make inferences about past occurrences of fault types on the basis of the observed events. In order to solve this problem the system model is converted into a diagnoser.
The diagnoser $Diag(M) = (Q_{Diag}, \Sigma_{Diag}, T_{Diag}, q_{0\,Diag})$ is a deterministic finite state machine built from the system model M (For more details see [8]). It can be used for on-line diagnosis and/or diagnosability analysis. Here, the diagnoser is used to perform the diagnosability analysis.

Definition 10 *Given a diagnoser state $q_{Diag} \in Q_{Diag}$, this state is F_i-uncertain if F_i does not belong to all the labels of the state whereas F_i belongs to at least one label of the state.*

Theorem 2 *The system M is not diagnosable* [5] *[8] if and only if the associated diagnoser $Diag(M)$:*

- *contains an uncertain cycle, i.e. a cycle in which there is at least one F_i-uncertain diagnoser state for some F_i.*
- *the states of the original system involved in the different diagnoser cycling states also define a cycle in the original system M.*

5 Diagnosability of Hybrid Systems

Diagnosing a hybrid system consists on tracking the system mode by using both continuous and discrete observable behaviors. The hybrid system is diagnosable if and only if the occurrence of any unobservable fault event is detected with a finite number of discrete event and continuous observations. The behavior of the hybrid system is the result of continuous and discrete behaviors. Hence, the hybrid diagnosability analysis must call upon both discrete event and continuous informations. A common framework is required in order to combine these informations. In the next subsection we aim at combining continuous and discrete knowledge in a unified model by abstracting the change of continuous dynamics in terms of discrete events.

[5] Under the liveness hypothesis of the discrete automaton.

5.1 Abstraction of the continuous dynamics in terms of discrete events

We assume that the dynamics of the discrete control inputs are slower than the dynamics of residual generators (mode signatures establish between two consecutive discrete events).
We define a function f_{CS_DES}, that for each mode transition of the underlying DES, associates an event issued from the continuous domain, which represents the change of the mode signature.
This function aims to define Σ^{Sig}, as the set of discrete events issued from the abstraction of continuous dynamics of the multimode system.

$$f_{CS\_DES} : Q \times T(Q, \Sigma) \longrightarrow \Sigma^{Sig}$$

$$(q_i, q_j) \longmapsto \begin{cases} Ro_{ij} \in \Sigma_o^{Sig} \text{ if } Sig(q_i) \neq Sig(q_j) \\ Ruo_{ij} \in \Sigma_{uo}^{Sig} \text{ if } Sig(q_i) = Sig(q_j) \end{cases}$$

- Σ_o^{Sig} is a set of observable events, generated when the mode signature of the source mode is different from the mode signature of the destination mode.
- Σ_{uo}^{Sig} is a set of unobservable events generated when the mode signature of the source mode is equal to the mode signature of the destination mode.
- Σ^{Sig} is defined as $\Sigma_o^{Sig} \cup \Sigma_{uo}^{Sig}$.

5.2 Hybrid Language and Hybrid Trajectories

The abstraction of the continuous dynamics changes in terms of discrete events allows us to define the language of the hybrid system, which describes the evolution of the system behavior. We denote by $\Sigma_{hybrid} = \Sigma \cup \Sigma^{Sig}$ the alphabet that contains "natural" discrete events and events modeling mode switches. We model the behavior of the hybrid system as a prefix closed language $L(S) \subset \Sigma_{hybrid}^*$ over the event alphabet Σ_{hybrid}, where Σ_{hybrid}^* denotes the set of all finite strings of elements of the set Σ_{hybrid} including the empty string (Σ_{hybrid}^* is called the Kleene Closure of Σ_{hybrid} [7]). A trajectory of the hybrid system is represented by a string of events of the hybrid alphabet Σ_{hybrid}.

5.3 Behavior Automaton

The hybrid language $L(S)$ can be generated by its finite state generator representation [7]. In this paper, this automaton is called *the behavior automaton* and mixes both "natural" discrete events and signature switches.

5.3.1 Properties of the hybrid language

The hybrid language $L(S) \subset \Sigma_{hybrid}^*$ mixes "natural" discrete events from Σ and events issued from the abstraction of the continuous dynamics Σ^{Sig}. Hence, some specific properties can be stated (see Figure 1).

Property 1 $\forall w \in L(S), w = e'.R'.w'$, *where* $e' \in \Sigma, R' \in \Sigma^{Sig}, w' \in L(S)$.

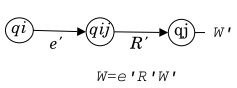

Figure 1. Property of the hybrid language

5.3.2 Hybrid diagnosability

The diagnosability of a hybrid system is defined as follows:

Definition 11 *A fault event f is diagnosable if its occurrence can always be detected after a finite set of continuous and discrete observations i.e. after a finite sequence of observable events and a finite set of continuous variable observations. The system is said to be diagnosable if and only if all the anticipated faults are diagnosable.*

This definition provides the following result in the hybrid language framework :

Proposition 2 *The hybrid system is diagnosable if $\forall f_i$, $\exists n_i \in \mathbb{N}$ such as: $\forall s_{F_i} t \in L(S)$, such that s_{F_i} ends with the occurrence of f_i, and $t \in L(S)$ is a continuation of s_{F_i}, $||t|| \geq n_i \Rightarrow (\forall w \in L(S) : P_{\Sigma_{hybrid_o}}(w) = P_{\Sigma_{hybrid_o}}(s_{F_i} t) \Rightarrow f_i \in w)$, where $P_{\Sigma_{hybrid_o}}$ is the projection operator on the set of observable events of Σ_{hybrid} i.e. $\Sigma_{hybrid_o} = \Sigma_o \cup \Sigma_o^{Sig}$.*

5.4 DES Sufficient Criterion

Theorem 3 *The hybrid system $S = (\zeta, Q, \Sigma, T, C, (\zeta_0, q_0))$ is diagnosable if its underlying discrete event system $M = (Q, \Sigma, T, q_0)$ is diagnosable.*

Proof 2 *Given a Hybrid System $S = (\zeta, Q, \Sigma, T, C, (\zeta_0, q_0))$, such that the underlying discrete event system $M = (Q, \Sigma, T, q_0)$ is diagnosable. Given a fault $f \in \Sigma_F$, given $s_F t \in L(S)$ such that $s_F \in L(S)$ ends with the occurrence of f, and $t \in \Sigma^*_{hybrid}$ is a continuation of s_F (see Figure 2).*
We denote $s'_F = P_\Sigma(s_F)$ and $t' = P_\Sigma(t)$, where P_Σ is the projection on the set of discrete events Σ.
We have $s'_F \in L(M)$ ends with $f \in \Sigma_{uo} \subset \Sigma$, and $t' \in \Sigma^*$ is a continuation of s'_F.
Since, $M = (Q, \Sigma, T, q_0)$ is diagnosable then there exists an integer n' such that: $||t'|| \geq n' \Rightarrow \forall w' \in L(M), (P_{\Sigma_o}(w') = P_{\Sigma_o}(s'_F t') \Rightarrow f \in w')$ (proposition 1).
We consider the integer $n = 2n' + 1$, then from property 1 we have

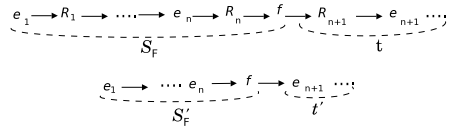

Figure 2. Composition of a hybrid fault trajectory and its projection into the discrete event set Σ

$||t|| \geq n \Rightarrow ||t'|| \geq n'$
$\forall w \in L(S)$ *such that* $P_{\Sigma_{hybrid_o}}(w) = P_{\Sigma_{hybrid_o}}(s_F t)$, *we consider* $w' = P_\Sigma(w)$
$P_{\Sigma_{hybrid_o}}(w) = P_{\Sigma_{hybrid_o}}(s_F t) \Rightarrow P_{\Sigma_o}(w') = P_{\Sigma_o}(s'_F t')$
$\Rightarrow f \in w'$ *thus* $f \in w$
and consequently the hybrid system S is diagnosable w.r.t. proposition 2. □

The above result provides a sufficient condition for hybrid diagnosability that is solely based on the underlying DES. In practice, the underlying DES is rarely diagnosable because it does not include explicit information about the events that occur after the occurrence of a fault. Diagnosability can only be decided on the basis of the observation of discrete control inputs and discrete sensor outputs.

5.5 CS Sufficient criterion

Theorem 4 *The hybrid system $S = (\zeta, Q, \Sigma, T, C, (\zeta_0, q_0))$ is diagnosable if the underlying continuous system $\Xi = (\zeta, Q, C, \zeta_0)$ is diagnosable.*

Proof 3 *Consider a Hybrid System $S = (\zeta, Q, \Sigma, T, C, (\zeta_0, q_0))$, such that the underlying multimode system $\Xi = (\zeta, Q, C, \zeta_0)$ is diagnosable and a fault $f \in \Sigma_F$, given $s_F t \in L(S)$ such that $s_F \in L(S)$ ends with the occurrence of f. Let $q_c(q_f)$ be the mode of the system before (after) the occurrence of the fault event f (see Figure 3).*
Since the underlying multimode system is diagnosable then $\forall q_i \neq q_j, Sig(q_i) \neq Sig(q_j)$, therefore $\Sigma^{Sig}_{uo} = \emptyset$ and in addition, all the observable events $R_{o_{ij}}$ are different.
Let $t \in \Sigma^*_{hybrid}$ be a continuation of s_F such that $||t|| \geq 1$. $\forall w \in L(S)$ such that $P_{\Sigma_{hybrid_o}}(w) = P_{\Sigma_{hybrid_o}}(s_F t) = P_{\Sigma_{hybrid_o}}(s_F) R_{o_{cf}} w'$ (where $w' \in \Sigma^*_{hybrid_o}$) (this is guaranteed by the property 1). The observation of the event $R_{o_{cf}}$ means

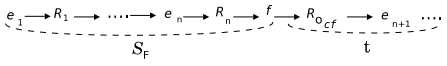

Figure 3. Composition of a hybrid fault trajectory

that the system has transited from the current mode q_c to the fault mode q_f, thus $f \in w$. Hence, the hybrid system S is diagnosable. □

Corollary 1 *Two modes q_i and q_j, $i \neq j$ of the hybrid system S are diagnosable if $Sig(q_i) \neq Sig(q_j)$. If all pairs of modes (q_i, q_j), $i \neq j$ of the hybrid system are diagnosable then the hybrid system is diagnosable.*

This is again only a sufficient condition in terms of the underlying multimode system. As a matter of fact, the next section shows that continuous and discrete information are required to achieve a necessary and sufficient condition.

5.6 Necessary and sufficient condition

We build the hybrid system diagnoser, by considering the behavior automaton defined in 5.3. The diagnosability property of the hybrid system is analyzed on this diagnoser by extending the DES diagnosability theorem [8] (theorem 2) to hybrid systems.

Proposition 3 *The hybrid system $S = (\zeta, Q, \Sigma, T, C, (\zeta_0, q_0))$ is not diagnosable if and only if:*

- *the associated diagnoser computed from the corresponding behavior automaton contains an uncertain cycle, i.e. a cycle in which there is at least one F_i-uncertain diagnoser state for some F_i.*
- *the states of the behavior automaton involved in the different diagnoser cycling states also define a cycle in the behavior automaton.*

6 Illustrative example

Consider the circuit modeled by a hybrid automaton represented in Figure 4. The nominal modes are q and q' that represent the configurations $sw = on$ and $sw = off$, respectively. For sake of simplicity, only single faults are modeled. Events f_1 and f_2 model the occurrence of faults: "R_1 broken" (R_1 opened circuit) and "R_2 broken" (R_2 opened circuit) respectively. The fault events f_1 (f_2) can occur

in configuration $sw = on$ or $sw = off$ and lead to fault modes qf_1 (qf_2) or $q'f_1$ ($q'f_2$) respectively.
The control events on and off and the observation events $o_1$and o_2 (the lamp lights/doesn't light) are observable. Fault events f_1 and f_2 are not observable. Voltages V and E and the current I are the continuous observable variables. The consistency indicators are derived

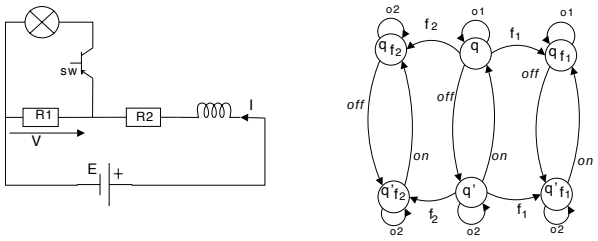

Figure 4. The Hybrid System

from the underlying continuous behavior in every mode. A consistency indicator r_k is associated with a constraint C^k_{obs}.

$$\begin{cases} \{q, qf_1\}\text{:}C^1_{obs} : E - V - R_2 I = 0\,(r_1) \\ \{q'\}\text{:}C^2_{obs} : E - V - R_2 I = 0\,(r_2),\ C^3_{obs} : V - R_1 I = 0\,(r_3) \\ \{q'f_1\}\text{:}C^4_{obs} : I = 0\,(r_4),\ C^5_{obs} : E - V = 0\,(r_5) \\ \{qf_2, q'f_2\}\text{:}C^6_{obs} : V = 0\,(r_6),\ C^7_{obs} : I = 0\,(r_7) \end{cases}$$

The underlying DES automaton is shown in Figure 4. Diagnosability

$Sig(q) = Sig(qf_1) = [0,0,1,1,1,1,1]^T$	$Sig(q') = [0,0,0,1,1,1,1]^T$
$Sig(qf_2) = Sig(q'f_2) = [1,1,1,0,1,0,0]^T$	$Sig(q'f_1) = [1,1,1,0,0,1,0]^T$

Table 1. Mode Signatures of the underlying continuous system Ξ[6]

analysis is performed below and summarized in table 2.
Consider the multimode system and the table of mode signatures [6] given in table 1. Constraints in modes q and qf_1 (qf_2 and $q'f_2$) are the same. Hence, mode signatures of q and qf_1 (qf_2 and $q'f_2$) are identical and consequently the two modes q and qf_1 (qf_2 and $q'f_2$) are not diagnosable. The other pairs of modes are diagnosable. Hence, the underlying multimode system is not diagnosable.
Consider the underlying DES. Notice that when the observable event o_1 occurs infinitely, the occurrence of the fault event f_1 cannot be decided (the system may be in mode q or in mode qf_1). The same happens for q', $q'f_2$ and qf_2 with respect to o_2. Then the underlying DES is not diagnosable (this can be shown by building the diagnoser of the underlying DES).

CS criterion	DES criterion	Hybrid criterion
$\{q, qf_1\}$	$\{q, qf_1\}$	$\{q, qf_1\}$
$\{qf_2, q'f_2\}$	$\{q', q'f_1, q'f_2\}$	

Table 2. Non diagnosable mode sets w.r.t CS, DES and Hybrid criteria

As a result, the diagnosability of the hybrid system cannot be decided using the CS (DES) sufficient conditions for diagnosability. The necessary and sufficient criterion of hybrid diagnosability is required. For this, the diagnoser of the hybrid system is built from the behavior automaton (but not provided due to the space limitation). It

[6] For sake of concision, identical residuals are represented by one single entry in the mode signatures.

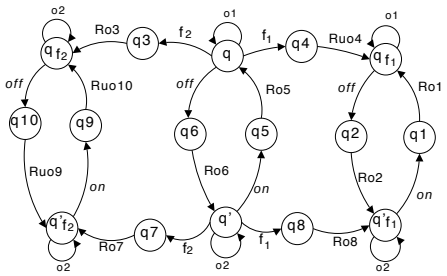

Figure 5. The behavior automaton

shows that the only modes that are non diagnosable are q and qf_1, manifested by an uncertain cycle (o_1) containing the uncertain diagnoser state $\{(q, \{\}), (qf_1, \{f_1, Ruo_4\}\}$.

7 Conclusion

In this paper a theoretical framework is proposed to analyze the diagnosability of multimode and hybrid systems. It leads to the introduction of the new concepts of mirror, reflexive and mode signatures. Based on these concepts, a characterization of diagnosability for multimode systems is achieved. Then, hybrid diagnosability is defined and associated conditions are provided. The difference between diagnosability of multimode systems and hybrid systems is clarified. By abstracting the continuous dynamics in terms of discrete events, a general framework for analyzing hybrid systems diagnosability is proposed, that builds upon existing work on DES and CS diagnosability. The system being decomposed into CS and DES underlying systems, we offer the possibility to use DES and CS techniques for hybrid diagnosability analysis and for on-line hybrid state tracking [1]. Future works will be based on these results and consider active diagnosis and reconfiguration [10] guided by diagnosability properties of the hybrid system.

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